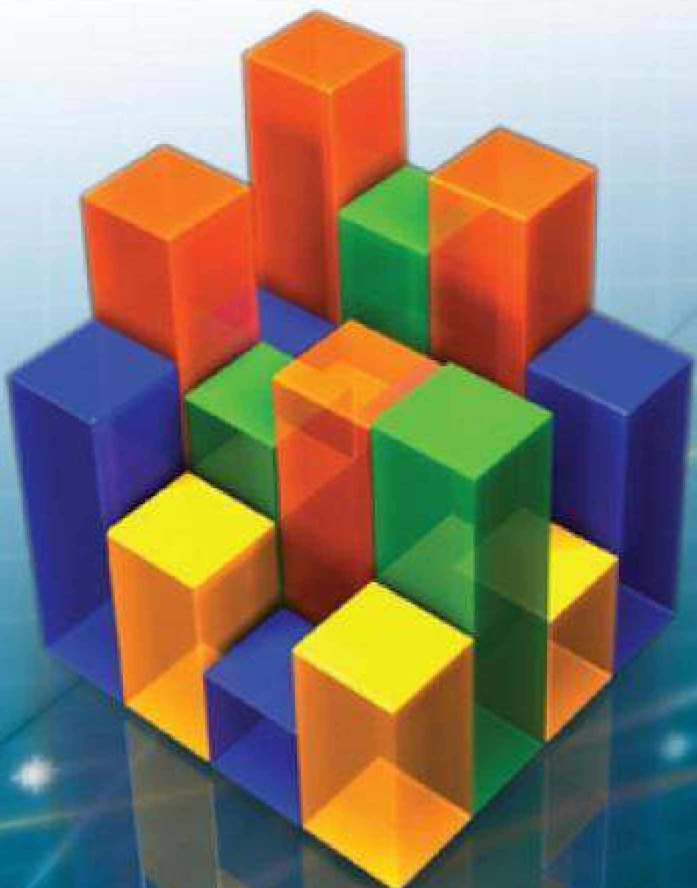
I GUSTI NGURAH AGUNG

Time Series Data Analysis Using EViews



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I Gusti Ngurah Agung

Graduate School Of Management
Faculty Of Economics University Of Indonesia

Ph.D. in Biostatistics and
MSc. in Mathematical Statistics from
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Dedicated to my wife
Anak Agung Alit Mas,
children
Ningsih A. Chandra, Ratna E. Lefort, and Dharma Putra,
sons in law
Aditiawan Chandra, and Eric Lefort,
daughter in law
Refiana Andries, and
all grand children
Indra, Rama, Luana, Leonard, and Natasya

Contents

Preface

Time series data, growth, or change over time can be observed and recorded in all their biological and nonbiological aspects. Therefore, the method of time series data analysis should be applicable not only for financial economics but also for solving all biological and nonbiological growth problems. Today, the availability of statistical package programs has made it easier for each researcher to easily apply any statistical model, based on all types of data sets, such as cross-section, time series, cross-section over time and panel data. This book introduces and discusses time series data analysis, and represents the first book of a series dealing with data analysis using EViews.

After more than 25 years of teaching applied statistical methods and advising graduate students on their theses and dissertations, I have found that many students still have difficulties in doing data analysis, specifically in defining and evaluating alternative acceptable models, in theoretical or substantial and statistical senses. Using time series data, this book presents many types of linear models from a large or perhaps an infinite number of possible models (see Agung, 1999a, 2007). This book also offers notes on how to modify and extend each model. Hence, all illustrative models and examples presented in this book will provide a useful additional guide and basic knowledge to the users, specifically to students, in doing data analysis for their scientific research papers.

It has been recognized that EViews is an excellent interactive program, which provides an excellent tool for us to use to do the best detailed data analyses, particularly in developing and evaluating models, in doing residual analysis and in testing various hypothesis, either univariate or multivariate hypotheses. However, it has also been recognized that for selected statistical data analyses, other statistical package programs should be used, such as SPSS, SAS, STATA, AMOS, LISREL and DEA.

Even though it is easy to obtain the statistical output from a data set, we should always be aware that we never know exactly the true value of any parameter of the corresponding population or even the true population model. A population model is defined as the model that is assumed or defined by a researcher to be valid for the corresponding population. It should be remembered that it is not possible to represent what really happens in the population, even though a large number of variables are used. Furthermore, it is suggested that a person's best knowledge and experience should be used in defining several alternative models, not only one model, because we can never obtain the best model out of all possible models, in a statistical sense. To obtain the truth about a model or the best population model, read the following statements:

> *Often in statistics one is using parametric models .... Classical (parametric) statistics derives results under the assumption that these models are strictly true. However, apart from simple discrete models perhaps, such models are never exactly true (Hample, 1973, quoted by Gifi, 1990, p. 27).*

Corresponding to this statement, Agung (2004, 2006) has presented the application of linear models, either univariate or multivariate, starting from the simplest linear model, i.e. the cell-means models, based on either a single factor or multifactors. Even though this cell-means model could easily be justified to represent the true population model, the corresponding estimated regression function or the sample means greatly depends on the sampled data.

> *In data analysis we must look on a very heavy emphasis on judgment (Tukey, 1962, quoted by Gifi, 1990, p. 23).*

Corresponding to this statement, there should be a good or strong theoretical and substantial base for any proposed model specification. In addition, the conclusion of a testing hypothesis cannot be taken absolutely or for granted in order to omit or delete an exogenous variable from a model. Furthermore, the exogenous variables of a growth or time series model could include the basic or original independent variables, the time *t*-variable, the lagged of dependent or independent variables and their interaction factors, with or without taking into account the autocorrelation or serial correlation and heterogeneity of the error terms. Hence, there is a very large number of choices in developing models. It has also been known that based on a time series data set, many alternative models could be applied, starting with the simplest growth models, such as the geometric and exponential growth models up to the VAR (*Vector Autoregression*), VEC (*Vector Error Correction*), System Equation in general and GARTH (*Generalized Conditional Heteroskedasticity*) models.

The main objective of this book is to present many types of time series models, which could be defined or developed based on only a set of three or five variables. The book also presents several examples and notes on unestimable models, especially the nonlinear models, because of the *overflow* of the iteration estimation methods. To help the readers to understand the advantages and disadvantages of each of the models better, notes, conclusions and comments are also provided. These illustrative models could be used as good basic guides in defining and evaluating more advanced time series models, either univariate or multivariate models, with a larger number of variables.

This book contains eleven chapters as follows.

Chapter 1 presents the very basic method in EViews on how to construct an EViews workfile, and also a descriptive statistical analysis, in the form of summary tables and graphs. This chapter also offers some remarks and recommendations on how to use scatter plots for preliminary analysis in studying relationships between numerical variables.

Chapter 2 discusses continuous growth models with the numerical time *t* as an independent variable, starting with the two simplest growth models, such as the geometric and exponential growth models and the more advanced growth models, such as a group of the general univariate and multivariate models, and the S-shape *vector autoregressive* (VAR) growth models, together with their residual analyses. This chapter also presents growth models, which could be considered as an extension or modification of the Cobb–Douglas and the CES (Constant Elasticity of Substitution) production functions, models with interaction factors and trigonometric growth models. For alternative estimation methods, this chapter offers examples using the White and the Newey–West HAC estimation methods.

Chapter 3 presents examples and discussions on discontinuous growth models with the numerical time *t* and its defined or certain dummy variable(s) as independent variables of the models. This chapter provides alternative growth models having an interaction factor(s) between their exogenous variable(s) with the time *t* as an independent variable(s). Corresponding to the discontinued growth models, this chapter also presents examples on how to identify breakpoints, by using Chow's Breakpoint Test.

Chapter 4 discusses the time series models without the numerical time *t* as an independent variable, which are considered as *seemingly causal models* (*SCM*) for time series. For illustrative purposes, alternative representation of a model using dummy time variables and three-piece autoregressive SCMs are discussed based on a hypothetical data set, with their residual plots. This chapter also provides examples of the discontinued growth models, as well as models having an interaction factor(s).

Chapter 5 covers special cases of regression models based on selected data sets, such as the POOL1 and BASIC workfiles of the EViews/Examples Files, and the US Domestic Price of Copper, 1951–1980, which is presented as one of the exercises in Gujarati (2003, Table 12.7, p. 499). The BASIC workfile is discussed specifically to present good illustrative examples of nonparametric growth models.

Chapter 6 describes illustrative examples of multivariate linear models, including the VAR and SUR models, and the structural equation model (SEM), by using the symbol *Y* for the set of endogenous variables and the symbol *X* for the set of exogenous variables. The main idea for using these symbols is to provide illustrative general models that could be applied on any time series in all biological and nonbiological aspects or growth. As examples to illustrate, three *X* and two *Y* variables are selected or derived from the US Domestic Price of Copper data, which were used for linear model presentation in the previous chapters. All models presented there as examples could be used for any time series data. Analysts or researchers could replace the *X* and *Y* variables by the variables that are relevant to their field of studies in order to develop similar models.

Chapter 7 covers basic illustrative instrumental variables models, which could be easily extended using all types of models presented in the previous chapters, either with or without the time *t*-variable as an independent variable.

Chapter 8 presents the autoregressive conditional heteroskedasticity (ARCH) models, generalized ARCH (GARCH), threshold ARCH (TARCH) and exponential ARCH (E_GARCH) models, either additive or interaction factor models.

In addition to the Wald tests, which have been applied in the previous chapters for various testing hypotheses, Chapter 9 explores some additional testing hypotheses, such as the unit root test, the omitted and redundant variables tests, the nonnested test and Ramsy's RESET tests, with special comments on the conclusion of a testing hypothesis.

Chapter 10 introduced a general form of nonlinear time series model, which could also represent all time series models presented in the previous chapters. For illustrative examples, this chapter discusses models that should be considered, such as the *Generalized Cobb-Douglas* (G_CD) model and the *Generalized Constant Elasticity of Substitution* (G_CES) model.

Finally, Chapter 11 presents nonparametric estimation methods, which cover the classical or basic moving average estimation method and the *k-Nearest Forecast* (*k*-NF), which can easily be calculated manually or by using Microsoft Excel, and the smoothing techniques (Hardle, 1999), such as the Nearest Neighbor and Kernel Fit Models, which should be done using EViews.

In addition to these chapters, the theoretical aspects of the basic estimation methods based on the time series data are presented in four appendices. In writing these appendices I am indebted to Haidy A. Pasay, Ph.D, lecturer in Microeconomics and Econometrics at the Graduate Program of Economics, the Faculty of Economics, University of Indonesia, who are the coauthors of my book on Applied Microeconomics (Agung, Pasay and Sugiharso, 1994). They spent precious time reading and making detailed corrections on mathematical formulas and econometric comprehension.

I express my gratitude to the Graduate School of Management, Faculty of Economics, University of Indonesia, for providing a rich intellectual environment and facilities indispensable for the writing of this text, as well as other published books in Indonesian.

In the process of writing this applied statistical book in English, I am indebted to Dr Anh Dung Do, the President of PT Kusuma Raya (Management, Financing and Investment Advisory Services) and Lecturer in Strategic Management at the Master Program of the Faculty of Economics, University of Indonesia. Dr Do motivated and supported me in the completion of this book. He spent a lot of his precious time in reading and making various corrections to my drafts.

I am also deeply indebted to my daughter, Martingsih Agung Chandra, BSPh, MSi, The Founder and Director of NAC Consultant Public Relations, and my son, Dharma Putra, MBA, Director of the PURE Technology, PT. Teknologi Multimedia Indonesia, for all their help in reading and making corrections to my drafts.

Puri AGUNG
Jimbaran, Bali

1

EViews workfile and descriptive data analysis

1.1 What is the EViews workfile?

The EViews workfile is defined as a file in EViews, which provides many convenient visual ways, such as (i) to enter and save data sets, (ii) to create new series or variables from existing ones, (iii) to display and print series and (iv) to carry out and save results of statistical analysis, as well as each equation of the models applied in the analysis. By using EViews, each statistical model that applied previously could be recalled and modified easily and quickly to obtain the best fit model, based on personal judgment using an interactive process. Corresponding to this process, the researcher could use a specific name for each EViews workfile, so that it can be identified easily for future utilization.

This chapter will describe how to create a workfile in a very simple way by going through Microsoft Excel, as well as other package programs, if EViews 5 or 6 are used. Furthermore, this chapter will present some illustrative statistical data analysis, mainly the descriptive analysis, which could also be considered as an exploration or an evaluation data analysis.

1.2 Basic options in EViews

It is recognized that many students have been using EViews 4 and 5. For this reason, in this section the way to create a workfile using EViews 4 is also presented, as well as those using EViews 5 and 6. However, all statistical results presented as illustrative examples use EViews 6.

Figure 1.1 presents the toolbar of the EViews main menus. The first line is the Title Bar, the second line is the Main Menus and the last space is the Command Window and the Work Area.

Then all possible selections can be observed under each of the main menus. Two of the basic options are as follows:

(1) To create a workfile, click *File/New*, which will give the options in Figure 1.2.

Time Series Data Analysis Using EViews IGN Agung

Figure 1.1 The toolbar of the main menus

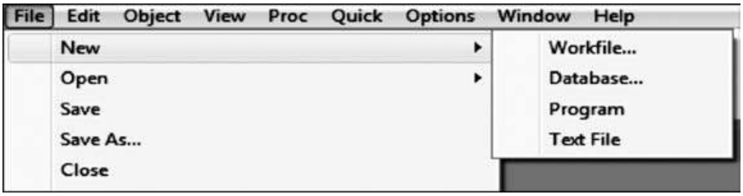


Figure 1.2 The complete options of the new file in EViews 4, 5 and 6

(2) To open a workfile, click *File/Open*, which will give the options in Figure 1.3 using EViews 4. Using EViews 5 or 6 gives the options in Figure 1.4.

Note that by using EViews 5 or 6, 'Foreign Data as Workfile. . .' can be opened. By selecting the option '*Foreign Data as Workfile. . .*' and clicking the '*All files* (*.*)' option, all files presented in Figure 1.5 can be seen, and can be opened as workfiles. Then a workfile can be saved as an EViews workfile.

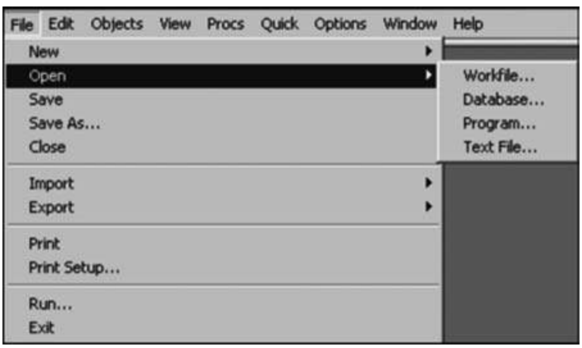


Figure 1.3 The complete options of the open file in EViews 4

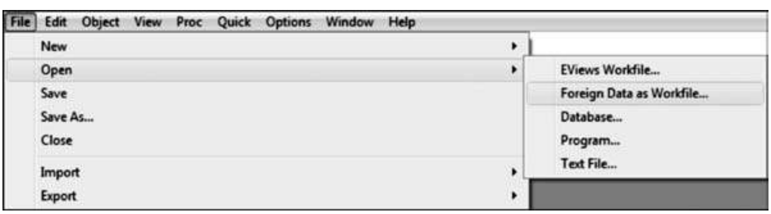


Figure 1.4 The complete options of the open file in EViews 5 and 6

Figure 1.5 All files that can be opened as a workfile using EViews 5 and 6

1.3 Creating a workfile

1.3.1 Creating a workfile using EViews 5 or 6

Since many '*Foreign Data as Workfile...*' can be opened using EViews 5, as well as EViews 6, as presented in Figures 1.3 and 1.4, there are many alternative ways that can be used to create an EViews workfile. This makes it easy for a researcher to create or derive new variables, indicators, composite indexes as well as latent variables (unmeasurable or unobservable factors) by using any one of the package programs presented in Figure 1.4, which is very convenient for the researcher. Then he/she can open the whole data set as a workfile.

1.3.2 Creating a workfile using EViews 4

By assuming that creating an Excel datafile is not a problem for a researcher, only the steps required to copy Data.xls to an EViews workfile will be presented here. As an illustration and for the application of statistical data analysis, the data in Demo.xls will be used, which are already available in EViews 4.

To create the desired workfile, the steps are as follows:

(1) If EViews 4 is correctly installed, by clicking *My Documents...*, the directory 'EViews Example Files' will be seen in My Documents, as presented in Figure 1.6.
(2) Double click on the EViews Example Files, then double click on the data and the window in Figure 1.7 will appear. Then the file Demo.xls can be seen, in addition to several workfiles and programs. From now on, Demo.xls will be used.
(3) Double click on Demo.xls; a time series data set having four variables will be seen: *GDP*, *PR*, *NPM* and *RS* in an Excel spreadsheet, as shown in Figure 1.8. For further demonstrations of data analysis, three new variables are created in the spreadsheet: (i) t as the time variable having values from 1 up to 180, (ii) *Year* having values from 1952 up to 1996 and (iii) Q as a quarterly variable having values 1, 2, 3 and 4 for each year (see the spreadsheet below).

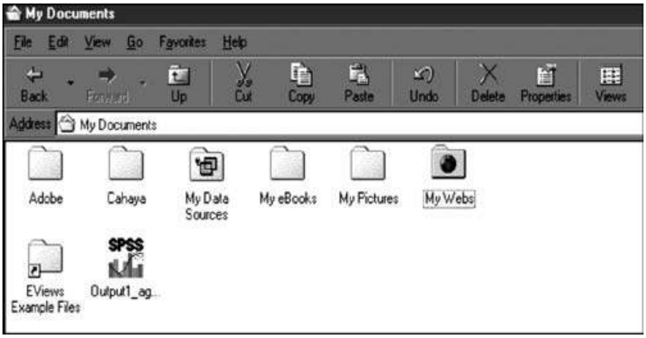


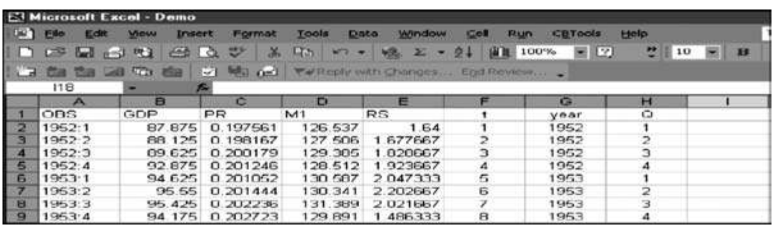
Figure 1.6 The EViews example files in My Documents

Figure 1.7 List of data that are available in EViews 4

(4) Block Demo.xls and then click *Edit/Copy*. . . .

(5) Open Eviews and then click *File/New/Workfile* . . . . This gives the window in Figure 1.9, showing the quarterly data set with starting and ending dates in Demo.xls. The rules for describing the dates are as follows:

- *Annual:* specify the year. Years from 1930 to 2029 may be identified using either 2- or 4-digit identifiers (e.g. ‘32’ or ‘1932’). All other years must be identified with full year identifiers.
- *Quarterly:* the year followed by a colon or the letter ‘Q,’ and then the quarter number. Examples: ‘1932 : 3,’ ‘32 : 3’ and ‘2003Q4.’



	A	B	C	D	E	F	G	H	I
1	OBS	GDP	PR	M1	RS	t	year	Q	
2	1952:1	87.875	0.197561	126.537	1.64	1	1952	1	
3	1952:2	88.125	0.198167	127.506	1.677667	2	1952	2	
4	1952:3	89.625	0.200179	129.305	1.820667	3	1952	3	
5	1952:4	92.875	0.201246	128.512	1.923667	4	1952	4	
6	1953:1	94.625	0.201052	130.587	2.047333	5	1953	1	
7	1953:2	95.55	0.201444	130.341	2.202667	6	1953	2	
8	1953:3	95.425	0.202236	131.389	2.021667	7	1953	3	
9	1953:4	94.175	0.202723	129.891	1.486333	8	1953	4	

Figure 1.8 A part of data in Demo.xls

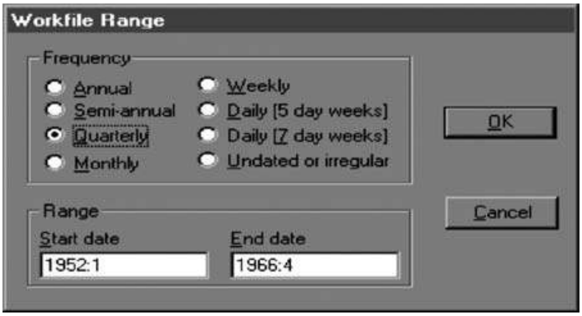


Figure 1.9 The workfile frequency and range

- *Monthly:* the year followed by a colon or the letter 'M,' and then the month number. Examples: '1932M9' and '1939 : 11.'
- *Semiannual:* the year followed a colon of the letter 'S,' and then either '1' or '2' to denote the period. Examples: '1932 : 2' and '1932S2.'
- *Weekly and daily:* by *default*, these dates should be specified as month number, followed by a colon, then followed by the day number, then followed by a colon, followed by the year. For example, entering '4 : 13 : 60' indicates that the workfile begins on April 13, 1960.
- Alternatively, for quarterly, monthly, weekly and daily data, just the year can be entered and EViews will automatically specify the first and the last observation.
- For other types of data, '*Undated or irregular*' is selected.

(6) Click *OK* produces the space or window, as presented in Figure 1.10. For every new data set or workfile at this stage, the window always shows a parameter vector 'C' and a space 'RESID,' which will be used to save the parameter and the residuals of the models used in an analysis.
(7) Click *Quick/Empty Group*... brings up the spreadsheet in Figure 1.11 on the screen. Put the cursor in the second column of the OBS indicator and then click so that the second column will block or darken.

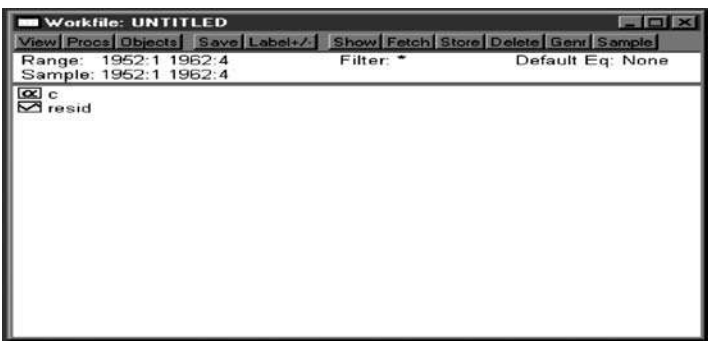


Figure 1.10 A workfile space of quarterly data in Demo.xls

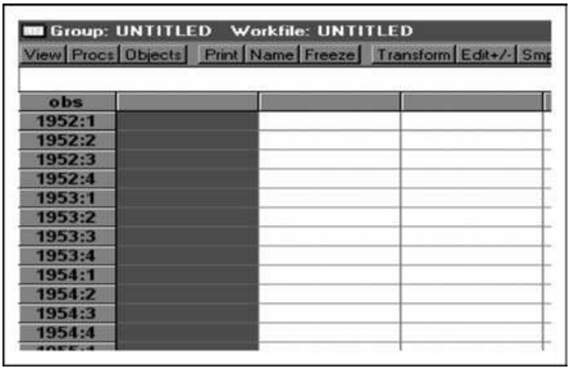


Figure 1.11 The group space to insert Demo.xls

	A	B	C	D	E	F	G	H	I
1	OBS	GDP	PR	M1	RS	t	Year	q	
2	1952:1	87.875	0.197561	126.537	1.64	1	52	1	
3	1952:2	88.125	0.198167	127.506	1.677667	2	52	2	
4	1952:3	89.625	0.200179	129.385	1.828667	3	52	3	
5	1952:4	92.875	0.201246	128.512	1.923667	4	52	4	
6	1953:1	94.625	0.201052	130.587	2.047333	5	53	1	
7	1953:2	95.55	0.201444	130.341	2.202667	6	53	2	
8	1953:3	95.425	0.202236	131.389	2.021667	7	53	3	
9	1953:4	94.175	0.202723	129.891	1.486333	8	53	4	

Figure 1.12 Demo.xls with additional data of the variables *t*, *Year* and *Q*

(8) Put the cursor again in column 2 and click the right button of the mouse; then click *Paste*. The spreadsheet in Figure 1.12 will be seen. In fact, additional variables, such as the variables *t*, *Year* and *Q* (quarter), can be created, entered or defined in the Excel spreadsheet, before the data set needs to be copied.

(9) Click *File/Saved As...* and then identify a name for the workfile. In this case, Demo_Modified is used, as shown in the following window (Figure 1.13).

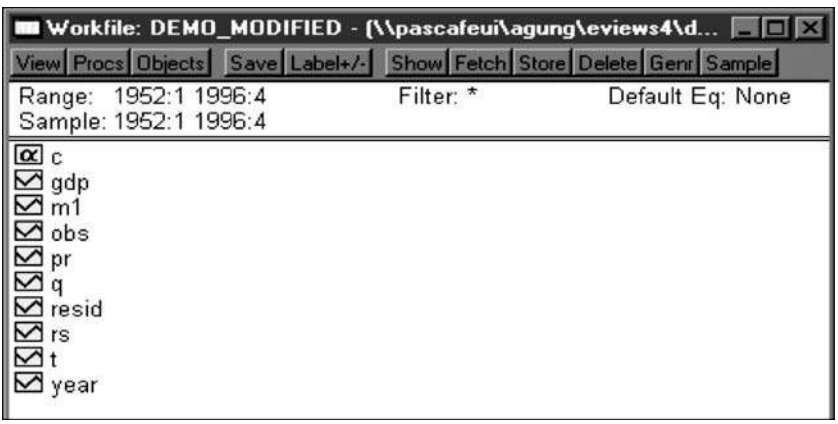


Figure 1.13 List of variables in the Demo_Modified workfile

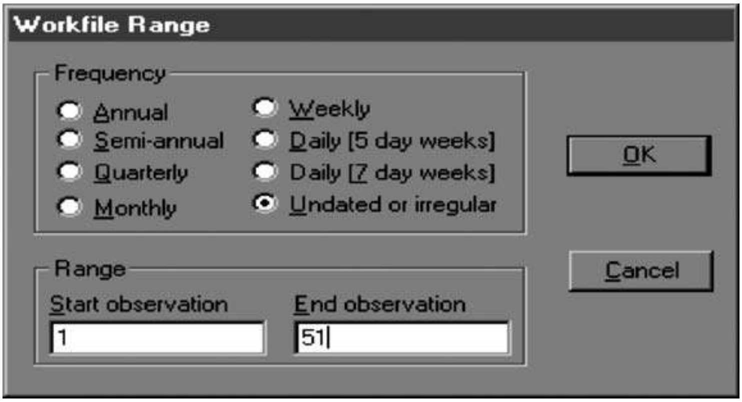


Figure 1.14 The option for creating a workfile based on an undated or irregular data set with 51 observations

1.3.2.1 Creating a workfile based on an undated data set

Figure 1.14 shows an example that can be used to create a workfile based on an undated data set. Using the same process as in the previous subsection, the workfile is created from an Excel datafile having 51 lines. The first line shows the names of the variables and the next 50 lines are the observation units.

1.4 Illustrative data analysis

The examples of the descriptive data analysis, as well as the inferential data analysis presented in this book, will be done using EViews 6. With reference to descriptive data analysis, it has been known that the statistical results are in the form of summary statistical tables and graphs. However, they have a very important role in data evaluation and policy analysis or decision making. Agung (2004) pointed out that summary descriptive statistics are one of the best supporting data for policy analysis. He also presented illustrative examples in selecting specific indicators, factors or variables, to show causal models in the form of summary tables.

However, in this chapter only a few methods are demonstrated in doing a statistical analysis, mainly a descriptive analysis using EViews 6 based on Demo_Modified.wf1.

1.4.1 Basic descriptive statistical summary

The summary statistics of the four numerical variables *GDP*, *M*1, *PR* and *RS* in Demo_Modified can be presented using the following steps:

(1) After opening the workfile, click the variable *GDP*; then by pressing the '*CTRL*' button click the variable *M*1. Make similar executions for the variables *PR* and *RS*; the result is that the four variables are blocked, as shown in Figure 1.15.
(2) Click *OK*...; the four variables will be seen on the screen, as presented in Figure 1.16. Then by clicking *OK* ..., the data of the four variables will be seen on

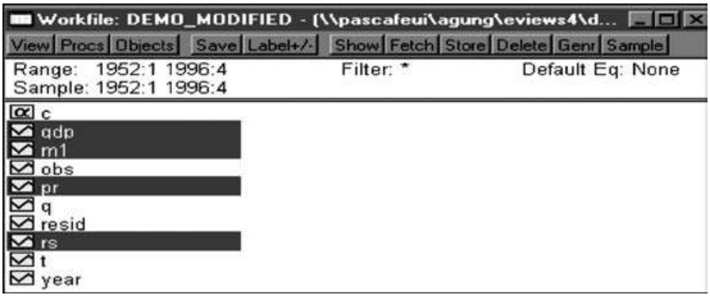


Figure 1.15 Blocked or selected variables that will be analyzed

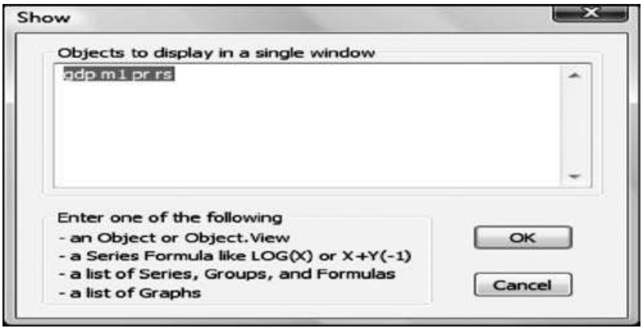
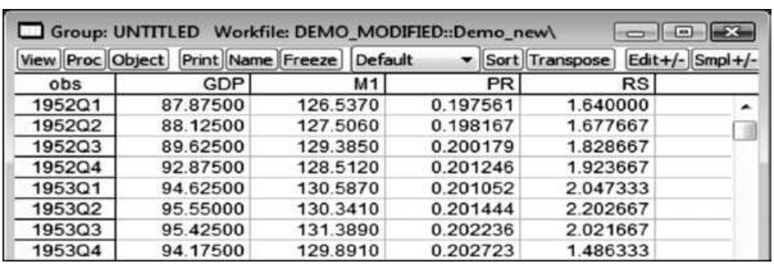


Figure 1.16 The variables whose data will be presented on the screen

the screen, as presented in Figure 1.17. This window should be used as a preliminary data evaluation, particularly for identifying new created variables and/or to edit selected values/scores, if it is needed.

(3) By clicking *View*..., the options in Figure 1.18 can be seen, which shows (14 + 2) alternative options, including two options for Descriptive Stats.

(4) Click *View/Descriptive Stats/Individual Samples*...; the summary descriptive statistics in Figure 1.19 are obtained. Selected computation formulas based on a


Group: UNTITLED Workfile: DEMO_MODIFIED::Demo_new\

View Proc Object Print Name Freeze Default Sort Transpose Edit+/- Smpl+/-

obs	GDP	M1	PR	RS
1952Q1	87.87500	126.5370	0.197561	1.640000
1952Q2	88.12500	127.5060	0.198167	1.677667
1952Q3	89.62500	129.3850	0.200179	1.828667
1952Q4	92.87500	128.5120	0.201246	1.923667
1953Q1	94.62500	130.5870	0.201052	2.047333
1953Q2	95.55000	130.3410	0.201444	2.202667
1953Q3	95.42500	131.3890	0.202236	2.021667
1953Q4	94.17500	129.8910	0.202723	1.486333

Figure 1.17 The screen shot of the data of selected variables in Figure 1.5

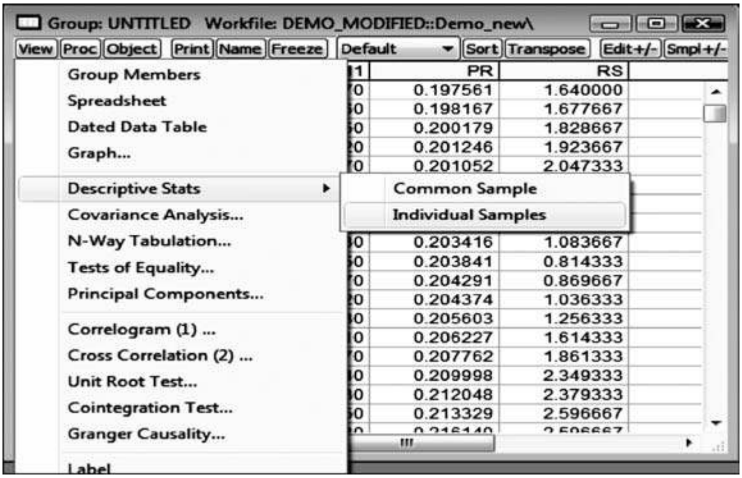
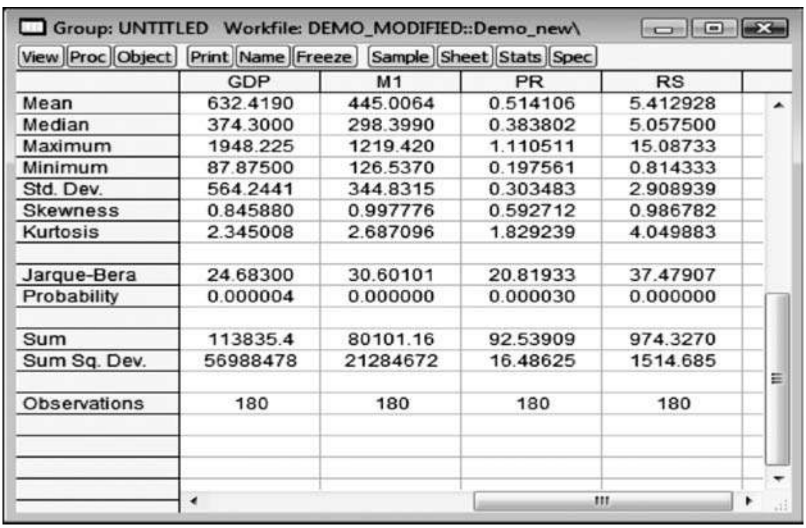


Figure 1.18 The Proc options and the Descriptive Stats options

time series are presented in Table 1.1. In this section the advantages of presenting a summary descriptive statistics will be discussed, as well as the use of the Jarque–Bera statistic, which is included in the descriptive statistics.

1.4.1.1 The Advantages of presenting summary descriptive statistics

The advantages of presenting summary descriptive statistics for all variables in a data set are as follows:



	GDP	M1	PR	RS
Mean	632.4190	445.0064	0.514106	5.412928
Median	374.3000	298.3990	0.383802	5.057500
Maximum	1948.225	1219.420	1.110511	15.08733
Minimum	87.87500	126.5370	0.197561	0.814333
Std. Dev.	564.2441	344.8315	0.303483	2.908939
Skewness	0.845880	0.997776	0.592712	0.986782
Kurtosis	2.345008	2.687096	1.829239	4.049883
Jarque-Bera	24.68300	30.60101	20.81933	37.47907
Probability	0.000004	0.000000	0.000030	0.000000
Sum	113835.4	80101.16	92.53909	974.3270
Sum Sq. Dev.	56988478	21284672	16.48625	1514.685
Observations	180	180	180	180

Figure 1.19 The descriptive statistics of *GDP*, *M*1, *PR* and *RS*

Table 1.1 A list of statistics as a function of $\{y_1, y_2, \ldots, y_T\}$

Name	Statistics/functions
Mean	$\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$
Standard deviation	$s = \sqrt{\sum_{t=1}^{T}(y_t - \bar{y})^2/(T-1)}$
Population variance	$\sigma^2 = \frac{T-1}{T}s^2$
Standard error (Std Err)	$\text{Std Err} = s/\sqrt{T}$
Skewness	$S = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{y_t - \bar{y}}{s\sqrt{(T-1)/T}}\right)^3$
Kurtosis	$K = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{y_t - \bar{y}}{s\sqrt{(T-1)/T}}\right)^4$
Jarque–Bera	$JB = \frac{T}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$, where $S =$ skewness and $K =$ kurtosis
Standard normal	$z = \frac{\bar{y} - \mu_0}{\sigma/\sqrt{T}}$
Chi-squared-statistic	$\chi^2 = \frac{(T-1)s^2}{\sigma^2}$, with d$f = T - 1$
F-statistic	$F = s_1^2/s_2^2$, with $df = (T_1 - 1, T_2 - 1)$
Autocorrelation y at lag k, in EViews 6	$\rho_k = \frac{\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y}_{t-k})/(T-k)}{\sum_{t=1}^{T}(y_t - \bar{y})^2/T}$ where $\bar{y}_{t-k} = \sum y_{t-k}/(T-k)$
Partial autocorrelation y at lag k	Regressed y_t on C, $y_{t-1}, \ldots, y_{t-k}$

(i) To evaluate the scores/measurements of each variable for further or a more advanced statistical analysis. For example, by observing the minimum and maximum scores, it is possible to know whether or not the observed scores are within the expected range. A data set has been observed showing a mother giving birth at the age of 80. This score indicates a typing error. Another case is presented by one of the author's students, Suk (2006), where two numerical variables, %ASTINDO and % BLOCKA, have minimum values = medians = 0. This indicates that at least 50% of their observed values are zeros. As a result, he could not present a linear model based on the whole data set by using either one or both variables in the model.

(ii) The summary statistics, in the form of tables and/or graphs, can easily be understood by a lot more people, compared to the inferential statistics. On the other hand, under the assumption that the data used are valid and reliable, then the summary descriptive statistics would be true statistical values for all individuals in the sample (Agung, 1992, p. 21). As a result, a relevant summary statistics would become an excellent input for policy makers (Agung, 2000a, 2004).

(iii) A positive skewness indicates that observed values of the variable have a long tail to the right, large values or a positive side.

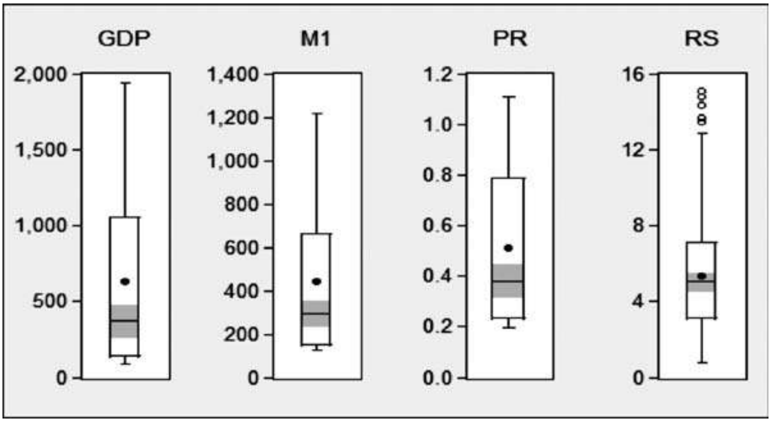


Figure 1.20 Multiple box plots of the variables *GDP*, *M*1, *PR* and *RS*

1.4.1.2 The use of the Jarque–Bera statistic

This statistic can be used to test a null hypothesis where each variable is considered to have a normal distribution. The results in Figure 1.19 show that the data do not support the supposition that each variable has a normal distribution, since the null hypothesis that each variable has a normal distribution is rejected based on a p-value $= 0.0000$. For a detailed discussion on the normality test, refer to Section 2.14.

1.4.2 Box plots and outliers

Selecting *Graph... → Basic Graphs/Boxplot/Multiple Graphs → OK* gives the graphs in Figure 1.20. These graphs can directly present the type of outliers, as presented in the following options.

Note that the box plot of *RS* shows that it has near and far outliers. Furthermore, corresponding to the positive skewness of each variable, as presented in Figure 1.19, these box plots present long vertical lines above each box. The box portion represents 50% of the nonparametric range from the first to the third quartiles (i.e. $Q1$ to $Q3$). The difference between those quartiles represents the *interquartile range* (*IQR*), as presented in Figure 1.21.

The *inner fences* are defined by $Q1 - 1.5^{*}IQR$ and $Q3 + 1.5^{*}IQR$. The data points outside the inner fences are known as outliers, as presented by the box plot of *RS*.

The median is depicted using a line through the centre of the box, while the mean is presented as a symbol or large bold point. Each of the graphs shows that the mean of each variable is greater than its median, which corresponds to its positive skewness, as presented in Figure 1.19.

The bounds of the shaded area are defined by Median $\pm 1.57{*}IQR/\sqrt{T}$.

1.4.3 Descriptive statistics by groups

Since the Demo_Modified contains a group of dated variables, such as *Year* and quartile-*Q*, by clicking *View/Dated Data Table...* the summary statistics by

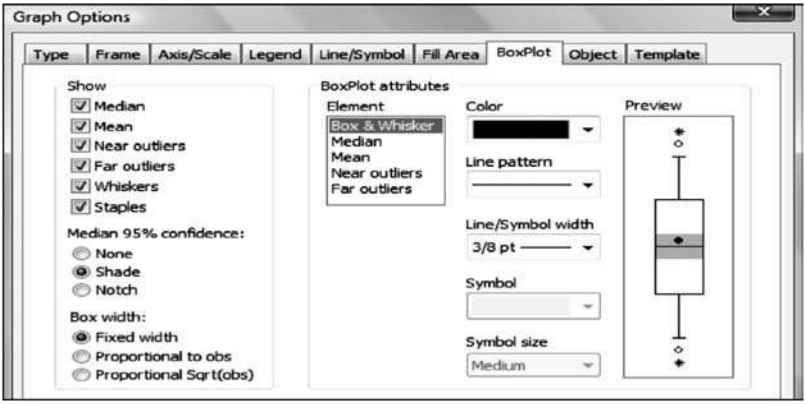


Figure 1.21 The graph options for BoxPlot

categorical variables *Year* and *Q* are obtained, as shown in Figure 1.22. This figure shows the averages of the four variables by *Year* and *Q*, but only presents the summary for the first two years of observations.

1.4.4 Graphs over times

1.4.4.1 Growth curves

Figure 1.23 presents two alternative sets of options for constructing graphical representation of variables, namely the basic and categorical graphs.

In order to have growth curves for each of the four variables considered, click *View/ Graph . . .* → *Lines and Symbol/Multiple Graph* → *OK* to find the growth curves of the four numerical variables *GDP*, *NPM*, *PR* and *RS*, by time, as presented in Figure 1.24. Based on these graphs the following notes and conclusions can be obtained:

(1) These graphs are in fact the bivariate graphs between each of the four variables and the time *t*-variable.

	Q1	Q2	Q3	Q4	Year
	Q1	Q2	Q3	Q4	Year
		1952			1952
GDP	87.9	88.1	89.6	92.9	89.6
M1	126.5	127.5	129.4	128.5	128.0
PR	0.20	0.20	0.20	0.20	0.20
RS	1.6	1.7	1.8	1.9	1.8
		1953			1953
GDP	94.6	95.6	95.4	94.2	94.9
M1	130.6	130.3	131.4	129.9	130.6
PR	0.20	0.20	0.20	0.20	0.20
RS	2.0	2.2	2.0	1.5	1.9

Figure 1.22 The means of *GDP*, *M*1, *PR* and *RS* by quarter and year

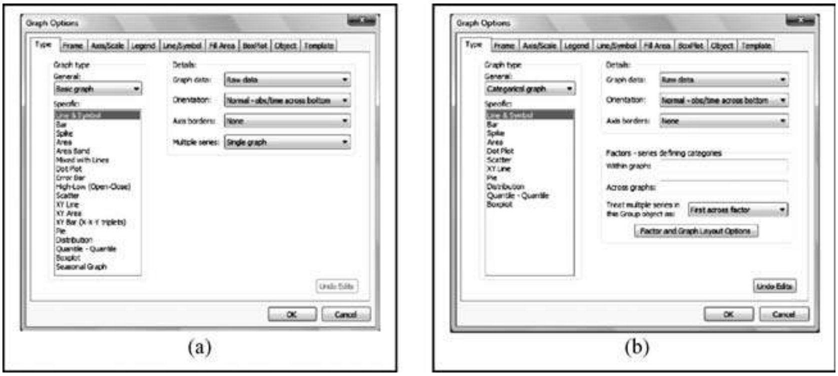


Figure 1.23 The basic graph options (a) and the categorical graph options (b)

(2) The graphs of *GDP*, *M*1 and *PR* clearly show that they have a positive growth rate. However, the graph of *RS* shows a positive growth rate, say, for $t < t_1$ and a negative growth rate for $t \geq t_1$, where the maximum values of *RS* are achieved at $t = t_1 = 119$.

(3) Corresponding to point (2), a conclusion is reached that *RS* should not be used as a predictor of the variables *GDP* and *M*1, as well as *PR*. Moreover, it cannot be considered as a cause factor of the other variables. Note that a causal relationship between two variables should be identified based on a theoretical and substantial basis, supported by their graphical representation(s).

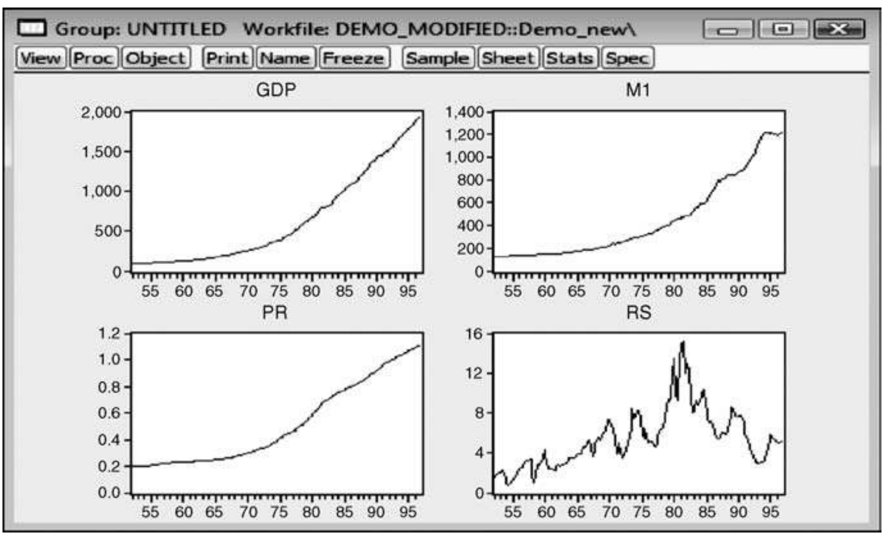


Figure 1.24 Growth curves of the variables *GDP*, *M*1, *PR* and *RS*

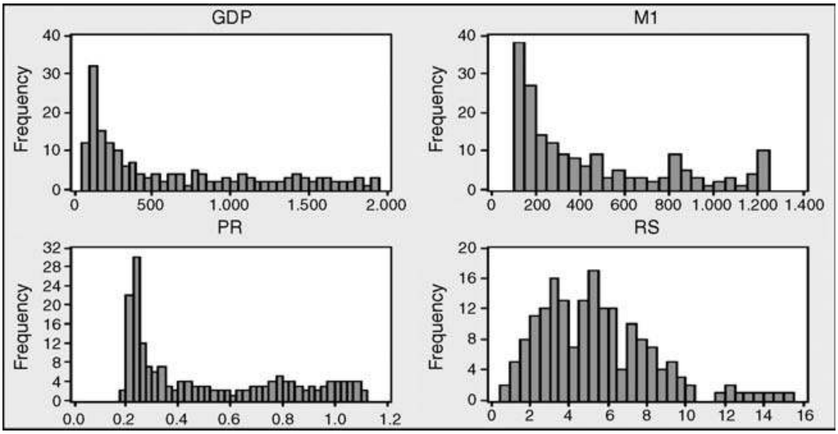


Figure 1.25 Histograms of the variables *GDP*, *M*1, *PR* and *RS*

1.4.4.2 Multiple distributions over time

Alternative distributions of each variable over time can be developed by selecting *Graph . . . /Distribution* and then each of the options (i) Histogram, (ii) Kernel Density and (iii) Theoretical Distribution with a '*Multiple Graphs*' option. The graphs are presented in the following three figures (Figures 1.25 to 27).

Based on these graphs the following notes and comments are made:

(i) The histogram, as well as the kernel density, shows that the observed values of each variable are skewed to the right. As the data are not normally distributed, this is common in general. The discussion should not be about a normal

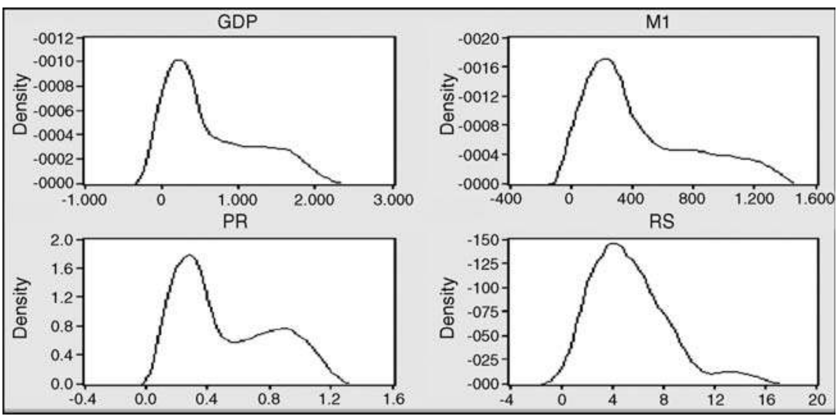


Figure 1.26 Kernel density of the variables *GDP*, *M*1, *PR* and *RS*

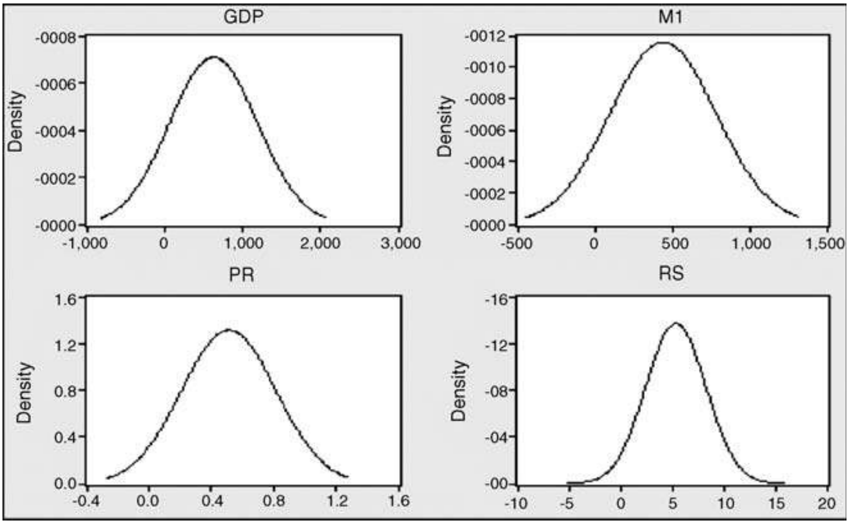


Figure 1.27 Theoretical distributions of the variables *GDP*, *M*1, *PR* and *RS*

distribution of a sample data set, but only the sampling distribution or the distribution of a statistic, as a real-valued function, based on a random sample.

(ii) Figure 1.27 presents theoretical distributions of the four variables *GDP*, *M*1 and *PR*, as well as *RS*, which are normal distributions using the default option. These theoretical normal distributions are not observable distributions. They are in fact the distributions of the mean statistics or the sample space of means of all possible random samples of a fixed size that could be selected from a defined population. These theoretical normal distributions are supported by the *Central Limit Theorem*. For additional and more detailed notes and comments, refer to Sections 1.5 and 2.14.

(iii) Since EViews provides many smoothing graphs, as well as theoretical distributions, and it is never known which one is the best alternative graph, it is suggested that the default option should be used.

1.4.5 Means seasonal growth curve

By clicking *Graph* . . . , selecting *Basic Graph/Seasonal Graph* and then clicking *OK*, graphs of the means of the variables by season can be obtained, as shown in Figure 1.28.

1.4.6 Correlation matrix

By clicking *Views/Covariance Analysis* . . . , the options presented in Figure 1.29 are obtained. Note that these options are not available in EViews 4 and 5.

By selecting the options Covariance, Correlation, *t*-statistic and Probability, the correlation matrix presented in Figure 1.30 is obtained. Based on this figure the following notes and comments are made:

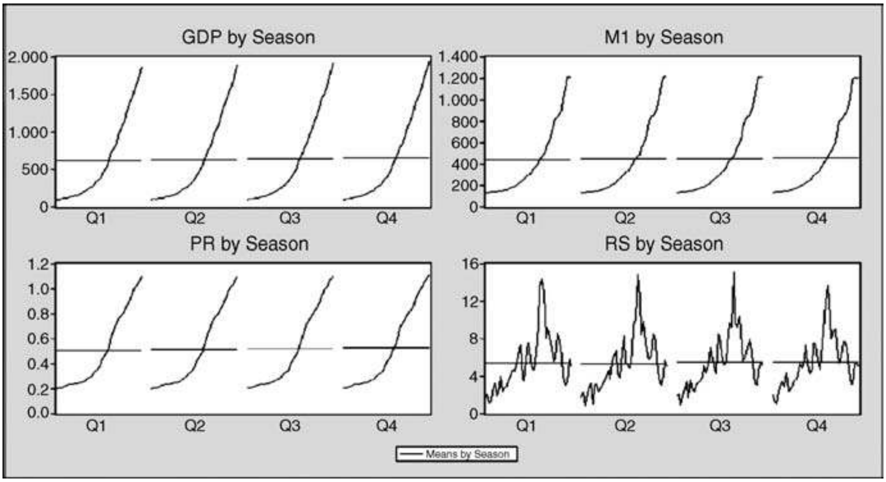


Figure 1.28 Graphs of means of the variables *GDP*, *M*1, *PR* and *RS*, by season

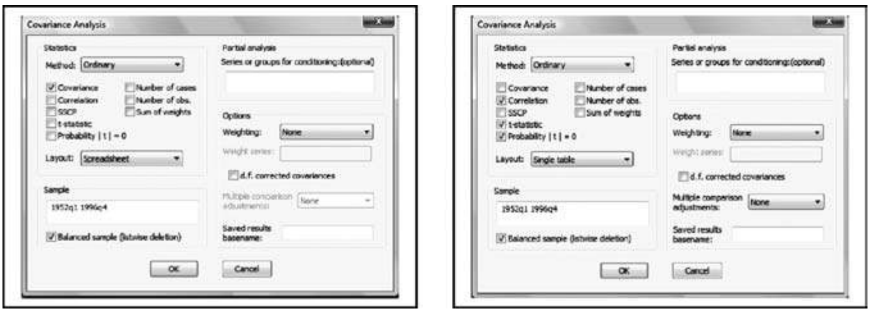


Figure 1.29 Selected options to construct a correlation matrix with the *t*-statistic

Covariance Analysis: Ordinary
Date: 10/14/07 Time: 06:31
Sample: 1952Q1 1996Q4
Included observations: 180

Correlation t-Statistic Probability	GDP	M1	PR	RS
GDP	1.000000 ----- -----			
M1	0.995197 135.6364 0.0000	1.000000 ----- -----		
PR	0.992475 108.1367 0.0000	0.980402 66.39448 0.0000	1.000000 ----- -----	
RS	0.333494 4.719553 0.0000	0.270059 3.742084 0.0002	0.412471 6.040862 0.0000	1.000000 ----- -----

Figure 1.30 A correlation matrix of the variables *GDP*, *M*1, *PR* and *RS*

(1) The p-value of the t-statistic presented is for the two-sided hypothesis. However, it can also be used to test a one-sided hypothesis. In this case, since the observed correlation of each pair is positive, it can be concluded that each pair of the variables *GDP*, *M*1, *PR* and *RS* (in the corresponding population) has a significant positive correlation with a p-value $= 0.0000/2 = 0.0000$.

(2) These coefficients of correlation can also represent the statistical results of the standardized simple linear regressions, with the following equation:

$$ZY_t = \beta^* ZX_t + \mu_t = \rho^* ZX_t + \mu_t \tag{1.1}$$

where *ZX* and *ZY* are the *Z*-scores of the variables *X* and *Y* respectively and ρ is the correlation parameter of (X,Y) in the population. For this reason, the bivariate correlation could also be used to learn or to test a linear causal effect of a source (an independent or explanatory) variable on a downstream (dependent or impact) variable. However, at the first stage, the causal relationship between a pair of variables should be defined based on a theoretical and substantive basis.

(3) The variance, covariance and the moment product correlation based on the time series X_t and Y_t are defined as follows:

$$\mathrm{Var}(X) = \frac{1}{T-1}\sum_{t=1}^{T}(X_t - \bar{X})^2 \tag{1.2}$$

$$\mathrm{Var}(Y) = \frac{1}{T-1}\sum_{t=1}^{T}(Y_t - \bar{Y})^2 \tag{1.3}$$

$$\mathrm{Cov}(X,Y) = \frac{1}{T-1}\sum_{t=1}^{T}(X_t - \bar{X})(Y_t - \bar{Y}) \tag{1.4}$$

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X).\mathrm{Var}(Y)}} \tag{1.5}$$

1.4.7 Autocorrelation and partial autocorrelation

For a time series data set, the autocorrelation and partial autocorrelation coefficients (*AC* and *PAC*) of each dated variable can also be identified. The sample autocorrelation function of a dated variable Y_t at lag k is computed as follows:

$$\begin{aligned} \hat{\rho}_k &= \hat{\gamma}_k / \hat{\gamma}_0 \\ \hat{\gamma}_k &= \sum (Y_t - \bar{Y})(Y_{1-k} - \bar{Y}_{t-k})/T \\ \hat{\gamma}_0 &= \sum (Y_t - \bar{Y})^2/T \end{aligned} \tag{1.6}$$

To obtain a more precise estimate of the PAC, simply run the regression:

$$Y_t = C(1) + C(2)Y_{t-1} + \cdots + C(k-1)Y_{t-(k-1)} + \rho_k Y_k + e_t \tag{1.7}$$

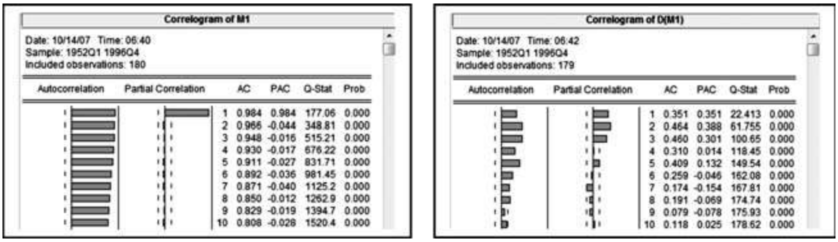


Figure 1.31 Correlograms of the variables $M1$ and $D(M1)$

In addition to the AC and PAC, there is also a Q-statistic, which is a test statistic for the *joint hypothesis*, which stipulates that all of the γ_k up to a certain lag are simultaneously equal to zero. The Q-statistic is defined as

$$Q = T\sum_{k=1}^{m}\rho_k^2 \tag{1.8}$$

where $T =$ sample size and $m =$ lag length.

A variant of this Q-statistic is the *Ljung–Box* (*LB*)-statistic, which is defined as

$$LB = T(T+2)\sum_{k=1}^{m}\left(\frac{\hat{\rho}_k^2}{n-k}\right) \approx \chi^2(m) \tag{1.9}$$

It has been found that the LB-statistic has better (or more powerful, statistically speaking) small properties than the Q-statistic.

Figure 1.31 shows the correlograms of the variable $M1$ and its first-difference $D(M1)$ respectively. The dotted lines in the plots of the partial correlation are the approximate two standard error bounds computed as $\pm 2/\sqrt{T}$. The Q-statistic is presented together with its probability. These results can be obtained by selecting *View/Correlogram* . . . . This matter will be discussed in more detail later.

1.4.8 Bivariate graphical presentation with regression

The relationship between pairs of variables, including a causal relationship, can also be presented using graphs. For an illustration, the scatter graph with regression of $M1$ on *GDP*, as well as $M1$ on *RS*, will be presented. The stages of data analysis are as follows:

(1) At the first stage, the data of the variables $M1$ on *GDP* are presented on the screen by blocking the variables and then clicking *Show* → *OK*.
(2) Select *View/Graph/Scatter* with *Regression Line* as presented in Figure 1.32 and then click *OK*, which will give the graph in Figure 1.33(a).
(3) By doing the same process the graph with regression of $M1$ on *RS* will be obtained, as shown in Figure 1.33(b). Based on this graph, it can be concluded that $M1$ and *RS* do not have a linear relationship. In other words, *RS* should not be used as a linear predictor of $M1$ and moreover as a cause factor of $M1$. In fact, this condition can already be identified by observing their growth graphs in Figure 1.24.

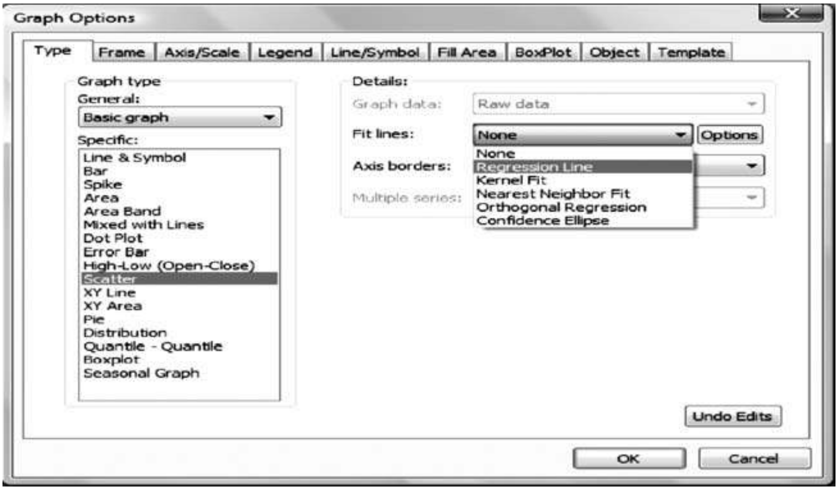


Figure 1.32 The scatter graph option with regression line

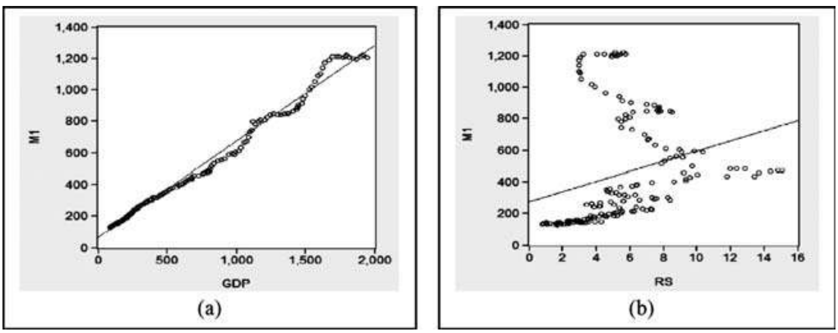


Figure 1.33 Scatter graphs with regression lines of (a) $M1$ on GDP and (b) $M1$ on RS

(4) Another type of graph can be presented by using an '*Orthogonal Regression*' as the fit-lines. This gives the graphs in Figure 1.34. The orthogonal regression is defined by using the horizontal distances of the observed values (points) to the regression line, while the general regression is defined by using the vertical distances of the points to the regression line. Note that the regressions of $M1$ on RS in Figures 1.33 and 1.34 are quite different. Which linear regression do you think is a better graph for representing the data?

1.5 Special notes and comments

In this section the use of scatter plots will be discussed as a preliminary analysis for studying relationships between numerical variables. It also offers some recommendations.

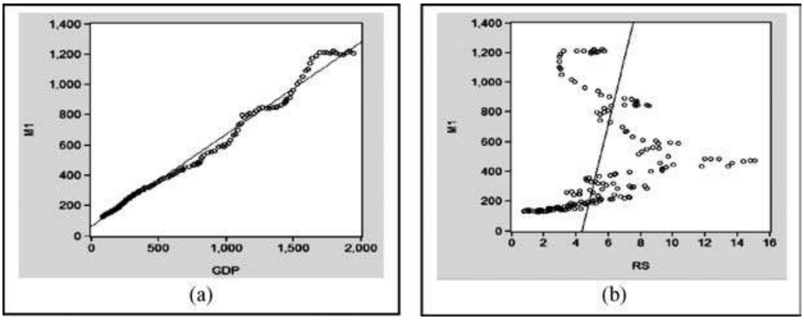


Figure 1.34 Scatter plots with orthogonal regressions of (a) *M*1 on *GDP* and (b) *M*1 on *RS*

(1) Graph representation between a pair of variables, as well as their correlation coefficient, could not be used to derive a conclusion that both variables have a causal relationship. A causal relationship between variables should be identified using a theoretical and substantive basis. For example, if variables *X* and *Y* both have monotonic growth curves by time, this does not directly mean that they have a causal relationship. On the other hand, if a respond variable *Y* has a monotonic growth curve, but *X* does not, it is almost certain that *X* is not a cause factor of *Y*. For example, based on the growth curves of *GDP*, *M*1, *PR* and *RS* in Figure 1.24, there can be every confidence that the variable *RS* is not a cause factor for the other three variables, *NPM, GDP* and *PR*, since *RS* does not have the same pattern of growth as the other variables, but has a maximum value at a time point $t = t_1 = 119$.
(2) In many cases, when based on the whole sample data, a bivariate scatter plot cannot give a good picture that both variables have either a linear or nonlinear relationship. However, within some subsamples, they could. Agung (2006, p. 312) proposed three methods for defining subsamples based on a numerical variable, using (i) nonparametric statistics, such as median, quartiles and percentiles, (ii) parametric statistics, such as sample mean and its standard deviation, and (iii) subjective or expert judgment. This technique had been presented in a dissertation of the author's student, Do Anh Dung (2006). He was constructing four subsamples to show that OCB-I has a positive and significant effect on the Company Performance within a relevant subsample.
(3) On the other hand, Wilson and Keating (1994, p. 161) present an illustration of the scatter plots of four bivariate $\{X, Y\}$ data sets that have very similar statistical properties, but are visually quite different. They show that each of the data sets has the same OLS simple linear regression equation, that is $Y = 3 + 0.5X$.
(4) Furthermore, in other cases, a bivariate scatter plot could demonstrate that it is impossible for someone to find or define a specific regression model, especially a continuous regression model, that could have a good fit to the sample data. As an example, refer to the scatter plot with regression of (*M*1, *RS*) presented in Figure 1.33, as well as the following graphs, which are presented by the author's students, Narindra (2006) and Gunawan (2005) respectively. In these cases, nonparametric regression models should be used, which will be presented in Chapter 11.

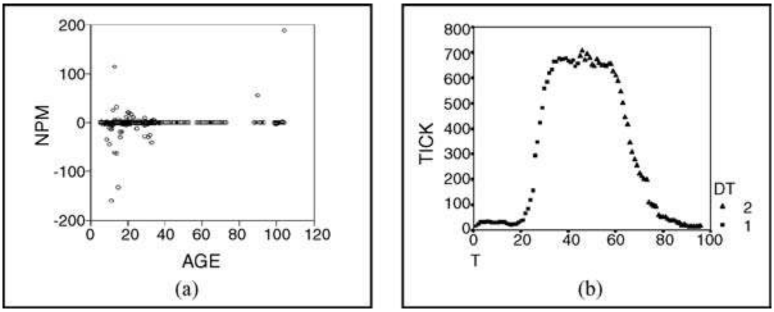


Figure 1.35 Illustrative scatter plots of (a) Narindra's data set and (b) Gunawan's data set

Note that the scatter graph in Figure 1.35(a) shows several outliers and there are very small and very large observed values of *NPM*. These could make a subset of many observed values, represented by the horizontal thick line. In order to obtain a good model, a sub-data set should be used without the outliers. On the other hand, even the scatter graph in Figure 1.35(b) does not show any outlier, so it is very difficult to define a smooth or continuous regression model.

(5) Figure 1.36 presents illustrative graphs of the four selected time series or dated variables, *PPI*, *FF*, *URATE* and *Y*, in BASICS.wf1 of the EViews examples. Compared to the graph of *PPI*, it is very difficult or impossible to define a smooth growth model based on each of the other three variables.

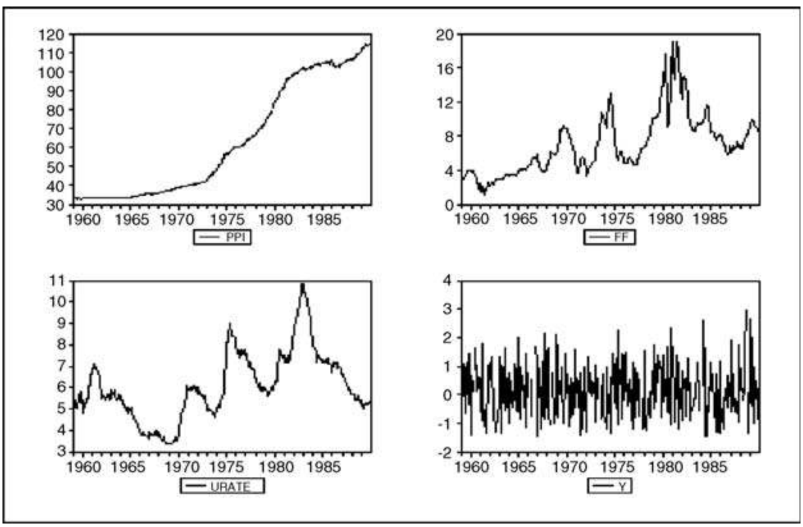


Figure 1.36 Growth curves of the variables *PPI*, *FF*, *URATE* and *Y* in BASICS.wf1 of EViews

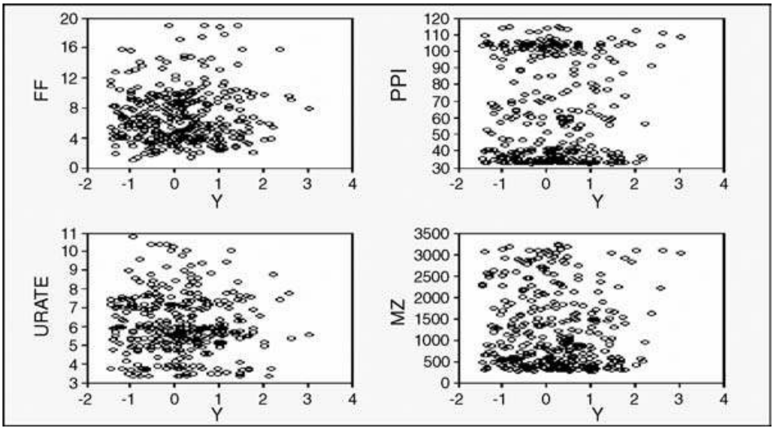


Figure 1.37 Bivariate scatter plots between the variables *FF*, *PPI*, *URATE*, *M*2 and *Y*

Furthermore, suppose that the variable Y is considered as an endogenous variable. Then by taking each of the variables *FF*, *PPI*, *URATE* and *M*2 as exogenous, there will be four bivariate scatter graphs as presented in Figure 1.37. What type of model with an endogenous variable Y could or would be proposed?

(6) These illustrations clearly show that a scatter plot has a very important role in developing an empirical statistical model.

(7) It should be noted that EViews provides many options for doing a descriptive data analysis, in particular for graphical presentation either in parametric or nonparametric techniques. However, to select the best option for a specific data set is not an easy task. The above illustrative graphs show that there is a need to evaluate case by case in order to define an empirical model. Several scatter graphs between a selected endogenous variable and each of the numerical exogenous variables should be used in making the best possible model selection, even though it would be very subjective. In addition to these graphical representations, many alternative equations or types of time series models exist and will be presented in the following chapters.

1.6 Statistics as a sample space

Corresponding to a time series $\{y_t, t = 1, 2, \ldots, T\}$, a statistic is defined as a *real valued function* of $\{y_1, y_2, \ldots, y_T\}$, namely $f\{y_1, y_2, \ldots, y_T\}$ Note that this statistic is not a number or constant value, but represents a set of values based on all possible samples of size T, which could be selected or observed from the series, random variable or population Y. A set of those real values or all possible values of $f\{y_1, y_2, \ldots, y_T\}$ is called the *sample space* of the corresponding statistic; specifically it is the *real-valued sample space*. As a result, a sample space can never be observed, for only a constant number is a member or an element of the corresponding sample space. For example, in practice, there is only a sampled mean, as well as other statistical values based on a set of observed values $\{y_1, y_2, \ldots, y_T\}$. Table 1.1 presents selected statistics as a function of the series $\{y_t,\ t = 1,\ 2, \ldots,\ T\}$; it is not a score or statistical value computed based on a sampled data set.

Furthermore, it is well known that a statistic or a real-valued sample space has a theoretical distribution. The very basic and important distribution is the normal distribution of the mean statistic, namely $\bar{y} = (y_1 + y_2 + \ \ldots \ + y_T)/T$, if and only if $\{y_1, y_2, \ldots, y_T\}$ is a random sample, as stated in the *Central Limit Theorem*. Refer to the further notes and comments presented in Section 2.14.

2

Continuous growth models

2.1 Introduction

Time series data are used in all fields of studies, including economics and finance. Hence, the growth models presented in this chapter could apply to all studies by using time series data sets. It has been well recognized that the unit of observations, as well as the unit of data analysis, is a discrete time variable, say t, for $t = 1, 2, \ldots, T$. However, in growth models, the t-variable can be used as an independent variable, but not as a cause factor.

Furthermore, the time series data analysis should have at least three main objectives, namely (i) to present growth models of specific numerical macroindicators using the time t-variable as an independent variable, (ii) to present models without using the time t-variable as an independent variable, in other words, to study the possible causal relationship between dated indicators or variables and (iii) to forecast.

In the time series data analysis, the simplest growth models to be considered are the two classical growth curve models, such as the geometric and the exponential growth models, based on a bivariate indicators, say (Y_t, t). The data analysis will be presented based on the data in workfile 'Demo_Modified', as discussed in Chapter 1.

2.2 Classical growth models

The classical growth models are the geometric growth model, which can be presented by an equation:

$$Y_t = Y_0(1 + r_g)^t \tag{2.1}$$

and the exponential growth model:

$$Y_t = Y_0 \exp(r_e t) \tag{2.2}$$

For estimation or projection purposes, both models could be estimated using a semilog (i.e. semilogarithmic) regression model as follows:

$$\log(Y_t) = \alpha + \beta \cdot t + \varepsilon_t \tag{2.3a}$$

Time Series Data Analysis Using EViews IGN Agung

However, in EViews the model will be presented and saved in the following form, with the $\log(Y_t)$ indicating the natural logarithm of Y_t:

$$\log(Y_t) = C(1) + C(2) \cdot t + \mu_t \tag{2.3b}$$

Note that the corresponding regression function of the model in (2.3) can be written as

$$\log(Y_t) = a + b \cdot t \tag{2.4}$$

which is a continuous function of the time t-variable, with $d\log(Y)/dt = b$. For this reason, all models presented in this book having the time t as an independent variable will be considered as continuous growth models.

The following sections only present statistical results based on various continuous growth models, with special notes and comments. The theoretical concept of the Ordinary Least Squares (OLS) estimation method is presented in Appendices A, B, and C.

Example 2.1. (Basic regression model in (2.3)) Considering the variable $M1$ (money supply) in the Demo_Modified workfile, the steps of data analysis using the growth model in (2.3) are as follows:

(1) Having the 'Demo_Modified' workfile on the screen, click *Quick/Estimate Equation* ...; the options in Figure 2.1 will be seen on the screen. Then enter a series of variables in the space of the '*Equation specification*,' as shown in the window (note the form of the explicit equation). This gives a growth model having $\log(m1)$ as a dependent variable, $C(1)$ as the intercept parameter and $C(2)$ as the slope parameter. Note that this table presents the options of the least squares (LS) estimation method (NLS and ARMA), as well as the sample used in this analysis. This may be modified, depending on the need.

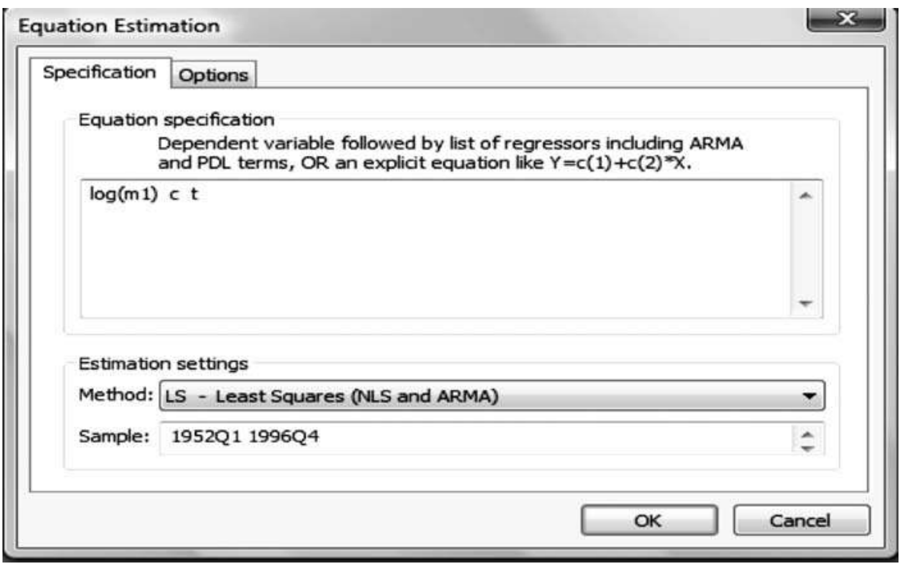


Figure 2.1 Equation specification, estimation settings and options for doing univariate regression analysis

Equation: UNTITLED Workfile: DEMO_MODIFIED::Demo_ne...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 17:08
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.517962	0.018429	245.1609	0.0000
T	0.014290	0.000177	80.92125	0.0000

R-squared	0.973537	Mean dependent var	5.811220
Adjusted R-squared	0.973388	S.D. dependent var	0.754650
S.E. of regression	0.123108	Akaike info criterion	-1.340464
Sum squared resid	2.697683	Schwarz criterion	-1.304987
Log likelihood	122.6418	Hannan-Quinn criter.	-1.326080
F-statistic	6548.249	Durbin-Watson stat	0.015856
Prob(F-statistic)	0.000000		

Figure 2.2 Statistical results based on a growth model of $M1$

(2) Click *OK*; the results shown in Figure 2.2 will appear.

Based on these results, comments on some of the basic statistics can be presented as follows:

(1) *R-squared*. For the time series models having k exogenous variables, the (centered) R^2 is the coefficient of determination, and in EViews is computed as

$$R^2 = 1 - \frac{e'e}{(y-\bar{y})'(y-\bar{y})}, \; \bar{y} = \sum_{t=1}^{T} y_t/T \qquad (2.5)$$

with $0 \le R^2 \le 1$, where $\mathbf{y} = (y_1, \ldots, y_T)'$ and $\mathbf{e} = (e_1, \ldots, e_T)'$ are the vectors of the observed values and the estimated error terms respectively. If $y_t = \bar{y}, \forall t$ (i.e. the coefficient of all independent variables is equal to zero) then $R^2 = 0$, and $R^2 = 1$ if and only $y_t = \hat{y}, \forall t$ (i.e. all observations fall directly on the fitted response surface). The positive square root of R^2, namely R, is the coefficient of multiple correlations between all independent variables with the dependent variable. Furthermore, for $k = 1$, then R^2 will be reduced to the coefficient of simple determination, namely r^2, and r is a bivariate (simple) coefficient of correlation with $-1 \le r \le +1$.

(2) *Adjusted R-squared.* The adjusted R^2 is measured as

$$R_a^2 = 1 - (1 - R^2)\frac{T-1}{T-k} \qquad (2.6)$$

where k is the number of model parameters. The adjusted R-squared value is never larger than R^2, can decrease as independent variables are added and, for poorly fitting models, it may be negative (EViews 4 User's Guide, p. 265).

(3) *Large values of* R^2. In this case, there is a very large $R^2 = 0.973\,537$, and it may be concluded that the model is a very good fit for estimating the growth curve of the observed scores of $M1$. This number indicates that 97.3537% of the total variation of log($m1$) can be explained by the time t. However, note that a large R^2 does not directly imply that the model is a good or useful one. By observing a very small DW (Durbin–Watson)-statistic of 0.015 856, in fact this is an autocorrelation problem with the error terms of the model. Therefore, this model is not an appropriate model for statistical inference and so should be revised or modified, as will be presented in the following examples.
(4) *Small values of* R^2. Even though a value of R^2 is (very) small, the model could be an acceptable one, in a statistical sense, whenever the scatter plot of the error terms represent a tape along the line $e = 0$.
(5) *The F- and t-statistics*. In general, the F-statistic will be used to test the joint effects of all exogenous variables and the t-statistic will be used to test the adjusted effect of an exogenous variable on the corresponding endogenous variable. Note that the t-statistic presented in the output can also be used to test the one-sided hypothesis.

By assuming that the model in this example is an acceptable model, since $k = 1$, then the F- and t-statistics can be used to test the two-sided hypothesis, i.e. the effect of the time t on log($m1$). The null hypothesis H_0: $C(2) = 0$ is rejected based on the F-statistic when $F_0 = 6548.249$ with $df = (1, 178) = (k, T - (k + 1))$ and the p-value $= 0.0000$ or based on the t-statistic when $t_0 = 80.92$ with $df = 178$ and the p-value $= 0.0000$. Note that $F_0 = t_0^2$, for $k = 1$. In this case, $(80.921\,25)^2 = 6548.249 = F_0$. However, the t-statistic can also be used for testing a one-sided hypothesis. For example, if a hypothesis is proposed that $M1$ has a positive growth rate, then a statistical hypothesis would be

$$H_0 : C(2) \leq 0 \text{ versus } H_1 : C(2) > 0 \tag{2.7}$$

For this hypothesis, since $t_0 > 0$, there will be a t-statistic with a p-value $= 0.0000/2 = 0.0000$, and the null hypothesis is rejected. As a result, it can be concluded that '*the data supports the proposed hypothesis*,' or $M1$ has a significant positive growth rate, or the time t has a significant positive effect on log($m1$).

(6) *AIC and SC.* Finally, the Akaike Information Criterion (AIC) is used in model selection for nonnested alternatives, with smaller values of AIC preferred. The Schwarz Criterion (SC) is an alternative to the AIC and imposes a larger penalty for an additional coefficient. Two models are considered as nonnested models if and only if the set of exogenous or independent variables of the first model is not the subset or upper set of the other model. Since in this example there is only one model, these statistics will not be used.
(7) *Residual graph.* The residual graph should be used to study visually the autoregressive part of the defined model. The residual graphs in Figure 2.3 can be obtained by clicking *View/Actual*, *Fitted*, *Residual/Actual*, *Fitted*, *Residual Graph*. Note that the graph shows that the sign (±) of the estimated error terms

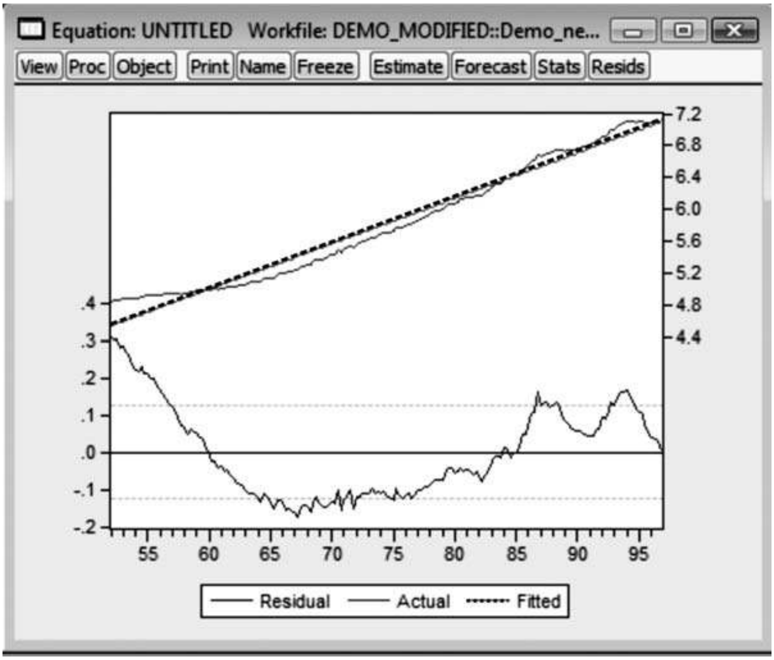


Figure 2.3 Residual graphs of the *M*1 growth model

has systematic changes along the line $e = 0$, which indicates that the error terms are serially correlated. Therefore, there is a need to consider using autoregressive models, as presented in the following section. □

2.3 Autoregressive growth models

An autoregressive growth model is defined as a growth model that takes into account the serial correlation of the error terms in the growth model (2.1). Hence, there should be an appropriate regression model to do the statistical inferences. The following subsections will present autoregressive growth models, starting with the simplest one.

2.3.1 First-order autoregressive growth models

The simplest first-order autoregressive growth model, say AR(1)_GM, could be presented as

$$\begin{aligned} \log(Y_t) &= C(1) + C(2).t + \mu_t \\ \mu_t &= \rho.\mu_{t-1} + \varepsilon_t \end{aligned} \tag{2.8}$$

where $-1 < \rho < +1$ is the first-order serial correlation or autocorrelation coefficient between the error terms μ_t, that is the correlation between μ_t and μ_{t-1}. For this model it is expected or assumed that the error term ε_t is the stochastic term of the AR(1)_GM, so that it can satisfy the standard *OLS* assumptions, namely

$$\begin{aligned} &E(\varepsilon_t) = 0 \\ &\mathrm{Var}(\varepsilon_t) = \sigma_\varepsilon^2 \\ &\mathrm{Cov}(\varepsilon_t, \varepsilon_{t+s}) = 0, \quad s \neq 0 \end{aligned} \tag{2.9}$$

Note that the residual series in the second line of (2.8) can be extended as

$$\begin{aligned} \mu_t &= \rho^2 \cdot \mu_{t-2} + \varepsilon_t + \rho \cdot \varepsilon_{t-1} = \rho^3 \cdot \mu_{t-2} + \varepsilon_t + \rho \cdot \varepsilon_{t-1} + \rho^2 \cdot \varepsilon_{t-2} \\ &= \rho^h \cdot \mu_{t-h} + \varepsilon_t + \rho \cdot \varepsilon_{t-1} + \rho^2 \cdot \varepsilon_{t-2} + \ldots + \rho^h \cdot \varepsilon_{t-h} \end{aligned} \tag{2.10}$$

Since $|\rho| < 1$, then

$$\lim_{h \to \infty} \rho^h = 0 \tag{2.11}$$

2.3.2 AR(p) growth models

A more general autoregressive growth model is the *p*th-order autoregressive growth model, namely AR(p)_GM, which can be presented as

$$\begin{aligned} &\log(Y_t) = C(1) + C(2) \cdot t + \mu_t \\ &\mu_t = \rho_1 \cdot \mu_{t-1} + \ldots + \rho_p \cdot \mu_{t-p} + \varepsilon_t \end{aligned} \tag{2.12}$$

where ρ_p is a partial autocorrelation or serial correlation coefficient between μ_t and μ_{t-p}.

This model could also have the following form:

$$\begin{aligned} &\log(Y_t) = C(1) + C(2) \cdot t + \mu_t \\ &\Delta\mu_t = \rho_1 . \Delta\mu_{t-1} + \ldots + \rho_p \cdot \Delta\mu_{t-p} + \Delta\varepsilon_t \end{aligned} \tag{2.13}$$

where $\Delta\mu_t = \mu_t - \mu_{t-1}$ is the first-difference of the residual term μ_t.

Example 2.2. (AR(1) growth model) Here, the AR(1) growth model of $M1$ is considered, with the following equation:

$$\log(m1_t) = C(1) + C(2) \cdot t + [AR(1) = C(3)] + \varepsilon_t \tag{2.14a}$$

or

$$\log(m1_t) = C(1) + C(2) \cdot t + C(3)^* \mu_{t-1} + \varepsilon_t \tag{2.14b}$$

The statistical results in Figure 2.4 can be obtained by entering the variables

$$\log(M1) C\ T\ AR(1) \tag{2.15}$$

in the '*Equation specification*' window.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 17:34
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.155760	0.198262	20.96100	0.0000
T	0.016575	0.001168	14.19412	0.0000
AR(1)	0.974460	0.009047	107.7095	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Akaike info criterion	-5.563732
Sum squared resid	0.038862	Schwarz criterion	-5.510312
Log likelihood	500.9540	Hannan-Quinn criter.	-5.542071
F-statistic	228602.4	Durbin-Watson stat	2.168644
Prob(F-statistic)	0.000000		

Figure 2.4 Statistical results based on the growth model in (2.8)

Based on these results, the following conclusions may be derived:

(1) The growth rate of the money supply, $M1$, is $\hat{C}(2) = 0.016\,575$ and the time t-variable has a significant effect on log($m1$) with a p-value of 0.0000.
(2) The null hypothesis of no first-order autocorrelation, H_0: $\rho = 0$, is rejected with a p-value of 0.0000 and its point estimator is $\hat{\rho} = 0.974\,460$ with Std Err $= 0.009\,047$.
(3) The DW-statistic $= 2.17$, which indicates that this model is better than the growth model (2.14). Also note that the comments presented in the following example are based on its residual graph.
(4) The statistical result in Figure 2.5 can be obtained by selecting *View/Representations . . ..* This figure shows the estimation command and equation, as well as the regression function.
(5) By clicking *View/Actual*, *Fitted*, *Residual/Actual*, *Fitted*, *Residual Graph*, the residual graph in Figure 2.6 is presented. This residual graph should be used to

```
Estimation Command:
=========================
LS LOG(M1) C T AR(1)

Estimation Equation:
=========================
LOG(M1) = C(1) + C(2)*T + [AR(1)=C(3)]

Substituted Coefficients:
=========================
LOG(M1) = 4.15575954915 + 0.0165752230174*T + [AR(1)=0.9744597
```

Figure 2.5 The estimation command and equation of the model in (2.8)

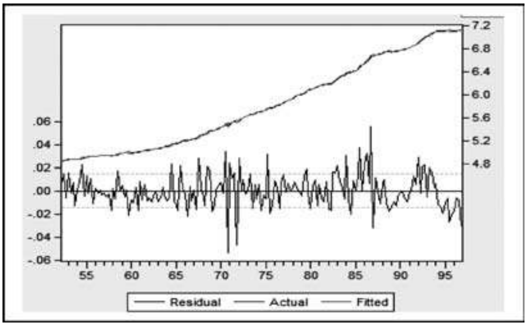


Figure 2.6 Residual graph of the model in (2.8)

evaluate, visually, the correctness of the model and, in particular, whether the data support the error term assumptions. Note that the residual graph in Figure 2.6 does not show that the residual term ε_t has systematic changes in its signs (±) along the line $e = 0$. Hence, this model is better than the previous model. □

2.4 Residual tests

With either the statistical results or the residual graph on the screen, select *View/ Residuals Tests* and a complete list of alternative residual tests will appear, as presented in Figure 2.7. Note that this screen shot is obtained after having the

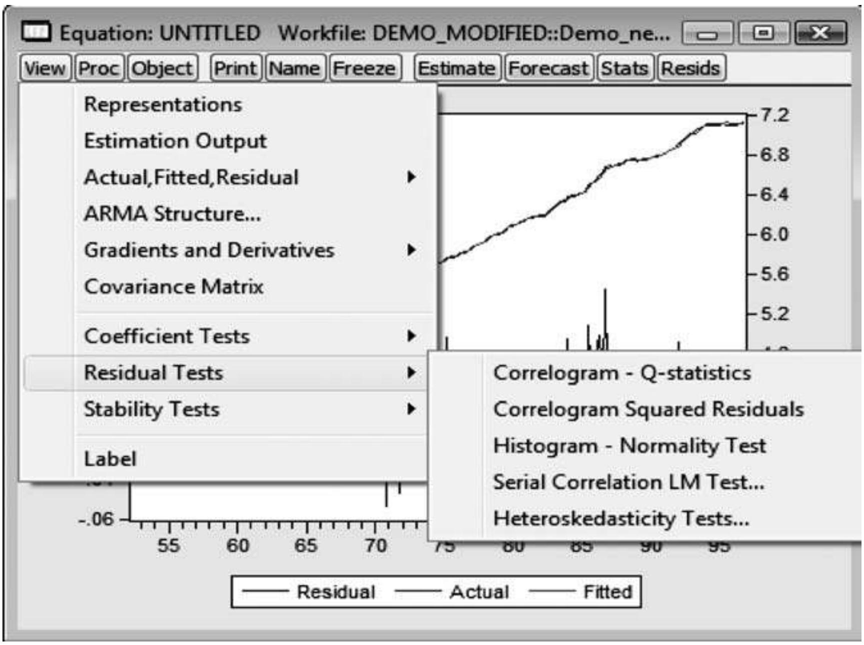


Figure 2.7 The options of residual tests, after having the residual graph

residual graph on the screen. For illustrative purposes, this section will present several statistics and comments corresponding to the model presented in Example 2.1.

2.4.1 Hypothesis of no serial correlation

By clicking *View/Residual Tests/Serial Correlation M Test* ... and entering 1 (one) for the number of lagged variables, the BG serial correlation LM test shown in Figure 2.8 will appear. This test shows that the hypothesis of no serial correlation is accepted for the AR(1) growth model, based on the chi-square-statistic (Obs*R-squared $= T^*R^2$) of 1.770 743 with $df = 1$ and a p-value $= 0.1833$ or the F-statistic of $F_0 = 1.748\,470$ with $df = (1, 175)$ and a p-value $= 0.1878$.

Furthermore, the following regression with the t-statistic in [·] is obtained:

$$\hat{Resid} = \underset{[-0.1417]}{-0.028\,195} + \underset{[0.1316]}{0.000\,154t} + \left[AR(1) = \underset{[0.1610]}{0.001\,464}\right] - \underset{[-1.322397]}{0.101\,491}\,Resid(-1)$$

where the first lagged variable of the residual $Resid(-1) = Resid_{t-1}$ has an insignificant adjusted effect on $Resid_t$ based on the t-statistic, namely $t_0 = -1.322\,397$ with $df = 1$ and a p-value $= 0.187\,81$. Corresponding to $F_0 = 1.748\,470$ with $df = (1, 175)$, in a theoretical sense, then $(t_0)^2 = F_0$.

Breusch-Godfrey Serial Correlation LM Test:

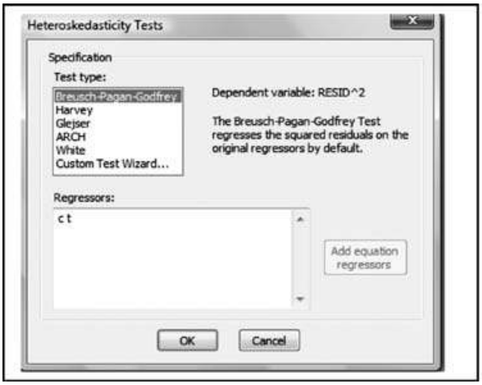
F-statistic	1.748470	Prob. F(1,175)	0.1878
Obs*R-squared	1.770743	Prob. Chi-Square(1)	0.1833

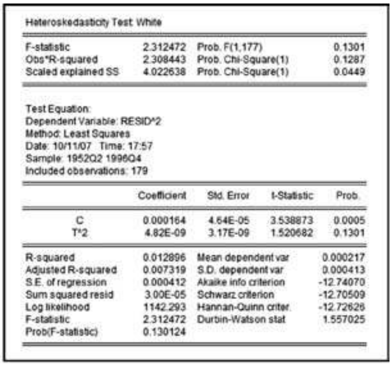
Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 10/11/07 Time: 17:44
Sample: 1952Q2 1996Q4
Included observations: 179
Presample missing value lagged residuals set to zero.

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.028195	0.198987	-0.141691	0.8875
T	0.000154	0.001171	0.131623	0.8954
AR(1)	0.001464	0.009096	0.160961	0.8723
RESID(-1)	-0.101491	0.076753	-1.322297	0.1878

R-squared	0.009892	Mean dependent var	4.88E-11
Adjusted R-squared	-0.007081	S.D. dependent var	0.014776
S.E. of regression	0.014828	Akaike info criterion	-5.562501
Sum squared resid	0.038477	Schwarz criterion	-5.491274
Log likelihood	501.8438	Hannan-Quinn criter.	-5.533619
F-statistic	0.582823	Durbin-Watson stat	1.940320
Prob(F-statistic)	0.627031		

Figure 2.8 Statistical results based on the BG serial correlation LM test




Heteroskedasticity Test: White

F-statistic	2.312472	Prob. F(1,177)	0.1301
Obs*R-squared	2.308443	Prob. Chi-Square(1)	0.1287
Scaled explained SS	4.022638	Prob. Chi-Square(1)	0.0449

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 10/11/07 Time: 17:57
Sample: 1952Q2 1996Q4
Included observations: 179

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000164	4.64E-05	3.538873	0.0005
T^2	4.82E-09	3.17E-09	1.520682	0.1301

R-squared	0.012896	Mean dependent var	0.000217
Adjusted R-squared	0.007319	S.D. dependent var	0.000413
S.E. of regression	0.000412	Akaike info criterion	-12.74070
Sum squared resid	3.00E-05	Schwarz criterion	-12.70509
Log likelihood	1142.293	Hannan-Quinn criter.	-12.72626
F-statistic	2.312472	Durbin-Watson stat	1.557025
Prob(F-statistic)	0.130124		

Figure 2.9 The options and statistical results of the White heteroskedasticity

2.4.2 *Hypothesis of the homogeneous residual term*

By selecting *View/Residual Tests/White Heteroskedasticity* . . ., EViews 6 provides the options in Figure 2.9 on the left-hand side and the statistical results on the right-hand side.

The White test shows that the hypothesis of the homogeneous residual term is accepted, at a significant level of 0.10, based on the chi-square- or F-statistics. Hence, based on the two residual tests, it may be concluded that the data supports the OLS assumptions (2.9). Note that the heteroskedasticity problem should be found in cross-section data, as well as cross-section over times and panel data, and not only in time series data.

In fact, in theoretical statistics, the null hypothesis of $\mathrm{Var}(\varepsilon_i) = \sigma^2$, for all $i = 1, 2, \ldots, n$, or $\mathrm{Var}(\varepsilon_t) = \sigma^2$, for all $t = 1, 2, \ldots, T$, could not be tested, because there is only one observation for each parameter ε_i (ε_t). Hence, someone should use his/her broad experience and knowledge to make a best possible judgment whether a problem indicator has a stable variance or not. Talking about the researcher's judgment, Tukey (1962, in Gifi, 1990, p. 22) stated: 'In data analysis we must look to a very heavy emphasis on judgment.'

For example, the number of children by age of mothers, salaries by length of education or working times (in years), and the number of errors by time of measurements (observations) should not have a stable (or constant) variance in the corresponding population or universe. However, the null hypothesis of constant variances could be accepted based on the sample that happens to be selected (Agung, 2006). If this is the case, the regression analysis could be performed twice, by using the linear model with and without taking into account the residual heterogeneity.

2.4.3 *Hypothesis of the normality assumption*

By clicking *Histogram-Normality Test* . . ., the summary descriptive statistics of the error terms, ε_t, of the model (2.8) in Figure 2.10 is obtained. Note that this figure also shows the Jarque–Bera statistic for testing the normality assumption. However, this

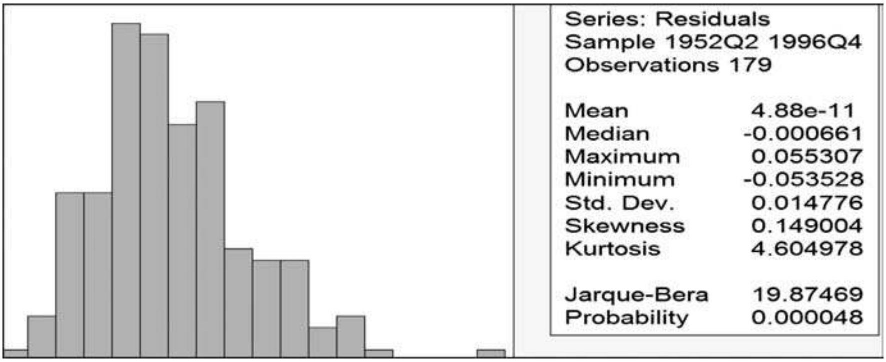


Figure 2.10 Residual histogram of the model in (2.8) and its statistics

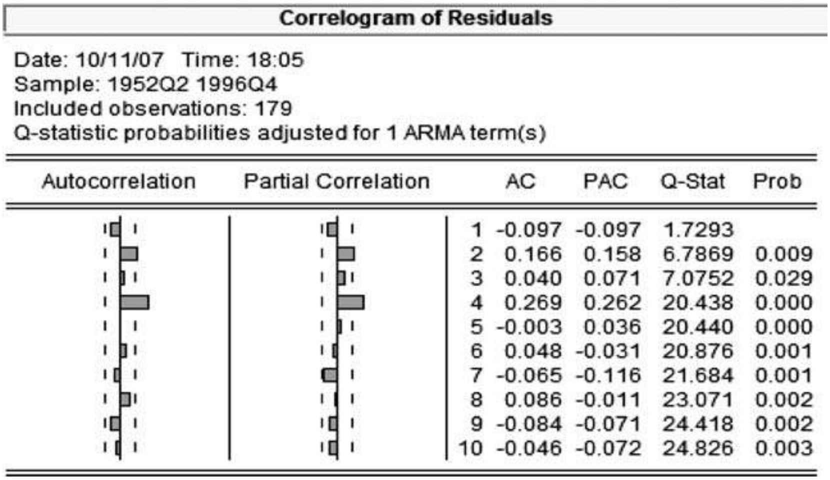
test is presented for specific discussion only, and not for use in any model selection, for the following reasons:

(i) It is believed that conducting an hypothesis test on the distribution of a random variable, including the normal distribution, does not have any concrete result and will be a circular problem, because any statistics used for the testing also depend on the assumption that the statistical test should have a specific distribution function. Should this assumption also be tested? These activities indicate a circular problem.

(ii) In mathematical statistics, it is known that a statistic has a certain (approximately) distribution function, not a sampled data set. A statistic is defined as a function of a random sample of size n, namely Y_1, Y_2, ..., Y_n; then the mean statistic is a function $\bar{Y} = (Y_1 + Y_2 + \ldots + Y_n)/n$. It has been recognized that a normal distribution of the mean statistics in any sample space is based on or supported by the *central limit theorem* (*CLT*). The sample space of the means is defined as a set of means computed using all possible random samples having the same size, which can be selected from a defined population. Furthermore, the distributions of the basic statistic tests, the *t test*, *F test* and *chi-square test*, are also developed based on the *CLT* (Garybill, 1976). In an extreme case, Shewart Shewart,(1931, p. 60) demonstrated that a set of 1000 sample means of size four has approximately a normal distribution, using two universes with uniform and triangular distributions. Enders (2004, p. 85) wrote: 'Although it is common practice to assume that the $\{\varepsilon_t\}$ sequence is normally distributed, it is not necessarily the case that the forecast errors are normally distributed with a mean of zero.'

2.4.4 Correlogram Q-statistic

Figure 2.11 shows three statistics: (i) the AC (autocorrelation coefficient), (ii) the PAC (partial autocorrelation coefficient) and (iii) a Box–Pierce Q-statistic with its probability. Note that the dot lines in the graphs of AC and PAC are the approximate two standard error bounds computed as $\pm 2/\sqrt{T}$. The graphs show that at lagged $k = 4$, the



Correlogram of Residuals

Date: 10/11/07 Time: 18:05
Sample: 1952Q2 1996Q4
Included observations: 179
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.097	-0.097	1.7293	
		2	0.166	0.158	6.7869	0.009
		3	0.040	0.071	7.0752	0.029
		4	0.269	0.262	20.438	0.000
		5	-0.003	0.036	20.440	0.000
		6	0.048	-0.031	20.876	0.001
		7	-0.065	-0.116	21.684	0.001
		8	0.086	-0.011	23.071	0.002
		9	-0.084	-0.071	24.418	0.002
		10	-0.046	-0.072	24.826	0.003

Figure 2.11 Correlogram of residuals of the model in (2.8) with Q-statistic probabilities

hypothesis of no autocorrelation is rejected. It is noted that if there is no serial correlation in the residuals, the autocorrelations and partial autocorrelations at all lags should be zero, and all Q-statistics should be insignificant with large p-values (EViews 4 User's Guide, p. 301). Compare this with the AR(4) and AR(3) models presented in following example.

The Q-statistic is a test statistic for the *joint hypothesis* that all of the autocorrelation coefficients ρ_k up to certain lagged values are simultaneously equal to zero. The result above shows that H_0: $\rho_1 = \ldots = \rho_k = 0$ is rejected for all k. If the mean equation is correctly specified, all Q-statistics should not be significant. However, there remains the practical problem of choosing the order of the lagged variables to be utilized for the test.

Example 2.3. (Higher-order AR growth models) Figures 2.12 and 2.13 present the statistical results based on two higher-order AR growth models, namely AR(4)_GM and AR(3)_GM respectively, together with the descriptive statistics of their residuals. Note that the data do not support the assumption that the residuals have a normal distribution, based on the Jarque–Bera statistic with a p-value $= 0.000\,000$.

Furthermore, note the following statements and conclusions, which are in relation to the autoregressive model only:

(1) The first model, using AR(1) up to AR(4), shows that the indicator AR(4) is not statistically significant, with a p-value $= 0.3086$. Hence, it is suggested (*based on a rule of thumb*) that an attempt should be made to apply a lower AR model, i.e. the AR(3) model.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:07
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.213965	0.172514	24.42673	0.0000
T	0.016275	0.001092	14.90589	0.0000
AR(1)	0.878173	0.077444	11.33951	0.0000
AR(2)	0.263954	0.102322	2.579640	0.0107
AR(3)	-0.090678	0.102315	-0.886263	0.3767
AR(4)	-0.077464	0.075858	-1.021159	0.3086

R-squared	0.999624	Mean dependent var	5.833023
Adjusted R-squared	0.999613	S.D. dependent var	0.748997
S.E. of regression	0.014731	Akaike info criterion	-5.564181
Sum squared resid	0.036892	Schwarz criterion	-5.456096
Log likelihood	495.6479	Hannan-Quinn criter.	-5.520342
F-statistic	90444.06	Durbin-Watson stat	2.011707
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97	.53	-.31+.24i	-.31-.24i

	RESID01
Mean	0.013272
Median	-7.86E-05
Maximum	0.610295
Minimum	-0.050838
Std. Dev.	0.089459
Skewness	6.226422
Kurtosis	40.91412
Jarque-Bera	11944.15
Probability	0.000000
Sum	2.388906
Sum Sq. Dev.	1.432514
Observations	180

Figure 2.12 Statistical results of an AR(4) growth model of *M*1 and the descriptive statistics of its residual

(2) It could happen, in the second model, that the indicator AR(3) is statistically significant with a p-value = 0.0323. Therefore, the procedure could stop when there are three AR growth models of $M1$, namely the AR(1) model in Example 2.2 and the AR(3) and AR(4) models in this example. Among these models, it could be said that the AR(4) is not an acceptable model, in a statistical sense, since the indicators AR(3) and AR(4) are insignificant. Hence, either the AR(1)_GM or AR (3)_GM should be chosen.

(3) Based on the AR(3)_GM, the average of the error terms is not equal to zero, namely 0.010 397. Note that this value is an observed statistical value and is not the expected value or the mean of the residual or error term ε_t in the corresponding population, which is assumed to be $E(\varepsilon_t) = 0$. □

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:11
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.193861	0.168843	24.83887	0.0000
T	0.016400	0.001052	15.58970	0.0000
AR(1)	0.888324	0.076016	11.68597	0.0000
AR(2)	0.245382	0.100279	2.446997	0.0154
AR(3)	-0.161104	0.074655	-2.157986	0.0323

R-squared	0.999625	Mean dependent var	5.827503
Adjusted R-squared	0.999617	S.D. dependent var	0.750468
S.E. of regression	0.014695	Akaike info criterion	-5.574716
Sum squared resid	0.037144	Schwarz criterion	-5.484994
Log likelihood	498.3623	Hannan-Quinn criter.	-5.538328
F-statistic	114706.7	Durbin-Watson stat	1.995183
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97	.37	-.45

	RESID02
Mean	0.010397
Median	-0.000174
Maximum	0.630273
Minimum	-0.050510
Std. Dev.	0.081373
Skewness	7.187832
Kurtosis	54.45726
Jarque-Bera	21408.82
Probability	0.000000
Sum	1.871507
Sum Sq. Dev.	1.185263

Figure 2.13 Statistical results of an AR(3) growth model of *M*1 and the descriptive statistics of its residual

So far, there have been three growth models for the variable $M1$. More models will be introduced having $M1$ or $\log(M1)$ as dependent variables, in the following sections. It will be more and more difficult to select which one is the best. The question is whether the observed statistical values should be trusted or whether a person's own judgment should be used.

To answer this question, the following statements can be considered:

The hallmark of good science is that it uses models and theory but never believes them (Wilk, in Gifi, 1990, p. 27).

Classical (parametric) statistics derives results under the assumption that these models are strictly true. However apart from some discrete simple models, such models are never true (Hampel, 1973, in Gifi, 1990, p. 27).

One reason it is not desirable to have an over parameterized model is that forecast error variance increases as a result of error arising from parameter estimation. In other words, small models tend to have better out of sample performance than large models. Moreover, the more parameters estimated, the greater the parameter uncertainty (Enders, 2004, p. 106).

In view of the above, there should be confidence in using knowledge and experience to present simple models, supported by relevant references, to make the best judgment or possible choice. Corresponding to the statement of Hampel above, the simplest model should be chosen, namely the AR(1)_GM, as the final model.

2.5 Bounded autoregressive growth models

Agung, Pasay and Sugiharso (1994) and Agung (2006) proposed bounded growth models having a general form as follows:

$$\log\left(\frac{Y_t-\beta}{\alpha-Yt}\right) = C(1) + C(2)\cdot f(t) + \varepsilon_t \tag{2.16}$$

where α is an upper bound of Y_t, β is the lower bound and $f(t)$ is a specific defined function of the time t-variable. It does not contain any parameter. The upper and lower bounds should be constant numbers, to be selected and defined by the observed problem indicator. It could also use trial-and-error methods.

Note that, in some cases, the lower bound could be a negative number, e.g. if Y_t is a rate indicator or a profit/loss variable. One of the author's students, Kernen (Kernen, 2003, p. 273), presented several time series models having the dependent variable log $[(ROA_{it} - \beta)/(\alpha - ROA_{it})]$, with $\beta < 0$, specifically $\alpha = 0.8386$ and $\beta = -2.1699$.

Similarly for the $f(t)$ function, Agung (1999a, 2007) proposed $f(t) = (t-\theta)^2(t-\delta)$, and by using trial-and-error methods for various values of θ and δ, estimated or forecasted the growth of the *GDP* by provinces in Indonesia, before and after the monetary crises.

Since the graphs of each result could easily be compared, a best possible growth model can be anticipated. However, in this section only autoregressive growth models are considered.

Furthermore, note the following special cases:

(a) If Y_t is a proportion, with $0 < Y_t < 1$ for all t, then the logistic growth model is as follows:

$$\log\left(\frac{Y_t}{1-Y_t}\right) = C(1) + C(2)\cdot f(t) + \varepsilon_t \tag{2.17}$$

(b) If Y_t is a percentage, with $0 < Y_t < 100$ for all t, then the modified logistic growth model is as follows:

$$\log\left(\frac{Y_t}{100-Y_t}\right) = C(1) + C(2)\cdot f(t) + \varepsilon_t \tag{2.18}$$

Example 2.4. (Autoregressive bounded growth models and outliers) For illustration purposes, experimentation has been done by entering a series of variables:

$$\log\left(\frac{m1-125}{1250-m1}\right) c\; t\; AR(1)\; \ldots\; AR(p) \tag{2.19}$$

in the '*Equation specification*' window for $p = 1, 2, 3$ and 4. This model will be called an AR(p) bounded growth model, namely AR(p)_BGM. The upper and lower bounds are selected based on the minimum and maximum observed values of $M1$. It have been found that the AR(1) and AR(3) models are the best for illustration purposes, as presented in Figure 2.14, since the other models have insignificant partial autocorrelation(s).

Which one is the better model? Since the AR(3)_BGM has smaller values of AIC (Akaike information criteria), SC (Schwarz criteria) and HQC (Hannan–Quinn criteria) than the AR(1)_BGM, then the AR(3)_BGM is preferred, in a statistical sense, under the assumption that they are nonnested models, since they have the same independent variable, namely the time t.

Dependent Variable: LOG((M1-125)/(1250-M1))
Method: Least Squares
Date: 11/15/07 Time: 08:06
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.219179	0.487662	-10.70245	0.0000
T	0.042582	0.003767	11.30361	0.0000
AR(1)	0.953435	0.020801	45.83542	0.0000

R-squared	0.997291	Mean dependent var	-1.582881
Adjusted R-squared	0.997260	S.D. dependent var	2.298827
S.E. of regression	0.120330	Akaike info criterion	-1.380548
Sum squared resid	2.548337	Schwarz criterion	-1.327128
Log likelihood	126.5591	Hannan-Quinn criter.	-1.358887
F-statistic	32395.17	Durbin-Watson stat	1.776461
Prob(F-statistic)	0.000000		

Inverted AR Roots	.95

(a) AR(1)_BGM

Dependent Variable: LOG((M1-125)/(1250-M1))
Method: Least Squares
Date: 11/15/07 Time: 08:03
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-6.070229	1.081728	-5.611604	0.0000
T	0.049011	0.007981	6.141158	0.0000
AR(1)	0.892652	0.074951	11.90982	0.0000
AR(2)	0.236979	0.108111	2.191986	0.0297
AR(3)	-0.154680	0.073391	-2.107599	0.0365

R-squared	0.997700	Mean dependent var	-1.534958
Adjusted R-squared	0.997647	S.D. dependent var	2.266507
S.E. of regression	0.109953	Akaike info criterion	-1.549686
Sum squared resid	2.079420	Schwarz criterion	-1.459965
Log likelihood	142.1473	Hannan-Quinn criter.	-1.513299
F-statistic	18653.18	Durbin-Watson stat	1.685841
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97	.36	-.44

(b) AR(3)_BGM

Figure 2.14 Statistical results based on AR(p)_BGM of the variable $M1$: (a) the AR(1)_BGM and (b) the AR(3)_BGM

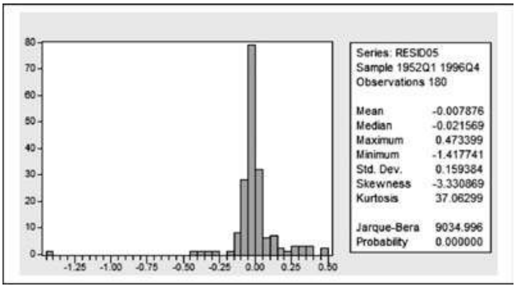


Figure 2.15 Residual histogram of the AR(1)_BGM

On the other hand, the AR(1)_BGM has greater values of the *F*- and *Durbin–-Watson*-statistics. Hence, it may be said that the effect of the time t on log(m1) is stronger or higher based on this model compared to the AR(3)_BGM. In addition, in general based on *a rule of thumb*, the simplest model that can be statistically accepted should be presented. For this reason, in practice, the AR(1)_BGM should be selected as the final model for further analysis and discussions.

For further discussion, Figures 2.15 and 2.16 present the residual histogram and graph of the AR(1)_BGM respectively. Based on this residual analysis some limitations of the model are noted as follows:

(1) The residual histogram, as well as the residual graph, show that there are some outliers that can easily be identified by looking at the original data. This suggests that other types of data analysis should be undertaken, such as (i) by the smoothing process, where the outlier(s) will be replaced by the means of the neighbor observations, and (ii) by doing data analysis based on the subset of data without the outlier(s). However, note that by deleting an outlier, there would be two sub-time series. For example, if the y_j are deleted from a time series $\{y_1, y_2, \ldots, y_T\}$, then two time series $\{y_1, y_2, \ldots, y_{j-1}\}$ and $\{y_{j+1}, y_{j+2}, \ldots, y_T\}$ should be considered, which could have different patterns of growth curves, as well as their relationships with the exogenous variables. Therefore, a model with dummy variables may be applied, which will be presented in Chapter 3. Do this as an exercise.
(2) The residual graph also shows the heteroskedasticity of the error terms. Hence, it is suggested that the weighted least squares (WLS), the White or the Newey–West

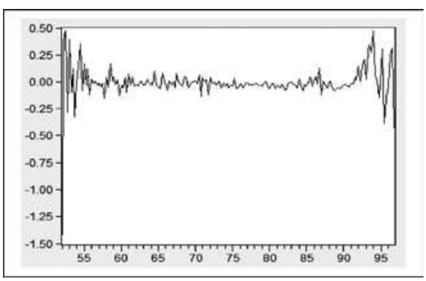


Figure 2.16 Residual graph of the AR(1)_BGM

estimation methods should be applied, as well as using other alternative models, which will be presented in the following sections and examples.

(3) The negative skewness in Figure 2.15 indicates that the residual is skewed to the left, which can easily be identified based on the histogram or the residual box plot (refer to Section 1.4.2). □

2.6 Lagged variables or autoregressive growth models

This section first presents examples of data analysis based on alternative growth models, starting with the simplest lagged variables or autoregressive growth models of the money supply, $M1$, with the following equations:

(a) The LV(1)_GM (i.e. the first lagged-variable growth model):

$$\log(m1_t) = C(1) + C(2)^*\log(m1_{t-1}) + C(3)^*t + \varepsilon_t \tag{2.20}$$

(b) The AR(1)_GM (i.e. the first-order autoregressive growth model):

$$\begin{aligned} \log(m1_t) &= C(1) + C(2)^*t + \mu_t \\ \mu_t &= \rho_1\mu_{t-1} + \varepsilon_t \end{aligned} \tag{2.21}$$

Example 2.5. (Comparison between the LV(1) and AR(1) growth models) The data analysis based on the LV(1)_GM can be obtained by entering the series of variables:

$$\log(m1)\ c\ t\log(m1(-1)) \tag{2.22}$$

in the '*Equation specification*' window. The result and its residual plot of this model are presented in Figure 2.17. In fact, this model could be considered as the *first lagged dependent variable model with trend* (Enders, 2004, p. 156).

These statistical results will be compared with the results in Figure 2.4 based on the AR(1)_GM, as represented above for a better identification. The equation specification used is

$$\log(m1)\ c\ t\ ar(1) \tag{2.23}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:24
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.122291	0.040807	2.996781	0.0031
T	0.000423	0.000131	3.230176	0.0015
LOG(M1(-1))	0.974460	0.009047	107.7095	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Akaike info criterion	-5.563732
Sum squared resid	0.038862	Schwarz criterion	-5.510312
Log likelihood	500.9540	Hannan-Quinn criter.	-5.542071
F-statistic	228602.4	Durbin-Watson stat	2.168644
Prob(F-statistic)	0.000000		

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 11/14/07 Time: 15:03
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.155760	0.198262	20.96100	0.0000
T	0.016575	0.001168	14.19412	0.0000
AR(1)	0.974460	0.009047	107.7095	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Akaike info criterion	-5.563732
Sum squared resid	0.038862	Schwarz criterion	-5.510312
Log likelihood	500.9540	Hannan-Quinn criter.	-5.542071
F-statistic	228602.4	Durbin-Watson stat	2.168644
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97

Figure 2.17 Statistical results based on the models in (2.22) and (2.23)

By observing the results in Figures 2.17 and 2.4, the following findings can be observed:

(1) The coefficient of $\log(m1(-1))$ in the LV(1)_GM is exactly the same as the coefficient of AR(1) in the AR(1)_GM. This also applies to the values of R-squared, the DW-statistic, AIC and SC. Therefore, the best model cannot be chosen based on these statistics.
(2) The two regression functions can be presented as

$$\log(m1) = 0.122\,291 + 0.000\,423{*}t + 0.974\,460{*}\log(m1(-1)) \qquad (2.24)$$

$$\log(m1) = 4.155\,760 + 0.016\,575{*}t + [AR(1) = 0.974\,460] \qquad (2.25)$$

(3) Note that both models show the same estimated first-order autocorrelation, which is equal to 0.974 460. However, it is not very clear whether they should be equal in a theoretical sense.
(4) On the other hand, the coefficient of the time t-variable in the first model 0.000 423 and in the second model = 0.016 575, which indicate that the two models give quite different growth rates of $M1$ during the time of observation: 1952 : 2 to 1964 : 4. This indicates that the two models should use different assumptions or preconditions, and they should be considered as two distinct models.
(5) Therefore, there is a problem in choosing the best model, because the true growth rate of the corresponding population is never known. Furthermore, compare with the results in the following example. □

2.6.1 The white estimation method

The White estimation method (in EView's 6 User's Guide, 1980) provides correct estimates of the coefficient covariance in the presence of heteroskedasticity of unknown form, as presented in the following block/window (see Figure 2.18).

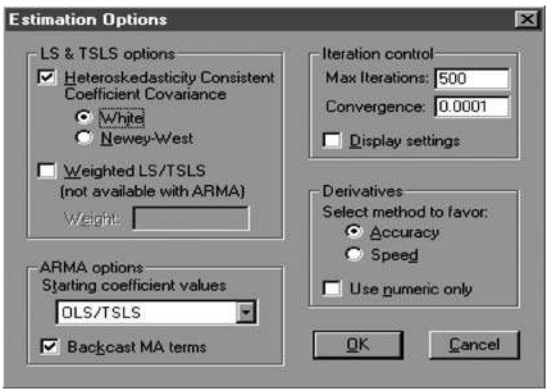


Figure 2.18 The estimation options

Example 2.6. (The white estimation method) This example presents the statistical results in Figure 2.19 based on the two models in (2.22) and (2.23) respectively, using the White heteroskedasticity estimation method. Note their differences. □

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:34
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.122291	0.039307	3.111193	0.0022
T	0.000423	0.000118	3.598333	0.0004
LOG(M1(-1))	0.974460	0.008569	113.7212	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Akaike info criterion	-5.563732
Sum squared resid	0.038862	Schwarz criterion	-5.510312
Log likelihood	500.9540	Hannan-Quinn criter.	-5.542071
F-statistic	228602.4	Durbin-Watson stat	2.168644
Prob(F-statistic)	0.000000		

(a) Based On Model (2.22)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:32
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 4 iterations
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.155760	0.208633	19.91903	0.0000
T	0.016575	0.001257	13.18134	0.0000
AR(1)	0.974460	0.008569	113.7212	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Akaike info criterion	-5.563732
Sum squared resid	0.038862	Schwarz criterion	-5.510312
Log likelihood	500.9540	Hannan-Quinn criter.	-5.542071
F-statistic	228602.4	Durbin-Watson stat	2.168644
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97		

(b) Based On Model (2.23)

Figure 2.19 Statistical results based on the model in (2.22) and (2.23), using the White heteroskedasticity estimation method

2.6.2 The Newey–West HAC estimation method

Note that the window in Figure 2.18 also presents another estimation option, i.e. the Newey–West estimation method. This estimation method would take into account the unknown serial correlation, as well as heteroskedasticity, of the error terms.

Example 2.7. (The Newey–West HAC estimation) Figure 2.20 presents statistical results based on the models in (2.22) and (2.23) by using the Newey–West estimation method.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:35
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.122291	0.036251	3.373422	0.0009
T	0.000423	0.000103	4.126490	0.0001
LOG(M1(-1))	0.974460	0.007896	123.4093	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Akaike info criterion	-5.563732
Sum squared resid	0.038862	Schwarz criterion	-5.510312
Log likelihood	500.9540	Hannan-Quinn criter.	-5.542071
F-statistic	228602.4	Durbin-Watson stat	2.168644
Prob(F-statistic)	0.000000		

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:37
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 4 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.155760	0.214529	19.37151	0.0000
T	0.016575	0.001567	10.57836	0.0000
AR(1)	0.974460	0.007896	123.4093	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Akaike info criterion	-5.563732
Sum squared resid	0.038862	Schwarz criterion	-5.510312
Log likelihood	500.9540	Hannan-Quinn criter.	-5.542071
F-statistic	228602.4	Durbin-Watson stat	2.168644
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97		

Figure 2.20 Statistical results based on the model in (2.22) and (2.23), using the Newey–West estimation method

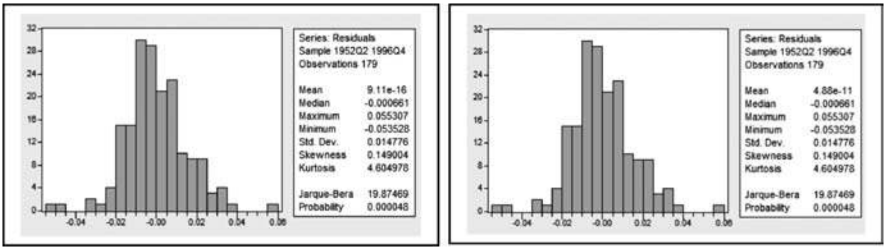


Figure 2.21 Residual histograms of the two regressions in Figure 2.20

Note that the White and Newey–West estimation methods present exactly the same estimated values of the model parameters. Therefore, the White and Newey–West estimation methods will give the same equation of regression functions, as well as the residual graphs. However, these estimation methods present different estimated values of the standard errors of the model parameters, which lead to different values of t-statistics, as well as the p-values of the corresponding hypothesis tests.

For further comparison, Figure 2.21 presents the residual histograms of both regressions, which are very similar. Besides the means of residuals, all other statistics are equal.

Based on the six statistical results presented in the last three examples, problems would be encountered in choosing the best model. Since, in general or in most cases, the form of the true heteroskedasticity and serial correlation is never known, the Newey–West estimation method should be chosen as the best estimation method.

On the other hand, by considering the differences between the LV(1) and AR(1) growth models, it might be preferable to present both regression functions as the final results.

Furthermore, a researcher would have more difficulty in selecting the best growth model of $m1$, because such a large number of models could be presented with $\log(m1)$ as an endogenous variable. Many possible models will be presented in the following examples and sections, as well as in the following chapters. □

2.6.3 The Akaike Information and Schwarz Criterions

In addition to the previous statistics, there are two other statistics that should be taken into consideration in the printout, which are the *Akaike Information Criterion* (*AIC*) and the *Schwarz Criterion* (*SC*). Both could be used to select nonnested models. A model is called nested of a second model, if and only if the set of independent variables of the first model is a subset of the independent variables of the second model. In a statistical sense, it is suggested that the nonnested model should be selected to have smaller values of AIC or SC.

Note that the six previous statistical results present the same values of AIC and SC. Hence, in this case, these statistics cannot be used to select the best possible model.

2.6.4 Mixed lagged-variable autoregressive growth models

As an extension of the LV(1) and AR(1) growth models presented in the previous examples, in this subsection, an LVAR_GM (i.e. lagged-variable autoregressive growth model) or LVAR_T (i.e. lagged-variable autoregressive model with trend) is proposed with the following general equation:

$$\begin{aligned} \log(Y_t) &= \beta_0 + \beta_1 \log(Y_{t-1}) + \ldots + \beta_p \log(Y_{t-p}) + r^* t + u_t \\ u_t &= \rho_1 u_{t-1} + \ldots + \rho_q u_{t-q} + \varepsilon_t \end{aligned} \tag{2.26a}$$

In EViews this model will be filed as follows:

$$\begin{aligned} \log(Y_t) &= c(1) + c(2)^* t + c(3)^* \log(Y_{t-1}) + \ldots \\ &\quad + c(p+2)^* \log(Y_{t-p}) + u_t \\ u_t &= \rho_1 u_{t-1} + \ldots + \rho_q u_{t-q} + \varepsilon_t \end{aligned} \tag{2.26b}$$

This model will be presented as an LVAR(p,q) growth model. For $q = 0$, the LV(p) growth model is obtained, and the AR(q) growth model if $p = 0$. If $p = q = 1$, the simplest mixed lagged-variable autoregressive growth model LVAR(1,1)_GM is obtained.

Note that for any selected values of p and q, all statistical results and testing hypotheses presented in the previous examples could easily be obtained. The following examples present comparisons between selected growth models.

Example 2.8. (Higher-order lagged-variable growth models) Figure 2.22 presents statistical results based on the two models in (2.26), with ($p = 2$, $q = 0$) and ($p = 3$, $q = 0$) respectively. In EViews, the models have the following equations:

$$\begin{aligned} \log(Y_t) &= C(1) + C(2)^* t + C(3)^* \log(Y_{t-1}) \\ &\quad + C(4)^* \log(Y_{t-2}) + u_t \end{aligned} \tag{2.27}$$

and

$$\begin{aligned} \log(Y_t) &= C(1) + C(2)^* t + C(3)^* \log(Y_{t-1}) \\ &\quad + C(4)^* \log(Y_{t-2}) + c(5)^* \log(Y_{t-3}) + u_t \end{aligned} \tag{2.28}$$

Based on these results, the following notes and comments are presented:

(1) The model in (2.27) has estimated partial autocorrelation coefficients of ρ_1 0.874 364 and $\rho_2 = 0.096\,023$. However, the model in (2.28) has $\rho_1 = 0.888\,324$, $\rho_2 = 0.245\,382$ and a negative value of $\rho_3 = -0.161\,104$.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:42
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.132002	0.038947	3.389259	0.0009
T	0.000461	0.000111	4.143990	0.0001
LOG(M1(-1))	0.874384	0.110536	7.910413	0.0000
LOG(M1(-2))	0.098023	0.109797	0.892768	0.3732

R-squared	0.999616	Mean dependent var	5.822083
Adjusted R-squared	0.999609	S.D. dependent var	0.751831
S.E. of regression	0.014858	Akaike info criterion	-5.558286
Sum squared resid	0.038414	Schwarz criterion	-5.486785
Log likelihood	498.6874	Hannan-Quinn criter.	-5.529290
F-statistic	151001.2	Durbin-Watson stat	1.942204
Prob(F-statistic)	0.000000		

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:44
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.129590	0.036501	3.550282	0.0005
T	0.000449	0.000109	4.111909	0.0001
LOG(M1(-1))	0.888324	0.099542	8.924132	0.0000
LOG(M1(-2))	0.245382	0.099671	2.461924	0.0148
LOG(M1(-3))	-0.161104	0.091306	-1.764436	0.0794

R-squared	0.999625	Mean dependent var	5.827503
Adjusted R-squared	0.999617	S.D. dependent var	0.750468
S.E. of regression	0.014695	Akaike info criterion	-5.574716
Sum squared resid	0.037144	Schwarz criterion	-5.484994
Log likelihood	498.3623	Hannan-Quinn criter.	-5.538328
F-statistic	114706.7	Durbin-Watson stat	1.995183
Prob(F-statistic)	0.000000		

Figure 2.22 Statistical results based on the model in (2.27) and (2.28), using the Newey–West estimation method

(2) Corresponding to the basic regression, a partial autocorrelation coefficient can be considered as an adjusted effect of an independent variable on the dependent variable. For example, the first lagged endogenous variable log($m1(-1)$) has a significant adjusted effect on the dependent variable log($m1$) with p-values 0.0000, based on both models.

(3) On the other hand, log($m1(-2)$) has an insignificant adjusted effect on log($m1$) with a p-value = 0.3732, based on the model in (2.27), but based on the model in (2.28) it has a significant adjusted effect. Note that the model in (2.28) has more independent variables than in (2.27). The inconsistency of the results has been known as the effects of multicollinearity between the independent variables. Even the bivariate correlation between the lagged variables in the model in (2.28) should have an effect on the estimated values of the model parameters. It could be said that no model can exist without having an empirical coefficient bivariate correlation between the independent or exogenous variables, even though a pair of independent variables is not correlated, in a theoretical sense. Hence, in selecting an acceptable model personal judgment should be used (Tukey, 1962, in Gifi, 1990, p. 22).

(4) If only one of these two models can be chosen, the LV(2) model in (2.28) should be chosen, for two reasons: (i) each independent variable has a significant adjusted effect and (ii) it has a sufficient value of the DW-statistic of 1.996 183. Would you choose the other model? If so, why?

(5) However, further analysis on the error terms should be done in order to find out the limitations of the model. Do this as an exercise. □

Example 2.9. (LVAR(2,1) growth model) This example presents a comparison between two types of equation specifications of the same growth model. As an illustration, Figures 2.23 and 2.24 present statistical results based on an LVAR (2,1) growth model, that is the model in (2.26) for $p = 2$ and $q = 1$.

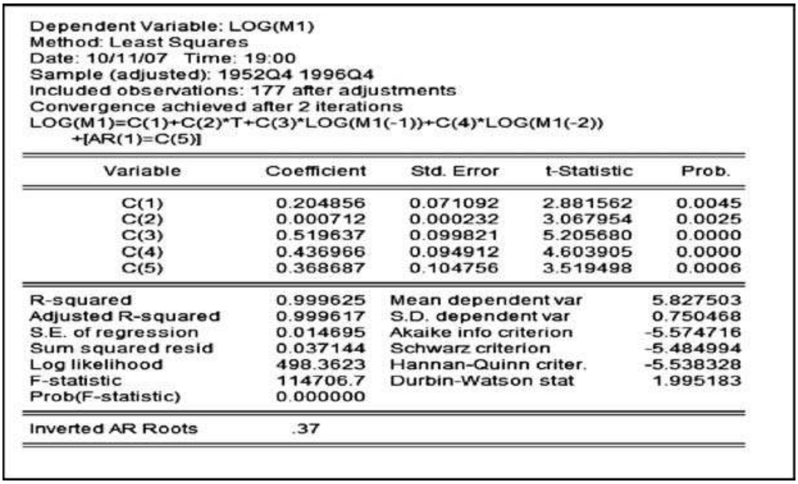
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 18:53
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 7 iterations
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.204856	0.067018	3.056728	0.0026
T	0.000712	0.000206	3.455802	0.0007
LOG(M1(-1))	0.519637	0.105558	4.922773	0.0000
LOG(M1(-2))	0.436966	0.101622	4.299919	0.0000
AR(1)	0.368687	0.117612	3.134765	0.0020

R-squared	0.999625	Mean dependent var	5.827503
Adjusted R-squared	0.999617	S.D. dependent var	0.750468
S.E. of regression	0.014695	Akaike info criterion	-5.574716
Sum squared resid	0.037144	Schwarz criterion	-5.484994
Log likelihood	498.3623	Hannan-Quinn criter.	-5.538328
F-statistic	114706.7	Durbin-Watson stat	1.995183
Prob(F-statistic)	0.000000		

Inverted AR Roots .37

Figure 2.23 Statistical results based on the model in (2.29), where convergence is achieved after eight iterations


Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 19:00
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 2 iterations
LOG(M1)=C(1)+C(2)*T+C(3)*LOG(M1(-1))+C(4)*LOG(M1(-2))
+[AR(1)=C(5)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.204856	0.071092	2.881562	0.0045
C(2)	0.000712	0.000232	3.067954	0.0025
C(3)	0.519637	0.099821	5.205680	0.0000
C(4)	0.436966	0.094912	4.603905	0.0000
C(5)	0.368687	0.104756	3.519498	0.0006

R-squared	0.999625	Mean dependent var	5.827503
Adjusted R-squared	0.999617	S.D. dependent var	0.750468
S.E. of regression	0.014695	Akaike info criterion	-5.574716
Sum squared resid	0.037144	Schwarz criterion	-5.484994
Log likelihood	498.3623	Hannan-Quinn criter.	-5.538328
F-statistic	114706.7	Durbin-Watson stat	1.995183
Prob(F-statistic)	0.000000		

Inverted AR Roots .37

Figure 2.24 Statistical results based on the model in (2.30), where convergence is achieved after two iterations

Two types of the statistical results can be obtained by entering the series of variables

$$\log(m1)\ c\ t\ \log(m1(-1)\ \log(m1(-2))\ ar(1) \tag{2.29}$$

and the following equation respectively

$$\begin{aligned}\log(m1) &= c(1) + c(2)^*t + c(3)^*\log(m1(-1)) \\ &\quad + c(4)^*\log(m1(-2)) + [ar(1) = c(5)]\end{aligned} \tag{2.30}$$

Based on these results, the following notes and conclusions are obtained:

(1) Both models have equal values of the Akaike information criterion, Schwarz criterion and Durbin–Watson statistics.
(2) Note all differences between the results in Figures 2.23 and 2.24 as follows:

- Even though the same option has been used, Figure 2.24 does not present the White heteroskedasticity statement.
- Based on the input in (2.29) convergence is achieved after seven iterations, but convergence is achieved after two iterations based on the input in (2.30).
- The standard errors of the coefficients, as well as the t-tests and their p-values, are unequal for both equations.
- Based on these findings, it could be said with confidence that EViews should use different computational or estimation processes for the two different input equation specifications, even though the same regression is being considered, in a statistical sense.

- Note that both statistical results show equal values of the AIK and SC, as well as the Hannan–Quinn Criterion (HQC). It can therefore be concluded that both models have the same quality based on these statistics.
- However, since both outputs are the results of a single defined model, then personal best judgment should be used (Tukey, 1962, in Gifi, 1990, p. 22) to select one as a final statistical result. □

2.6.5 Serial correlation LM test for LV(2,1)_GM

This example presents two alternative serial correlation LM tests, as presented in Figure 2.25, based on the LVAR(2,1)_GM with (2.29) as the equation specification. Based on these results, the following notes and conclusions are given:

(1) The results in Figure 2.25(a) show that the null hypothesis of the first-order serial correlation of the error terms, that is H_0: $\rho_1 = 0$, is accepted based on the chi-squared-statistic (Obs*R-squared $= T^* R^2$) of 0.590 742 with a p-value $= 0.4421$. Hence, it can be concluded that the model is an acceptable AR(1) model, in a statistical sense. Furthermore, note that these results also present the growth model of the error terms, *Resid*, which can be written as

$$\begin{aligned} Resid = c(1) + c(2)t + c(3)\log(m1(-1)) + c(4)\log(m1(-2)) \\ + [ar(1) = c(5)] + c(6)Resid(-1) \end{aligned} \tag{2.31}$$

A similar model could easily be written based on the results in Figure 2.25(b).

(2) The results in Figure 2.25(b) show that the null hypothesis of the second-order serial correlation of the error terms, that is H_0: $\rho_1 = \rho_2 = 0$, is rejected based on the chi-squared-statistic (Obs*R-squared $= T^* R^2$) of 10.062 56 with a p-value 0.0065. Since the error terms have a significant second-order serial correlation,

Breusch-Godfrey Serial Correlation LM Test

F-statistic	0.572628	Prob. F(1,171)	0.4503
Obs*R-squared	0.590742	Prob. Chi-Square(1)	0.4421

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 10/11/07 Time: 19:38
Sample: 1952Q4 1996Q4
Included observations: 177
Presample missing value lagged residuals set to zero.

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.006620	0.071716	-0.092307	0.9266
T	-2.42E-05	0.000234	-0.103041	0.9181
LOG(M1(-1))	0.129827	0.198554	0.653863	0.5141
LOG(M1(-2))	-0.128597	0.194705	-0.660470	0.5098
AR(1)	0.177485	0.256929	0.690796	0.4906
RESID(-1)	-0.317748	0.419900	-0.756722	0.4503

R-squared	0.003338	Mean dependent var	6.42E-14
Adjusted R-squared	-0.025805	S.D. dependent var	0.014527
S.E. of regression	0.014714	Akaike info criterion	-5.566759
Sum squared resid	0.037021	Schwarz criterion	-5.459093
Log likelihood	498.6582	Hannan-Quinn criter.	-5.523094
F-statistic	0.114526	Durbin-Watson stat	2.005967
Prob(F-statistic)	0.989047		

(a) First Order SC LM Test

Breusch-Godfrey Serial Correlation LM Test

F-statistic	5.123791	Prob. F(2,170)	0.0069
Obs*R-squared	10.06295	Prob. Chi-Square(2)	0.0065

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 10/11/07 Time: 19:41
Sample: 1952Q4 1996Q4
Included observations: 177
Presample missing value lagged residuals set to zero.

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.212479	0.099359	2.138495	0.0339
T	0.000739	0.000336	2.201895	0.0290
LOG(M1(-1))	-1.059202	0.429062	-2.468648	0.0146
LOG(M1(-2))	1.013026	0.413762	2.448330	0.0154
AR(1)	1.793538	0.577566	3.105339	0.0022
RESID(-1)	-0.754884	0.433175	-1.742676	0.0832
RESID(-2)	-1.165548	0.375281	-3.105801	0.0022

R-squared	0.056853	Mean dependent var	6.42E-14
Adjusted R-squared	0.023565	S.D. dependent var	0.014527
S.E. of regression	0.014355	Akaike info criterion	-5.610650
Sum squared resid	0.035033	Schwarz criterion	-5.485039
Log likelihood	503.5425	Hannan-Quinn criter.	-5.559707
F-statistic	1.707930	Durbin-Watson stat	1.978250
Prob(F-statistic)	0.121874		

(b) Second Order SC LM Test

Figure 2.25 Two serial correlation (SC) LM tests, based on the model in (2.29), where convergence is achieved after two iterations then it suggests that the LVAR(2,1)_GM should be modified to LVAR(2, q)_GM for a certain or preferable value of q. Do this as an exercise.

2.7 Polynomial growth model

2.7.1 Basic polynomial growth models

A basic polynomial growth model, which is the semilog (i.e. semilogarithmic) polynomial model, based on a bivariate (Y_t, t) has the flowing general equation

$$\ln(Y_t) = \beta_0 + \sum_{k=1}^{K} \beta_k t^k + \varepsilon_t \tag{2.32}$$

However, in EViews, it will be saved or filed as

$$\log(Y_t) = C(1) + \sum_{k=1}^{K} C(k+1)^* t^k + \mu_t \tag{2.33}$$

This model is a polynomial growth model of degree k in time t. By using the same stages of process, it is easy to apply this model based on a time series data set. However, the corresponding scatter plot should be observed in order to identify what degree of the polynomial growth model can be used.

Example 2.10. (The use of observed scatter plots) Note that the growth curve of the variable *RS* in the Demo_Modified workfile shows a nonlinear curve. The simplest polynomial model should be (at least) a quadratic growth model. Hence, if the following series of variables is entered:

$$\log(RS)\ c\ t\ t^2 \tag{2.34}$$

in the '*Equation specification*' window, then the statistical results in Figure 2.26 and its residual graph in Figure 2.27 will be obtained.

Note that Figure 2.26 shows that each of the time variables t and t^2 has a significant adjusted effect on log(*RS*), with a sufficiently large value of R-squared, but with a very low value of the DW-statistic. The structure of the residual, actual and fitted graphs

Dependent Variable: LOG(RS)
Method: Least Squares
Date: 10/11/07 Time: 19:46
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.144517	0.068258	2.117225	0.0356
T	0.031731	0.001741	18.22320	0.0000
T^2	-0.000136	9.32E-06	-14.57797	0.0000

R-squared	0.732526	Mean dependent var	1.536879
Adjusted R-squared	0.729503	S.D. dependent var	0.580420
S.E. of regression	0.301872	Akaike info criterion	0.458901
Sum squared resid	16.12946	Schwarz criterion	0.512117
Log likelihood	-38.30107	Hannan-Quinn criter.	0.480478
F-statistic	242.3729	Durbin-Watson stat	0.226516
Prob(F-statistic)	0.000000		

Figure 2.26 Statistical results based on the model in (2.34)

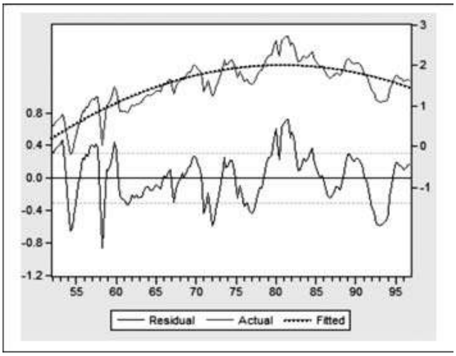


Figure 2.27 Residual graph of the model in (2.34)

should also be taken into consideration. Should a higher-degree polynomial regression be tried? Do this as an exercise.

In fact, this basic linear model is not an appropriate model for statistical inferences, since it concerns time series data. Previous examples already show that an AR, an LV or an LVAR growth model should be applied. Do this as an exercise. □

Example 2.11. (A cubic polynomial for the first difference, dlog(m1)) As an illustration, a series or a dated variable $d\log(m1) = \log(m1_t) - \log(m_{t-1})$ is generated, as well as the scatter graph of $(t, d\log(m1))$ with its kernel fit in Figure 2.28. This graph clearly shows that a linear growth model with the dependent variable $d\log(m1)$ is not an appropriate model, and nor is the quadratic growth model. Therefore, a third-degree polynomial is tried, giving the statistical results in Figure 2.29.

Note that the results in Figure 2.29 show that $DW = 2.3$ with a small value of R-squared; each of the independent variables t and t^2 is insignificant. However, the

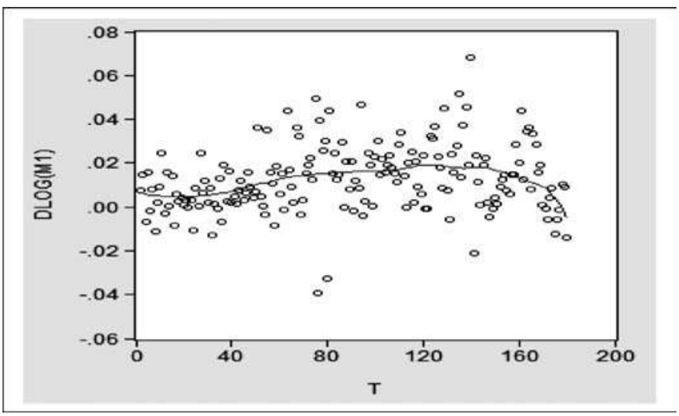


Figure 2.28 Scatter graph of $d\log(m1)$ and its kernel fit

Dependent Variable: DLOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 19:59
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004893	0.004625	1.058033	0.2915
T	-7.43E-05	0.000218	-0.340631	0.7338
T^2	4.22E-06	2.77E-06	1.522107	0.1298
T^3	-2.14E-08	1.00E-08	-2.139163	0.0338

R-squared	0.125672	Mean dependent var	0.012577
Adjusted R-squared	0.110684	S.D. dependent var	0.015406
S.E. of regression	0.014529	Akaike info criterion	-5.603271
Sum squared resid	0.036940	Schwarz criterion	-5.532044
Log likelihood	505.4927	Hannan-Quinn criter.	-5.574389
F-statistic	8.384591	Durbin-Watson stat	2.338992
Prob(F-statistic)	0.000031		

Figure 2.29 Third-degree polynomial growth model of $d\log(m1)$

joint effects of t and t^2 is significant based on the Wald test, i.e. the chi-squared-variable of 24.133 88 with $df = 2$ and the p-value $= 0.000$, as presented in Figure 2.30. □

Example 2.12. (Possible reduced models) In a statistical sense, two reduced models can be presented based on the cubic polynomial presented in the Figure 2.29, since the joint effect of t and t^2 is insignificant. Those reduced models are obtained by deleting either the time t or t^2 respectively. Hence, two statistical results should be considered, as presented in Figures 2.31 and 2.32, with their residual graphs in Figures 2.33 and 2.34 respectively.

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	12.06694	(2, 175)	0.0000
Chi-square	24.13388	2	0.0000

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(2)	-7.43E-05	0.000218
C(3)	4.22E-06	2.77E-06

Restrictions are linear in coefficients.

Figure 2.30 Statistical results for testing H_0: $c(2) = c(3) = 0$, i.e. the joint effects of t and t^2 on $d\log(m1)$

Dependent Variable: DLOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 20:09
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000502	0.002982	-0.168307	0.8665
T	0.000248	5.33E-05	4.653499	0.0000
T^3	-6.39E-09	1.65E-09	-3.874369	0.0002

R-squared	0.114097	Mean dependent var	0.012577
Adjusted R-squared	0.104030	S.D. dependent var	0.015406
S.E. of regression	0.014583	Akaike info criterion	-5.601292
Sum squared resid	0.037429	Schwarz criterion	-5.547872
Log likelihood	504.3156	Hannan-Quinn criter.	-5.579631
F-statistic	11.33369	Durbin-Watson stat	2.308790
Prob(F-statistic)	0.000023		

Figure 2.31 Growth model of $d\log(m1)$ with linear and cubic trends

Dependent Variable: DLOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 20:06
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003508	0.002194	1.598577	0.1117
T^2	3.30E-06	6.73E-07	4.913155	0.0000
T^3	-1.83E-08	3.91E-09	-4.676993	0.0000

R-squared	0.125093	Mean dependent var	0.012577
Adjusted R-squared	0.115150	S.D. dependent var	0.015406
S.E. of regression	0.014492	Akaike info criterion	-5.613781
Sum squared resid	0.036965	Schwarz criterion	-5.560361
Log likelihood	505.4334	Hannan-Quinn criter.	-5.592120
F-statistic	12.58207	Durbin-Watson stat	2.337503
Prob(F-statistic)	0.000008		

Figure 2.32 Growth model of $d\log(m1)$ with quadratic and cubic trends

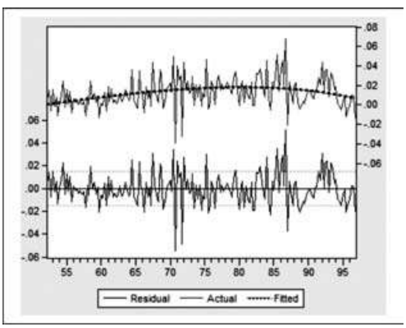


Figure 2.33 Residual graph of the model in Figure 2.29

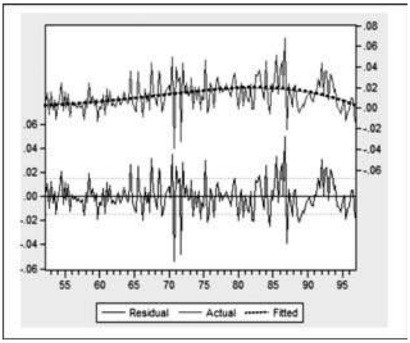


Figure 2.34 Residual graphs of the model in Figure 2.30

Based on the statistical results of the three growth models of $d\log(m1)$, the following questions, notes and conclusions are presented:

(1) Which one is considered to be the best model? Since, in a statistical sense, the first model should be reduced, then one of the two reduced models should be selected. This again is a problem, because each independent variable in both models has a significant adjusted effect. In both models, the independent variables also have a joint significant effect, based on the F-statistic.
(2) Considering a higher value of R-squared and that lower values of the AIC and SC statistics are preferred, the model in Figure 2.32 should be selected as the best model of the three considered models. However, this might be a higher-degree polynomial growth model. Do this as an exercise.
(3) On the other hand, the residual graph shows an indication of outliers or break-points, since there are some long/high vertical lines presented by the residual graph, as well as the graph of actual values. For a further explanation, refer to the notes in Example 2.4. □

Example 2.13. (The White heteroskedasticity test) Considering the scatter plot of $d\log(m1)$ by the time t in Figures 2.33 and 2.34, a suggestion is made to test the null hypothesis of no heterogeneity of the residual or the error terms. To illustrate this, the test will be conducted for the model in Figure 2.32, namely the growth model having t^2 and t^3 as independent variables. By selecting *View/Residual Tests/White H* ... (*no cross term*), the result in Figure 2.35 is obtained.

This figure shows that the null hypothesis of no heteroskedasticity of the error is accepted, based on the chi-squared-statistic (Obs*R-square $= T^*R^2$) of 7.990 311 with a p-value $= 0.1568$. However, as expected, the null hypothesis of no first-order serial correlation is rejected based on the chi-squared-statistic of 5.423 776 with a p-value 0.0199, since time series data are used and the model in Figure 2.30 does not take into account the autocorrelations of the error terms. Hence, it is suggested that an LVAR_GM should be applied, in order to obtain a better growth model, as well as an acceptable time series model. The following example presents alternative modified models. □

Heteroskedasticity Test: White

F-statistic	1.616661	Prob. F(5,173)	0.1580
Obs*R-squared	7.990311	Prob. Chi-Square(5)	0.1568
Scaled explained SS	14.29258	Prob. Chi-Square(5)	0.0139

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 10/11/07 Time: 20:13
Sample: 1952Q2 1996Q4
Included observations: 179

	Coefficient	Std. Error	t-Statistic	Prob.
C	6.49E-05	0.000109	0.598265	0.5504
T^2	-1.13E-07	3.93E-07	-0.287833	0.7738
(T^2)^2	-1.34E-10	1.63E-10	-0.821253	0.4126
(T^2)*(T^3)	8.53E-13	8.95E-13	0.953166	0.3418
T^3	8.09E-09	1.30E-08	0.622131	0.5347
(T^3)^2	-1.88E-15	1.80E-15	-1.048418	0.2959

R-squared	0.044639	Mean dependent var	0.000207
Adjusted R-squared	0.017027	S.D. dependent var	0.000398
S.E. of regression	0.000395	Akaike info criterion	-12.80264
Sum squared resid	2.70E-05	Schwarz criterion	-12.69580
Log likelihood	1151.837	Hannan-Quinn criter.	-12.75932
F-statistic	1.616661	Durbin-Watson stat	1.526096
Prob(F-statistic)	0.157999		

Figure 2.35 The White heteroskedasticity test for the model in Figure 2.30 for the growth model of $d\log(m1)$ with quadratic and cubic trends

Example 2.14. (Modified models of the cubic polynomial model in Figure 2.32, Example 2.13) Figure 2.36 presents the statistical results based on LV(1) and AR(1) growth models with quadratic and cubic trends, which are two modified

Dependent Variable: DLOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 20:20
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004041	0.002214	1.824929	0.0697
T^2	3.87E-06	7.10E-07	5.454435	0.0000
T^3	-2.14E-08	4.10E-09	-5.219548	0.0000
DLOG(M1(-1))	-0.174795	0.074979	-2.331241	0.0209

R-squared	0.151493	Mean dependent var	0.012605
Adjusted R-squared	0.136864	S.D. dependent var	0.015445
S.E. of regression	0.014350	Akaike info criterion	-5.627978
Sum squared resid	0.035828	Schwarz criterion	-5.556477
Log likelihood	504.8900	Hannan-Quinn criter.	-5.598982
F-statistic	10.35539	Durbin-Watson stat	1.960272
Prob(F-statistic)	0.000003		

(a) LV (1)_GM Of Dlog (M1)

Dependent Variable: DLOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 20:21
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 1 iteration

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003411	0.002239	1.523793	0.1294
T^2	3.33E-06	6.83E-07	4.875698	0.0000
T^3	-1.84E-08	3.97E-09	-4.646939	0.0000
AR(1)	0.002498	0.076177	0.032791	0.9739

R-squared	0.124228	Mean dependent var	0.012605
Adjusted R-squared	0.109128	S.D. dependent var	0.015445
S.E. of regression	0.014578	Akaike info criterion	-5.596350
Sum squared resid	0.036980	Schwarz criterion	-5.524849
Log likelihood	502.0751	Hannan-Quinn criter.	-5.567354
F-statistic	8.227273	Durbin-Watson stat	2.342494
Prob(F-statistic)	0.000038		
Inverted AR Roots	.00		

(b) AR (1)_GM Of Dlog (M1)

Figure 2.36 Statistical results based on the LV(1) and AR(1) growth models of the model in Figure 2.32

models of the model in Figure 2.32. Note their differences. Based on these results, the following notes and conclusions are presented:

(1) In a statistical sense, model LV(1)_GM in Figure 2.36(a) should be considered a better model than model AR(1)_GM in Figure 2.36(b), because $d(\log(m1(-1))$ has a significant effect, but the indicator AR(1) is insignificant with a large p-value.
(2) Further experimentation or exercises could be carried out using an LV(p)_GM or an AR(q)_GM or the mixed LVAR(p, q)_GM for some p and q. Do this as an exercise.
(3) Furthermore, note that more alternative growth models could be defined or proposed having $\log(m1)$ or $d\log(m1)$ as a dependent variable. Hence, problems will always be faced in selecting an acceptable or the best possible model based on personal judgment, which can be very subjective. Some additional examples will be presented in the following sections and chapters by using pure exogenous (independent) variables. □

Example 2.15. (A cubic polynomial model by Enders (2004)) For illustrative comparison purposes, the polynomial growth model presented by Enders Enders (2004, p. 157) should be considered. He presents a cubic polynomial growth function for the time series of real *GDP* $\{rgdp_t\}$, as follows:

$$rgdp_t = \underset{(61.27)}{2.224} + \underset{(20.89)}{0.385t} - \underset{(-6.78)}{0.0002t^2} + \underset{(-12.17)}{1.85E{-}6t^3} \tag{2.35}$$

As in the previous examples, this model does not take into account the autocorrelations between its error terms. To explain this, Enders stated: 'Regardless of the t-statistics, the use of such a model for trend of real GDP is problematic. Since there is no stochastic component in the trend, the function above implies that there is a deterministic (and accelerating) long-run growth rate of the real economy.'

Hence, for further analysis, the model should take into account the autocorrelations of the error terms, as well as their heteroskedasticity. □

2.7.2 *Special polynomial growth models*

Agung, Pasay and Sugiharso (1994) and Agung (1999a, 2007) proposed a special third-degree polynomial growth model, called the *generalized exponential growth functions*, having the general form:

$$\log\left(\frac{Y_t-\beta}{\alpha-Y_t}\right) = C(1) + C(2)*(t-\theta)^2(t-\delta) + \mu_t \tag{2.36}$$

where α and β are fixed defined values of the upper and lower bounds of all possible observed or theoretical values of the dependent variable Y, and δ and θ are also fixed values selected corresponding to possible values of time t, where the Y-variable is predicted to reach its relative extreme values, either minimum or maximum values.

An advantage of applying this model is that it can be considered as a simple linear regression model with the independent variable $f(t) = (t-\theta)^2(t-\delta)$. Then, using

Microsoft Excel, a set of many regression functions could easily be produced, together with their graphs, having an intercept $C(1)$ and a slope $C(2)$. Based on the set of those graphs, one could be selected that is considered as the best third-degree polynomial model to be used for estimation or forecasting.

Furthermore, the independent variable $f(t) = (t - \theta)^2(t - \delta)$ can also be used for bounded growth models of the proportion variable Y_t in (2.17), as well as the percentage variable in (2.18).

2.8 Growth models with exogenous variables

The growth models presented in the previous sections could easily extend to a general growth model with multidimensional exogenous variables as follows:

$$\log(Y_t) = c(1) + c(2)^*t + \sum_{k=1}^{K} c(k+2)^*X_{k,t} + \mu_t \tag{2.37}$$

where Y_t is an endogenous time series (variable), $X_{k,t}$ is the kth exogenous time series, $k = 1, \ldots, K$, and μ_t is the error term of the model. This model can be considered as a semilog model with trend and a multidimensional exogenous variable.

Note that the exogenous variables could be pure exogenous variables, other endogenous variables, lagged variables of the endogenous or exogenous variables or the interaction factors of selected independent variables, including the time t, as well as dummy variables and the transformation of the original variables, such as log (X_k) and $(X_k)^\alpha$. Furthermore, this model can be extended to the bounded growth models presented in Section 2.5.

Hence, there could be a very large number of possible growth models or there might be an infinite number of possible growth models based on a limited number of variables, as mentioned in the Preface. For example, if there are only two endogenous variables and three pure exogenous variables, how many possible growth models could be developed or defined? Find the many alternative models presented in the following sections. For this reason, the best growth model, or statistical and econometric models in general, could never be presented because all possible models are never considered that would be acceptable models in a statistical sense.

Example 2.16. (AR additive growth models) This example presents an additional illustrative growth model for the time series $M1$, which is an additive growth model having selected exogenous variables with equation specification as follows:

$$\begin{aligned}\log(m1) = C(1) + C(2)^*t + C(3)^*\log(gdp) \\ + C(4)^*\log(pr) + [AR(1) = C(5)]\end{aligned} \tag{2.38}$$

The statistical results in Figure 2.37, can easily be obtained by entering this equation or the following series of variables in the '*Equation specification*' window:

$$\log(m1)\ c\ t\ \log(gdp)\ \log(pr)\ ar(1) \tag{2.39}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 20:31
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 12 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.972707	0.755639	3.934029	0.0001
T	0.011411	0.002802	4.072085	0.0001
LOG(GDP)	0.275941	0.112556	2.451598	0.0152
LOG(PR)	-0.014651	0.198280	-0.073890	0.9412
AR(1)	0.975404	0.010372	94.04605	0.0000

R-squared	0.999630	Mean dependent var	5.816642
Adjusted R-squared	0.999621	S.D. dependent var	0.753241
S.E. of regression	0.014656	Akaike info criterion	-5.580394
Sum squared resid	0.037375	Schwarz criterion	-5.491361
Log likelihood	504.4452	Hannan-Quinn criter.	-5.544291
F-statistic	117499.1	Durbin-Watson stat	2.164391
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 2.37 Statistical results based on an AR(1) translog linear growth model in (2.39)

Note that this model will be considered as a modification of the Cobb–Douglas production function with two input variables, namely *gdp* and *pr*, which has the characteristics: (i) *increasing return-to-scale*, if and only if $C(3) + C(4) > 1$; (ii) *constant return-to-scale*, if and only if $C(3) + C(4) = 1$; and (iii) *decreasing return-to-scale*, if and only if $C(3) + C(4) < 1$. The estimate of the parameter $C(2)$ provides an estimate of the annual percentage of change resulting from technological change, adjusted for log(*gdp*) and log(*pr*), as well as the AR(1).

By using the Wald test, the null hypothesis H_0: $c(3) + c(4) = 1$ is rejected based on the *F*-statistic of 14.620 14 with $df = (1, 174)$ and the *p*-value = 0.0002; or the chi-square-statistic of 14.620 14 with $df = 1$ and the *p*-value = 0.0001. This test can be conducted by selecting *View/Coefficient Tests/Wald-Coefficient restriction* ... and entering $C(3) + C(4) = 1$ in the '*Coefficient restrictions*' window; then click *OK*.

Since log(*pr*) has an insignificant effect, with such a large *p*-value and a negative coefficient, it is proposed that a reduced model should be used, which is a nested model as follows:

$$\log(m1) = C(1) + C(2)^*t + C(3)^*\log(gdp) + [AR(1) = C(4)] \tag{2.40a}$$

which can also be presented as

$$\begin{aligned} \log(m1_t) &= C(1) + C(2)^*t + C(3)^*\log(gdp_t) + \mu t \\ \mu_t &= \rho\mu_{t-1} + \varepsilon_t \end{aligned} \tag{2.40b}$$

The statistical results are presented in Figure 2.38, with its residual graph in Figure 2.39, p. 58. □

Example 2.17. (LVAR(1,1) growth model) As a modification of the reduced model in the previous example, namely the AR(1) model in (2.40), here a mixed

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 20:34
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.017338	0.521171	5.789537	0.0000
T	0.011278	0.002260	4.990839	0.0000
LOG(GDP)	0.272944	0.106023	2.574376	0.0109
AR(1)	0.975310	0.010364	94.10968	0.0000

R-squared	0.999630	Mean dependent var	5.816642
Adjusted R-squared	0.999624	S.D. dependent var	0.753241
S.E. of regression	0.014614	Akaike info criterion	-5.591536
Sum squared resid	0.037376	Schwarz criterion	-5.520309
Log likelihood	504.4424	Hannan-Quinn criter.	-5.562654
F-statistic	157560.9	Durbin-Watson stat	2.164460
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 2.38 Statistical results based on a reduced model in (2.40)

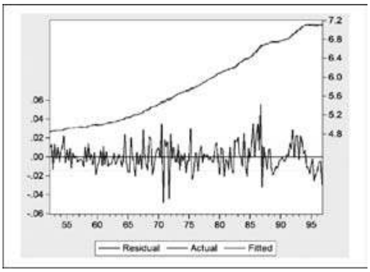


Figure 2.39 Residual graph of the model in (2.40)

lagged-variables autoregressive growth model, the LVAR(1,1)_GM, is considered, with the statistical results presented in Figure 2.40 and its residual graph in Figure 2.41.

Based on this figure, the following notes and conclusions can be presented:

(1) The time t has an insignificant adjusted effect, with a sufficiently large p-value. If this independent variable is deleted in order to obtain a reduced model, it will not produce a growth model or a model with a linear trend (*a model with trend*). Models without the time t as an independent variable will be presented in Chapter 4.
(2) The indicator AR(1) is insignificant at the significant level of 10%. However, it is significant if a left-hand-side hypothesis is considered: $H_0 : \rho_1 \geq 0$ versus $H_1 : \rho_1 < 0$. Since the t-statistic is $-1.620\,646$ with the p-value $= 0.106\,69/2 = 0.053\,345$, then the null hypothesis is rejected at the significant level of 10%.
(3) If the AR(1) is deleted, then in general there will be a first lagged-variable growth model, which is the LV(1)_GM, or specifically the first lagged-variable model with trend, namely the LV(1)_T, with a pure exogenous variable. □

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 20:39
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.119981	0.037893	3.166298	0.0018
T	0.000419	0.000122	3.424300	0.0008
LOG(M1(-1))	0.974928	0.008413	115.8835	0.0000
AR(1)	-0.100544	0.076793	-1.309288	0.1922

R-squared	0.999616	Mean dependent var	5.822083
Adjusted R-squared	0.999609	S.D. dependent var	0.751831
S.E. of regression	0.014858	Akaike info criterion	-5.558286
Sum squared resid	0.038414	Schwarz criterion	-5.486785
Log likelihood	498.6874	Hannan-Quinn criter.	-5.529290
F-statistic	151001.2	Durbin-Watson stat	1.942204
Prob(F-statistic)	0.000000		

Inverted AR Roots -.10

Figure 2.40 Statistical results based on an LVAR(1,1)_GM of log($m1$)

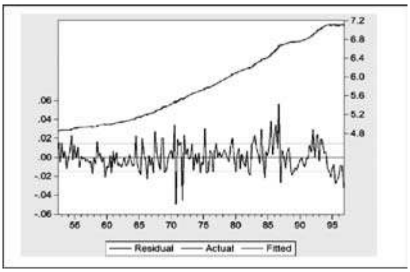


Figure 2.41 Residual graph of the model in Figure 2.37

2.9 A Taylor series approximation model

An extension of the translog growth model presented in the previous examples is a Taylor series approximation model, which is derived from the *constant elasticity of substitution* (*CES*) production function. For example, with two exogenous variables, X_1 and X_2, the model has the following general equation:

$$\begin{aligned}\log(Y_t) \quad = \quad & C(1) + C(2)^*\log(X_1) + C(3)^*\log(X_2) + C(4)^*\log(X_1)^2 \\ & + C(5)^*\log(X_1)^*\log(X_2) + C(6)^*\log(X_2)^2 + C(7)^*t + \mu_t\end{aligned} \quad (2.41)$$

where the estimated value of parameter $C(7)$ provides the exponential growth rate of Y_t adjusted for all other independent variables. As a modified model, log(t) may be used as an independent variable, instead of the time t.

On the other hand, Coelli, Prasada Rao and Battese (2001, p. 36) present an alternative model as follows:

$$\begin{aligned}\log(Y_t) \quad = \quad & C(1) + C(2)^*\log(X_1) + C(3)^*\log(X_2) + C(4)^*\log(X_1)^2 \\ & + C(5)^*\log(X_1)^*\log(X_2) + C(6)^*\log(X_2)^2 \\ & + C(7)^*t + C(8)^*t^2 + \mu_t\end{aligned} \quad (2.42)$$

Note that by using several or many independent variables in a model, statistical results would frequently be produced, with several independent variables having insignificant adjusted effects. This problem arises due to the multicollinearity between the independent variables, which always exists even though the independent variables are uncorrelated in a theoretical sense (further notes and comments are given in Section 2.14).

As a result, it is not an easy task to obtain an acceptable reduced model. Do it as an exercise and see how to develop possible reduced models, which has been presented in Example 2.12.

2.10 Alternative univariate growth models

2.10.1 A more general growth model

Based on a time series $\{t, y_t\}$, Gourierroux and Manfort (1997, pp. 12–13) presented a general growth model or a general model with the time t as an exogenous variable, called an *adjusted model*, as follows:

$$y_t = f(t, \mu_t) \tag{2.43}$$

where f is a function characterized by a finite number of unknown parameters and μ_t is a zero mean random variable.

This model can be extended to a more general model as follows:

$$g(y_t) = f(t, x_t, \mu_t) \tag{2.44}$$

where $g(y_t)$ is a defined function of an endogenous variable y_t without a parameter of the endogenous variable and $f(t, \boldsymbol{x}_t, \mu_t)$ is a function of the time t and a multidimensional exogenous variable $\boldsymbol{x}_t = (x_t, x_2, \ldots, x_{k)t}$ having a finite number of unknown parameters. The simplest model of the model in (2.44) is an *additive model* as follows:

$$g(y_t) = f(t, x_t, \mu_t) = f_1(t) + f_2(x_t) + \mu_t \tag{2.45}$$

where $f_1(t)$ is a function of the time t and $f_2(\boldsymbol{x}_t)$ is a function of a multidimensional exogenous variable $\boldsymbol{x}_t$ having a finite number of unknown parameters.

2.10.2 Translog additive growth models

Corresponding to the model (2.43), the simplest translog (i.e. translogarithmic) linear model might be

$$\ln(Y_t) = C(1) + C(2)^*\ln(t) + \mu_t \tag{2.46}$$

Note that this model is derived from the Cobb–Douglas production function: $Q = AK^{\alpha}$, which has specific characteristics or classification, such as an *increasing return-to-scale* model if $\alpha > 1$, a *constant return-to-scale* model if $\alpha = 1$ and a *decreasing return-to-scale* model if $\alpha < 1$.

If there is an additional numerical independent variable X, then the following additive translog growth model may be obtained, which could be considered also as the Cobb–Douglas (CD) growth model.

$$\ln(Y_t) = C(1) + C(2)^*\ln(X) + C(3)^*\ln(t) + \mu_t \tag{2.47}$$

Furthermore, if there are multivariate independent variables, say $X_1, X_2, \ldots, X_K$, then a CD growth modelis formed:

$$\ln(Y_t) = C(1) + C(2)^*\ln(t) + \sum_{k=1}^{K} C(k+2)^*\ln(X_k) + \mu_t \tag{2.48}$$

Example 2.18. (Translog linear growth models) One of the main objectives in applying the model in (2.48) is to test the null hypothesis of constant return-to-scale, that is H_0: $C(2) + C(3) + \cdots C(K+2) = 1$, which is a special linear combination of the model parameters. The following model is a translog linear growth model:

$$\begin{aligned} \log(m1) = \; & C(1) + C(2)^*\log(t) + C(3)^*\log(gdp) \\ & + C(4)^*\log(pr) + [AR(1) = C(5)] \end{aligned} \tag{2.49}$$

Entering the series of variables

$$\log(m1)\; c \log(t) \log(gdp) \log(pr)\; ar(1) \tag{2.50}$$

in the '*Equation specification*' window gives the statistical results in Figure 2.42 and its residual graph in Figure 2.43. Note that the AR(1) model is used directly because of the time series data.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 21:25
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 20 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.112956	0.781357	3.984040	0.0001
LOG(T)	0.015703	0.024149	0.650261	0.5164
LOG(GDP)	0.498609	0.102703	4.854871	0.0000
LOG(PR)	0.467547	0.170817	2.737122	0.0068
AR(1)	0.973140	0.015104	64.43001	0.0000

R-squared	0.999595	Mean dependent var	5.816642
Adjusted R-squared	0.999586	S.D. dependent var	0.753241
S.E. of regression	0.015328	Akaike info criterion	-5.490683
Sum squared resid	0.040883	Schwarz criterion	-5.401650
Log likelihood	496.4161	Hannan-Quinn criter.	-5.454581
F-statistic	107413.4	Durbin-Watson stat	1.979118
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97

Figure 2.42 Statistical results based on the model in (2.50)

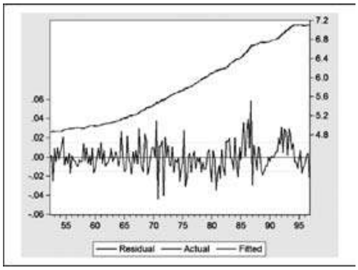
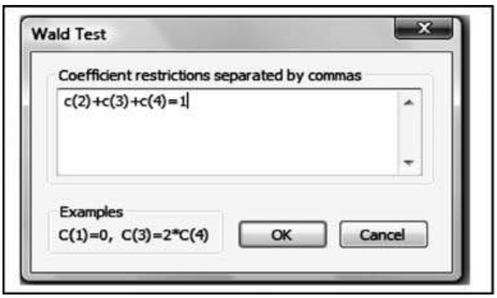


Figure 2.43 Residual graph of the model in (2.50)

Based on this figure, it is easy to test several one-sided hypotheses by using the *t*-test, such as the adjusted effects of each independent variable, as well as the autocorrelation coefficient. Do this as an exercise.

For illustration purposes, corresponding to the CD production function, the null hypothesis H_0: $C(2) + C(3) + C(4) = 1$ will be tested. The processes to test this hypothesis are as follows:



Wald Test
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	0.039850	(1, 174)	0.8420
Chi-square	0.039850	1	0.8418

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2) + C(3) + C(4)	-0.018141	0.090877

Restrictions are linear in coefficients.

Figure 2.44 An illustrative example in testing a linear combination of the model parameters, using the Wald test

(1) In the equation box, click *View/Coefficient Tests/Wald-Coefficient restriction* …. This gives the Wald test window in Figure 2.44 on the left-hand side.
(2) Entering the equation $C(2) + C(3) + C(4) = 1$ and then clicking *OK* gives the result in Figure 2.44 on the right-hand side. The null hypothesis is accepted based on the chi-squared-statistic with a *p*-value $= 0.8418$.
(3) Note that the *F*-statistic presented can be used under the basic assumptions of the error terms of the model, which are having a zero mean, constant variance and identical independent normal distributions. Refer to the notes in Section 2.14 concerning problems in testing these assumptions. □

Example 2.19. (A reduced model of the model in (2.50)) Since the previous example shows that log(t) has an insignificant effect, a reduced model may be applied

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 21:34
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 20 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.279737	0.761798	4.305255	0.0000
LOG(GDP)	0.485966	0.103300	4.704397	0.0000
LOG(PR)	0.474420	0.169475	2.799350	0.0057
AR(1)	0.978000	0.014786	66.14546	0.0000

R-squared	0.999594	Mean dependent var	5.816642
Adjusted R-squared	0.999587	S.D. dependent var	0.753241
S.E. of regression	0.015300	Akaike info criterion	-5.499823
Sum squared resid	0.040966	Schwarz criterion	-5.428597
Log likelihood	496.2342	Hannan-Quinn criter.	-5.470942
F-statistic	143748.4	Durbin-Watson stat	1.984622
Prob(F-statistic)	0.000000		

Inverted AR Roots .98

Figure 2.45 Statistical results based on a reduced model of the model (2.50)

without the log(t) as an independent variable, which will be considered as a see mingly causal model (*SCM*) or *neo-classical growth model*, because it is not a pure causal or growth model. This model will be discussed in more detail in Chapter 4.

The statistical results based on a reduced model of the model in (2.50) is presented in Figure 2.45 and a Wald test is presented in Figure 2.46. This figure shows that the null hypothesis H_0: $C(2) + C(3) = 1$ is accepted, with a p-value of 0.6686. Hence, the regression function obtained can be considered as a *constant return-to-scale* production function. □

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	0.183897	(1, 175)	0.6686
Chi-square	0.183897	1	0.6680

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2) + C(3)	-0.039614	0.092377

Restrictions are linear in coefficients.

Figure 2.46 Wald test based on the model in Figure 2.42

2.10.3 Some comments

So far, there have been many growth models for the indicator money supply $M1$, and many other alternative additive models could have been obtained by using additional independent variables, higher-order autoregressive coefficients and lagged variables. In the following subsection, additional growth models are presented having interaction factors as independent variables. It is certain that every researcher will encounter problems in selecting a model that can be considered to be the best.

Personal best judgment should be used in selecting a group of growth models, particularly for the statistical models. Then a choice could be made as to which one is the best in the group of models, even though this would be quite a subjective choice. It is difficult for anyone to present all possible models based on even a small group of variables, and it could never be a certainty that any defined model is strictly true for the corresponding population.

2.10.4 Growth model having interaction factors

2.10.4.1 The simplest growth model with two independent variables

The simplest interaction growth model is defined as

$$\ln(Y_t) = (C(1) + C(2)^*X) + (C(3) + C(4)^*X)^*t + \mu_t \tag{2.51}$$

where X is a numerical independent dated variable. Note that this model has the following characteristics:

- The corresponding regression function presents a curve or surface in a three-dimensional coordinate system with X, $\ln(Y)$ and t axes.
- For each fixed value of X, say X_0, the corresponding regression function presents a straight line with an *intercept* $= [C(1) + C(2)X_0]$ and a *slope* $= [C(3) + C(4)X_0]$ in a two-dimensional coordinate system with $\ln(Y)$ and t axes. Hence, for all possible values of the independent variable X, the model (2.51) can be presented as a set of straight lines. As an illustration, Alternative 1 in Figure 2.47 shows a set of lines $\log(y) = a + (bx).t$, for $x < 0$, $x = 0$ and $x > 0$, which could be extended for $-\infty < x < \infty$.
- Based on this type of model, a statement could be made that the effect of the time t-variable depends on the X-variable.

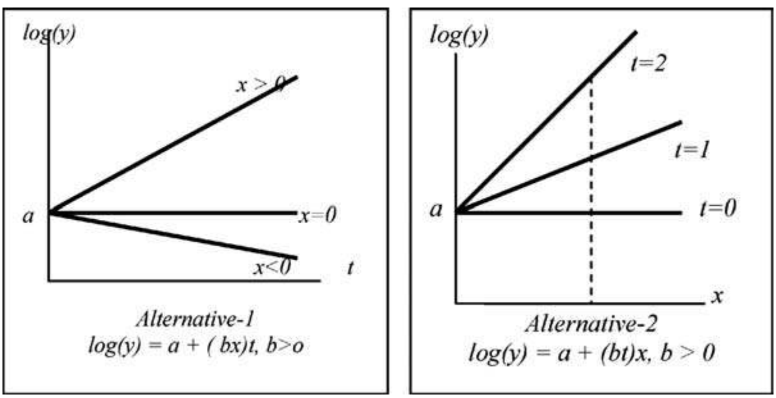


Figure 2.47 Illustrative sets of heterogeneous lines with an intercept

- Note that the model (2.51) can also be written as

$$\ln(Y_t) = (C(1) + C(3)^*t) + (C(2) + C(4)^*t)^*X + \mu_t \tag{2.52}$$

- Based on this equation, it could be said that the effect of the X-variable depends on the time t-variable. The corresponding regression function, for all possible values of t, will present a set of straight lines in a two-dimensional coordinate system with ln (Y) and X axes. As an illustration, Alternative 2 in Figure 2.47 shows a set of straight lines $\log(y) = a + (bt)x$, for $t = 0$, $t = 1$ and $t = 2$, $b > 0$ and $x > 0$, which can be extended for all $t \geq 0$.
- This model will be considered as a *two-way interaction model*, since it has a two-way interaction factor as an independent variable.

2.10.4.2 Additional growth models with interaction factors

In this subsection, only two models will be presented having three numerical independent variables X_1, X_2 and the time t-variable. The two following models could be easily extended for multivariate numerical independent variables, besides the t-variable, as well as for the translog growth models (see Agung, 2006):

$$\begin{aligned}\ln(Y_t) = {} & (C(1) + C(2)^*X_1 + C(3)^*X_2) \\ & + (C(4) + C(5)^*X_1 + C(6)^*X_2)^*t + \mu_t\end{aligned} \tag{2.53}$$

$$\begin{aligned}\ln(Y_t) = {} & (C(1) + C(2)^*X_1 + C(3)^*X_2 + C(4)^*X_1{}^*X_2) \\ & + (C(5) + C(6)^*X_1 + C(7)^*X_2 + C(8)^*X_1{}^*X_2)^*t + \mu_t\end{aligned} \tag{2.54}$$

Note that the model in (2.53) shows that the effect of the time t-variable depends on an additive function defined as $\{c(3) + c(4)X_1 + c(5)X_2\}$, but the model in (2.54) shows that it depends on the function having an interaction factor, namely $\{c(5) + c(6)X_1 + c(7)X_2 + c(8)X_1X_2\}$. The model in (2.53) is a two-way interaction model and the model in (2.54) is a three-way interaction model, since it has a three-way interaction factor, $X_1{}^*X_2{}^*t$, as an independent variable. These types of model could be considered as time series models with *linear trend and time-related effects* (Bansal, 2005).

Example 2.20. (Growth model having interaction factors) Based on the data in Demo_Modified.wf1, the following translog growth model with an interaction factor together with its corresponding AR(1)_GM is applied:

$$\begin{aligned}\log(M1_t)) = {} & C(1) + C(2)^*\log(GPD) + C(3)^*(\text{year-52}) \\ & + C(4)^*\log(GDP)^*(\text{year-52}) + \mu_t\end{aligned} \tag{2.55}$$

where $t = (\text{year} - 52)$, so that the time t-variable has values 0 up to 44.

The results of the analysis should be obtained by entering a series of variables:

$$\log(m1)\ c\ \log(gdp)\ (\text{year-52})\ \log(gdp)^*(\text{year-52}) \tag{2.56}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 21:44
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.050176	0.141340	21.58034	0.0000
LOG(GDP)	0.397584	0.032099	12.38622	0.0000
YEAR-52	-0.038366	0.002447	-15.67791	0.0000
LOG(GDP)*(YEAR-52)	0.008450	0.000271	31.22284	0.0000

R-squared	0.997616	Mean dependent var	5.811220
Adjusted R-squared	0.997576	S.D. dependent var	0.754650
S.E. of regression	0.037158	Akaike info criterion	-3.725328
Sum squared resid	0.243000	Schwarz criterion	-3.654373
Log likelihood	339.2795	Hannan-Quinn criter.	-3.696559
F-statistic	24552.28	Durbin-Watson stat	0.196419
Prob(F-statistic)	0.000000		

Figure 2.48 Statistical results based on the model in (2.56)

in the '*Equation specification*' window. Its corresponding AR(1) model can be easily found as in the previous examples, by entering the following series:

$$\log(m_1)\ c\ \log(gdp)\ (\text{year-52})\ \log(gdp)*(\text{year-52})\ ar(1) \tag{2.57}$$

The results based on these two models are presented in Figures 2.48 and 2.49 respectively. Based on these results, the following notes and conclusions are presented:

(1) Since this concerns time series data, then the growth model in (2.56) is not appropriate to use for testing the hypothesis, because of the serial correlation or autocorrelation of the error terms. Its residual graph is shown in Figure 2.50. However, because of such a very large value of R-squared (99.8%), it could be said that this model is a good model for estimation and forecasting.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 21:47
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 13 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.092510	0.390897	5.353097	0.0000
LOG(GDP)	0.600107	0.070705	8.487458	0.0000
YEAR-52	-0.025349	0.012524	-2.024078	0.0445
LOG(GDP)*(YEAR-52)	0.004662	0.002013	2.316460	0.0217
AR(1)	0.956618	0.025109	38.09816	0.0000

R-squared	0.999589	Mean dependent var	5.816642
Adjusted R-squared	0.999579	S.D. dependent var	0.753241
S.E. of regression	0.015452	Akaike info criterion	-5.474638
Sum squared resid	0.041544	Schwarz criterion	-5.385605
Log likelihood	494.9801	Hannan-Quinn criter.	-5.438536
F-statistic	105703.1	Durbin-Watson stat	2.047610
Prob(F-statistic)	0.000000		
Inverted AR Roots	.96		

Figure 2.49 Statistical results based on the model in (2.57)

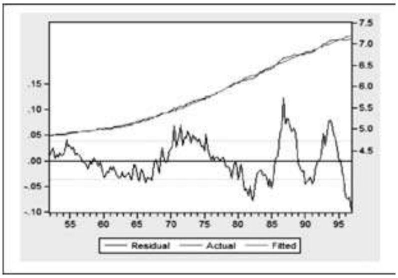


Figure 2.50 Residual graph of the model in (2.56)

(2) The AR(1) model in (2.57) should be a better model to be used for testing the hypothesis. Its residual graph is shown in Figure 2.51. This model shows that the interaction factor log(*GDP*)* *t* = log(*GDP*)*(Year − 52) has a significant effect on log(*M*1) with a *p*-value of 0.0217. It may be concluded that the effect of log(*GDP*) on log(*M*1) is significant and depends on the time *t*-variable.

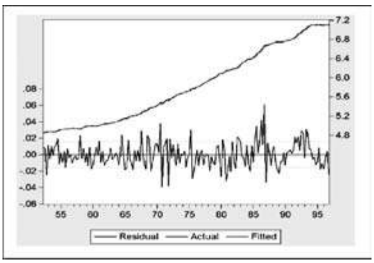


Figure 2.51 Residual graph of the model in (2.57)

(3) The equation of the AR(1) regression function can be written as

$$\begin{aligned}\log(m_1) &= 2.093+0.600^*\log(gdp)-0.025^*t+0.005^*\log(gdp)^*t+[ar(1)=0.957]\\ &= \{2.093-0.025^*t\}+\{0.600+0.005^*t\}^*\log(gdp)+[ar(1)=0.957]\end{aligned} \tag{2.58}$$

This function shows that log(*gdp*) has a positive effect on log(*m*1), because $\{0.600 + 0.005t\} > 0$. For all possible values of *t*, this function will present a set of straight lines in a two-dimensional coordinate system with log(*m*1) and log (*gdp*) axes, as illustrated in Figure 2.47. □

Example 2.21. (Advanced growth model having interaction factors) The result in Figure 2.52 is obtained by entering a series of variables

$$\begin{aligned}&\log(m_1)\ c\ \log(gdp)\ \log(pr)\ \log(gdp)^*\log(pr)\\ &\qquad t\ t^*\log(gdp)\ t^*\log(pr)\ t^*\log(gdp)^*\log(pr)\ ar(1)\end{aligned} \tag{2.59}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 22:00
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 65 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.718440	2.169070	1.253275	0.2118
LOG(GDP)	0.084468	0.330608	0.255493	0.7987
LOG(PR)	-1.431575	1.334767	-1.072528	0.2850
LOG(GDP)*LOG(PR)	0.053331	0.212688	0.250748	0.8023
T	0.030581	0.019792	1.545067	0.1242
T*LOG(GDP)	-0.001180	0.002559	-0.461316	0.6452
T*LOG(PR)	0.039503	0.012253	3.223863	0.0015
T*LOG(GDP)*LOG(PR)	-0.005258	0.001552	-3.386997	0.0009
AR(1)	0.925730	0.035516	26.06520	0.0000

R-squared	0.999654	Mean dependent var	5.816642
Adjusted R-squared	0.999638	S.D. dependent var	0.753241
S.E. of regression	0.014337	Akaike info criterion	-5.602996
Sum squared resid	0.034943	Schwarz criterion	-5.442736
Log likelihood	510.4681	Hannan-Quinn criter.	-5.538012
F-statistic	61396.08	Durbin-Watson stat	2.194756
Prob(F-statistic)	0.000000		
Inverted AR Roots	.93		

Figure 2.52 Statistical results based on the model in (2.59)

in the '*Equation specification*' window. This is an application of the model in (2.54) with $X_1 = \log(gdp)$ and $X_2 = \log(pr)$. Based on this result, the following notes and conclusions may be produced:

(1) All independent variables have a joint significant effect on $\log(m1)$, based on the F-test with a p-value = 0.0000. However, five out of eight independent variables have insignificant adjusted effects. This situation indicates an unpredictable impact of the multicollinearity coefficient on the independent variables. As a result, an attempt should be made to find a reduced model or a modified model.
(2) To obtain a reduced model, an attempt should always first be made to delete or omit the main factor because the two-way interaction(s) should be used to indicate that the effect of a factor on the dependent variable is most likely to be dependent on the other factor.
(3) By omitting $\log(pr)$ and then the t-variable, the reduced model presented in Figure 2.53 was obtained; $\log(gdp)$ was still used as an independent variable, although it does not have a significant effect on $\ln(m1)$ because the corresponding test has a p-value < 0.25, as suggested by Hosmer and Lemesshow (Hosmer and Lemesshow, 2000, p. 95). On the other hand, at a significant level $\alpha = 0.10$, in fact, $\log(gdp)$ has a significantly negative effect on $\log(m1)$ with a p-value = 0.1543/2 = 0.077 15.
(4) Based on the reduced model, the following equation is obtained:

$$\begin{aligned}\ln(m_1) \quad = \quad & \{C(1) + C(2)*\log(gdp) + C(3)*\log(gdp)*\log(pr)\} \\ & + \{C(4)*\log(gdp) + C(5)*\log(pr) + C(6)*\log(gdp)*\log(pr)\}*\ t \\ & + [ar(1) = C(7)]\end{aligned} \tag{2.60}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/11/07 Time: 22:03
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 20 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	5.576502	0.864080	6.453689	0.0000
LOG(GDP)	-0.275464	0.192546	-1.430644	0.1543
LOG(GDP)*LOG(PR)	-0.103444	0.050017	-2.068161	0.0401
T*LOG(GDP)	0.002787	0.000579	4.815393	0.0000
T*LOG(PR)	0.029453	0.010391	2.834600	0.0051
T*LOG(GDP)*LOG(PR)	-0.004357	0.001478	-2.947889	0.0036
AR(1)	0.942013	0.035216	26.74954	0.0000

R-squared	0.999650	Mean dependent var	5.816642
Adjusted R-squared	0.999637	S.D. dependent var	0.753241
S.E. of regression	0.014344	Akaike info criterion	-5.612661
Sum squared resid	0.035389	Schwarz criterion	-5.488015
Log likelihood	509.3332	Hannan-Quinn criter.	-5.562118
F-statistic	81780.48	Durbin-Watson stat	2.158438
Prob(F-statistic)	0.000000		
Inverted AR Roots	.94		

Figure 2.53 Statistical results based on a reduced model of the model in Figure 2.48

This model shows that the effect of the time t on the money supply, namely log ($m1$), is significantly dependent on the values of the function

$$\{C(4)^*\log(gdp) + C(5)^*\log(pr) + C(6)^*\log(gdp)^*\log(pr)\} \qquad (2.61)$$

since each of the interactions $t^*\log(gdp)$, $t^*\log(pr)$ and $t^*\log(gdp)^*\log(pr)$ has a significant adjusted effect on $\log(m1)$.

Furthermore, note that the time t-variable could be considered as representing other variables of the model that have significant positive correlations with t. □

2.10.5 Trigonometric growth models

The trigonometric growth models are derived from the trigonometric models presented in Thomopolous (1980, pp. 37–38). Three basic models are presented below. The extensions of these models are their corresponding autoregressive models, semilog growth model and translog growth models, as shown in the previous subsections.

Furthermore, note that the following models are the specific or special forms of the additive adjusted model in (2.43).

- The *three-term growth model*:

$$Y_t = C(1) + C(2)^*\sin(\omega^*t) + C(3)^*\cos(\omega^*t) + \mu_t \qquad (2.62)$$

- The *four-term growth model*:

$$Y_t = C(1) + C(2)^*t + C(3)^*\sin(\omega^*t) + C4^*\cos(\omega^*t) + \mu_t \qquad (2.63)$$

- The *six-term growth model*:

$$\begin{aligned} Y_t \;=\; & C(1) + C(2)*t + C(3)*\sin(\omega*t) + C(4)*\cos(\omega*t) \\ & + C(5)*\sin(2*\omega*t) + C(6)*\cos(2*\omega*t) + \mu_t \end{aligned} \tag{2.64}$$

where $\omega = 2\pi/M$ and M is a *cycle length*. Note that $\sin(\omega t)$ has a minimum value of -1 and a maximum value of $+1$, with the average $\sin(\omega t) = 0$. The same is true for cos (ωt).

2.11 Multivariate growth models

2.11.1 The classical multivariate growth model

The classical multivariate growth model, in fact, is a set or system of the simple growth models in (2.3). Hence, the following general system of equations is presented:

$$\log(Y_{gt}) = C(1g) + C(2g)*t + \mu_{gt}, \quad \text{for} \quad g = 1, 2, \ldots, G. \tag{2.65}$$

For time series data, the multivariate first-order autoregressive growth model, namely MAR(1)_GM, will be considered, as follows:

$$\begin{aligned} & \log(Y_{gt}) = C(g1) + C(g2)*t + \mu_{gt} \\ & \mu_{gt} = \rho_g \mu_{g(t-1)} + \varepsilon_{gt} \quad \text{for} \quad g = 1, 2, \ldots, G \end{aligned} \tag{2.66}$$

Since this model has a set of dependent variables or a vector of dependent variables, it is also called the first-order vector autoregressive (VAR) model. However, in EViews, the term 'VAR' is used to representing a special type of multivariate time series model, so here it is proposed that the term MAR is used to represent the general multivariate autoregressive model, where the VAR models are special cases of the MAR models. The VAR models will be presented in Chapter 6.

Example 2.22. (The simplest bivariate AR(1) growth model) The simplest bivariate AR(1) growth model using variables in the Demo_Modified workfile considered is

$$\begin{aligned} & \ln(m1) = C(11) + C(12)*t + [ar(1) = C(13)] \\ & \ln(gdp) = C(21) + C(22)*t + [ar(1) = C(23)] \end{aligned} \tag{2.67}$$

Note that double subscripts are used for the model parameters, namely $C(ij)$, because this makes it easier to produce modified models using the method presented in the previous examples, especially for a large number of exogenous variables.

The process of analysis can be done as follows:

(1) After opening the workfile and click *Object/New Object…*; the window in Figure 2.54 will then appear on the screen. By selecting the object '*System*' and

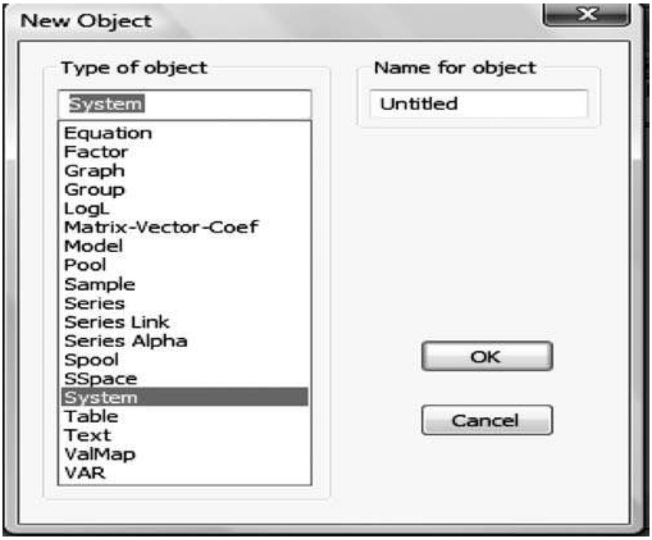


Figure 2.54 Type of new objects available in EViews 6

clicking *OK*, the window space in Figure 2.55 can be seen, where the system equations in (2.67) can be entered.

(2) Then click *Estimate* . . ., which gives the options in Figure 2.56 on the screen. In this case, there are three possible selections of estimation methods: OLS, WLS and SUR. The other options will be presented later.

(3) Figure 2.57 presents the statistical results using the iteration least squares (ILS) estimation method. This table shows that the second regression has a small value of the DW-statistic of 1.213 396. This model should therefore be modified by using higher-order autoregressive model(s), producing the model presented in Figure 2.58. It could be said that this model is a better bivariate growth model and could be the best model in presenting the growth rate of *M*1 and *GDP* as a basic bivariate model.

However, further analysis should be done, residual analysis in particular, to study or explore the limitation or weakness of the statistical results. For illustration purposes, Figure 2.59 presents the residual box plots, as well as the residual graphs of the model.

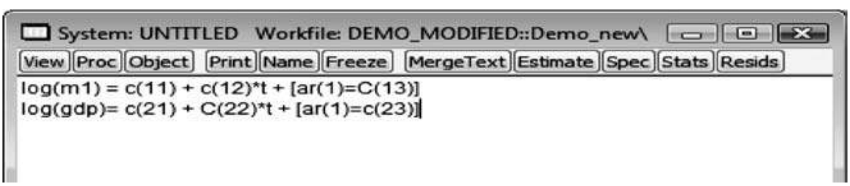


Figure 2.55 The input of the system equation in (2.67)

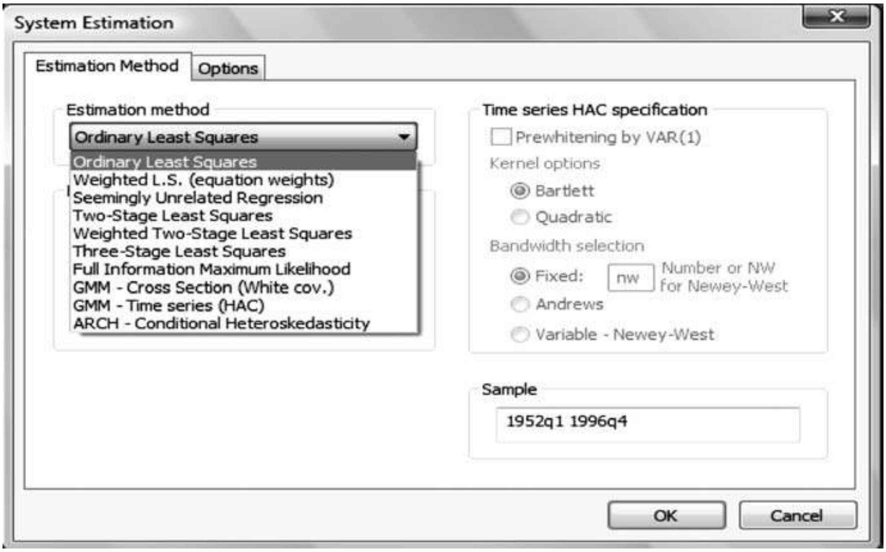
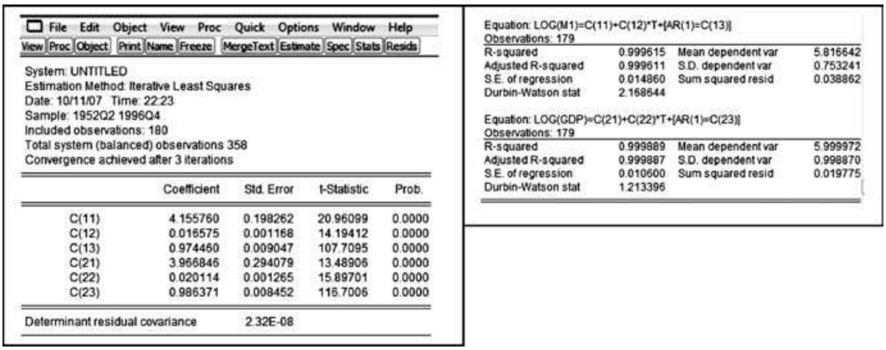


Figure 2.56 The estimation method options for the system equation



File Edit Object View Proc Quick Options Window Help

View Proc Object Print Name Freeze MergeText Estimate Spec Stats Resids

System: UNTITLED
Estimation Method: Iterative Least Squares
Date: 10/11/07 Time: 22:23
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (balanced) observations 358
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.155760	0.198262	20.96099	0.0000
C(12)	0.016575	0.001168	14.19412	0.0000
C(13)	0.974460	0.009047	107.7095	0.0000
C(21)	3.966846	0.294079	13.48906	0.0000
C(22)	0.020114	0.001265	15.89701	0.0000
C(23)	0.986371	0.008452	116.7006	0.0000
Determinant residual covariance		2.32E-08		

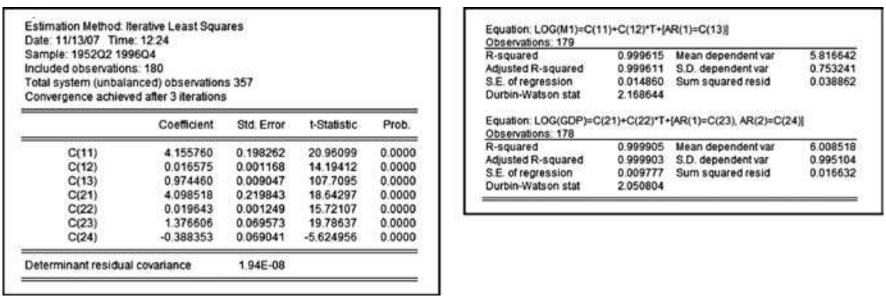
Equation: LOG(M1)=C(11)+C(12)*T+[AR(1)=C(13)]
Observations: 179

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Sum squared resid	0.038862
Durbin-Watson stat	2.168644		

Equation: LOG(GDP)=C(21)+C(22)*T+[AR(1)=C(23)]
Observations: 179

R-squared	0.999889	Mean dependent var	5.999972
Adjusted R-squared	0.999887	S.D. dependent var	0.998870
S.E. of regression	0.010600	Sum squared resid	0.019775
Durbin-Watson stat	1.213396		

Figure 2.57 Statistical results based on the model in (2.67)



Estimation Method: Iterative Least Squares
Date: 11/13/07 Time: 12:24
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (unbalanced) observations 357
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.155760	0.198262	20.96099	0.0000
C(12)	0.016575	0.001168	14.19412	0.0000
C(13)	0.974460	0.009047	107.7095	0.0000
C(21)	4.098518	0.219843	18.64297	0.0000
C(22)	0.019643	0.001249	15.72107	0.0000
C(23)	1.376606	0.069573	19.78637	0.0000
C(24)	-0.388353	0.069041	-5.624956	0.0000
Determinant residual covariance		1.94E-08		

Equation: LOG(M1)=C(11)+C(12)*T+[AR(1)=C(13)]
Observations: 179

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Sum squared resid	0.038862
Durbin-Watson stat	2.168644		

Equation: LOG(GDP)=C(21)+C(22)*T+[AR(1)=C(23), AR(2)=C(24)]
Observations: 178

R-squared	0.999905	Mean dependent var	6.008518
Adjusted R-squared	0.999903	S.D. dependent var	0.995104
S.E. of regression	0.009777	Sum squared resid	0.016632
Durbin-Watson stat	2.050804		

Figure 2.58 Statistical results based on a modified model of (2.67)

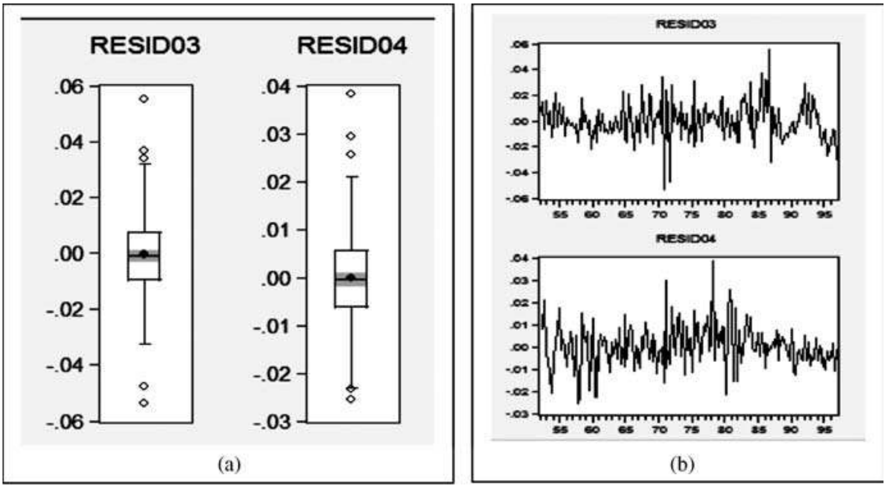


Figure 2.59 (a) Residual box plots and (b) graphs of the model in Figure 2.53

Refer to the characteristics of the box plots presented in Section 1.4.2. Note that both the residual box plots and graphs indicate the existence of some outliers. Corresponding to the problems of outliers, refer to the notes in Example 2.4. □

2.11.1.1 The WLS and SUR estimates

For a comparison study, Figure 2.60 presents two sets of statistical results based on the same model in (2.67), by using the WLS and SUR estimation methods respectively.

System: UNTITLED
Estimation Method: Weighted Least Squares
Date: 10/29/07 Time: 11:17
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (balanced) observations 358
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 3 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.155760	0.196593	21.13888	0.0000
C(12)	0.016575	0.001158	14.31458	0.0000
C(13)	0.974460	0.008971	108.6236	0.0000
C(21)	3.966846	0.291604	13.60354	0.0000
C(22)	0.020114	0.001255	16.03192	0.0000
C(23)	0.986371	0.008381	117.6910	0.0000

Determinant residual covariance 2.32E-08

Equation: LOG(M1)=C(11)+C(12)*T+[AR(1)=C(13)]
Observations: 179

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Sum squared resid	0.038862
Durbin-Watson stat	2.168644		

Equation: LOG(GDP)=C(21)+C(22)*T+[AR(1)=C(23)]
Observations: 179

R-squared	0.999889	Mean dependent var	5.999972
Adjusted R-squared	0.999887	S.D. dependent var	0.998870
S.E. of regression	0.010600	Sum squared resid	0.019775
Durbin-Watson stat	1.213396		

(a)

System: UNTITLED
Estimation Method: Seemingly Unrelated Regression
Date: 10/29/07 Time: 11:18
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (balanced) observations 358
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 5 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.045662	0.301909	13.40029	0.0000
C(12)	0.017108	0.001587	10.77826	0.0000
C(13)	0.979286	0.008884	110.2280	0.0000
C(21)	4.021727	0.209237	19.22096	0.0000
C(22)	0.019966	0.001022	19.53459	0.0000
C(23)	0.983813	0.008300	118.5327	0.0000

Determinant residual covariance 2.31E-08

Equation: LOG(M1)=C(11)+C(12)*T+[AR(1)=C(13)]
Observations: 179

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999610	S.D. dependent var	0.753241
S.E. of regression	0.014872	Sum squared resid	0.038925
Durbin-Watson stat	2.175522		

Equation: LOG(GDP)=C(21)+C(22)*T+[AR(1)=C(23)]
Observations: 179

R-squared	0.999889	Mean dependent var	5.999972
Adjusted R-squared	0.999887	S.D. dependent var	0.998870
S.E. of regression	0.010603	Sum squared resid	0.019785
Durbin-Watson stat	1.209669		

(b)

Figure 2.60 Statistical results based on the model in (2.67) by using the (a) WLS and (b) SUR estimation methods

Compared to the statistical results in Figure 2.57, using the ILS (iteration least squares) estimation method, the following findings can be obtained:

(1) The parameter estimates using the ILS and WLS are equal, but they are different from those using the SUR estimation method.
(2) The three estimation methods give different values of the standard error estimates. Hence, they will present different values of the t-statistic.

2.11.1.2 Testing hypotheses

Corresponding to the model in (2.67), the univariate hypothesis may be tested as well as the multivariate hypothesis, which can easily be tested using the Wald test, as follows.

Univariate hypothesis
The hypothesis on the growth rate of each endogenous variable $M1$ and GDP, with the null hypothesis H_0: $C(12) = 0$ and H_0: $C(22) = 0$ respectively; in other words, the hypothesis on the effect of the time t on each of the endogenous variables $\log(M1)$ and $\log(GDP)$.

Multivariate hypothesis

- The hypothesis on the effect of the time t on both endogenous variables, with the null hypothesis H_0: $C(12) = C(22) = 0$.
- The hypothesis on whether $M1$ and GDP have different growth rates, with the null hypothesis H_0: $C(12) = C(22)$.

2.11.2 Modified multivariate growth models

The previous example shows that a simple bivariate AR(1) growth model is not appropriate for the variables $m1$ and gdp. It is expected that, in most cases, the simple models are not necessarily good models either. Hence, this subsection will present a method showing how to modify a multivariate growth model. However, the trial-and-error methods should be used.

Example 2.23. (A modified bivariate growth model) Since the result above shows that the second regression has a DW-statistic of 1.2, then by '*rule of thumb*' an attempt should be made to modify the second regression by using the lagged variable $\log(gdp(-1))$. It happens that an acceptable model can be obtained directly, in a statistical sense. To modify the model, the following steps should be used:

System: UNTITLED
Estimation Method: Iterative Least Squares
Date: 10/11/07 Time: 22:44
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (unbalanced) observations 357
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.155760	0.198262	20.96099	0.0000
C(12)	0.016575	0.001168	14.19412	0.0000
C(13)	0.974460	0.009047	107.7095	0.0000
C(21)	0.098985	0.056874	1.740431	0.0827
C(22)	0.000382	0.000259	1.474668	0.1412
C(23)	0.980548	0.013427	73.02660	0.0000
C(24)	0.396057	0.071484	5.540525	0.0000
Determinant residual covariance		1.94E-08		

Equation: LOG(M1)=C(11)+C(12)*T+[AR(1)=C(13)]
Observations: 179

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999611	S.D. dependent var	0.753241
S.E. of regression	0.014860	Sum squared resid	0.038862
Durbin-Watson stat	2.168644		

Equation: LOG(GDP)=C(21)+C(22)*T+C(23)*LOG(GDP(-1)) + [AR(1)=C(24)]
Observations: 178

R-squared	0.999905	Mean dependent var	6.008518
Adjusted R-squared	0.999903	S.D. dependent var	0.995104
S.E. of regression	0.009777	Sum squared resid	0.016632
Durbin-Watson stat	2.050804		

Figure 2.61 Statistical results based on a modified model in (2.68)

(1) Click *View/System specification* ... and then enter the following system equations:

$$\begin{aligned} \log(m1) &= c(11) + c(12)*t + [ar(1) = c(13)] \\ log(gdp) &= c(21) + c(22)*t + c(23)*\log(gdp(-1)) + [ar(1) = c(24)] \end{aligned} \tag{2.68}$$

(2) Click *Estimate* ...; three alternative options then appear on the screen, as mentioned above. Select the OLS option and then click *OK*. This gives the statistical results in Figure 2.61, which presents an acceptable model based on the DW-statistic, as well as the *t*-statistic of each independent variable. □

Example 2.24. (A case of polynomial bivariate AR(1) growth models) Figure 2.62 presents the residual graphs of the system equations

$$\begin{aligned} \log(pr) &= c(11) + c(12)*t + [ar(1) = c(13)] \\ \log(rs) &= c(21) + c(22)*t + c(23)*t^2 + [ar(1) = c(24)] \end{aligned} \tag{2.69}$$

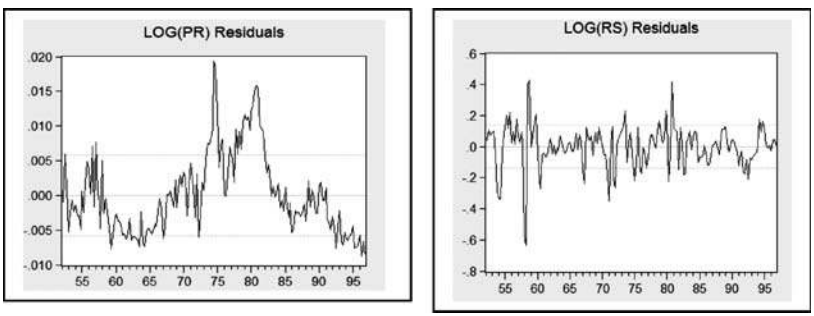


Figure 2.62 Residual graphs of the bivariate growth model in (2.69)

Here, a quadratic model is used in the time *t* for the dependent variable *RS*, which is supported by its growth curve, presented in Figure 1.24.

The statistical results show that both models have small values of DW-statistics (refer to the special notes in the previous example) and the residual plot of the first regression in Figure 2.62 gives a strong indication that a higher-order of autoregressive coefficients should be used. Hence, an attempt should be made to find a better fit growth model for each of the variables *PR* and *RS*. Refer to the following examples. □

Example 2.25. (Higher-order autoregressive bivariate bodel) Note that the residual plot of log(*pr*) in the previous example shows that a higher-order autoregressive model should be used. On the other hand, the residual plot of log(*rs*) is not very clear after using a higher-order autoregressive model.

After doing several exercises, an acceptable model was found with the statistical results presented in Figure 2.63 and its residual graph in Figure 2.64.

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 07:39
Sample: 1952Q3 1996Q4
Included observations: 180
Total system (unbalanced) observations 354
Convergence achieved after 2 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-2.028463	0.198635	-10.21199	0.0000
C(12)	0.012066	0.001250	9.655380	0.0000
C(13)	1.834979	0.040973	44.78541	0.0000
C(14)	-0.838986	0.040500	-20.71545	0.0000
C(21)	0.021032	0.184353	0.114083	0.9092
C(22)	0.034299	0.004482	7.652107	0.0000
C(23)	-0.000147	2.31E-05	-6.366726	0.0000
C(24)	1.440951	0.073292	19.66034	0.0000
C(25)	-0.958711	0.123048	-7.791373	0.0000
C(26)	0.650054	0.122975	5.286054	0.0000
C(27)	-0.299892	0.072894	-4.114089	0.0000
Determinant residual covariance		1.31E-07		

Equation: LOG(PR)=C(11)+C(12)*T+[AR(1)=C(13), AR(2)=C(14)]
Observations: 178

R-squared	0.999972	Mean dependent var	-0.832770
Adjusted R-squared	0.999972	S.D. dependent var	0.593951
S.E. of regression	0.003164	Sum squared resid	0.001741
Durbin-Watson stat	2.491693		

Equation: LOG(RS)=C(21)+C(22)*T+C(23)*T^2+[AR(1)=C(24), AR(2)=C(25), AR(3)=C(26), AR(4)=C(27)]
Observations: 176

R-squared	0.958075	Mean dependent var	1.558911
Adjusted R-squared	0.956586	S.D. dependent var	0.567913
S.E. of regression	0.118330	Sum squared resid	2.366340
Durbin-Watson stat	2.018999		

Figure 2.63 Higher-order autoregressive bivariate model of the model in (2.69)

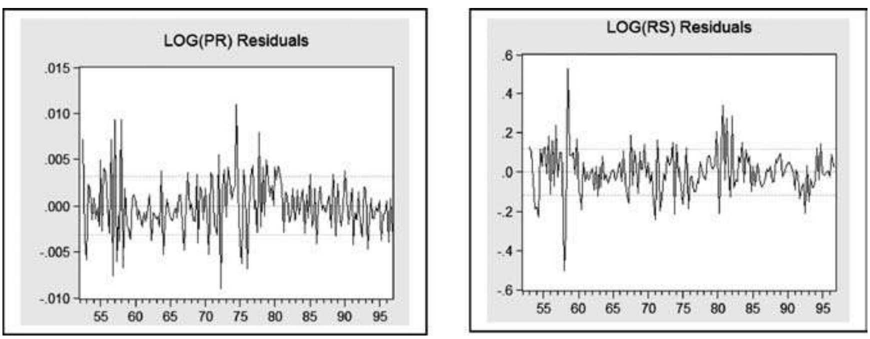


Figure 2.64 Residual graphs of the regression in Figure 2.63

Based on this result, the following notes and conclusions are presented:

(1) The first equation of the bivariate growth model is an AR(2) model with a linear trend and the second is an AR(4) model with a quadratic trend. The bivariate model has the following equation:

$$\begin{aligned} \log(pr_t) &= c(11)+c(12)*t+[ar(1)=c(13),ar(2)=c(14)]+\varepsilon_{1t} \\ \log(rs_t) &= c(21)+c(22)*t+c(23)*t^2 \\ &\quad +[ar(1)=c(24),ar(2)=c(25),ar(3)=c(26),ar(4)=c(27)]+\varepsilon_{2t} \end{aligned} \tag{2.70}$$

(2) Each of the autocorrelations or serial correlations is significant, which is indicated by the p-values of the t-statistics, corresponding to the parameters $c(13)$, $c(14)$, $c(24)$, $c(25)$, $c(26)$ and $c(27)$.
(3) The adjusted exponential growth rate of pr (= 1.21%) is significant.
(4) The endogenous variable rs has a significant quadratic growth rate with a very small p-value = 0.0000.
(5) Compared to the AR(1) model in the previous example, the residual graphs of this model show that it is a better model, with DW-statistics of 2.49 and 2.02 respectively. However, the residual graphs show the pattern of heteroskedasticity. Therefore, is suggested that the WLS, White or Newey–West estimation methods should be applied, or perhaps other modified model(s). Do this as an exercise. □

Example 2.26. (Lagged-variable autoregressive bivariate growth model) Figure 2.65 presents the statistical results based on a lagged-variable autoregressive bivariate growth model, say an LVAR(1,q) bivariate growth model, which can be considered as an alternative model of the models in the previous example.

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 08:27
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (unbalanced) observations 355
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.048293	0.040681	-1.187126	0.2360
C(12)	0.000357	0.000245	1.455488	0.1464
C(13)	0.970420	0.021292	45.57611	0.0000
C(14)	0.864560	0.051581	16.76109	0.0000
C(21)	0.025250	0.040646	0.621214	0.5349
C(22)	0.005831	0.002250	2.591732	0.0100
C(23)	-2.59E-05	1.01E-05	-2.566437	0.0107
C(24)	0.826016	0.060474	13.65911	0.0000
C(25)	0.545282	0.087752	6.213905	0.0000
C(26)	-0.288373	0.079139	-3.643884	0.0003
Determinant residual covariance		1.44E-07		

Equation: LOG(PR)=C(11)+C(12)*T+C(13)*LOG(PR(-1))+[AR(1)=C(14)]
Observations: 178

R-squared	0.999972	Mean dependent var	-0.832770
Adjusted R-squared	0.999972	S.D. dependent var	0.593951
S.E. of regression	0.003164	Sum squared resid	0.001741
Durbin-Watson stat	2.491693		

Equation: LOG(RS)=C(21)+C(22)*T+C(23)*T^2+C(24)*LOG(RS(-1))
+[AR(1)=C(25),AR(2)=C(26)]
Observations: 177

R-squared	0.954474	Mean dependent var	1.553799
Adjusted R-squared	0.953143	S.D. dependent var	0.570365
S.E. of regression	0.123464	Sum squared resid	2.606601
Durbin-Watson stat	1.859522		

Figure 2.65 Statistical results based on an LVAR(1, q) bivariate growth model, by using the iterative least squares estimation method of the model in (2.77)

Note that the equation of the bivariate model can easily be written based on the output. The following notes and conclusions can be made based on the results given in the figure:

(1) The first regression is an LVAR(1,1)_GM of the variable *PR*. At a significant level of $\alpha = 0.10$, the time t has a significant positive linear effect on log(*pr*), based on the t-statistic with a p-value $= 0.1464/2 = 0.0732 < \alpha = 0.10$. In other words, it could be said that the slope of log(*PR*) with respect to the time t is significantly positive at 0.000 357 or that *PR* has a significantly positive growth rate at 0.0357% during the observation time period.
(2) On the other hand, the second regression is an LVAR(1,2) quadratic growth model of the variable *RS*, where the time t^2 has a significantly negative effect on log(*rs*), based on the t-statistic with a p-value $= 0.0107/2 = 0.005\,35 < \alpha = 0.01$. Therefore, it can be concluded that the growth rate of the series *RS* is significantly dependent on the time t. Based on the regression function in Figure 2.65 gives the result $\partial \log(rs)/\partial t = 0.005\,81 + 2(-2.50\text{e} - 05)t$, which indicates that the adjusted effect of the time t on log(*rs*) is dependent on t. By looking at the growth curve of the time series *RS* in Figure 1.24, it is very clear that log(*rs*) and t have a nonlinear relationship. Other alternative model(s) using the series *RS* will be presented in the following chapter.
(3) For a comparison, the statistical results can also be obtained by using other estimation methods, such as the weighted least squares and seemingly unrelated regression. Do this as an exercise. □

2.11.3 AR(1) multivariate general growth models

As an extension of the classical exponential multivarite growth model in (2.66), a general multivariate AR(1) growth model should have the following equation:

$$\begin{aligned}\log(Y_{gt}) &= \left\{\sum_{k=1}^{K} C(gk)^* X_{gk}\right\} + \Re_g{}^* t + \mu_{gt} \\ \mu_{gt} &= \rho_g \mu_{g(t-1)} + \varepsilon_{gt}\end{aligned} \qquad (2.71)$$

where $X_{g1}, X_{g2}, \ldots, X_{gK}$ are multivariate independent or cause variables with $X_{g1} = 1$ for all g and $\Re_g$ is the adjusted growth rate of the endogenous variable Y_g or the trend (time) effect. Note that the sets of exogenous variables $\{X_{g1}, X_{g2}, \ldots, X_{gk}\}$ could be unequal sets of any types of variables for all $g = 1, \ldots, G$.

This time series model can be called a *first-order autoregressive mutivariate model with trend*, namely MAR(1)_T.

Note that even though the time t-variable is a discrete variable, its corresponding regression functions should be considered as differentiable functions with respect to time t for each g. Under the assumption that all exogenous variables are numerical variables, this model and all models presented in this chapter should be considered as continuous growth models, because their corresponding estimated regression functions would give the following partial derivatives:

$$\frac{\partial \log(\hat{Y}_g)}{\partial X_{gk}} = \hat{C}(gk) \quad \text{and} \quad \frac{\partial \log(\hat{Y}_g)}{\partial t} = \hat{\Re}_g \tag{2.72}$$

with finite or fixed values for each $g = 1,2, \ldots, G$ and $k = 1,2, \ldots, K$.

2.11.4 The S-shape multivariate AR(1) general growth models

A further extension of the classical exponential growth model is an S-shape AR(1) multivariate growth model, which can be easily derived from the model in (2.71). The system of bounded growth models has the following general equation:

$$\begin{aligned} \log\left(\frac{Y_{gt}-L_g}{U_g-Y_{gt}}\right) &= \left\{\sum_{k=1}^{K} C(gk)^* X_{gk}\right\} + \Re_g{}^* t + \mu_{gt} \\ \mu_{gt} &= \rho_g \mu_{g(t-1)} + \varepsilon_{gt} \quad \text{for} \quad g = 1, 2, \ldots, G \end{aligned} \tag{2.73}$$

where $X_{g1}, X_{g2}, \ldots, X_{gK}$ are multivariate independent or cause variables with $X_{g1} = 1$ for all g, L_g and U_g are lower and upper bounds of all possible values of the random variable Y_g respectively and $\Re_g$ is the adjusted growth rate of the respond variable Y_g. The values of L_g and U_g should be subjectively selected by researchers.

Further extension of the AR multivariate growth models in (2.71) and (2.73) could easily be developed, such as the translog growth models, the polynomial growth models and the growth models using interaction factors between the X-variables, as well as between the t-variable and the X-variables.

2.12 Multivariate AR(*p*) GLM with trend

Note that the model in (2.71) can also be considered as a multivariate AR(1) model with trend, namely MAR(1)_T. As an extension of this model, a more general multivariate autoregressive model with trend, namely MAR(p)_T, could be considered, as follows:

$$\begin{aligned} Y_{gt} &= \left\{\sum_{k=1}^{K} C(gk)^* X_{gk}\right\}\} + \Re_g{}^* t + \mu_{gt} \\ \mu_{gt} &= \rho_{g1} \mu_{g(t-1)} + \cdots + \rho_{gp} \mu_{g(t-p)} + \varepsilon_{gt} \end{aligned} \tag{2.74}$$

where Y_{gt} can be the original or any transformed endogenous variables, such as in the bounded growth model in (2.36), and the set of exogenous variables $\{X_{gk}\}$ for all g and k could be the pure exogenous variables, other endogenous variables, their lagged variables and their interactions as well as their power.

Note that this model could be extended by using the transformation or function of the time t, such as $\log(t), f(t) = (t - \delta)(t - \theta)^2$ for selected fixed values of δ and θ as presented in the model in (2.36), and other functions of t, which does not have a parameter. Furthermore, refer to the general models in (2.44) and (2.45).

On the other hand, also note that the time t-variable can be considered as representing technology improvement, as well as other variables of the system in (2.74) that have a high or significant positive correlation with the time t.

Example 2.27. (A bivariate model with trend) This example presents an illustrative general method on how to write or input the equation specification in order to obtain statistical results based on a model either in (2.66), (2.71) or (2.74). For a bivariate AR(1) model with trend in (2.74), the following equation specification should be used or entered:

$$\begin{aligned} y1 &= c(11) + c(12)^{*}t + c(13)^{*}x11 + \ldots + c(1k)^{*}x1k + [ar(1) = c(1)] \\ y2 &= c(21) + c(22)^{*}t + c(23)^{*}x21 + \ldots + c(2k)^{*}x2k + [ar(1) = c(2)] \end{aligned} \tag{2.75}$$

Hence, based on the estimated regression function, the following partial derivatives are found:

$$\frac{\partial \hat{y}1}{\partial t} = \hat{c}(12) \quad \text{and} \quad \frac{\partial \hat{y}2}{\partial t} = \hat{c}(21) \tag{2.76}$$

Note that, if log(y1) and log(y2) are used instead of y1 and y2 as dependent variables, then a bivariate growth model is obtained. Then the partial derivatives in (2.76) can be considered as the adjusted exponential growth rates of y1 and y2 respectively. □

Example 2.28. (Lagged-variable AR(1) bivariate model with trend) Based on the data set in Demo.wf1, for an empirical example the lagged-variable autoregressive bivariate model with trend is considered, which has the path diagram presented in Figure 2.66.

Based on this theoretical proposed path diagram, the equations of a bivariate model with trend are as follows:

$$\begin{aligned} m1_t &= c(11) + c(12)^{*}t + c(13)^{*}m1_{t-1} + c(14)^{*}gdp_t + C(15)^{*}gdp_{t-1} + \mu 1_t \\ gdp_t &= c(21) + c(22)^{*}t + c(23)^{*}m1_{t-1} + c(14)^{*}gdp_{t-1} + \mu 2_t \end{aligned} \tag{2.77}$$

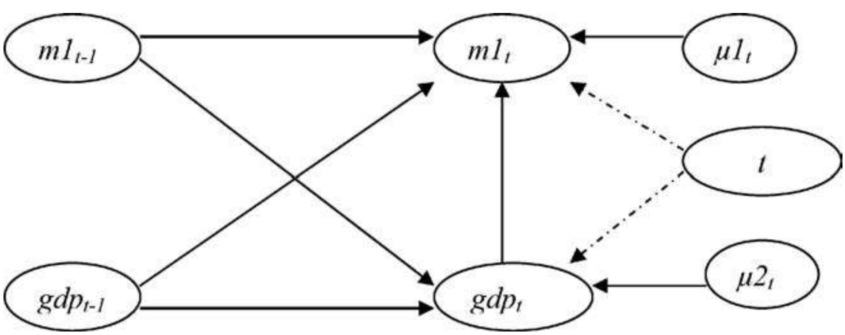


Figure 2.66 Path diagram of the endogenous and exogenous variables

Note that this path diagram is presented under the following assumptions:

(1) The exogenous variables $m1_{t-1}$, gdp_t and gdp_{t-1} have direct effects on the endogenous variable $m1_t$. However, note that the statement 'direct effect' may not indicate a pure causal effect, but a relationship between an independent or source variable and a dependent or downstream variable.
(2) Considering gdp_t as an endogenous variable in the second equation, the exogenous variables $m1_{t-1}$ and gdp_{t-1} have direct effects on gdp_t.
(3) Even though the time t-variable cannot be considered as a cause factor, the arrows with broken lines from t to both endogenous variables, $m1_t$ and gdp_t, are used to represent the trend effects.
(4) Since gdp_t is assumed to be a source factor of $m1_t$, note that the relationships between $m1_{t-1} and\ gdp_{t-1}$ should exist. However, their relationship is not taken into account in this bivariate model.

By doing some experimentation, a lagged-variable AR(1) bivariate model was found as the first full model that should be presented, with its statistical results presented in Figure 2.67. Since several exogenous variables have insignificant adjusted effects, further analysis should be done to develop a reduced model.

By deleting either gdp_t or gdp_{t-1} from the first equation and deleting $m1_{t-1}$ from the second equation, there would be two alternative acceptable reduced models, in a statistical sense. However, for illustration purposes, gdp_t should be kept as a source factor of $m1_t$ in the first equation, because the second equation will represent gdp_{t-1} as the cause factor of gdp_t. The statistical results are presented in Figure 2.68.

Note that based on the results presented in Figures 2.67 and 2.68, it is easy to write the corresponding regression functions, as well as their models.

In relation to the bivariate model presented in Figure 2.68, the path diagram presented in Figure 2.69 is obtained. Furthermore, based on the p-values of the t-statistics, the following notes and conclusions are given:

(1) At the level of significance $\alpha = 0.05$ and gdp_t has a significant positive effect on $m1_t$, based on the p-value $= 0.0815/2 = 0.040\,75$.

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 08:56
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2.523330	3.249818	0.776453	0.4380
C(12)	0.947173	0.028587	33.13244	0.0000
C(13)	0.041016	0.120436	0.340559	0.7336
C(14)	-0.007707	0.121621	-0.063373	0.9495
C(15)	0.059980	0.053182	1.127819	0.2602
C(16)	0.228870	0.083867	2.728957	0.0067
C(21)	-3.181415	2.071138	-1.536071	0.1254
C(22)	0.009462	0.017594	0.537793	0.5911
C(23)	0.995507	0.012070	82.47811	0.0000
C(24)	0.134117	0.033499	4.003562	0.0001
C(25)	0.262836	0.074765	3.515500	0.0005
Determinant residual covariance	2158.768			

Equation: M1=C(11)+C(12)*M1(-1)+C(13)*GDP+C(14)*GDP(-1)+C(15)*T +[AR(1)=C(16)]
Observations: 178

R-squared	0.999380	Mean dependent var	448.5793
Adjusted R-squared	0.999362	S.D. dependent var	345.1043
S.E. of regression	8.717290	Sum squared resid	13070.48
Durbin-Watson stat	2.071468		

Equation: GDP=C(21)+C(22)*M1(-1)+C(23)*GDP(-1)+C(24)*T +[AR(1)=C(25)]
Observations: 178

R-squared	0.999907	Mean dependent var	538.5360
Adjusted R-squared	0.999905	S.D. dependent var	564.4308
S.E. of regression	5.499915	Sum squared resid	5233.089
Durbin-Watson stat	2.039709		

Figure 2.67 Statistical results based on an AR(1) model in (2.77)

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 08:59
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2.522086	3.239166	0.778622	0.4367
C(12)	0.947011	0.028277	33.49102	0.0000
C(13)	0.033440	0.019142	1.746989	0.0815
C(15)	0.060735	0.051787	1.172785	0.2417
C(16)	0.228727	0.083431	2.741503	0.0064
C(21)	-2.384972	1.431886	-1.665616	0.0967
C(23)	1.001806	0.002868	349.3561	0.0000
C(24)	0.127935	0.031152	4.106769	0.0001
C(25)	0.257597	0.073467	3.506310	0.0005
Determinant residual covariance		2162.386		

Equation: M1=C(11)+C(12)*M1(-1)+C(13)*GDP+C(14)*T+[AR(1)=C(15)]
Observations: 178

R-squared	0.999380	Mean dependent var	448.5793
Adjusted R-squared	0.999366	S.D. dependent var	345.1043
S.E. of regression	8.692160	Sum squared resid	13070.78
Durbin-Watson stat	2.071513		

Equation: GDP=C(21)+C(22)*GDP(-1)+C(23)*T+[AR(1)=C(24)]
Observations: 178

R-squared	0.999907	Mean dependent var	638.5360
Adjusted R-squared	0.999905	S.D. dependent var	564.4308
S.E. of regression	5.488738	Sum squared resid	5241.967
Durbin-Watson stat	2.035759		

Figure 2.68 Statistical results based on a reduced model of the model in Figure 2.67

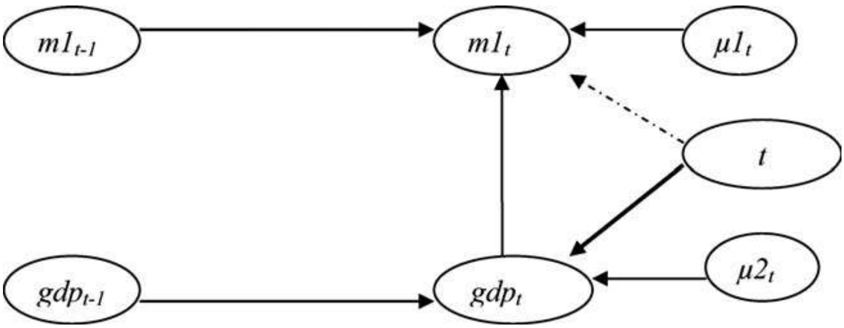


Figure 2.69 Path diagram of the bivariate regression function in Figure 2.68

(2) The time t has an insignificant effect on $m1_t$, but it has a significant effect on gdp_t. Corresponding to this condition, the effect of the time t on the bivariate ($m1_t$, gdp_t) can be tested using the Wald test. To continue further, the null multivariate hypothesis H_0:$c(14) = c(23) = 0$ is rejected, based on the chi-squared-statistic of 18.240 96 with $df = 2$ and the p-value = 0.0001. Hence it can be concluded that the time t has a significant effect on the endogenous bivariate ($m1_t$, gdp_t).

(3) Through gdp_t, gdp_{t-1} has a significant positive indirect effect on $m1_t$.

(4) The relationship between gdp_{t-1} and $m1_{t-1}$ is not presented in the diagram, but it could be said that their relationship has been represented by the causal relationship between the two endogenous variables gdp_t and $m1_t$. □

Example 2.29. (An advanced bivariate model with trend) This example and some of the following examples will demonstrate alternative bivariate or trivariate linear models based on the three variables, $m1_t$, gdp_t and pr_t, in Demo.wf1. Since their lagged variables and the time t could also be used in the model, and by considering their possible causal relationships, many multivariate models with trends could be obtained. Many of those models could be acceptable models, in a statistical sense.

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 09:14
Sample: 1952Q4 1996Q4
Included observations: 179
Total system (balanced) observations 354
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-13.00367	8.717392	-1.491693	0.1367
C(12)	0.069947	0.077271	0.905214	0.3660
C(13)	0.961346	0.047745	20.13496	0.0000
C(14)	-0.023278	0.043682	-0.532884	0.5945
C(15)	-882.6488	360.1089	-2.451061	0.0147
C(16)	978.2713	364.7853	2.681773	0.0077
C(17)	0.094401	0.091813	1.028179	0.3046
C(18)	0.288105	0.088908	3.240467	0.0013
C(21)	-13.13309	4.519341	-2.905975	0.0039
C(22)	0.019191	0.046544	0.412308	0.6804
C(23)	0.053898	0.022048	2.444594	0.0150
C(24)	0.945932	0.021399	44.20519	0.0000
C(25)	614.8745	232.7309	2.641997	0.0086
C(26)	-559.2854	237.6513	-2.353387	0.0192
C(27)	0.214695	0.080395	2.670506	0.0079
C(28)	0.108694	0.082268	1.321215	0.1873
Determinant residual covariance		1668.445		

Equation: M1=C(11)+C(12)*T+C(13)*M1(-1)+C(14)*GDP+C(15)*PR+C(16)
*PR(-1)+[AR(1)=C(17),AR(2)=C(18)]
Observations: 177

R-squared	0.999471	Mean dependent var	450.3826
Adjusted R-squared	0.999449	S.D. dependent var	345.2413
S.E. of regression	8.101283	Sum squared resid	11091.60
Durbin-Watson stat	2.066976		

Equation: GDP=C(21)+C(22)*T+C(23)*M1+C(24)*GDP(-1)+C(25)*PR
+C(26)*PR(-1)+[AR(1)=C(27),AR(2)=C(28)]
Observations: 177

R-squared	0.999916	Mean dependent var	641.6372
Adjusted R-squared	0.999912	S.D. dependent var	564.5092
S.E. of regression	5.286528	Sum squared resid	4723.107
Durbin-Watson stat	1.966404		

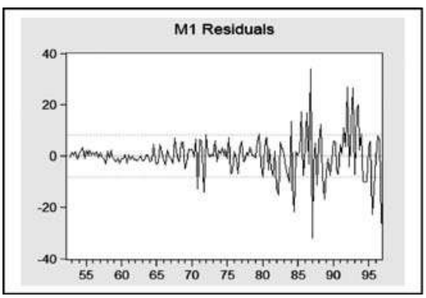
Figure 2.70 Statistical results based on model with trend in (2.78)

For illustrative purposes, supposed that the following bivariate AR(2) model with trend exists:

$$\begin{aligned}
m1_t &= c(11) + c(12)^*t + c(13)^*m1_{t-1} + c(14)^*gdp_t \\
&\quad + c(15)^*pr_t + c(16)^*pr_{t-1} + \mu 1_t \\
\mu 1_t &= \rho_{11}\mu 1_{t-1} + \rho_{12}\mu 1_{t-2} + \varepsilon_{1t} \\
gdp_t &= c(21) + c(22)^*t + c(23)^*m1_t + c(24)^*gdp_{t-1} \\
&\quad + c(25)^*pr_t + c(26)^*pr_{t-1} + \mu 2_t \\
\mu 2_t &= \rho_{21}\mu 2_{t-1} + \rho_{12}\mu 2_{t-2} + \varepsilon_{2t}
\end{aligned} \tag{2.78}$$

In most cases, it has been recognized that an analyst would directly apply her/his proposed or defined model without considering or discussing the limitations or assumptions of the model, including the basic assumptions. For the first stage of this discussion, the model in (2.78) is applied directly. This would give the statistical results in Figure 2.70, with their residual graphs in Figure 2.71.

Based on this model, several independent variables have insignificant adjusted effects. In general, an attempt would be made to obtain a reduced acceptable model by deleting the independent variables having large p-values.



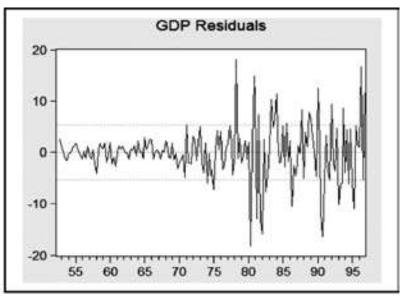


Figure 2.71 Residual graphs of the regressions in Figure 2.70

Estimation Method: Weighted Least Squares
Date: 10/12/07 Time: 09:24
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 6 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-18.49509	5.973298	-3.096295	0.0021
C(12)	0.061508	0.052401	1.173796	0.2413
C(13)	1.000182	0.028530	35.05709	0.0000
C(14)	-0.056294	0.028404	-1.981935	0.0483
C(15)	-873.1596	297.1815	-2.938136	0.0035
C(16)	988.7262	303.2981	3.259915	0.0012
C(17)	0.086024	0.081156	1.059987	0.2899
C(21)	-12.34243	3.880729	-3.180442	0.0016
C(22)	0.025554	0.039373	0.649023	0.5168
C(23)	0.047477	0.018736	2.533962	0.0117
C(24)	0.950897	0.018282	52.01364	0.0000
C(25)	549.2063	212.4677	2.584893	0.0102
C(26)	-496.1270	217.5346	-2.280682	0.0232
C(27)	0.232929	0.076645	3.039048	0.0026
Determinant residual covariance		1793.533		

Equation: M1=C(11)+C(12)*T+C(13)*M1(-1)+C(14)*GDP+C(15)*PR+C(16)*PR(-1)+[AR(1)=C(17)]
Observations: 178

R-squared	0.999434	Mean dependent var	448.5793
Adjusted R-squared	0.999414	S.D. dependent var	345.1043
S.E. of regression	8.350975	Sum squared resid	11925.33
Durbin-Watson stat	1.994709		

Equation: GDP=C(21)+C(22)*T+C(23)*M1+C(24)*GDP(-1)+C(25)*PR+C(26)*PR(-1)+[AR(1)=C(27)]
Observations: 178

R-squared	0.999915	Mean dependent var	638.5360
Adjusted R-squared	0.999912	S.D. dependent var	564.4308
S.E. of regression	5.283017	Sum squared resid	4772.655
Durbin-Watson stat	2.009066		

Figure 2.72 Statistical results based on the unexpected reduced model of (2.78) by using the WLS estimation method

Here, however, experimentation has been done to obtain other types of reduced models. A reduced model was found by deleting the indicator $ar(2)$ from both regressions, even though $ar(2) = \rho_{12}$ has a significant effect on the first regression. Based on the statistical results in Figure 2.72, this AR(1) bivariate model could be considered as an acceptable model, in a statistical sense, but it certainly would not be the best model. This reduced model would be considered as an unexpected reduced model.

Note that the important findings based on this model are the statistical results showing that gdp_t has a significant adjusted effect on m_t, based on the first regression, and m_t also has a significant adjusted effect on gdp_t, based on the second regression. Hence, based on this bivariate model, it may be concluded that m_t and gdp_t have simultaneous causal relationships. However, in practice, a simultaneous causality between a pair of variables should have been defined based on a theoretical basis, before doing the testing hypothesis.

On the other hand, even though the time t has insignificant effects on m_t and gdp_t, it is still kept in the model because there is a need to present the model with trend. If the time t is deleted, this would give a reduced model, which will be discussed and presented in Chapter 4.

Furthermore, note that the statistical results in Figure 2.72 are obtained by using the WLS (weighted least squares) estimation method, because the residual graphs indicate that the error terms of the bivariate model in (2.78) are heterogeneous.

To test an hypothesis by using the t-statistic presented in the printout, other hypotheses could be tested for each or both of the endogenous variables m_t and gdp_t by using the Wald tests. The following hypotheses are given as examples:

(i) *Univariate Hypotheses*

(1) The effects of all exogenous variables, $t, m1_{t-1}, gdp_t, pr_t$ and pr_{t-1}, as well as the indicator AR(1) on the endogenous variable $m1_t$, can be tested by entering the equation $c(12) = c(13) = c(14) = c(15) = c(16) = c(17) = 0$. The null

hypothesis is rejected based on the chi-squared-statistic of 263 316.3 with $df = 6$ and the p-value = 0.0000.

(2) The joint effects of (pr_t, pr_{t-1}) on the univariate m_t can be tested by entering the equation $c(15) = c(16) = 0$. The null hypothesis is rejected based on the chi-square-statistic of 20.431 86 with $df = 2$ and the p-value = 0.0000.

(ii) *Multivariate Hypotheses*

(1) The adjusted effect of time t on the bivariate $(m1_t, gdp_t)$ can be tested by entering the equation $c(12) = c(22) = 0$. The null hypothesis is accepted based on the chi-squared-statistic of 1.799 027 with $df = 2$ and the p-value = 0.4086.

(2) The adjusted joint effects of (pr_t, pr_{t-1}) on the bivariate $(m1_t, gdp_t)$ can be tested by entering the equation $c(15) = c(16) = c(25) = c(26) = 0$. The null hypothesis is rejected based on the chi-squared-statistic of 37.292 03 with $df = 4$ and the p-value = 0.0000. □

Example 2.30. (Residual analysis) This example presents an illustrative residual analysis based on the bivariate model presented in Figure 2.72. The main objective of the residual analysis is to find out the limitation of the model. For further information on the residual analysis refer to the notes and comments presented in Section 2.14.

In order to do the residual analysis in detail, first the observed error terms of both regressions should be generated, using the following steps.

(1) With the model or the statistical results on the screen, click *Proc/Make Residuals*, which brings up the options in Figure 2.73 on the screen.

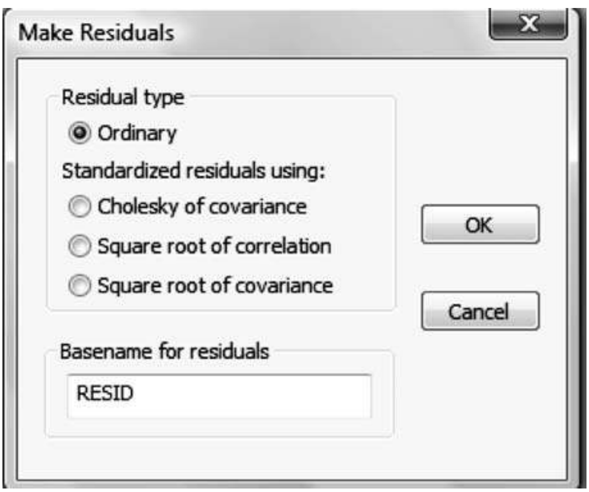
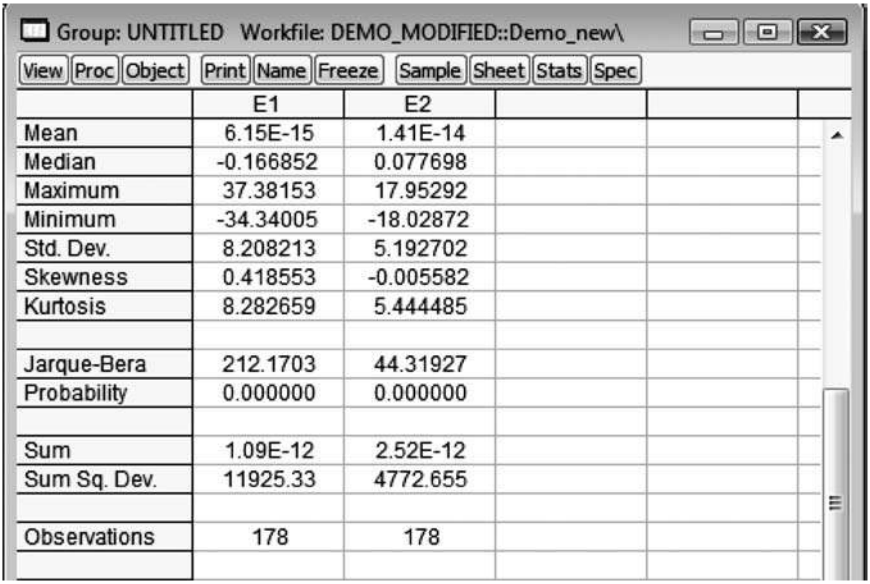


Figure 2.73 The options for making residuals



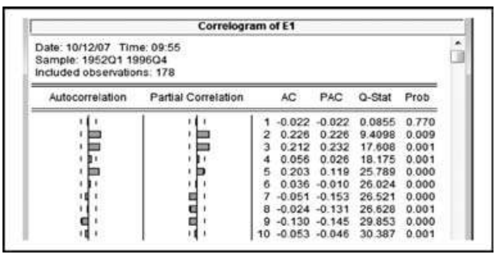
	E1	E2
Mean	6.15E-15	1.41E-14
Median	-0.166852	0.077698
Maximum	37.38153	17.95292
Minimum	-34.34005	-18.02872
Std. Dev.	8.208213	5.192702
Skewness	0.418553	-0.005582
Kurtosis	8.282659	5.444485
Jarque-Bera	212.1703	44.31927
Probability	0.000000	0.000000
Sum	1.09E-12	2.52E-12
Sum Sq. Dev.	11925.33	4772.655
Observations	178	178

Figure 2.74 Descriptive statistics for both residuals, E1 and E2

(2) Then by clicking OK, both residual variables will be obtained on the screen, namely *Resid*01 and *Resid*02. On the other hand, other symbols may be used as the base-name for residuals, for example '*E*', giving two series of residuals, namely $E1 = Resid01$ and $E2 = Resid02$. For illustrative purposes, based on these two residuals, *E*1 and *E*2, the following residual analysis could be performed.

(3) After presenting the variables *E*1 and *E*2 on the screen, click *View/Descriptive stat/Common Samples*, giving the descriptive statistics in Figure 2.74. These results show that the average (mean) values of both residuals are very close to zero. However, the normality assumption of each residual is rejected based on the Jarque–Bera test. Refer to the notes and comments in Section 2.14.

(4) In order to analyze each residue, first only one residue is shown on the screen, for example *E*1:
   - By selecting *View/Corre*log*ram/Level* . . . , the correlogram of the residual *E*1 in Figure 2.75 is obtained. This figure shows that the null hypothesis of no first-



	AC	PAC	Q-Stat	Prob
1	-0.022	-0.022	0.0855	0.770
2	0.226	0.226	9.4098	0.009
3	0.212	0.232	17.608	0.001
4	0.056	0.026	18.175	0.001
5	0.203	0.119	25.789	0.000
6	0.036	-0.010	26.024	0.000
7	-0.051	-0.153	26.521	0.000
8	-0.024	-0.131	26.628	0.001
9	-0.130	-0.145	29.853	0.000
10	-0.053	-0.046	30.387	0.001

Figure 2.75 Correlogram of E1

Null Hypothesis: E1 has a unit root
Exogenous: Constant
Lag Length: 1 (Fixed)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.250850	0.0000
Test critical values:	1% level	-3.467633	
	5% level	-2.877823	
	10% level	-2.575530	

*MacKinnon (1996) one-sided p-values.

Figure 2.76 A unit root test for E1

order autocorrelation is accepted based on the Q-statistic with a p-value $= 0.770$.

- By selecting *View/Unit Root Test/Augmented Dickey-Fuller Test/Level ...*, the unit root test in Figure 2.76 is obtained, which shows that the null hypothesis $E1$ has a unit root, but is rejected based on the t-statistic with a p-value $= 0.0000$.

(5) The first autocorrelation of the residual $E1$ could also be tested by using a simple linear regression of $E1(-1) = E1_{t-1}$ on $E1_t$. Then $t_0 = -0.294\,250$ with a p-value $= 0.7689$.

(6) Besides using the Jarque–Bera test for the normality of each residual, the empirical distribution of $E1$ can also be tested by selecting *View/Descriptive Statistics and Tests*, giving the options in Figure 2.77. Then by clicking the option *Empirical Distribution Tests ...*, the options in Figure 2.78 are obtained.

(7) By entering $\mu = 0$ without a value of σ, the statistical results in Figure 2.79 are obtained. Note that this table presents three statistics, namely W2, U2 and A2, with p-values < 0.0025 and a statistic SGMA with a p-value $= 0.0000$. Therefore, the data do not support the empirical normal distribution of $E1$. □

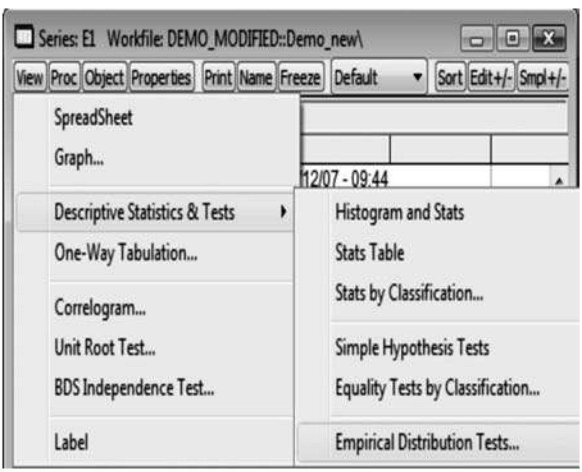


Figure 2.77 Options of descriptive statistics and tests

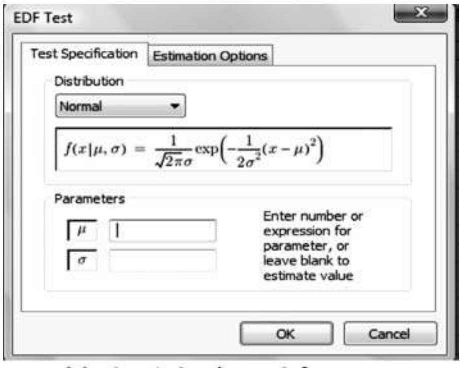


Figure 2.78 Options of the EDF test

Empirical Distribution Test for E1
Hypothesis: Normal
Date: 10/12/07 Time: 10:21
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments

Method	Value	Adj. Value	Probability
Cramer-von Mises (W2)	1.199394	NA	< 0.0025
Watson (U2)	1.195660	NA	< 0.0025
Anderson-Darling (A2)	6.461508	NA	< 0.0025

Method: Maximum Likelihood (Exact Solution)

Parameter	Value	Std. Error	z-Statistic	Prob.
MU	0.000000	*	NA	NA
SIGMA	8.185124	0.433811	18.86796	0.0000

Log likelihood	-626.7837	Mean dependent var.	6.15E-15
No. of Coefficients	1	S.D. dependent var.	8.208213

* Fixed parameter value

Figure 2.79 The empirical normal distribution tests for E1.

2.12.1 Kernel density and theoretical distribution

By selecting *View/Graph* ... the list of graph options in Figure 2.80 is obtained. Then select *Distribution/Kernel Density* and click '*Options*' in order to find the options for the kernel density, as presented in the window on the right box below. Figure 2.81 presents two selected graphs of the residual $E1$, namely the kernel density and its theoretical distribution. Note that there are many alternative graphs that can be

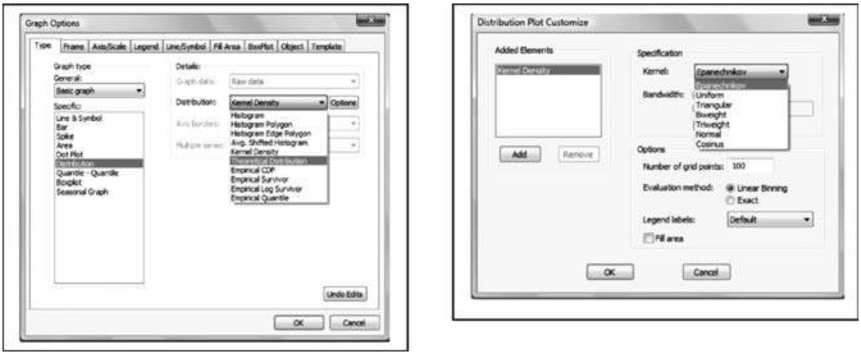


Figure 2.80 The kernel density options

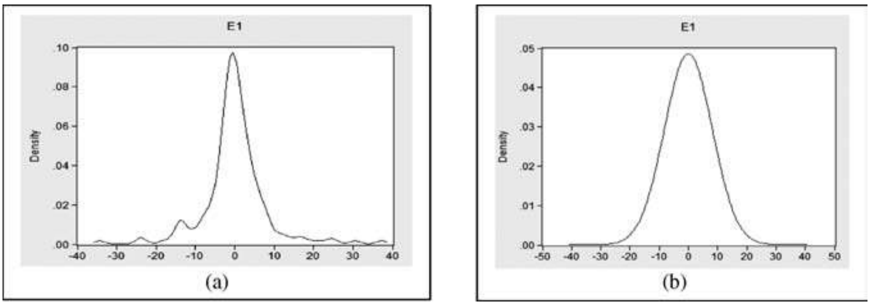


Figure 2.81 Kernel density with (a) normal bandwidth and (b) theoretical distribution of the residual E1

presented, but it is very difficult to select the graph that could be considered as the best one. Refer to the special notes and comments presented in Section 2.14.

Example 2.31. (Path diagram of the model in (2.78)) To study the limitation or the characteristics of the model in (2.78), as well as other multivariate general linear models, it is necessary to look at its path diagram. Corresponding to the model in (2.78), the path diagram or causal relationships between the endogenous and exogenous variables should be developed, as presented in Figure 2.82.

This diagram is constructed under the following conditions and assumptions:

(1) The endogenous variables $m1_t$ and gdg_t have simultaneous causal effects, because these variables have double status, endogenous and exogenous variables, as presented in the equation of the bivariate model in (2.78).
(2) The first regression in (2.78) is an additive model of the independent variables t, $m1_{t-1}$, gdp_t, pr_t and pr_{t-1}. Hence, these independent or exogenous variables could be considered as cause, source or explanatory variables of the endogenous variable $m1_t$. These relationships could be represented by an arrow from each

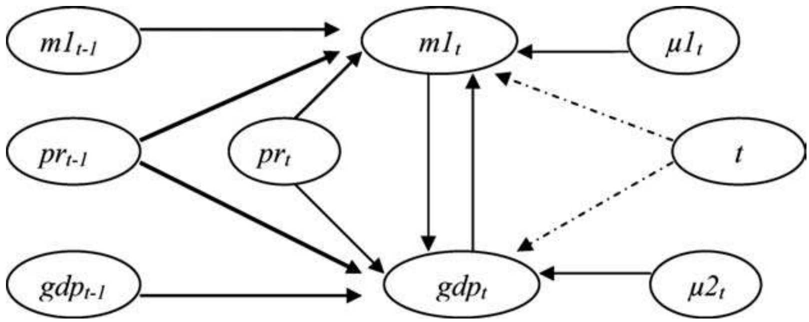


Figure 2.82 Path diagram of the endogenous and exogenous variables of the model in (2.78)

source variable to the endogenous or downstream variables. The same applies for the second regression.

(3) Variables t, pr_t and pr_{t-1} are considered as pure exogenous variables. Corresponding to these variables, several questions could be raised about their relationships. These should be considered as the limitations of the proposed model in (2.78). One of the questions is related to the status of the lagged variable pr_{t-1} and whether it could have a direct effect on m_t and gdp_t or an indirect effect through pr_t. It is probable that it would have an indirect effect. Find the result in the following example.

(4) The model in (2.78), as well as in Figure 2.82, does not take into account the possible causal effects between the pure exogenous or independent variables t, pr_t, pr_{t-1},$m1_{t-1}$ and gdp_{t-1}. However, the bivariate correlations, as well as their multicollinearity, should be taken into account in the estimation process.

(5) Note that the relationships between the lagged variables pr_{t-1},$m1_{t-1}$ and gdp_{t-1} should have been presented by the causal relationships between pr_t, $m1_t$ and gdp_t. □

Example 2.32. (A modified path diagram and trivariate model) Corresponding to the path diagram in Figure 2.82, a modified path diagram may be considered, as presented in Figure 2.83.

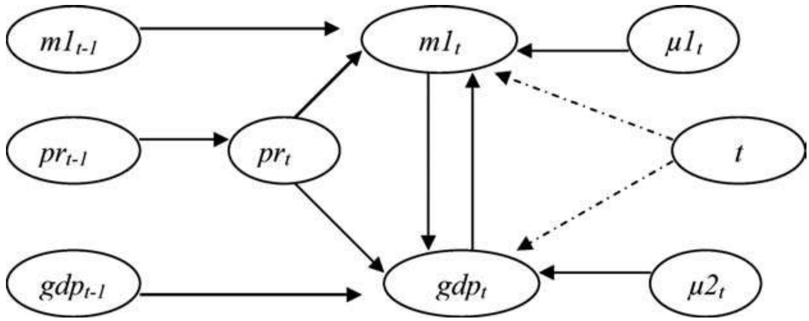


Figure 2.83 A modified path diagram in Figure 2.82

Based on this diagram, and the unexpected reduced model presented in Example 2.30, the following trivariate model would be obtained as an acceptable model, in a statistical sense:

$$\begin{aligned}
m1_t &= c(11) + c(12)^*t + c(13)^*m1_{t-1} + c(14)^*gdp_t \\
&\quad + c(15)^*pr_t + c(16)^*pr_{t-1} + \mu 1_t \\
\mu 1_t &= \rho_{11}\mu 1_{t-1} + \varepsilon_{1t} \\
gdp_t &= c(21) + c(22)^*t + c(23)^*m1 + c(24)^*gdp_{t-1} \\
&\quad + c(25)^*pr_t + c(26)^*pr_{t-1} + \mu 2_t \\
\mu 2_t &= \rho_{21}\mu 2_{t-1} + \varepsilon_{2t} \\
pr_t &= c(31) + c(32)^*pr_{t-1} + \mu 3_t \\
\mu 3_t &= \rho_{31}\mu 3_{t-1} + \rho_{32}\mu 3_{t-2} + \varepsilon_{3t}
\end{aligned} \tag{2.79}$$

By using the WLS estimation method, the statistical results of the model in (2.79), presented in Figure 2.84, show that pr_{t-1} has a significant positive effect on pr_t and the partial autocorrelations ρ_{31} and ρ_{32} are significantly positive. For illustrative purposes, the statistical results found by using the SUR estimation method are presented in Figure 2.84. Note that the indicator $ar(1)$ corresponding to $C(17)$, in the first regression, is insignificant with a p-value = 0.3017. Therefore, this may be a reduced model. Do this as an exercise. □

Estimation Method: Seemingly Unrelated Regression
Date: 10/12/07 Time: 11:30
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 7 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-17.91121	5.971702	-2.999348	0.0029
C(12)	0.062361	0.052298	1.192416	0.2339
C(13)	0.997668	0.028539	34.95755	0.0000
C(14)	-0.053419	0.028410	-1.880308	0.0609
C(15)	-874.4071	296.7512	-2.946600	0.0034
C(16)	987.2846	302.8434	3.260050	0.0012
C(17)	0.084093	0.081302	1.034327	0.3017
C(21)	-12.89197	3.896310	-3.308764	0.0010
C(22)	0.024256	0.039444	0.614936	0.5390
C(23)	0.051404	0.018843	2.727968	0.0067
C(24)	0.947553	0.018376	51.56519	0.0000
C(25)	559.6834	212.6880	2.631476	0.0089
C(26)	-504.7431	217.7482	-2.318012	0.0210
C(27)	0.234315	0.076917	3.046317	0.0025
Determinant residual covariance		1792.842		

Equation: M1=C(11)+C(12)*T+C(13)*M1(-1)+C(14)*GDP+C(15)*PR+C(16)*PR(-1)+[AR(1)=C(17)]
Observations: 178

R-squared	0.999434	Mean dependent var	448.5793
Adjusted R-squared	0.999414	S.D. dependent var	345.1043
S.E. of regression	8.351287	Sum squared resid	11926.22
Prob(F-statistic)	1.985003		

Equation: GDP=C(21)+C(22)*T+C(23)*M1+C(24)*GDP(-1)+C(25)*PR+C(26)*PR(-1)+[AR(1)=C(27)]
Observations: 178

R-squared	0.999915	Mean dependent var	638.5360
Adjusted R-squared	0.999912	S.D. dependent var	564.4308
S.E. of regression	5.283684	Sum squared resid	4773.862
Prob(F-statistic)	2.004441		

Figure 2.84 Statistical results based on the model in (2.79)

Example 2.33. (Further modified path diagram and trivariate model) Based on the path diagram in Figure 2.83, a further modified path diagram could be obtained, as presented in Figure 2.85.

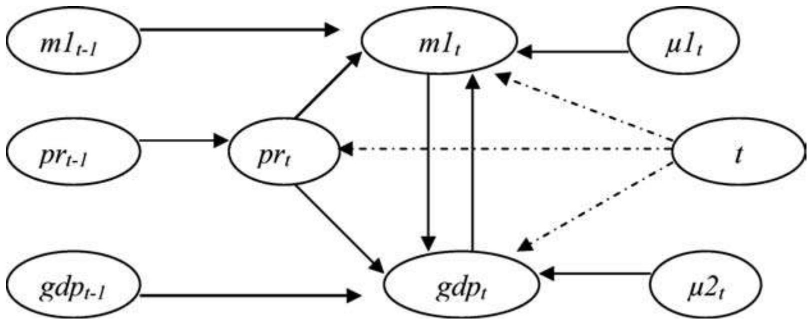


Figure 2.85 A modified path diagram in Figure 2.83

Then corresponding to this path diagram, there would be three autoregressive univariate linear regressions with trend, as follows:

$$\begin{aligned}
m1_t &= c(11) + c(12)*t + c(13)*m1_{t-1} + c(14)*gdp_t \\
&\quad + c(15)*pr_t + \mu1_t \\
\mu1_t &= \rho_{11}\mu1_{t-1} + \varepsilon_{1t} \\
gdp_t &= c(21) + c(22)*t + c(23)*m1 + c(24)*gdp_{t-1} \\
&\quad + c(25)*pr_t + \mu2_t \\
\mu2_t &= \rho_{21}\mu2_{t-1} + \varepsilon_{2t} \\
pr_t &= c(31) + c(32)*t + c(33)*pr_{t-1} + \mu3_t \\
\mu3_t &= \rho_{31}\mu3_{t-1} + \rho_{32}\mu3_{t-2} + \varepsilon_{3t}
\end{aligned} \tag{2.80}$$

By using the WLS estimation method, at a level of significance of $\alpha = 0.05$, the statistical results of the model in (2.80) show that the time t and pr_{t-1} have significant positive effects on pr_t, and the partial autocorrelations ρ_{31} and ρ_{32} are significantly positive. Hence this model can be considered as an acceptable model with trend, in a statistical sense.

As a further study, thought should be given to the possible effects of the lagged variables $m1_{t-1}$ and gdp_{t-1} on pr_t. For this reason another modified trivariate model is presented in the following example. □

Example 2.34. (Another modified path diagram and trivariate model) Based on the path diagram in Figure 2.85, another modified path diagram could be as pre- sented in Figure 2.86.

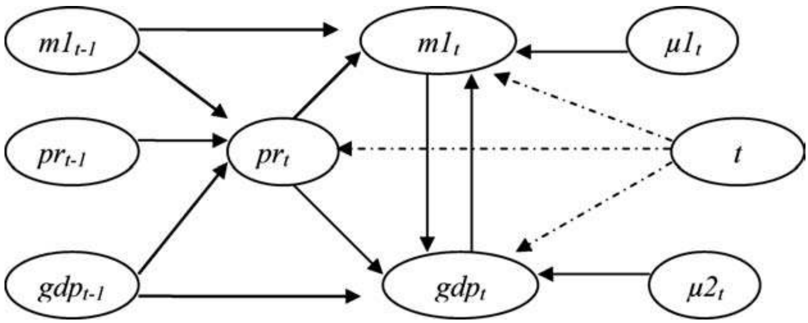


Figure 2.86 A modified path diagram in Figure 2.85

Corresponding to this path diagram, there would be three univariate autoregressive linear regressions with trend, as follows:

$$
\begin{aligned}
m1_t &= c(11) + c(12)^*t + c(13)^*m1_{t-1} + c(14)^*gdp_t \\
&\quad + c(15)^*pr_t + \mu 1_t \\
\mu 1_t &= \rho_{11}\mu 1_{t-1} + \varepsilon_{1t} \\
gdp_t &= c(21) + c(22)^*t + c(23)^*m1_t + c(24)^*gdp_{t-1} \\
&\quad + c(25)^*pr_t + \mu 2_t \\
\mu 2_t &= \rho_{21}\mu 2_{t-1} + \varepsilon_{2t} \\
pr_t &= c(31) + c(32)^*t + c(33)^*pr_{t-1} + c(34)^*m1_{t-1} \\
&\quad + c(35)^*gdp_{t-1} + \mu 3_t \\
\mu 3_t &= \rho_{31}\mu 3_{t-1} + \rho_{32}\mu 3_{t-2} + \varepsilon_{3t}
\end{aligned}
\tag{2.81}
$$

However, the statistical results based on this model show that each of the lagged variables, $m1_{t-1}$ and gdp_{t-1}, has an insignificant adjusted effect on pr_t with large p-values of 0.7864 and 0.8487 respectively. The joint effects of $m1_{t-1}$ and gdp_{t-1} on pr_t also have an insignificant effect, based on the chi-squared-statistic of 0.097 395 with $df = 2$ and the p-value = 0.9525.

Based on these findings, it can be concluded that the model in (2.81) is not an acceptable model, in a statistical sense. Considering the autoregressive trivariate models with trends presented in the last three examples, it could be said that the model in (2.80) is the best model. Note that the model in (2.80) consists of three additive multiple regression models.

Based on the same Figure 2.85, a trivariate model having a two-way interaction factor(s) might be considered, as presented in the following illustrative example. □

Example 2.35. (A trivariate model with interaction factors) Based on the path diagram in Figure 2.85, a trivariate model with interaction factors could be applied, as follows:

$$
\begin{aligned}
m1_t &= c(11) + c(12)^*t + c(13)^*m1_{t-1} + c(14)^*gdp_t \\
&\quad + c(15)^*pr_t + c(16)^*gdp_t{}^*pr_t + \mu 1_t \\
\mu 1_t &= \rho_{11}\mu 1_{t-1} + \varepsilon_{1t} \\
gdp_t &= c(21) + c(22)^*t + c(23)^*m1_t + c(24)^*gdp_{t-1} \\
&\quad + c(25)^*pr_t + c(26)^*m1_t{}^*pr_t + \mu 2_t \\
\mu 2_t &= \rho_{21}\mu 2_{t-1} + \varepsilon_{2t} \\
pr_t &= c(31) + c(32)^*t + c(33)^*pr_{t-1} + \mu 3_t \\
\mu 3_t &= \rho_{31}\mu 3_{t-1} + \rho_{32}\mu 3_{t-2} + \varepsilon_{3t}
\end{aligned}
\tag{2.82}
$$

By using the WLS estimation method, statistical results are obtained that show that the interaction factor $gdp_t{}^*pr_t$ has an insignificant adjusted effect on $m1_t$ based on the t-test with a p-value $= 0.5786$; the interaction factor $m_t{}^*pr_t$ also has an insignificant adjusted effect on gdp_t with a p-value $= 0.8330$.

However, by deleting the main factor pr_t from the first two regressions, an acceptable reduced model with interaction factors is obtained, as presented in Figure 2.87. Based on this figure, the following notes and conclusions could be presented:

(1) At the level of significance of $\alpha = 0.10$, the interaction factor $gdp_t{}^*pr_t$ has a significant negative effect on $m1_t$, based on the t-statistic of $t_0 = -1.458\,799$ with a p-value $= 0.1452/2 = 0.0726 < \alpha = 0.10$. The interaction factor $m1_t{}^*pr_t$ has a significant negative effect on gdp_t, based on the t-statistic of $t_0 = -2.590\,473$ with a p-value $= 0.0099/2 = 0.004\,95$.

(2) By using the Wald test, a conclusion can be made that the time t has an insignificant effect on the trivariate ($m1_t$, gdp_t, pr_t), based on the chi-squared-statistic of 3.330 221 with $df = 3$ and a p-value $= 0.3474$. Since, at the level of significance of

Estimation Method: Weighted Least Squares
Date: 10/12/07 Time: 11:48
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (unbalanced) observations 533
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 5 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.536392	3.622887	-0.148057	0.8824
C(12)	-0.035206	0.082057	-0.429049	0.6681
C(13)	0.976802	0.032544	30.01516	0.0000
C(14)	0.051161	0.021999	2.325552	0.0204
C(16)	-0.025311	0.017350	-1.458799	0.1452
C(17)	0.190530	0.082896	2.298424	0.0219
C(21)	-8.218049	2.704154	-3.039047	0.0025
C(22)	0.022010	0.051970	0.423520	0.6721
C(23)	0.073695	0.029571	2.492116	0.0130
C(24)	0.999310	0.010985	90.97296	0.0000
C(26)	-0.047736	0.018428	-2.590473	0.0099
C(27)	0.213040	0.075289	2.829645	0.0048
C(31)	-0.001143	0.002946	-0.388007	0.6982
C(32)	0.000208	0.000121	1.714281	0.0871
C(33)	0.973887	0.018721	52.02096	0.0000
C(34)	0.641158	0.076872	8.340565	0.0000
C(35)	0.250018	0.075699	3.302776	0.0010

Determinant residual covariance 0.003719

Equation: M1=C(11)+C(12)*T+C(13)*M1(-1)+C(14)*GDP+C(15)*GDP*PR +[AR(1)=C(17)]
Observations: 178

R-squared	0.999387	Mean dependent var	448.5793
Adjusted R-squared	0.999369	S.D. dependent var	345.1043
S.E. of regression	8.669030	Sum squared resid	12926.16
Durbin-Watson stat	2.056202		

Equation: GDP=C(21)+C(22)*T+C(23)*M1+C(24)*GDP(-1)+C(25)*M1*PR +[AR(1)=C(26)]
Observations: 178

R-squared	0.999910	Mean dependent var	638.5360
Adjusted R-squared	0.999908	S.D. dependent var	564.4308
S.E. of regression	5.420219	Sum squared resid	5053.149
Durbin-Watson stat	2.004384		

Equation: PR=C(31)+C(32)*T+C(33)*PR(-1)+[AR(1)=C(34), AR(2)=C(35)]
Observations: 177

R-squared	0.999979	Mean dependent var	0.519453
Adjusted R-squared	0.999978	S.D. dependent var	0.303227
S.E. of regression	0.001408	Sum squared resid	0.000341
Durbin-Watson stat	2.115655		

Figure 2.87 Statistical results based on a reduced model of the autoregressive model in (2.82)

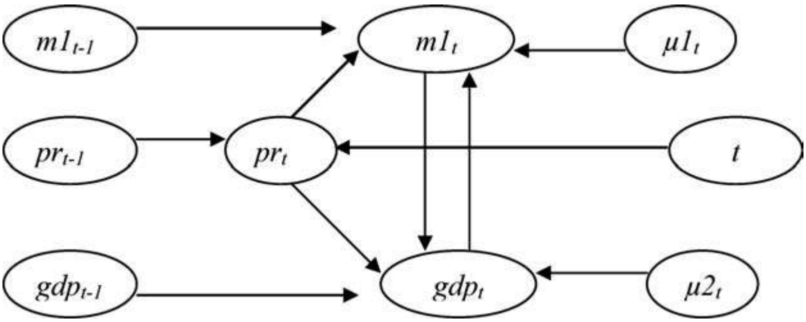


Figure 2.88 A reduced model path diagram of model (2.82)

$\alpha = 0.10$ the time t has a significant adjusted effect on pr_t, the time t can be deleted from the first two regression models to obtain a second reduced model. Do this as an exercise. However, if the time t is deleted from the first two regressions, then a model with the path diagram presented in Figure 2.88 would be obtained.

Note that, in this case, the time t could be considered to have indirect effects on both time series $m1_t$ and gdp_t going through the series pr. Furthermore, note that the effect of the time t in the system could be considered to represent the effect(s) of variables out of the system, which are highly or significantly positively correlated with the time t.□

2.13 Generalized multivariate models with trend

As an extension of the models based on three time series $m1_t$, gdp_t and pr_t presented in the previous section, here a set of six variables is considered: three exogenous variables, X_1, X_2 and X_3, two endogenous variables, Y_1 and Y_2, and the time t-variable. The main objective of using the symbols X and Y for the variables is to present illustrative models that would, in general, be applicable for various fields. Hence the data used for the illustration should be considered as a hypothetical data set. Note that the X and Y variables could be the original observable/measurable variables or their transformations, such as the logarithmic and exponential transformations, the first difference $dY_t = Y_t - Y_{t-1}$ and $d\log(Y_t) = \log(Y_t) - \log(Y_{t-1}) = R_t$, as well as the interactions between selected main factors or variables.

For illustrative purposes, Figure 2.89 presents a hypothetical path diagram of the six selected variables. Note that in this diagram there could be four downstream or dependent variables, which are Y_1, Y_2, X_1 and X_3, because the arrows are directed to these four variables. Hence, there would be a system of four multiple linear regressions, starting from the simplest or autoregressive multivariate additive model.

2.13.1 The simplest multivariate autoregressive model

Corresponding to the path diagram in Figure 2.89, the simplest *multivariate autoregressive* (MAR) linear model is defined as a set of four autoregressive additive regression models. Other authors use the name VAR (i.e. *vector autoregressive*) for

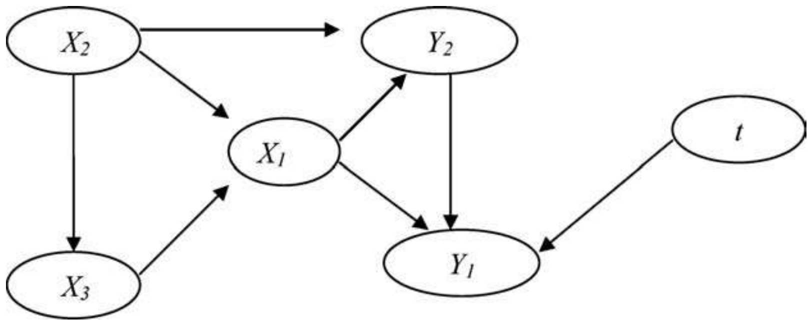


Figure 2.89 A hypothetical path diagram

the model. Since EViews uses the symbol or term 'VAR' for a special function or estimation method to present a special multivariate time series model, then the term 'MAR' will be used to present the general multivariate autoregressive model, and the VAR model is a special case of the MAR model. The VAR model will be presented in Chapter 6.

In order to perform the data analysis based on this MAR additive model, the following equation specification should be used:

$$\begin{aligned} y1 &= c(11) + c(12)^*t + c(13)^*y2 + c(14)^*x1 + [ar(1) = c(15), \ldots] \\ y2 &= c(21) + c(22)^*x1 + c(23)^*x2 + [ar(1) = c(24), \ldots] \\ x1 &= c(31) + c(32)^*x2 + c(33)^*x3 + [ar(1) = c(34), \ldots] \\ x3 &= c(41) + c(42)^*x2 + [ar(1) = c(43), \ldots] \end{aligned} \tag{2.83}$$

This model can also be considered as an *autoregressive structural equation model* (AR_SEM), specifically the simplest AR_SEM, that does not contain an interaction factor as an independent variable.

In relation to the independent variable of the time t in the first regression, it could also be used as an additional independent variable of the other regressions, if it is considered relevant (see the previous examples). Furthermore, note again that the time t could be considered as representing other variables out of the system equations that are highly or significantly linearly correlated with the time t.

Example 2.36. (Experimentation based on the model in (2.83)) Figure 2.90 presents the statistical results of a multivariate AR(1) model in (2.83). The equation of the regression functions can easily be written based on the printout, so will not be presented again. Those equations can easily be obtained by clicking *View/Representations*.

Even though some of the independent variables have insignificant adjusted effects with large p-values, this will not be considered as a problem. It is common for some of the independent variables to have insignificant adjusted effects if a model has several

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 15:22
Sample: 1968M02 1994M10
Included observations: 322
Total system (balanced) observations 1284
Convergence achieved after 10 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2309.623	1367.100	1.689432	0.0914
C(12)	-2.199491	5.013293	-0.438732	0.6609
C(13)	0.017013	0.027511	0.618415	0.5364
C(14)	0.179368	0.038319	4.680900	0.0000
C(15)	0.985669	0.010570	93.25579	0.0000
C(21)	-4668.858	16646.74	-0.280467	0.7792
C(22)	0.316158	0.047099	6.712608	0.0000
C(23)	0.283333	0.012978	21.83139	0.0000
C(24)	0.998276	0.008281	120.5509	0.0000
C(31)	21317.54	10356.24	2.058425	0.0398
C(32)	0.185695	0.012191	15.23255	0.0000
C(33)	0.016401	0.021447	0.764739	0.4446
C(34)	0.997286	0.002159	462.0249	0.0000
C(41)	34075.92	42618.68	0.799554	0.4241
C(42)	0.169752	0.030635	5.541058	0.0000
C(43)	0.998886	0.001864	535.8968	0.0000
Determinant residual covariance		8.45E+16		

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+[AR(1)=C(15)]
Observations: 321

R-squared	0.997540	Mean dependent var	4796.988
Adjusted R-squared	0.997509	S.D. dependent var	1564.584
S.E. of regression	78.09197	Sum squared resid	1927081.
Durbin-Watson stat	2.056187		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+[AR(1)=C(24)]
Observations: 321

R-squared	0.999772	Mean dependent var	14089.85
Adjusted R-squared	0.999770	S.D. dependent var	6599.741
S.E. of regression	100.0659	Sum squared resid	3174180.
Durbin-Watson stat	1.959514		

Equation: X1=C(31)+C(32)*X2+C(33)*X3+[AR(1)=C(34)]
Observations: 321

R-squared	0.999620	Mean dependent var	16076.63
Adjusted R-squared	0.999616	S.D. dependent var	6258.103
S.E. of regression	122.5788	Sum squared resid	4763100.
Durbin-Watson stat	1.581755		

Equation: X3=C(41)+C(42)*X2+[AR(1)=C(43)]
Observations: 321

R-squared	0.999279	Mean dependent var	16848.51
Adjusted R-squared	0.999274	S.D. dependent var	11982.89
S.E. of regression	322.8684	Sum squared resid	33149591
Durbin-Watson stat	1.467829		

Figure 2.90 Statistical results based on the multivariate AR(1) model in (2.83), using an hypothetical data set

or many independent variables. However, alternative reduced models can be developed if required. Do this as an exercise.

Other problems that should be considered are related to the error terms. Figure 2.91 presents the residual graphs of the four regression functions in Figure 2.92. These graphs, especially the X_3 residual graph, show the heterogeneity of the error terms. Therefore, it is suggested that the WLS estimation method should be used or applied instead of the OLS method. On the other hand, tests can be conducted on residuals in order to identify the limitation of the model, which have been presented in the previous examples. □

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 15:50
Sample: 1968M02 1994M10
Included observations: 322
Total system (unbalanced) observations 1282
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2309.623	1367.100	1.689432	0.0914
C(12)	-2.199491	5.013293	-0.438732	0.6609
C(13)	0.017013	0.027511	0.618415	0.5364
C(14)	0.179368	0.038319	4.680900	0.0000
C(15)	0.985669	0.010570	93.25579	0.0000
C(21)	-4668.520	16645.50	-0.280467	0.7792
C(22)	0.316158	0.047099	6.712602	0.0000
C(23)	0.283333	0.012978	21.83139	0.0000
C(24)	0.998276	0.008281	120.5512	0.0000
C(31)	18486.67	8964.765	2.062148	0.0394
C(32)	0.191625	0.011878	16.13271	0.0000
C(33)	0.022166	0.021759	1.018702	0.3085
C(34)	1.211858	0.055155	21.97195	0.0000
C(35)	-0.214459	0.055040	-3.896425	0.0001
C(41)	28154.69	32585.88	0.864015	0.3877
C(42)	0.167816	0.029016	5.783496	0.0000
C(43)	1.265014	0.054224	23.32945	0.0000
C(44)	-0.266123	0.054193	-4.910644	0.0000
Determinant residual covariance		7.66E+16		

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+[AR(1)=C(15)]
Observations: 321

R-squared	0.997540	Mean dependent var	4796.988
Adjusted R-squared	0.997509	S.D. dependent var	1564.584
S.E. of regression	78.09197	Sum squared resid	1927081.
Durbin-Watson stat	2.056187		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+[AR(1)=C(24)]
Observations: 321

R-squared	0.999772	Mean dependent var	14089.85
Adjusted R-squared	0.999770	S.D. dependent var	6599.741
S.E. of regression	100.0659	Sum squared resid	3174180.
Durbin-Watson stat	1.959514		

Equation: X1=C(31)+C(32)*X2+C(33)*X3+[AR(1)=C(34),AR(2)=C(35)]
Observations: 320

R-squared	0.999634	Mean dependent var	16109.83
Adjusted R-squared	0.999629	S.D. dependent var	6239.535
S.E. of regression	120.1658	Sum squared resid	4548540.
Durbin-Watson stat	2.065596		

Equation: X3=C(41)+C(42)*X2+[AR(1)=C(43), AR(2)=C(44)]
Observations: 320

R-squared	0.999328	Mean dependent var	16880.57
Adjusted R-squared	0.999322	S.D. dependent var	11987.86
S.E. of regression	312.1970	Sum squared resid	30799563
Durbin-Watson stat	2.147962		

Figure 2.91 Statistical results based on a modified model (2.83), using an hypothetical data set

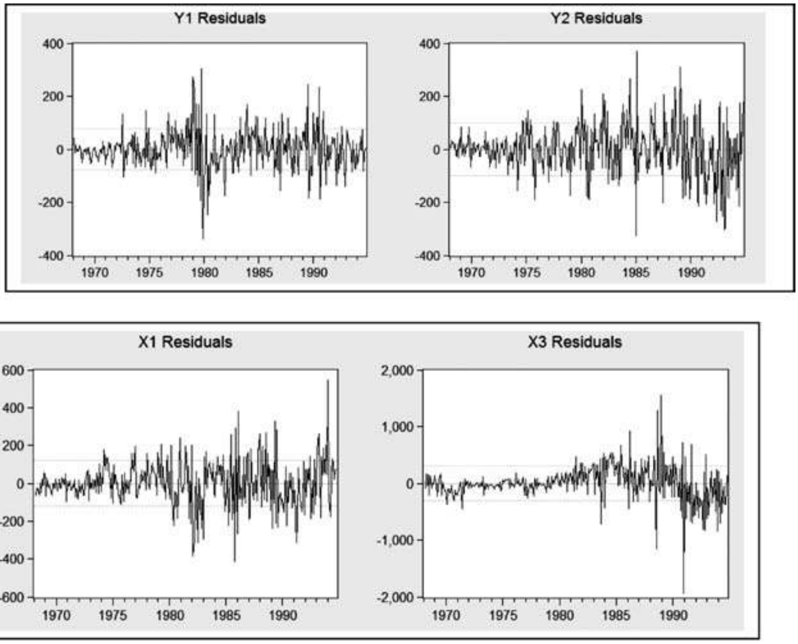


Figure 2.92 Residual graphs of the four regressions in Figure 2.91

Example 2.37. (Other modified models of the model in (2.83)) Another modified model of the model in (2.83) is the *lagged variable multivariate additive model*, namely the LV_M model. Figure 2.93 presents statistical results based on an acceptable LV_M model, where each regression has a good value of the DW-statistic. Furthermore, the LV_M model could be used with autoregressive errors.

System: SYS02
Estimation Method: Least Squares
Date: 10/29/07 Time: 14:12
Sample: 1968M02 1994M10
Included observations: 321
Total system (unbalanced) observations 1281

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	18.80046	23.67422	0.794132	0.4273
C(12)	-0.246670	0.251611	-0.980365	0.3271
C(13)	-0.007692	0.005129	-1.499645	0.1340
C(14)	0.016644	0.008261	2.014772	0.0441
C(15)	0.974000	0.013516	72.06012	0.0000
C(21)	-158.2712	46.14963	-3.429522	0.0006
C(22)	0.070252	0.012589	5.580237	0.0000
C(23)	0.012001	0.006383	1.880243	0.0603
C(24)	0.903349	0.016464	54.86808	0.0000
C(31)	50.10046	31.13500	1.609137	0.1078
C(32)	0.000229	0.003615	0.063335	0.9495
C(33)	-0.003693	0.001640	-2.252286	0.0245
C(34)	1.039525	0.056297	18.46516	0.0000
C(35)	-0.035628	0.057343	-0.621310	0.5345
C(41)	-160.5611	2190.196	-0.073309	0.9416
C(42)	-0.123787	0.101659	-1.217665	0.2236
C(44)	-51.99974	2.293525	-22.67241	0.0000
C(45)	10.18847	2.284064	4.460679	0.0000
Determinant residual covariance		7.83E+20		

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+C(15)*Y1(-1)
Observations: 321

R-squared	0.997152	Mean dependent var	4796.988
Adjusted R-squared	0.997116	S.D. dependent var	1564.584
S.E. of regression	84.02560	Sum squared resid	2231055.
Durbin-Watson stat	2.026439		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*Y2(-1)
Observations: 321

R-squared	0.998948	Mean dependent var	14089.85
Adjusted R-squared	0.998938	S.D. dependent var	6599.741
S.E. of regression	215.0256	Sum squared resid	14656809
Durbin-Watson stat	1.828322		

Equation: X1=C(31)+C(32)*X2+C(33)*X3+C(34)*X1(-1)+C(35)*X1(-2)
Observations: 320

R-squared	0.999313	Mean dependent var	16109.83
Adjusted R-squared	0.999304	S.D. dependent var	6239.535
S.E. of regression	164.6228	Sum squared resid	8536708.
Durbin-Watson stat	2.008198		

Equation: X3=C(41)+C(42)*X2+(43)*X3(-1)+C(44)*X3(-2)+C(45)*X3(-3)
Observations: 319

R-squared	-0.296208	Mean dependent var	16912.96
Adjusted R-squared	-0.308553	S.D. dependent var	11992.66
S.E. of regression	13718.64	Sum squared resid	5.93E+10
Durbin-Watson stat	2.170260		

Figure 2.93 Statistical results based on a lagged-variable multivariate additive model

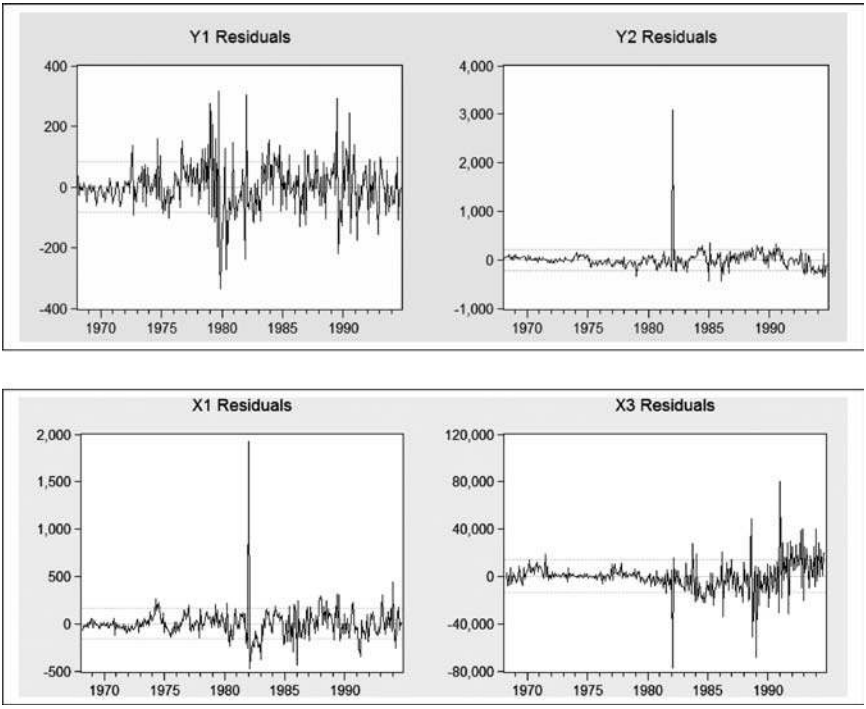


Figure 2.94 Residual graphs of the additive model in Figure 2.93

Based on the residual graphs in Figure 2.94, the following notes and conclusions can be presented:

(1) The estimated values of the error terms are very large. It is suggested that the logarithmic transformation should be used. The model of the tanslog linear model would certainly give a much smaller estimated value of the error terms. However, here the result will not be presented.

(2) The residual graphs also show the heterogeneity of the error terms. To overcome this problem it is suggested that the ARCH (i.e.autoregressive conditional heteroskedasticity) model(s) should be used. The ARCH models will be presented in Chapter 8.

(3) Note that these residual graphs, specifically the residual graph of Y_2 and X_1, also show the existence of a breakpoint or an outlier. By looking at the observed values of the endogenous variable, Y_2, the breakpoint(s) can be identified. Do this as an exercise. Based on those findings, a dummy variable of the time t should be used as an additional dependent variable. However, the model with dummy variable(s) will be presented in Chapter 3.

(4) On the other hand, if there is one outlier or more, the outliers should be handled as suggested in Example 2.4. □

2.13.2 Multivariate autoregressive model with two-way interactions

In fact, a two-way interaction model has been presented in Example 2.35. In this subsection, general two-way interaction multivariate models are considered, based on the path diagram in Figure 2.89. Two types of two-way interaction models will be presented.

The first type is constructed based on the model in (2.83) by adding the two-way interaction factor(s) of the independent variables within each regression, except the time t. Therefore, the following equation specification is obtained:

$$\begin{aligned} y1 &= c(11)+c(12)*t+c(13)*y2+c(14)*x1+c(15)*y2*x1+[ar(1)=c(16),\ldots] \\ y2 &= c(21)+c(22)*x1+c(23)*x2+c(24)*x1*x2+[ar(1)=c(24),\ldots] \\ x1 &= c(31)+c(32)*x2+c(33)*x3+c(34)*x2*x3+[ar(1)=c(35),\ldots] \\ x3 &= c(41)+c(42)*x2+[ar(1)=c(43),\ldots] \end{aligned} \tag{2.84}$$

For the second type, each of the other exogenous variables are considered that have an indirect effect on the corresponding endogenous variables. Hence, the following equation specification is obtained:

$$\begin{aligned} y1 &= c(11)+c(12)*t+c(13)*y2+c(14)*x1+c(15)*y2*x1 \\ &\quad +c(16)*y2*x2+c(17)*x1*x2+c(18)*x1*x3+[ar(1)=c(19),\ldots] \\ y2 &= c(21)+c(22)*x1+c(23)*x2+c(24)*x1*x2 \\ &\quad +c(25)*x1*x3+[ar(1)=c(26),\ldots] \\ x1 &= c(31)+c(32)*x2+c(33)*x3+c(34)*x2*x3+[ar(1)=c(35),\ldots] \\ x3 &= c(41)+c(42)*x2+[ar(1)=c(43),\ldots] \end{aligned} \tag{2.85}$$

Note that the first regression shows that the indirect effect of X_2 on Y_1 is going through Y_2 and X_1, and the indirect effect of X_3 on Y_1 is going through X_1. Similarly, the indirect effect of X_2 on Y_2, in the second regression, is going through X_1. Note that only the first regressions in (2.84) and (2.85) have the time t as an independent variable. These models can easily be extended to a multivariate model with all regressions having the time t as an independent variable. Further extension or modification could be done by using the transformed variables, as well as the lagged endogenous and exogenous variables.

Example 2.38. (Experimentation based on the model in (2.84)) Figure 2.95 presents statistical results based on the model in (2.84). Three of the independent variables in the first regression are insignificant with large p-values. In order to keep the two-way interaction in the model, then either one or both of the main factors should be deleted, and similarly for the second regression. After experimentation, an acceptable model is obtained, as presented in Figure 2.96.

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 17:18
Sample: 1968M02 1994M10
Included observations: 322
Total system (unbalanced) observations 1282
Convergence achieved after 13 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2688.876	2548.491	1.055086	0.2916
C(12)	-2.619399	6.097110	-0.429613	0.6676
C(13)	0.001642	0.088773	0.018501	0.9852
C(14)	0.165885	0.083173	1.994461	0.0463
C(15)	7.88E-07	4.35E-06	0.181338	0.8561
C(16)	0.986927	0.011177	88.29690	0.0000
C(21)	-5136.650	18327.10	-0.280276	0.7793
C(22)	0.333631	0.106937	3.119877	0.0019
C(23)	0.291059	0.042894	6.785604	0.0000
C(24)	-4.00E-07	2.14E-06	-0.187113	0.8516
C(25)	0.998240	0.008480	117.7172	0.0000
C(31)	13150.85	7844.697	1.676400	0.0939
C(32)	0.264815	0.022293	11.87895	0.0000
C(33)	0.264258	0.066843	3.953424	0.0001
C(34)	-4.78E-06	1.26E-06	-3.799139	0.0002
C(35)	1.256090	0.055001	22.83775	0.0000
C(36)	-0.258902	0.054919	-4.714240	0.0000
C(41)	28154.69	32585.95	0.864013	0.3877
C(42)	0.167816	0.029016	5.783496	0.0000
C(43)	1.265014	0.054224	23.32945	0.0000
C(44)	-0.266123	0.054193	-4.910644	0.0000

Determinant residual covariance 7.30E+16

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+C(15)*X1*Y2 +[AR(1)=C(16)]
Observations: 321

R-squared	0.997540	Mean dependent var	4796.988
Adjusted R-squared	0.997501	S.D. dependent var	1564.584
S.E. of regression	78.21270	Sum squared resid	1926926.
Durbin-Watson stat	2.056840		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X2+[AR(1)=C(25)]
Observations: 321

R-squared	0.999772	Mean dependent var	14089.85
Adjusted R-squared	0.999769	S.D. dependent var	6599.741
S.E. of regression	100.2183	Sum squared resid	3173813.
Durbin-Watson stat	1.958812		

Equation: X1=C(31)+C(32)*X2+C(33)*X3+C(34)*X2*X3+[AR(1)=C(35),AR(2) =C(36)]
Observations: 320

R-squared	0.999649	Mean dependent var	16109.83
Adjusted R-squared	0.999644	S.D. dependent var	6239.535
S.E. of regression	117.7662	Sum squared resid	4354829.
Durbin-Watson stat	2.076641		

Equation: X3=C(41)+C(42)*X2+[AR(1)=C(43), AR(2)=C(44)]
Observations: 320

R-squared	0.999328	Mean dependent var	16880.57
Adjusted R-squared	0.999322	S.D. dependent var	11987.86
S.E. of regression	312.1970	Sum squared resid	30799563
Durbin-Watson stat	2.147962		

Figure 2.95 Statistical results based on the two-way interaction model in (2.84)

Note that Figure 2.96 shows that each of the interaction factors, as an independent variable, has a significant adjusted effect on the corresponding dependent variable. As an illustration, based on the first regression, the following equation is obtained:

$$Y_1 = C(11) + C(12)^*T + C(13)^*Y2 + C(15)^*X_1{}^*Y_2 + [AR(1) = C(16)] \quad (2.86)$$

with a partial derivative

$$\frac{\partial Y_1}{\partial Y_2} = c(13) + c(15)^*X_1 \quad (2.87)$$

Estimation Method: Iterative Least Squares
Date: 10/12/07 Time: 17:30
Sample: 1968M02 1994M10
Included observations: 322
Total system (unbalanced) observations 1282
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	13969.94	27984.58	0.499202	0.6177
C(12)	-20.25346	46.59683	-0.434653	0.6639
C(13)	-0.142088	0.060788	-2.337456	0.0196
C(15)	8.86E-06	2.03E-06	4.363814	0.0000
C(16)	0.995628	0.006878	144.7638	0.0000
C(21)	-5136.693	18326.44	-0.280289	0.7793
C(22)	0.333631	0.106937	3.119876	0.0019
C(23)	0.291059	0.042894	6.785601	0.0000
C(24)	-4.00E-07	2.14E-06	-0.187114	0.8516
C(25)	0.998240	0.008480	117.7172	0.0000
C(31)	34060.94	14569.13	2.337884	0.0195
C(33)	-0.331950	0.051926	-6.392774	0.0000
C(34)	7.81E-06	7.87E-07	9.912059	0.0000
C(35)	1.135183	0.055909	20.30422	0.0000
C(36)	-0.137390	0.055795	-2.462418	0.0139
C(41)	28154.69	32585.95	0.864013	0.3877
C(42)	0.167816	0.029016	5.783496	0.0000
C(43)	1.265014	0.054224	23.32945	0.0000
C(44)	-0.266123	0.054193	-4.910644	0.0000

Determinant residual covariance 1.07E+17

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(15)*X1*Y2+[AR(1)=C(16)]
Observations: 321

R-squared	0.997515	Mean dependent var	4796.988
Adjusted R-squared	0.997484	S.D. dependent var	1564.584
S.E. of regression	78.48183	Sum squared resid	1946370.
Durbin-Watson stat	2.037383		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X2+[AR(1)=C(25)]
Observations: 321

R-squared	0.999772	Mean dependent var	14089.85
Adjusted R-squared	0.999769	S.D. dependent var	6599.741
S.E. of regression	100.2183	Sum squared resid	3173813.
Durbin-Watson stat	1.958812		

Equation: X1=C(31)+C(32)*X2+C(33)*X3+C(34)*X2*X3+[AR(1)=C(35),AR(2) =C(36)]
Observations: 320

R-squared	0.999649	Mean dependent var	16109.83
Adjusted R-squared	0.999644	S.D. dependent var	6239.535
S.E. of regression	117.7662	Sum squared resid	4354829.
Durbin-Watson stat	2.076641		

Equation: X3=C(41)+C(42)*X2+[AR(1)=C(43), AR(2)=C(44)]
Observations: 320

R-squared	0.999328	Mean dependent var	16880.57
Adjusted R-squared	0.999322	S.D. dependent var	11987.86
S.E. of regression	312.1970	Sum squared resid	30799563
Durbin-Watson stat	2.147962		

Figure 2.96 Statistical results based on a reduced interaction model in (2.84)

This finding or equation, in a mathematical sense, indicates that the (partial) effect of Y_2 on Y_1 is dependent on X_1. On the other hand, there is also the following partial derivative, which indicates that the effect of X_1 on Y_1 is dependent on Y_2:

$$\frac{\partial Y_1}{\partial X_1} = c(15)^* Y_2 \tag{2.88}$$

□

2.13.3 Multivariate autoregressive model with three-way interactions

In fact, a three-way interaction model has been presented in Example 2.36 as an AR(1) model with trend and time-related effects. Based on the theoretical causal model in Figure 2.89, the set of three variables Y_2, X_1 and X_2 may have either pairwise or complete associations. Similarly for the three variables, X_1, X_2 and X_3. If they have a complete association, then using a model with three-way interaction(s) could be considered. Hence, corresponding to the path diagram in Figure 2.89, a multivariate autoregressive model with three-way interactions may also be obtained, as follows:

$$\begin{aligned}
y1 &= c(11) + c(12)^*t + c(13)^*y2 + c(14)^*x1 + c(15)^*y2^*x1 \\
&\quad + c(16)^*y2^*x2 + c(17)^*x1^*x2 + c(18)^*x1^*x3 \\
&\quad + c(19)^*y2^*x1^*x2 + c(100)^*x1^*x2^*x3 + [ar(1) = c(101), \ldots] \\
y2 &= c(21) + c(22)^*x1 + c(23)^*x2 + c(24)^*x1^*x2 \\
&\quad + c(25)^*x1^*x3 + c(26)^*x1^*x2^*x3 + [ar(1) = c(27), \ldots] \\
x1 &= c(31) + c(32)^*x2 + c(33)^*x3 + c(34)^*x2^*x3 + [ar(1) = c(35), \ldots] \\
x3 &= c(41) + c(42)^*x2 + [ar(1) = c(43), \ldots]
\end{aligned} \tag{2.89}$$

Note that the three variables Y_2, X_1 and X_3 cannot have a complete association, because the path diagram shows that there is no direct association between Y_2 and X_3. Hence, the three-way interaction of the variables Y_2, X_1 and X_3 is not used as an independent variable of the first regression.

Example 2.39. (Experimentation based on the model in (2.89)) Figure 2.97 presents statistical results based on the model in (2.89). Note that some of the independent variables of the first regression are insignificant, so an attempt should be made to obtain a reduced model, by using a similar process to that presented in the previous example. By using the trial-and-error method, an acceptable reduced model can certainly be found with a three-way interaction factor.

Note that the results in Figure 2.97 already show that the three-way interaction $X1^*X2^*X3$ has a significant adjusted effect on $Y2$ with a p-value $= 0.0003$, corresponding to the parameter $C(26)$. On the other hand, at a significant level of 0.10, $X1^*X2^*Y2$ has a significant positive adjusted effect on $Y1$ with a p-value $= 0.1169/2 = 0.05\,845$, corresponding to the parameter $C(19)$.

Estimation Method: Weighted Least Squares
Date: 10/13/07 Time: 08:06
Sample: 1968M02 1994M10
Included observations: 322
Total system (unbalanced) observations 1282
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 44 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-2107.556	1302.396	-1.618215	0.1059
C(12)	-2.115262	2.347785	-0.900961	0.3678
C(13)	0.088243	0.263066	0.335442	0.7373
C(14)	0.829430	0.163418	5.075503	0.0000
C(15)	-2.59E-05	1.63E-05	-1.589218	0.1123
C(16)	3.12E-06	4.27E-06	0.731373	0.4647
C(17)	-1.14E-05	2.62E-06	-4.352358	0.0000
C(18)	2.81E-06	3.72E-06	0.755750	0.4499
C(19)	4.05E-10	2.58E-10	1.569970	0.1167
C(100)	-3.66E-11	6.99E-11	-0.523185	0.6009
C(101)	0.962630	0.016150	59.60621	0.0000
C(21)	-875.1904	2632.415	-0.332467	0.7396
C(22)	0.049000	0.095196	0.514730	0.6068
C(23)	0.268387	0.019981	13.43237	0.0000
C(24)	3.65E-06	1.53E-06	2.391846	0.0169
C(25)	1.25E-05	2.86E-06	4.369020	0.0000
C(26)	-1.91E-10	5.27E-11	-3.622100	0.0003
C(27)	0.994662	0.012251	81.18810	0.0000
C(31)	12852.17	7596.796	1.691788	0.0909
C(33)	0.272540	0.062661	4.349416	0.0000
C(34)	-4.95E-06	1.16E-06	-4.265762	0.0000
C(35)	1.258349	0.054365	23.14627	0.0000
C(36)	-0.261190	0.054282	-4.811739	0.0000
C(41)	28154.69	32381.65	0.869464	0.3848
C(42)	0.167816	0.028834	5.819985	0.0000
C(43)	1.265014	0.053884	23.47664	0.0000
C(44)	-0.266123	0.053853	-4.941626	0.0000

Determinant residual covariance 6.68E+16

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+C(15)*X1*Y2+C(16)*X2
*Y2+C(17)*X1*X2+C(18)*X1*X3+C(19)*X1*X2*Y2+ C(100)*X1*X2*X3
+[AR(1)=C(101)]
Observations: 321

R-squared	0.997729	Mean dependent var	4796.988
Adjusted R-squared	0.997656	S.D. dependent var	1564.584
S.E. of regression	75.75691	Sum squared resid	1779124.
Prob(F-statistic)	2.091635		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X2+C(25)*X1*X3+C(26)
*X1*X2*X3+[AR(1)=C(27)]
Observations: 321

R-squared	0.999788	Mean dependent var	14089.85
Adjusted R-squared	0.999784	S.D. dependent var	6599.741
S.E. of regression	97.07415	Sum squared resid	2958945.
Prob(F-statistic)	2.027472		

Equation: X1=C(31)+C(23)*X2+C(33)*X3+C(34)*X2*X3+ [AR(1)=C(35),
AR(2)=C(36)]
Observations: 320

R-squared	0.999649	Mean dependent var	16109.83
Adjusted R-squared	0.999644	S.D. dependent var	6239.535
S.E. of regression	117.7708	Sum squared resid	4355167.
Prob(F-statistic)	2.077280		

Equation: X3=C(41)+C(42)*X2+[AR(1)=C(43), AR(2)=C(44)]
Observations: 320

R-squared	0.999328	Mean dependent var	16880.57
Adjusted R-squared	0.999322	S.D. dependent var	11987.86
S.E. of regression	312.1970	Sum squared resid	30799563
Prob(F-statistic)	2.147952		

**** Prob(F-stat) should be the DW-stat.***

Figure 2.97 Statistical results based on the three-way interaction model in (2.86)

Note that this statistical result needs to be presented by using the previous version of EViews 6 with a statistical error of Prob(F-statistic), since the latest version of EViews 6 (which was received on 29 October 2007) presents the 'Near singular matrix' error message, and special notes on the three-way interaction model need to be presented. For this reason, the trial-and-error method is used to obtain an alternative three-way interaction model, as presented in Figure 2.98.

Estimation Method: Iterative Least Squares
Date: 11/05/07 Time: 05:33
Sample: 1968M02 1994M10
Included observations: 322
Total system (unbalanced) observations 1283
Convergence achieved after 12 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	869.4003	1843.074	0.471712	0.6372
C(12)	-1.403083	3.859361	-0.363553	0.7163
C(13)	0.336424	0.241410	1.393584	0.1637
C(14)	0.179167	0.091804	1.951623	0.0512
C(15)	-8.93E-06	1.17E-05	-0.763534	0.4453
C(16)	-5.44E-06	3.66E-06	-1.485525	0.1377
C(17)	1.91E-10	1.77E-10	1.075076	0.2825
C(18)	0.982651	0.013081	75.12168	0.0000
C(21)	-6988.419	7663.897	-0.911862	0.3620
C(22)	0.348459	0.097781	3.563676	0.0004
C(23)	0.291049	0.029956	9.715956	0.0000
C(24)	7.45E-06	3.31E-06	2.250981	0.0246
C(25)	2.67E-06	1.70E-06	1.570349	0.1166
C(26)	-2.06E-10	5.58E-11	-3.685493	0.0002
C(27)	0.997104	0.005686	175.3572	0.0000
C(31)	18255.96	10849.17	1.682706	0.0927
C(32)	0.238427	0.021952	10.86118	0.0000
C(33)	0.191215	0.064362	2.970944	0.0030
C(34)	-3.51E-06	1.22E-06	-2.874444	0.0041
C(35)	0.997304	0.002436	409.3761	0.0000
C(41)	28154.69	32585.95	0.864013	0.3877
C(42)	0.167816	0.029016	5.783496	0.0000
C(43)	1.265014	0.054224	23.32945	0.0000
C(44)	-0.266123	0.054193	-4.910644	0.0000

Determinant residual covariance 7.45E+16

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+C(15)*X1*Y2+C(16)*X2
*Y2+C(17)*X1*X2*Y2+[AR(1)=C(18)]
Observations: 321

R-squared	0.997575	Mean dependent var	4796.988
Adjusted R-squared	0.997521	S.D. dependent var	1564.584
S.E. of regression	77.90214	Sum squared resid	1899517.
Durbin-Watson stat	2.052494		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X3+C(25)*X2*X3+C(26)
*X1*X2*X3+[AR(1)=C(27)]
Observations: 321

R-squared	0.999789	Mean dependent var	14089.85
Adjusted R-squared	0.999785	S.D. dependent var	6599.741
S.E. of regression	96.86226	Sum squared resid	2946041.
Durbin-Watson stat	2.053839		

Equation: X1=C(31)+C(32)*X2+C(33)*X3+C(34)*X2*X3+[AR(1)=C(35)]
Observations: 321

R-squared	0.999630	Mean dependent var	16076.63
Adjusted R-squared	0.999625	S.D. dependent var	6258.103
S.E. of regression	121.1982	Sum squared resid	4641723.
Durbin-Watson stat	1.523586		

Equation: X3=C(41)+C(42)*X2+[AR(1)=C(43), AR(2)=C(44)]
Observations: 320

R-squared	0.999328	Mean dependent var	16880.57
Adjusted R-squared	0.999322	S.D. dependent var	11987.86
S.E. of regression	312.1970	Sum squared resid	30799563
Durbin-Watson stat	2.147962		

Figure 2.98 Statistical results based on the three-way interaction model, which is a reduced model of (2.89)

Estimation Method: Iterative Least Squares
Date: 11/05/07 Time: 06:21
Sample: 1968M02 1994M10
Included observations: 322
Total system (unbalanced) observations 1282
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	3334.576	1358.768	2.454117	0.0143
C(12)	0.111627	4.730975	0.023595	0.9812
C(13)	1.14E-05	2.84E-06	4.000124	0.0001
C(17)	-9.93E-11	3.92E-11	-2.535166	0.0114
C(18)	0.985316	0.010079	97.76234	0.0000
C(21)	-6988.388	7663.662	-0.911886	0.3620
C(22)	0.348459	0.097781	3.563675	0.0004
C(23)	0.291049	0.029956	9.715956	0.0000
C(24)	7.45E-06	3.31E-06	2.250982	0.0246
C(25)	2.67E-06	1.70E-06	1.570349	0.1166
C(26)	-2.06E-10	5.58E-11	-3.685493	0.0002
C(27)	0.997104	0.005686	175.3565	0.0000
C(31)	13150.83	7844.663	1.676405	0.0939
C(32)	0.264815	0.022293	11.87895	0.0000
C(33)	0.264259	0.066843	3.953429	0.0001
C(34)	-4.78E-06	1.26E-06	-3.799143	0.0002
C(35)	1.256090	0.055001	22.83776	0.0000
C(36)	-0.258902	0.054919	-4.714245	0.0000
C(41)	28154.69	32585.95	0.864013	0.3877
C(42)	0.167816	0.029016	5.783496	0.0000
C(43)	1.265014	0.054224	23.32945	0.0000
C(44)	-0.266123	0.054193	-4.910644	0.0000

Determinant residual covariance 7.09E+16

Equation: Y1=C(11)+C(12)*T+C(13)*X1*Y2+C(17)*X1*X2*Y2+[AR(1)=C(18)]
Observations: 321

R-squared	0.997524	Mean dependent var	4796.988
Adjusted R-squared	0.997493	S.D. dependent var	1564.584
S.E. of regression	78.34084	Sum squared resid	1939383.
Durbin-Watson stat	2.041112		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X3+C(25)*X2*X3+C(26)*X1*X2*X3+[AR(1)=C(27)]
Observations: 321

R-squared	0.999789	Mean dependent var	14089.85
Adjusted R-squared	0.999785	S.D. dependent var	6599.741
S.E. of regression	96.86226	Sum squared resid	2946041.
Durbin-Watson stat	2.053839		

Equation: X1=C(31)+C(32)*X2+C(33)*X3+C(34)*X2*X3+[AR(1)=C(35),AR(2)=C(36)]
Observations: 320

R-squared	0.999649	Mean dependent var	16109.83
Adjusted R-squared	0.999644	S.D. dependent var	6239.535
S.E. of regression	117.7662	Sum squared resid	4354829.
Durbin-Watson stat	2.076641		

Equation: X3=C(41)+C(42)*X2+[AR(1)=C(43), AR(2)=C(44)]
Observations: 320

R-squared	0.999328	Mean dependent var	16880.57
Adjusted R-squared	0.999322	S.D. dependent var	11987.86
S.E. of regression	312.1970	Sum squared resid	30799563
Durbin-Watson stat	[illegible]		

Figure 2.99 Statistical results based on an alternative three-way interaction model

Based on this table the following notes and conclusions are produced:

(1) This model should be considered as an acceptable model, even though some of the independent variables are insignificant, specifically for the first regression.
(2) Corresponding to the parameter $C(26)$, the three-way interaction factor $X1^*X2^*X3$ has a significantly negative adjusted effect on $Y2$.
(3) The DW-statistic of the third regression could be increased or modified by using either a higher autoregressive model or adding the lag(s) of the endogenous variable $X1$ as an independent variable.
(4) For illustration purposes, Figure 2.99 presents an alternative model, which shows that the three-way interaction $X1^*X2^*Y2$ has a significantly negative effect on $Y1$. □

2.14 Special notes and comments

Corresponding to what has been done in experimentation with the multivariate models, there are various notes and comments that need to be presented.

2.14.1 The true population model

It is well known that researchers never know the true values of the population parameters, such as the means, standard deviations and other parameters, or the true population model. It is also recognized that any proposed model could be an estimable model, in a statistical sense, even though the model might not be an appropriate model.

For these reasons, best judgment should be used to define alternative statistical models, and not only one univariate linear model having all selected variables. This is supported by knowledge and experience in the particular field of study, as well as ‘a broad experience with how particular techniques of data analysis have worked out in a variety of fields of applications’ (Tukey, 1962, in Gifi, 1990, p. 23). Furthermore, Tukey stated: ‘In data analysis we must look to a very heavy emphasis on judgment.’

On the other hand, Hampel (1973, in Gifi, 1990, p. 27) stated:

> *Often in statistics one is using a parametric model, such as the common model of normally distributed errors, or that of exponentially distributed observations. Classical (parametric) statistics derives results under the assumptions that these models are strictly true. However, apart from some simple models perhaps, such models are never exactly true. We should also remember that we never know the exact distribution of ordinary data; and even if we did, or as far as we do, there remain serious questions about how to handle the excess knowledge of details. After all a statistical model has to be simple (where ‘simple’, of course has a relative meaning, depending on state and standards of the subject matter field); Ockham’s razor is an essential tool for the progress of science.*

2.14.2 Near singular matrix

The process of data analysis applying to any time series model is a straightforward method using EViews. However, note that there will always be problems in selecting the best acceptable model among such a large number of possible choices, as well as in selecting the appropriate estimation method(s).

Furthermore, note that the process of data analysis in selecting an acceptable model, in a theoretical and statistical sense, is really a trial-and-error process, and it is believed that the process cannot be generalized because it is highly dependent on the data that are available or used. Even though EViews can simplify the work to try many alternative models, in some cases the error message ‘*Near singular matrix*’ could be received.

This error message indicates that the independent variables of the model have (almost) a perfect multicollinearity based on the data sets used. However, there might be nothing wrong with the model, since it could be an estimable model based on other data sets. Hence, if the statistical results are obtained, then it is certain that the model should be a good model, in a statistical sense. Furthermore, if the model has been defined based on a good or strong theoretical base, then it may be concluded that the model is an acceptable model, in both a theoretical and statistical sense.

Talking about the error message or multicollinearity, Blanchard (in Gujarati, 2003, p. 263) stated that ‘Multicollinearity is God’s will, not a problem with OLS (ordinary least square) or statistical technique in general.’ Based on this statement, there should not be too much concern about multicollinearity, since the bivariate and multiple correlations between the independent variables always exist, even though some of them may not be correlated, in a theoretical sense. Additionally, their quantitative values are highly dependent on the data set used or available for the analysis.

Talking about a data set, Agung (2004) defined it as a set of multidimensional scores/measurements that happen to be collected or available for an analyst or a researcher. It is recognized that unexpected estimated values of the model parameters can be obtained, even though the model is a good one, because of the unpredictable effect(s) of multicollinearity between the independent variables of the model.

Unexpected statistical results have been presented in several dissertations of the author's students, such as those of Supriyono (2003), Ary Suta (2005) and Hamzal and Agung (2007). Example 2.40, as well as Example 2.41, in Section 2.14.3 below, present illustrative contradictory statistical results, as the cause of high or significant bivariate correlations between the independent variables. Note again that each of the independent variables always has quantitative multiple correlations with the other independent variables, even though some of those variables might not be correlated substantively.

On the other hand, it has been found that many papers in the international journals, such as the *Journal of Finance and Strategic Management*, do not discuss multicollinearity of the independent variables of their models, even though each paper presents several alternative models. As an extreme illustrative example, Coombs and Gilley (2005) present two sets of 12 regressions without considering the multicollinearity problem of their independent variables.

As a comparison, if EViews presents the 'Near singular matrix' error message, the SPSS could provide the VIF (i.e. variance inflation factor) of each independent variable of a multiple regression, but for a multivariate linear model. If an independent variable has $VIF > 10$, then the independent variable has a high multicollinearity with the others. However, corresponding to Blanchard's statement above, additional analysis is not required as long as best knowledge and experience has been used in defining the model.

To solve the 'Near singular matrix' error message, experimentation should be performed or trial-and-error methods used to delete an independent variable or two from the model, or additional variable(s) and/or serial correlation indicator(s) should be inserted. However, it is suggested that an independent variable of a model should not be deleted that is only based on the largest *p*-value. Talking about the *p*-value of an independent variable, Hosmer and Lemesshow (Hosmer and Lemesshow, 2000, p. 118) stated:

> *The choice of $p_E = 0.05$ is too stringent, often excluding important variables from the model. Choosing a value for p_E in the range from 0.15 to 0.20 is highly recommended. Sometimes the goal of the analysis may be broader, and models containing more variables are sought to provide a more complete picture of possible models. In these cases, use of $p_E = 0.25$ or even larger might be a reasonable choice.*

In many cases, it has been found that an important independent variable, in a theoretical sense, has an insignificant adjusted effect or a large *p*-value, but the other has a significant adjusted effect. Hence, the least important variable should be deleted from the model, even though it has a significant adjusted effect or a very small *p*-value.

Finally, considering EViews, experience with data analyses using EViews 4 and 5, as well as 6, showed that there was a problem. In fact, the first draft of this book was presented using EViews 4. In order to update the book using EViews 6, it was found that some of the multivariate autoregressive models or system of equations were estimable models based on EViews 4, but the latest EViews 6 produced the

'Near singular matrix'. To date, this problem has not yet been solved. Refer to the problems presented in the following Examples 2.40 and 2.41.

2.14.3 'To Test or Not' the assumptions of the error terms

In this subsection, the *white noise* process of the error terms $\{\varepsilon_t\}$ of the time series models is considered. The basic assumption of the error terms of the univariate model, i.e. the $\{\varepsilon_t\}$ sequence, is a white noise process such that

$$\begin{aligned} E(\varepsilon_t) &= E(\varepsilon_{t-1}) = \cdots = 0 \\ E(\varepsilon_t^2) &= E(\varepsilon_{t-1}^2) = \cdots = \sigma^2 \\ E(\varepsilon_t\,\varepsilon_{t-s}) &= E(\varepsilon_{t-j}\,\varepsilon_{t-j-s}) = 0 \quad \text{for all } j \text{ and } s \end{aligned} \tag{2.90}$$

for each time period t. Note that $E(\varepsilon_t^2) = \mathrm{Var}(\varepsilon_t)$ and $E(\varepsilon_t\varepsilon_{t-s}) = \mathrm{Cov}(\varepsilon_t, \varepsilon_{t-s})$.

Furthermore, note that the true values of $E(\varepsilon_t)$, $E(\varepsilon_t^2)$ and $E(\varepsilon_t\varepsilon_{t-s})$ are never known by the researchers. Hence, they could be considered as theoretical or abstract indicators. In practice, since only a single observation exists within each time period or at one time point t, then only one set of the estimated error terms, say $\{e_t, t = 1,2, \ldots, T\}$, is observed, where e_t is a constant or fixed number with $E(e_t) = e_t$, which highly depends on the sampled data and the model used in the analysis. Hence, there is not a sufficient number of observations to test the assumptions in (2.90) for each time point. As a result, these cannot be proven, but they should be assumed to be valid for the present model(s). By applying a lagged-variable or autoregressive model, which is either first- or higher-order autoregressive, it is common to assume that the error terms $\{\varepsilon_t\}$ are white noise processes.

On the under hand, in order for the error term ε_t to have an expected value of zero for each time point t, $E(\varepsilon_t) = 0$ in particular, an assumption should be used that ε_t has a certain density or distribution function. In general it is assumed that ε_t is normally distributed for each time point t. This normal density function also cannot be proved but is assumed.

Note that the normal distribution of various defined statistics, the mean statistic in particular, has been proven based on the *central limit theorem*. In practice, however, the sample space of the mean statistic, say $\bar{X}$, can be considered to have an approximate normal distribution, for a sample size of $n > 30$. Conover (1980, p. 444) stated that for $n > 20$, the rth quintile of a binomial random variable may be approximated using the rth quintile of a standard normal random variable. On the other hand, the set of numbers or scores $\{e_t, t = 1,2, \ldots, T\}$, which might be observed by a researcher, will not have a specific density function, including the normal distribution. Refer again to Shewhart's finding presented in Section 2.4.3.

Corresponding to the multivariate normal distribution of the error vector of a multivariate linear model, the three types of multivariate central limit theorems should be considered: (i) *multivariate central limit theorem I* (Lineberg–Levy), (ii) *multivariate central limit theorem II* (Wald and Wolfowitz, 1944) and (iii) *multivariate central limit theorem III* (presented in Puri and Sen, 1993, pp. 22–25).

Based on the information and statements above, as well as in Section 2.4.3, it can be concluded that the assumptions of the error terms $\{\varepsilon_t\}$ of a model do not need to be tested for the following summary reasons:

(1) The true sequence of $\{\varepsilon_t\}$ is never known, as well as the true population model.
(2) A sampled data is defined as a set of multidimensional scores/measurements, which happen to be selected by a researcher, with a single sample unit in each time point t. Hence, there is not a sufficient number of observations to do the testing.
(3) In order to test the assumptions, especially the normal distribution, a distribution function of a statistic should be used, such as a normal distribution, the chi-square distribution, Student's t-distribution and Snedecor's F-distribution, which is assumed again to be true. Such a situation would produce circular problems. Note that it has been proved that the chi-square distribution is derived from a set of independent normal density functions, the t-distribution is derived from two independent random variables, one having a normal distribution and the other a chi-square distribution, and the F-distribution is derived from two independent chi-square distributions (Garybill, 1976, pp. 63–66; Wilks, 1962, pp. 183–186; Parzen, 1960, pp. 325–326).
(4) On the other hand, if a test is performed, then the conclusion of the testing hypothesis cannot be taken for granted. For examples, some of the author's students, such as Lindawati (2002) and Alamsyah (2007), present hypotheses that an independent variable has a positive effect on a dependent variable, but their statistical results show that the independent variable has a negative effect. Considering the conclusion of a testing hypothesis, Freund, Williams and Peters (1993, p. 442) stated: 'If we say something is statistically significant, we do not mean to imply that it is necessarily of any practical significance or importance.'

Example 2.40. (Unexpected effect of multicollinearity) Suppose the relationship between the variables X_1, X_2, X_3, Y_1, Y_2 and the time t is to be studied by using the following AR(2) bivariate model with trend:

$$\begin{aligned} y1 &= c(11) \;+\; + c(12)^*t + c(13)^*y2 + c(14)^*x1 + c(15)^*x2 + c(16)^*x3 \\ &\qquad + [ar(1) = c(17), ar(2) = c(18)] \\ y2 &= c(21) \;+\; c(22)^*t + c(23)^*x1 + c(24)^*x2 + c(25)^*x3 \\ &\qquad + [ar(1) = c(26), ar(2) = c(27)] \end{aligned} \tag{2.91}$$

The statistical results in Figure 2.100 using EViews 4 should be presented, since EViews 6 gave the '*Near singular matrix*' error message. This gives the special notes and comments as follows:

(1) The output presents an error message '*Convergence not achieved after* 1 *weight matrix*, 1000 *total coef iterations*'. Even though this gives the statistical results,

Estimation Method: Weighted Least Squares
Date: 03/05/07 Time: 10:12
Sample: 1952:3 1996:4
Included observations: 180
Total system (balanced) observations 356
Iterate coefficients after one-step weighting matrix
Convergence not achieved after: 1 weight matrix, 1000 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2297.822	789.7258	2.909645	0.0039
C(12)	11.33706	4.598915	2.465160	0.0142
C(13)	-1.242690	0.346606	-3.585309	0.0004
C(14)	-0.025505	0.352714	-0.072311	0.9424
C(15)	-1190.901	1079.938	-1.102749	0.2709
C(16)	7.759072	4.264881	1.819294	0.0697
C(17)	0.957772	0.075708	12.65095	0.0000
C(18)	0.008566	0.075220	0.113885	0.9094
C(21)	6907.323	5371.884	1.285829	0.1994
C(22)	-30.39961	15.31528	-1.984920	0.0480
C(23)	0.112650	0.059575	1.890876	0.0595
C(24)	110.3739	384.4514	0.287095	0.7742
C(25)	0.094489	0.667091	0.141643	0.8874
C(26)	1.626391	0.062467	26.03580	0.0000
C(27)	-0.629174	0.062349	-10.09112	0.0000

Determinant residual covariance 88029.55

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+C(15)*X2+C(16)*X3+ [AR(1)=C(17),AR(2)=C(18)]
Observations: 178

R-squared	0.997348	Mean dependent var	945.2776
Adjusted R-squared	0.997239	S.D. dependent var	824.7950
S.E. of regression	43.34031	Sum squared resid	319325.0
Durbin-Watson stat	1.995902		

Equation: Y2=C(21)+C(22)*T+C(23)*X1+C(24)*X2 +C(25)*X3 +[AR(1)=C(26), AR(2)=C(27)]
Observations: 178

R-squared	0.999805	Mean dependent var	1469.344
Adjusted R-squared	0.999798	S.D. dependent var	523.9305
S.E. of regression	7.437783	Sum squared resid	9459.824
Durbin-Watson stat	2.344828		

Figure 2.100 Statistical results based on model (2.91), using EViews 4

specifically the parameter estimates, it could be said that the estimates are unacceptable estimates, in a statistical sense. In other words, the estimates are not optimal estimates.

(2) This error message does not directly mean that the proposed model is a bad model, since optimal estimates could be obtained by using other data sets. Refer to the special notes and comments on the true population models and multicollinearity problems presented in Sections 2.14.1 and 2.14.2.

(3) By using the latest version of EViews 6, which was received on 29 October 2007, the results in Figure 2.101 were obtained, which presents another statement of the error message, namely '*Convergence not achieved after* 500 *iterations.*'

System: SYS05
Estimation Method: Iterative Least Squares
Date: 10/29/07 Time: 15:34
Sample: 1952Q3 1996Q4
Included observations: 180
Total system (balanced) observations 356
Convergence not achieved after 500 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2297.822	808.0940	2.843508	0.0047
C(12)	11.33706	4.705881	2.409126	0.0165
C(13)	-1.242690	0.354668	-3.503814	0.0005
C(14)	-0.025505	0.360918	-0.070667	0.9437
C(15)	-1190.901	1105.056	-1.077684	0.2819
C(16)	7.759071	4.364078	1.777941	0.0763
C(17)	0.957772	0.077468	12.36339	0.0000
C(18)	0.008566	0.076969	0.111296	0.9114
C(21)	6907.323	5480.732	1.260292	0.2084
C(22)	-30.39961	15.62561	-1.945499	0.0525
C(23)	0.112650	0.060782	1.853323	0.0647
C(24)	110.3739	392.2413	0.281393	0.7786
C(25)	0.094489	0.680608	0.138830	0.8897
C(26)	1.626391	0.063733	25.51873	0.0000
C(27)	-0.629174	0.063613	-9.890711	0.0000

Determinant residual covariance 88029.55

Equation: Y1=C(11)+C(12)*T+C(13)*Y2+C(14)*X1+C(15)*X2+C(16)*X3 +[AR(1)=C(17), AR(2)=C(18)]
Observations: 178

R-squared	0.997348	Mean dependent var	945.2776
Adjusted R-squared	0.997239	S.D. dependent var	824.7950
S.E. of regression	43.34031	Sum squared resid	319325.0
Durbin-Watson stat	1.995902		

Equation: Y2=C(21)+C(22)*T+C(23)*X1+C(24)*X2+C(25)*X3+[AR(1)=C(26), AR(2)=C(27)]
Observations: 178

R-squared	0.999805	Mean dependent var	1469.344
Adjusted R-squared	0.999798	S.D. dependent var	523.9305
S.E. of regression	7.437783	Sum squared resid	9459.824
Durbin-Watson stat	2.344828		

Figure 2.101 Statistical results using the latest EViews 6, based on the same model as presented in Figure 2.100

Estimation Method: Iterative Least Squares
Date: 10/29/07 Time: 16:37
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (unbalanced) observations 356
Convergence achieved after 11 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-524.4506	112.3974	-4.666040	0.0000
C(12)	3.363623	3.662273	0.918452	0.3590
C(13)	2166.842	556.5439	3.893389	0.0001
C(14)	5.771168	4.290198	1.345199	0.1794
C(15)	0.925586	0.030161	30.68814	0.0000
C(21)	5580.693	5766.278	0.967815	0.3338
C(22)	-16.96721	15.02574	-1.129210	0.2596
C(23)	0.092885	0.059882	1.551142	0.1218
C(24)	-1074.454	350.0773	-3.069190	0.0023
C(25)	1.543669	0.065180	23.68331	0.0000
C(26)	-0.546683	0.065174	-8.388093	0.0000

Determinant residual covariance 77404.60

Equation: Y1=C(11)+C(12)*T+C(13)*X2+C(14)*X3+[AR(1)=C(15)]
Observations: 179

R-squared	0.997180	Mean dependent var	940.4891
Adjusted R-squared	0.997115	S.D. dependent var	824.9664
S.E. of regression	44.30695	Sum squared resid	341580.4
Durbin-Watson stat	2.010095		

Equation: Y2=C(21)+C(22)*T+C(23)*X1+C(24)*X2(-1)+[AR(1)=C(25),AR(2)=C(26)]
Observations: 177

R-squared	0.999814	Mean dependent var	1467.310
Adjusted R-squared	0.999809	S.D. dependent var	524.7115
S.E. of regression	7.260329	Sum squared resid	9013.816
Durbin-Watson stat	2.217052		

Figure 2.102 Statistical results based on a modified model in Figure 2.101

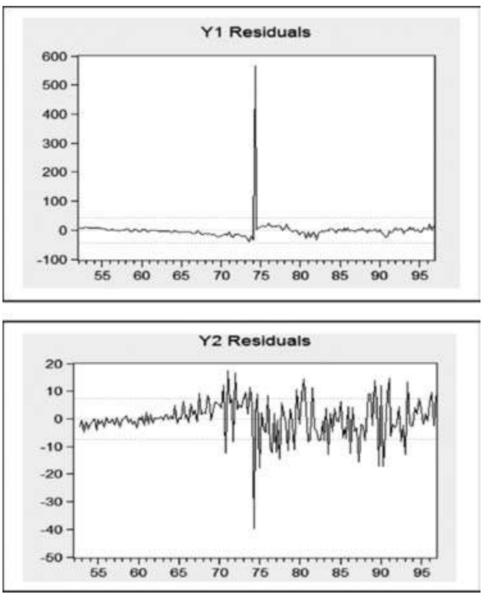


Figure 2.103 Residual graphs of the regressions in Figure 2.102

(4) Then for illustration purposes, after using the trial and error methods, the results in Figure 2.102 were obtained, with a note 'Convergence achieved after 11 iterations', and its residual graphs in Figure 2.103. Therefore, the corresponding model should be considered as a good fit model. However, it might not be the best fit model, since there could be many other alternative models giving optimal estimates. Based on this model the following notes and conclusions are derived:

- There is confidence that other modified models with trend can be found by using other types of independent variables, such as the transformed variables as well as the lags of endogenous or exogenous variables.
- The first regression has $X2$ as an independent variable, but the second regression has $X2(-1)$. This model should be considered as an unexpected model, since there is no good reason for selecting these independent variables. Note that either $X2$ or $X2(-1)$ may be used as an independent variable of both regressions.
- The residual graph of the first regression indicates that there is an outlier or a breakpoint. For this reason, it is suggested to do further data analysis (refer to the notes in Example 2.4).
- Corresponding to the statistical results in Figure 2.102, the associations between the variables can be presented as a path diagram, as in Figure 2.104. Note that the dotted lines represent the fact that the independent variables have insignificant effects on the corresponding dependent variable(s), at a significant level of 0.10.
- However, at a significant level of 0.10, in fact, $X3$ has a significantly positive effect on $Y1$, based on the t-statistic with a p-value $= 0.1794/2 = 0.0897 < 0.10$, and the time t has a significantly positive effect on $Y2$, based on the t-statistic with a p-value $= 0.1218/2 = 0.0609$.
- Furthermore, note that the model does not present the possible causal relationships between the independent variables t, $X1$, $X2$ and $X3$. However, their

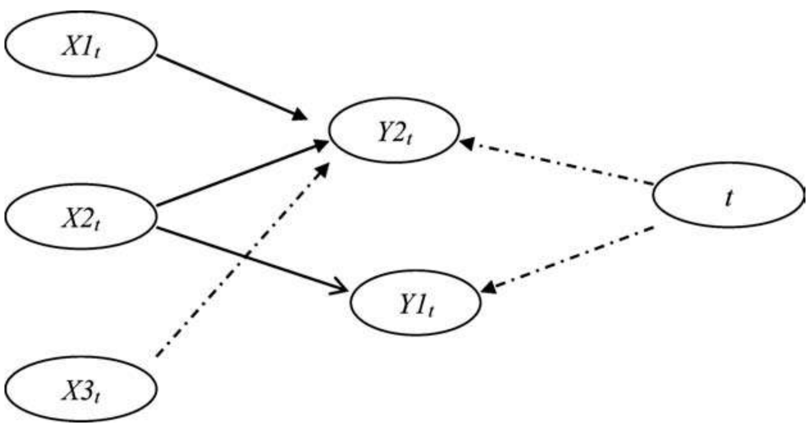


Figure 2.104 Path diagram based on statistical results in Figure 2.102

Covariance Analysis: Ordinary
Date: 11/03/07 Time: 08:16
Sample: 1952Q1 1996Q4
Included observations: 180

Correlation Probability	X1	X2	X3	Y1	Y2	T
X1	1.000000					

X2	0.980402	1.000000				
	0.0000	-----				
X3	0.270059	0.412471	1.000000			
	0.0002	0.0000	-----			
Y1	0.961217	0.989195	0.446274	1.000000		
	0.0000	0.0000	0.0000	-----		
Y2	-0.977783	-0.973796	-0.257958	-0.951496	1.000000	
	0.0000	0.0000	0.0005	0.0000	-----	
T	0.921329	0.955627	0.537062	0.956530	-0.872592	1.000000
	0.0000	0.0000	0.0000	0.0000	0.0000	-----

Figure 2.105 The correlation matrix of the selected six variables

coefficients of correlation or multicollinearity should be taken into account in the estimation process. Figure 2.105 presents the correlation matrix between the variables considered with their significant levels, which shows that they are highly correlated. This indicates that the independent variables of the model should have high multicollinearity. As a result, the parameter estimates could be contradictory with what would be expected, because of the unpredictable effects of multicollinearity, as presented above. □

Example 2.41. (Other unexpected effects of multicollinearity) One of the author's students, Hamsal (2006), in his dissertation, presents the following reduced regression function to test the hypothesis that the effect of strategic flexibility (X_1) on overall firm performance (Y) depends on strategic consistency (X_2) and perceived environment (X_3):

$$\begin{aligned} Y = 6.932 - &\underset{(0.006)}{2.490}\,X_2 - \underset{(0.020)}{1.892}\,X_3 + \underset{(0.011)}{0.240}\,X_1{*}X_2 + \underset{(0.111)}{0.143}\,X_1{*}X_3 \\ &+ \underset{(0.034)}{0.437}\,X_2{*}X_3 - \underset{(0.082)}{0.044}\,X_1{*}X_2{*}X_3 \qquad (2.92) \\ &F\text{-statistic} = 11.13 \qquad \text{Significant level} = 0.000 \qquad R^2 = 0.562 \end{aligned}$$

Note that, at a significant level of $\alpha = 0.10$, five out of the six independent variables have significant adjusted effects, with their p-values presented in parentheses. In fact, at a significant level of 0.10, $X_1{}^*X_3$ also has a positive significant adjusted effect on Y, based on the p-value $= 0.111/2 = 0.0555$. In a statistical sense, this regression function would be considered as a good or an acceptable regression.

Considering the bivariate correlation between the independent variables, Hamsal presents preliminary data analysis to show that each pair of all independent variables has significant positive correlations. Since each of the independent variables has a significant adjusted effect, these results should be considered as a contradiction to the results in the previous example, specifically Figures 2.101 and 2.102.

Hence, based on the last two examples, it could be concluded that the impact of multicollinearity or multiple correlations on the estimated values of the model parameters is unpredictable. □

2.15 Alternative multivariate models with trend

Based on the set of variables, many more models with trend could be defined. Thus an infinite number of lagged-variable autoregressive models might be produced by using lagged variables, either endogenous or exogenous variables, or both. For illustrative purposes, the following alternative models are presented, which could be extended to more complex models.

However, the empirical examples for all models will be presented with only a limited discussion, because the previous examples should be considered sufficient to represent the models.

2.15.1 The lagged endogenous variables: first autoregressive model with trend

This model can be presented in a matrix equation as follows:

$$\begin{aligned} \mathbf{Y_t} &= \mathbf{A} + \mathbf{B}^*\mathbf{t} + \mathbf{C}^*\mathbf{Y_{t-1}} + \boldsymbol{\mu}_\mathbf{t} \\ \boldsymbol{\mu}_\mathbf{t} &= \mathbf{D}^*\boldsymbol{\mu}_\mathbf{t-1} + \boldsymbol{\varepsilon}_\mathbf{t} \end{aligned} \tag{2.93}$$

where $\mathbf{Y_t}$ is a $K \times 1$ vector of endogenous variables, $\mu_\mathbf{t}$ is a $K \times 1$ vector of the corresponding residual terms, **A**, **B** and **C** are vectors or matrices of the model parameters and **D** is a diagonal matrix of the first serial correlation of the K regressions, namely $\mathbf{D} = \text{diag}(\rho_1, \rho_2, \ldots, \rho_K)$.

By using the two endogenous variables $Y1$ and $Y2$ presented above, two additive regressions are found, as follows:

$$\begin{aligned} y1_t &= c(11) + c(12)^*t + c(13)^*y1_{t-1} + c(14)^*y2_{t-1} + [ar(1) = c(15)] \\ y2_t &= c(21) + c(22)^*t + c(23)^*y1_{t-1} + c(24)^*y2_{t-1} + [ar(1) = c(25)] \end{aligned} \tag{2.94}$$

Corresponding to the model in (2.93), the matrix **D** of this model is $\text{diag}(\rho_1, \rho_2) = \text{diag}(c(15), c(25))$.

Furthermore, the multivariate $\boldsymbol{Y_t} = (X1,X2,X3,Y1,Y2)_t$ may also be used as the dependent variable of the model in (2.93).

Estimation Method: Iterative Least Squares
Date: 10/29/07 Time: 17:27
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-28.98915	44.81783	-0.646822	0.5182
C(12)	0.514974	0.252921	2.036102	0.0425
C(13)	0.980853	0.025381	38.64479	0.0000
C(14)	0.009154	0.023450	0.390359	0.6965
C(15)	-0.019734	0.079647	-0.247762	0.8045
C(21)	70.01101	8.688193	8.058179	0.0000
C(22)	0.173917	0.048897	3.556790	0.0004
C(23)	-0.042144	0.004812	-8.758172	0.0000
C(24)	0.962885	0.004505	213.7591	0.0000
C(25)	0.233159	0.075751	3.077965	0.0023

Determinant residual covariance 70464.97

Equation: Y1=C(11)+C(12)*T+C(13)*Y1(-1)+C(14)*Y2(-1)+[AR(1)=C(15)]
Observations: 178

R-squared	0.997059	Mean dependent var	945.2776
Adjusted R-squared	0.996991	S.D. dependent var	824.7950
S.E. of regression	45.23988	Sum squared resid	354069.9
Durbin-Watson stat	2.000522		

Equation: Y2=C(21)+C(22)*T+C(23)*Y1(-1)+C(24)*Y2(-1)+[AR(1)=C(25)]
Observations: 178

R-squared	0.999836	Mean dependent var	1469.344
Adjusted R-squared	0.999832	S.D. dependent var	523.9305
S.E. of regression	6.790676	Sum squared resid	7977.599
Durbin-Watson stat	2.026280		

(a) The Model (2.94)

Estimation Method: Iterative Least Squares
Date: 10/29/07 Time: 17:32
Sample: 1952Q2 1996Q4
Included observations: 179
Total system (unbalanced) observations 357
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-11.48307	9.235806	-1.243320	0.2146
C(12)	0.553349	0.220953	2.504372	0.0127
C(13)	0.972887	0.013988	69.55190	0.0000
C(21)	70.01101	8.688193	8.058179	0.0000
C(22)	0.173917	0.048897	3.556790	0.0004
C(23)	-0.042144	0.004812	-8.758172	0.0000
C(24)	0.962885	0.004505	213.7591	0.0000
C(25)	0.233159	0.075751	3.077965	0.0022

Determinant residual covariance 70195.73

Equation: Y1=C(11)+C(12)*T+C(13)*Y1(-1)
Observations: 179

R-squared	0.997073	Mean dependent var	940.4891
Adjusted R-squared	0.997040	S.D. dependent var	824.9564
S.E. of regression	44.88616	Sum squared resid	354599.1
Durbin-Watson stat	2.022607		

Equation: Y2=C(21)+C(22)*T+C(23)*Y1(-1)+C(24)*Y2(-1)+[AR(1)=C(25)]
Observations: 178

R-squared	0.999836	Mean dependent var	1469.344
Adjusted R-squared	0.999832	S.D. dependent var	523.9305
S.E. of regression	6.790676	Sum squared resid	7977.599
Durbin-Watson stat	2.026280		

(b) A Reduced Model of (2.94)

Figure 2.106 Statistical results based on (a) the model in (2.94) and (b) its reduced model

Example 2.42. (Application of the model in (2.94)) Figure 2.106 presents the statistical results based on the model in (2.94), as well as its reduced model.

Note that the reduced model is obtained by doing experimentation in order to delete either of the indicators $ar(1)$ or $y2(-1)$, or both, from the first regression. Three possible reduced models have therefore been observed, with the best one presented in Figure 2.106.

In fact, by considering the hypothesis H_0: $c(14) = c(15) = 0$, which is rejected based on the chi-square statistic of 0.177 511 with $df = 2$ and a very large p-value 0.9151, it can be concluded that the reduced model should be obtained by deleting both indicators $ar(1)$ and $y2(-1)$. □

2.15.2 The lagged endogenous variables: first autoregressive model with exogenous variables and trend

This model would be an extension of the model in (2.93), by adding a multivariate exogenous variable $\mathbf{X_t}$. The model can be presented in a matrix equation as follows:

$$\begin{aligned} \mathbf{Y_t} &= \mathbf{A} + \mathbf{B}^*\mathbf{t} + \mathbf{C}_1{}^*\mathbf{Y}_{\mathbf{t}-1} + \mathbf{C}_2{}^*\mathbf{X_t} + \boldsymbol{\mu}_\mathbf{t} \\ \boldsymbol{\mu}_\mathbf{t} &= \mathbf{D}^*\boldsymbol{\mu}_{\mathbf{t}-1} + \boldsymbol{\varepsilon}_\mathbf{t} \end{aligned} \quad (2.95)$$

By using the set of variables X_1, X_2, X_3, Y_1 and Y_2, and the time t-variable, a set of two additive regression models can be obtained, as follows:

$$\begin{aligned} y1_t &= c(11) + c(12)^*t + c(13)^*y1_{t-1} + c(14)^*y2_{t-1} \\ &\quad + c(15)^*x1_t + c(16)^*x2_t + c(17)^*x3_1 + [ar(1) = c(18)] \\ y2_t &= c(21) + c(22)^*t + c(23)^*y1_{t-1} + c(24)^*y2_{t-1} \\ &\quad + c(25)^*x1_1 + c(26)^*x2_t + c(27)^*x3_t + [ar(1) = c(28)] \end{aligned} \tag{2.96}$$

Example 2.43. (Application of the model in (2.96)) By using the equation specification in (2.96), the 'Near singular matrix' error message is obtained. Then by using the trial-and-error method, the model presented in Figure 2.107 is obtained. Based on this figure, the following notes and conclusions can be made:

Estimation Method: Iterative Least Squares
Date: 10/29/07 Time: 17:59
Sample: 1952Q2 1996Q4
Included observations: 179
Total system (unbalanced) observations 357
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-560.6763	150.5721	-3.723640	0.0002
C(12)	-0.960379	0.464038	-2.069615	0.0392
C(13)	0.929033	0.030037	30.92977	0.0000
C(14)	0.237771	0.066337	3.584269	0.0004
C(15)	0.125478	0.063660	1.971049	0.0495
C(16)	619.8114	187.5358	3.305029	0.0010
C(21)	40.28360	15.38110	2.619033	0.0092
C(22)	0.104190	0.056064	1.858394	0.0640
C(23)	-0.040385	0.004644	-8.696151	0.0000
C(24)	0.977725	0.007787	125.5613	0.0000
C(25)	0.027954	0.012069	2.316199	0.0211
C(26)	0.193725	0.076530	2.531348	0.0118
Determinant residual covariance		62576.49		

Equation: Y1=C(11)+C(12)*T+C(13)*Y1(-1)+C(14)*Y2(-1)+C(15)*X1+C(16)*X2
Observations: 179

R-squared	0.997290	Mean dependent var	940.4891
Adjusted R-squared	0.997212	S.D. dependent var	824.9664
S.E. of regression	43.56299	Sum squared resid	328308.0
Durbin-Watson stat	2.060604		

Equation: Y2=C(21)+C(22)*T+C(23)*Y1(-1)+C(24)*Y2(-1)+C(25)*X1+[AR(1)=C(26)]
Observations: 178

R-squared	0.999841	Mean dependent var	1469.344
Adjusted R-squared	0.999836	S.D. dependent var	523.9305
S.E. of regression	6.711784	Sum squared resid	7748.263
Durbin-Watson stat	2.008144		

Figure 2.107 Statistical results based on a reduced model of (2.96)

(1) The indicator $ar(1)$ in the first regression has a large p-value. Therefore, this indicator can be deleted from the first regression, since the lagged variable $Y1(-1)$ is already in the model in order to take into account the serial or autocorrelation problem.
(2) In the second regression, t also has a large p-value. However, since the model with trend is being considered, it should not be deleted from the regression.
(3) The two regressions have different sets of exogenous variables; namely the first regression does not have $X3$ and the second regression does not have $X2$ and $X3$. □

2.15.3 *The mixed lagged variables: first autoregressive model with trend*

This model can be considered as an extension of the model in (2.95), with the following matrix equation:

$$\begin{aligned} \mathbf{Y_t} &= \mathbf{A} + \mathbf{B}^*\mathbf{t} + \mathbf{C}_1{}^*\mathbf{Y_{t-1}} + \mathbf{C}_2{}^*\mathbf{X_t} + \boldsymbol{\mu}_\mathbf{t} \\ \boldsymbol{\mu}_\mathbf{t} &= \mathbf{D}^*\boldsymbol{\mu}_\mathbf{t-1} + \boldsymbol{\varepsilon}_\mathbf{1,t} \\ \mathbf{X_t} &= \mathbf{E} + \mathbf{F}^*\mathbf{X_{t-1}} + \boldsymbol{\nu}_\mathbf{t} \\ \boldsymbol{\nu}_\mathbf{t} &= \mathbf{G}^*\boldsymbol{\nu}_\mathbf{t-1} + \boldsymbol{\varepsilon}_\mathbf{2,t} \end{aligned} \tag{2.97}$$

By using the set of variables X_1, X_2, X_3, Y_1 and Y_2, and the time t-variable, a set of five regressions would be found, as follows:

$$\begin{aligned}
y1_t &= c(11) + c(12)*t + c(13)*y1_{t-1} + c(14)*y2_{t-1} \\
&\quad + c(15)*x1_t + c(16)*x2_t + c(17)*x3_1 + [ar(1) = c(18)] \\
y2_t &= c(21) + c(22)*t + c(23)*y1_{t-1} + c(24)*y2_{t-1} \\
&\quad + c(25)*x1_1 + c(26)*x2_t + c(27)*x3_t + [ar(1) = c(28)] \\
x1_t &= c(31) + c(32)*x1_{t-1} + c(33)*x2_{t-1} + c(34)*x3_{t-1} + [ar(1) = c(35)] \\
x2_t &= c(41) + c(42)*x1_{t-1} + c(43)*x2_{t-1} + c(44)*x3_{t-1} + [ar(1) = c(45)] \\
x3_t &= c(51) + c(52)*x1_{t-1} + c(53)*x2_{t-1} + c(54)*x3_{t-1} + [ar(1) = c(55)]
\end{aligned} \tag{2.98}$$

Note that the first two regressions are exactly the same as the model in (2.96) and show the effects of the bivariate $\{Y1_{t-1}, Y2_{t-1}\}$ on both $Y1_t$ and $Y2_t$. These two regressions also show that the four exogenous variables $X1_t, X2_t, X3_t$ and the time t have effects on both $Y1_t$ and $Y2_t$. The last three regressions show a trivariate model of the exogenous variables $X1$, $X2$ and $X3$ without trend. This multivariate model is an AR(1) additive linear model based on the model parameters $C(ij)$, and is also linear based on all independent variables.

The association patterns between the variables in this model are presented as a path diagram in Figure 2.108. Note the different sets of arrows, as follows:

(1) The first two regressions have dependent variables $Y1_t$ and $Y2_t$, with independent variables t, $Y1_{t-1}$ and $Y2_{t-1}$, and are presented by a set of six arrows in the right-hand box and a solid/thick arrow from a set of three exogenous variables, namely $X1_t$, $X2_t$ and $X3_t$, in the left-hand box.
(2) The last three regressions have dependent variables $X1_t$, $X2_t$ and $X3_t$, with their first lags, namely $X1_{t-1}$, $X2_{t-1}$ and $X3_{t-1}$, as independent variables and are presented in the left-hand box.
(3) It is well known that these five regressions do not consider the possible causal relationships between their independent variables. For example, the first two

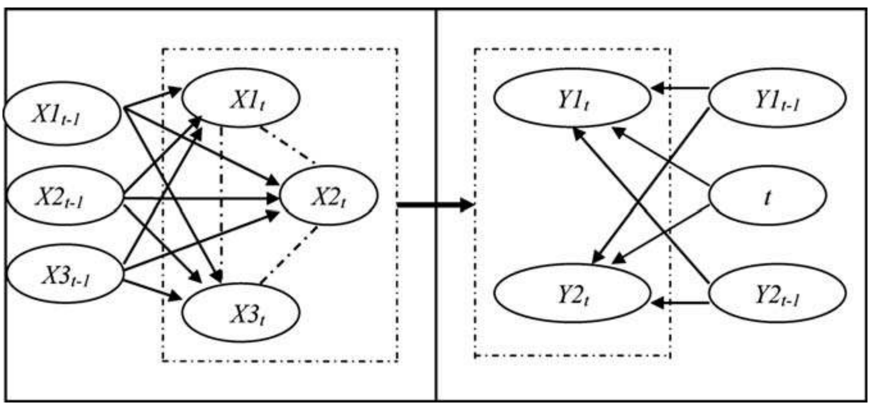


Figure 2.108 Path diagram of the model in (2.98)

regressions do not consider the type of relationships between the six independent variables $X1_t$, $X2_t$, $X3_t$, $Y1_{t-1}$, $Y2_{t-1}$ and t. Compare this with the interaction models presented in Sections 2.13.2 and 2.13.3. However, their quantitative coefficient of correlations or multicollinearity should have an unexpected impact on the estimates of the model parameters.

(4) The multivariate model or the system equations in (2.98) can easily be modified in order to produce many alternative time series models by using the transformed variables, such as their natural logarithms, their higher lagged variables, as well as their first differences.

Example 2.44. (Experimentation based on the model in (2.98)) Figure 2.109 presents the statistical results based on the model in (2.98). Based on this model, the following notes and conclusions are presented:

(1) The equations of each regression model can easily be written, as well as each regression function.

(2) By observing the probability of the t-statistic, it can be concluded whether a regression should be modified or not. The following are examples:

- Indicator AR(1) in the first regression has a large p-value = 0.6172; this indicator can be deleted to obtain a reduced model. The regression will become an LV(1) model with exogenous variables and trend.

Estimation Method: Iterative Least Squares
Date: 10/30/07 Time: 07:41
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 890
Convergence achieved after 14 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-522.7430	151.1793	-3.457768	0.0006
C(12)	-1.003591	0.460192	-2.180807	0.0295
C(13)	0.937533	0.031395	29.86247	0.0000
C(14)	0.222889	0.066227	3.365559	0.0008
C(15)	0.169073	0.078322	2.158700	0.0311
C(16)	523.2973	205.9769	2.540563	0.0112
C(17)	1.927748	2.155494	0.894341	0.3714
C(18)	-0.041215	0.082431	-0.499992	0.6172
C(21)	94.32897	29.99516	3.144807	0.0017
C(22)	0.230147	0.091847	2.505762	0.0124
C(23)	-0.033560	0.005837	-5.749668	0.0000
C(24)	0.955321	0.013155	72.62027	0.0000
C(25)	0.036076	0.015079	2.392423	0.0170
C(26)	-87.00120	39.91281	-2.179781	0.0295
C(27)	0.470303	0.404738	1.161995	0.2456
C(28)	0.195587	0.080677	2.424325	0.0155
C(31)	-6.053325	1.891646	-3.200031	0.0014
C(32)	0.917617	0.015369	59.70753	0.0000
C(33)	112.2713	18.38860	6.105485	0.0000
C(34)	-1.657484	0.387315	-4.279423	0.0000
C(35)	0.111939	0.079456	1.408815	0.1593
C(41)	-0.000745	0.001141	-0.652442	0.5143
C(42)	-1.70E-06	6.56E-06	-0.259086	0.7956
C(43)	1.007055	0.007761	129.7521	0.0000
C(44)	0.000547	0.000140	3.899580	0.0001
C(45)	0.735635	0.058408	12.59476	0.0000
C(51)	4.966381	3.178984	1.562254	0.1186
C(52)	0.007582	0.006974	1.087222	0.2772
C(53)	-7.915720	9.860126	-0.802801	0.4223
C(54)	0.246164	0.078569	3.133082	0.0018
C(55)	0.951781	0.031271	30.43693	0.0000
Determinant residual covariance		3.960940		

Equation: Y1=C(11)+C(12)*T+C(13)*Y1(-1)+C(14)*Y2(-1)+C(15)*X1+C(16)*X2+C(17)*X3+[AR(1)=C(18)]
Observations: 178

R-squared	0.997289	Mean dependent var	945.2776
Adjusted R-squared	0.997178	S.D. dependent var	824.7950
S.E. of regression	43.81723	Sum squared resid	326391.4
Durbin-Watson stat	2.001411		

Equation: Y2=C(21)+C(22)*T+C(23)*Y1(-1)+C(24)*Y2(-1)+C(25)*X1+C(26)*X2+C(27)*X3+[AR(1)=C(28)]
Observations: 178

R-squared	0.999845	Mean dependent var	1469.344
Adjusted R-squared	0.999839	S.D. dependent var	523.9305
S.E. of regression	6.649787	Sum squared resid	7517.343
Durbin-Watson stat	2.007020		

Equation: X1=C(31)++C(32)*X1(-1)+C(33)*X2(-1)+C(34)*X3(-1)+[AR(1)=C(35)]
Observations: 178

R-squared	0.999454	Mean dependent var	448.5793
Adjusted R-squared	0.999442	S.D. dependent var	345.1043
S.E. of regression	8.155296	Sum squared resid	11506.03
Durbin-Watson stat	2.002214		

Equation: X2=C(41)++C(42)*X1(-1)+C(43)*X2(-1)+C(44)*X3(-1)+[AR(1)=C(45)]
Observations: 178

R-squared	0.999978	Mean dependent var	0.517659
Adjusted R-squared	0.999977	S.D. dependent var	0.303315
S.E. of regression	0.001443	Sum squared resid	0.000360
Durbin-Watson stat	2.270595		

Equation: X3=C(51)++C(52)*X1(-1)+C(53)*X2(-1)+C(54)*X3(-1)+[AR(1)=C(55)]
Observations: 178

R-squared	0.932425	Mean dependent var	5.455109
Adjusted R-squared	0.930862	S.D. dependent var	2.897672
S.E. of regression	0.761915	Sum squared resid	100.4290
Durbin-Watson stat	1.869969		

Figure 2.109 Statistical results based on a reduced model of (2.98) by using the least squares estimation method

- On the other hand, the regression of $X3$ shows two parameters, namely $C(52)$ and $C(53)$, that have large p-values, corresponding to the independent variables $X1(-1)$ and $X2(-1)$. Even though $X3(-1)$ has a significant effect, a choice can be made to delete either one of the three independent variables, based on which variable is the least important variable, in a theoretical sense. Since the data is a hypothetical data set, do this as an exercise based on the empirical data set.
- For further illustration purposes, a test is carried out to discover the joint effects of the first lagged variables $X1(-1)$ and $X2(-1)$ on $X3$. It was found that the null hypothesis H_0: $C(52) = C(53) = 0$ is accepted based on the chi-square-statistic of 0.110 607 with $df = 2$ and a very large p-value = 0.9462. In a statistical sense, this indicates that both $X1(-1)$ and $X2(-2)$ can be deleted to obtain a reduced model.

(3) As a result, it could be said that many estimable five-dimensional multivariate models can be constructed based on any set of five-dimensional time series. □

2.16 Generalized multivariate models with time-related effects

A further extension of the multivariate models with trend is a multivariate model with *time-related effects*. In this type of model, there are two-way interactions between the time t and each of selected exogenous variables, as additional independent or exogenous variables of each regression in the system. Since the equation of this type of model could easily be derived from the previous illustrative models, they will not be presented again in detail. For example, based on the model in (2.95), the following general model may be obtained:

$$\begin{aligned} \mathbf{Y_t} &= \mathbf{A} + \mathbf{B}{*}\mathbf{t} + \mathbf{C}_1{*}\mathbf{Y}_{\mathbf{t}-1} + \mathbf{C}_2{*}\mathbf{X_t} + [\mathbf{C}_3{*}\mathbf{Y}_{\mathbf{t}-1} + \mathbf{C}{*}\mathbf{X_t}]{*}\mathbf{t} + \boldsymbol{\mu}_\mathbf{t} \\ \boldsymbol{\mu}_\mathbf{t} &= \mathbf{D}{*}\boldsymbol{\mu}_{\mathbf{t}-1} + \boldsymbol{\varepsilon}_\mathbf{t} \end{aligned} \tag{2.99}$$

Note that this model could easily be extended to more complex models, which can be derived from other models described in the previous subsections. As an illustrative example from International Journals, Bansal (2005) presents a data analysis based on a multiple regression or univariate linear model with trend and *time-related effects*. The following example presents a simple bivariate model with trend and time-related effects.

Example 2.45. (Bivariate model with time-related effects) Corresponding to the models with endogenous variables, Y_1 and Y_2, presented in the previous exam- ples, experimentation is performed in order to obtain a bivariate model with time-related effects, where each regression has a DW-statistic of around 2.0. Finally, a pair of regressions with trend and time-related effects was found, as presented in Figure 2.110, with DW-statistics of 2.01 and 2.07 respectively.

Based on this figure, the following notes and conclusions are presented:

(1) The first regression is a first-order lagged-variable regression, namely an LV(1) regression with trend and time-related effects, and the second regression is an LV_AR(1,1) model with trend and time-related effects.

Estimation Method: Weighted Least Squares
Date: 11/03/07 Time: 10:28
Sample: 1952Q2 1996Q4
Included observations: 179
Total system (unbalanced) observations 357
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 5 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	984.2853	516.1314	1.907044	0.0574
C(12)	0.448451	0.195555	2.293222	0.0224
C(13)	-0.536379	0.261001	-2.055081	0.0406
C(14)	0.383477	0.406564	0.943215	0.3462
C(15)	-13.00427	5.034747	-2.582903	0.0102
C(16)	0.004817	0.001850	2.603383	0.0096
C(17)	0.007061	0.002710	2.605482	0.0096
C(18)	-0.000657	0.002863	-0.229452	0.8187
C(21)	-622.0046	224.2482	-2.773733	0.0058
C(22)	1.283732	0.095269	13.47477	0.0000
C(23)	0.124130	0.092349	1.344142	0.1798
C(24)	421.1048	275.7166	1.527310	0.1276
C(25)	6.559586	1.917498	3.420908	0.0007
C(26)	-0.003172	0.000932	-3.403896	0.0007
C(27)	-0.001188	0.000615	-1.932153	0.0542
C(28)	-4.108435	2.035393	-2.018497	0.0443
C(29)	0.351847	0.074263	4.737873	0.0000
Determinant residual covariance		60079.47		

Equation: Y1=C(11)+C(12)*Y1(-1)+C(13)*Y2+C(14)*X1+C(15)*T+C(16)*T*Y1(-1)+C(17)*T*Y2+C(18)*T*X1
Observations: 179

R-squared	0.997248	Mean dependent var	940.4891
Adjusted R-squared	0.997135	S.D. dependent var	824.9664
S.E. of regression	44.15695	Sum squared resid	333422.1
Durbin-Watson stat	1.974969		

Equation: Y2=C(21)+C(22)*Y2(-1)+C(23)*X1+C(24)*X2+C(25)*T+C(26)*T*Y2(-1)+C(27)*T*X1+C(28)*T*X2+[AR(1)=C(29)]
Observations: 178

R-squared	0.999844	Mean dependent var	1469.344
Adjusted R-squared	0.999836	S.D. dependent var	523.9305
S.E. of regression	6.702661	Sum squared resid	7592.437
Durbin-Watson stat	2.096936		

Figure 2.110 Statistical results based on a model with trend and time-related effects

(2) Even though t^*X1 has an insignificant adjusted effect on $Y1$, the three interaction factors, namely $t^*Y1(-1)$, t^*Y2 and t^*X1 have a significant joint effect on $Y1$, based on the chi-squared-statistic of 9.883 673 with $df = 3$ and a p-value 0.0196. Therefore, this regression shows that the effect of t on $Y1$ is significantly dependent on the variables $Y1(-1)$, $Y2$ and $X1$; specifically it is dependent on the function $[-13.004 + 0.005^*Y1(-1) + 0.007^*\ Y2 - 0.001^*X1]$, as presented by the following regression function:

$$\begin{aligned}\hat{Y}1 &= 984.285 + 0.448*Y1(-1) - 0.536*Y2 + 0.383*X1 \\ &\quad + [-13.004 + 0.005*Y1(-1) + 0.007*Y2 - 0.001*X1]*t\end{aligned} \quad (2.100)$$

(3) On the other hand, note that $X1$ and t^*X1 do not have significant effects on $Y1$, so a reduced model may be produced by deleting either one or both of them. However, at a significant level of $\alpha = 0.10$, the joint effect of $X1$ and t^*X1 is significant when based on the chi-squared-statistic of 5.670 178 with $df = 2$ and a p-value $= 0.0587$.

(4) Therefore, there may be two alternative models. By deleting $X1$, the statistical results presented in Figure 2.111 are obtained and by deleting t^*X1 the statistical results in Figure 2.112 are obtained. Both of these models should be considered as good fit models.

(5) These findings indicate that even though two or more independent variables have insignificant adjusted effects, all of those variables should not be deleted in order to have a statistically acceptable reduced model, even though, in some cases, it has been recognized that an independent variable should be deleted that has a significant adjusted effect.

(6) Note that by deleting $X1$ from the model in Figure 2.110, the printout in Figure 2.111 is obtained without the parameter $C(14)$. Note that the symbols of the model parameters do not need to be modified. On the other hand, by deleting t^*X1, the printout in Figure 2.112 is obtained without the parameter $C(18)$.

Estimation Method: Weighted Least Squares
Date: 11/03/07 Time: 10:38
Sample: 1952Q2 1996Q4
Included observations: 179
Total system (unbalanced) observations 357
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 5 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	1254.329	430.5114	2.913578	0.0038
C(12)	0.485861	0.191966	2.530976	0.0118
C(13)	-0.661673	0.225226	-2.937817	0.0035
C(15)	-14.34665	4.841394	-2.963331	0.0033
C(16)	0.004323	0.001779	2.430056	0.0156
C(17)	0.007858	0.002581	3.044541	0.0025
C(18)	0.001914	0.000878	2.181027	0.0299
C(21)	-622.0046	224.2482	-2.773733	0.0058
C(22)	1.283732	0.095269	13.47477	0.0000
C(23)	0.124130	0.092349	1.344142	0.1798
C(24)	421.1048	275.7166	1.527310	0.1276
C(25)	6.559586	1.917498	3.420908	0.0007
C(26)	-0.003172	0.000932	-3.403896	0.0007
C(27)	-0.001188	0.000615	-1.932153	0.0542
C(28)	-4.108435	2.035393	-2.018497	0.0443
C(29)	0.351847	0.074263	4.737873	0.0000
Determinant residual covariance		60585.53		

Equation: Y1=C(11)+C(12)*Y1(-1)+C(13)*Y2+C(15)*T+C(16)*T*Y1(-1)
+C(17)*T*Y2+C(18)*T*X1
Observations: 179

R-squared	0.997234	Mean dependent var	940.4891
Adjusted R-squared	0.997137	S.D. dependent var	824.9664
S.E. of regression	44.13768	Sum squared resid	335079.2
Durbin-Watson stat	1.950194		

Equation: Y2=C(21)+C(22)*Y2(-1)+C(23)*X1+C(24)*X2+C(25)*T+C(26)*T
*Y2(-1)+C(27)*T*X1+C(28)*T*X2+[AR(1)=C(29)]
Observations: 178

R-squared	0.999844	Mean dependent var	1469.344
Adjusted R-squared	0.999836	S.D. dependent var	523.9305
S.E. of regression	6.702661	Sum squared resid	7592.437
Durbin-Watson stat	2.096936		

Figure 2.111 Statistical results based on a reduced model in Figure 2.110

(7) Since the three models considered are statistically acceptable models, then which one could be considered as the best model. There would be some reasons to select any of the models as the best one. However, from the author's point of view, the model in Figure 2.111 is considered as the best model, since the adjusted effect of t on $Y1$ is highly dependent on each of the other independent variables.

(8) At a significant level of $\alpha = 0.10$, Figure 2.111 shows that each of the independent variables $X1$ and $X2$ has an insignificant adjusted effect on $Y2$ with p-values of 0.1798 and 0.1276 respectively. A reduced model does not need to be constructed, since each of these variables in fact has a significant positive effect on $Y2$ with p-values of $0.1798/2 = 0.0899$ and $0.1276/2 = 0.0638$ respectively, which are less than 0.10. □

Estimation Method: Iterative Least Squares
Date: 11/03/07 Time: 10:46
Sample: 1952Q2 1996Q4
Included observations: 179
Total system (unbalanced) observations 357
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	1060.681	402.3883	2.635963	0.0088
C(12)	0.455759	0.196860	2.315147	0.0212
C(13)	-0.572801	0.211389	-2.709702	0.0071
C(14)	0.294658	0.126845	2.322986	0.0208
C(15)	-13.47356	4.693978	-2.870392	0.0044
C(16)	0.004706	0.001822	2.582551	0.0102
C(17)	0.007325	0.002503	2.926913	0.0037
C(21)	-622.0064	230.1354	-2.702784	0.0072
C(22)	1.283733	0.097770	13.13007	0.0000
C(23)	0.124127	0.094773	1.309721	0.1912
C(24)	421.1097	282.9550	1.488257	0.1376
C(25)	6.559610	1.967837	3.333411	0.0010
C(26)	-0.003172	0.000956	-3.316831	0.0010
C(27)	-0.001188	0.000631	-1.882698	0.0606
C(28)	-4.108472	2.088827	-1.966880	0.0500
C(29)	0.351848	0.076214	4.616588	0.0000
Determinant residual covariance		60094.15		

Equation: Y1=C(11)+C(12)*Y1(-1)+C(13)*Y2+C(14)*X1+C(15)*T+C(16)*T
*Y1(-1)+C(17)*T*Y2
Observations: 179

R-squared	0.997247	Mean dependent var	940.4891
Adjusted R-squared	0.997151	S.D. dependent var	824.9664
S.E. of regression	44.03488	Sum squared resid	333520.1
Durbin-Watson stat	1.968780		

Equation: Y2=C(21)+C(22)*Y2(-1)+C(23)*X1+C(24)*X2+C(25)*T+C(26)*T
*Y2(-1)+C(27)*T*X1+C(28)*T*X2+[AR(1)=C(29)]
Observations: 178

R-squared	0.999844	Mean dependent var	1469.344
Adjusted R-squared	0.999836	S.D. dependent var	523.9305
S.E. of regression	6.702661	Sum squared resid	7592.437
Durbin-Watson stat	2.096939		

Figure 2.112 Statistical results based on another reduced model

3

Discontinuous growth models

3.1 Introduction

In the previous chapter, continuous growth models were presented. Although the time t-variable is a discrete variable, the corresponding regression functions are differentiable with respect to the t-variable. However, in many cases, it was found that it is more appropriate to apply discontinuous growth models, either piecewise or step growth models, over the whole period of the observation time. For example, for the *RS* (retail sales) variable, its scatter plot supports the use of at least two pieces of a growth model. In some cases, a perfect judgment could be made using a discontinuous growth model, even before taking or having the corresponding time series data: for example, the growth of international tourists to Indonesia within three time periods, before the first Bali bomb in 2002, between the first and the second bomb in 2005 and after the second bomb. The impact of economic crises on Indonesia has been studied by the Demographic Institute, Faculty of Economics, University of Indonesia, and sponsored by the World Bank (Agung, dkk., 1999b).

This chapter will present illustrative examples of piecewise and step regression models, based on the data set in the Demo_Modified workfile. Note that a discontinuous growth model should consist of two or more pieces of a continuous growth model. As a result, each piece of the discontinuous growth model can easily be derived by using all continuous growth models presented in Chapter 2.

3.2 Piecewise growth models

Figure 3.1 presents illustrative graphs of four piecewise linear regression functions. The graphs in Figure 3.1(a) and (b) present two polygon graphs or broken lines at one point, say at $t = t_1$, and at two points, say $t = t_1$ and $t = t_2$ respectively, and the graphs in Figure 3.1 (c) and (d) present the step linear regression functions at one point and two points respectively.

In order to present the equations of discontinuous growth models, dummy variables of the time t-variable should be defined or generated.

Time Series Data Analysis Using EViews IGN Agung

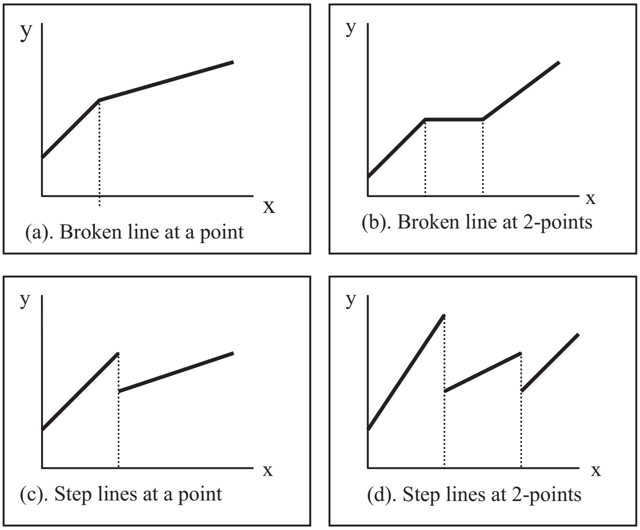


Figure 3.1 Illustrative piecewise simple regression functions

Piecewise regression models have been presented in several books in applied statistics, such as Neter and Wasserman (1974) and Agung (1998, 1992b, 1992c, and 1992d). Agung presented a special two-piece growth model called GPP (*Garis Patah Paritas*—Broken Line of Parities) by age of mothers, as well as three-piece regressions.

3.2.1 Two-piece classical growth models

Corresponding to the classical growth model in (2.3), it should be easy to derive a two-piece classical growth model by using a dummy variable of two defined time periods. For example, the two-piece growth model has one discontinuity or break point at time $t = t_1$; then there are two dummy variables defined as $D1 = 1$ if $t =< t_1$ and $D1 = 0$ if otherwise and $D2 = 0$ if $t =< t_1$ and $D2 = 1$ if otherwise. Hence, the following alternative piecewise growth models exist.

3.2.1.1 General two-piece growth model with an intercept

This model has a general form as follows:

$$\begin{aligned}\log(y_t) &= (C(1) + C(2)^* t) + C(3)^* D2 + C(4)^* t^* D2 + \mu_t \\ &= (C(1) + C(2)^* t) + (C(3) + C(4)^* t)^* D2 + \mu_t\end{aligned} \qquad (3.1)$$

In fact, this model represents the two classical growth models, as follows:

$$\log(y_t) = C(1) + C(2)^* t + \mu_t \quad \text{for} \quad t \leq t_1 \qquad (3.2a)$$

$$\log(y_t) = (C(1) + C(3)) + (C(2) + C(4))^* t + \mu_t \quad \text{for} \quad t > t_1 \qquad (3.2b)$$

Note that the model in (3.2a) shows that $C(2)$ is the growth rate of Y_t during the time period $t \leq t_1$ and $(C(2) + C(4))$ is the growth rate of Y_t during the time period $t > t_1$, based on the model in (3.2b).

The main objective of this model is to test the hypothesis of the growth rate difference between the two time periods considered. In general, the hypothesis should be the one-sided hypothesis. For example, for a right-hand hypothesis,

$$H_0 : C(4) \leq 0 \quad \text{versus} \quad H_1 : C(4) > 0 \tag{3.3}$$

3.2.1.2 General two-piece growth model without an intercept

This model has a general form as follows:

$$\log(y_t) = (C(1) + C(2)^* t)^* D1 + (C(3) + C(4)^* t)^* D2 + \mu_t \tag{3.4}$$

or

$$\begin{aligned} \log(y_t) &= C(1) + C(2)^* t \quad \text{for} \quad t \leq t_1 \\ \log(y_t) &= C(3) + C(4)^* t \quad \text{for} \quad t > t_1 \end{aligned} \tag{3.5}$$

The main objective of this model is to test the one-sided hypothesis on the growth rates of Y_t within each defined time period. For example: (i) H_0: $C(2) \leq 0$ versus H_1: $C(2) > 0$ for the growth rate of Y_t in the first time period and (ii) H_0: $C(4) \leq 0$ versus H_1: $C(4) > 0$ for the growth rate in the second time period.

Note that the model in (3.4) is called the model without intercept, corresponding to the two dummy independent variables. However, the two regressions in (3.5) within each time period represent models with intercepts $C(1)$ and $C(3)$ respectively.

3.2.1.3 Classical growth model having a corner point

A general piecewise growth model that has a corner point at time $t = t_1$ can be presented as

$$\log(y_t) = C(1) + C(2)^* t + (C(3)^* D2^* (t - t_1) + \mu_t \tag{3.6}$$

This model represents the following two regressions, with an intercept at $t = t_1$ and $\log(y_t) = C(1) + C(2)^* t_1$:

$$\begin{aligned} \log(y_t) &= C(1) + C(2)^* t + \mu_t, \quad \forall t \leq t_1 \\ \log(y_t) &= C(1) + C(2)^* t + C(3)^* (t - t_1) + \mu_t, \quad \forall t > t_1 \end{aligned} \tag{3.7}$$

where $C(2)$ and $[C(2) + C(3)]$ are the growth rates of Y_t in the first and second time periods respectively. Note that this model is in fact a special case of the models in (3.1) or (3.4). Furthermore, the two-piece growth model in (3.6) can also be presented by using the dummy variable $D1$, which gives the following equation:

$$\log(y_t) = C(1) + C(2)^* t + C(3)^* D1^* (t - t_1) + \mu_t \tag{3.8}$$

This model represents the following two regressions, with an intercept at $t = t_1$ and $\log\,(y_t) = C(1) + C(2)^* t_1$:

$$\begin{aligned} \log(y_t) &= C(1) + C(2)^* t + C(3)^* (t - t_1) + \mu_t, \quad \forall\, t \leq t_1 \\ \log(y_t) &= C(1) + C(2)^* t + \mu_t, \quad \forall\, t > t_1 \end{aligned} \tag{3.9}$$

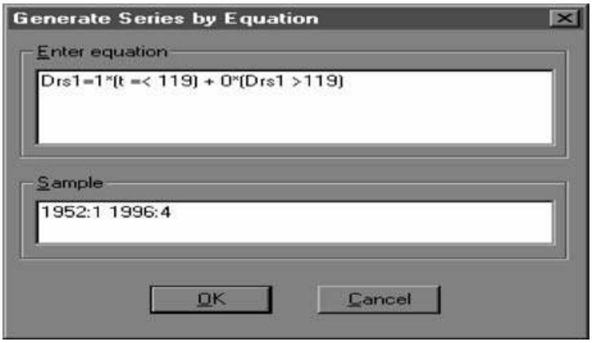


Figure 3.2 The window to generate series by equation

Example 3.1. (To generate dummy variables) Note that the graph of *RS* (retail sales) by time and its descriptive statistics show that a two-piece growth model can be presented with a breakpoint at time $t = 119$. Hence, in the first stage two dummy variables should be defined, namely *Drs*1 and *Drs*2, using the following steps:

(1) After opening the Demo_Modified workfile, click *Genr*...; the window in Figure 3.2 will be seen on the screen. Then enter the equation $Drs1 = 1^*(t \leq 119) + 0^*(t > 119)$ in the '*Enter Equation*' space.
(2) Click *OK* ... and an additional variable *Drs*1 will appear in the data set.
(3) Go through the same process to generate the second dummy variable. Again click *Genr*... and enter the equation $Drs2 = 1^*(Drs1 = 0) + 0^*(Drs1 = 1))$ in the '*Enter Equation*' window; then click *OK* ....
(4) In order to make sure that the correct dummy variables have been generated, block the variables and then click *View/Show* ... *OK*. □

Example 3.2. (Piecewise growth model for *RS*) Since this concerns time series data, an example is presented using an AR(1) growth model. Entering

$$\log(rs) = C(1) + C(2)^*t + C(3)^*Drs2 + C(4)^*Drs2^*t + [ar(1) = c(5)] \quad (3.10)$$

in the '*Equation specification*' window would give the results in Figure 3.3, with its residual graph in Figure 3.4.

Based on these results, the following conclusions can be derived:

(a) The growth rate of *RS* is $C(2) = 0.016\,755$ for $t \leq 119$ and $C(2) + C(4) = -0.012\,407$ for $t > 119$.

Dependent Variable: LOG(RS)
Method: Least Squares
Date: 10/15/07 Time: 11:30
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 5 iterations
LOG(RS)=C(1)+C(2)*T+C(3)*DRS2+C(4)*T*DRS2+[AR(1)=C(5)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.393028	0.206638	1.902012	0.0588
C(2)	0.016755	0.002645	6.334075	0.0000
C(3)	3.282094	0.821950	3.993058	0.0001
C(4)	-0.029162	0.006577	-4.433915	0.0000
C(5)	0.866467	0.038464	22.52657	0.0000

R-squared	0.943743	Mean dependent var	1.542701
Adjusted R-squared	0.942450	S.D. dependent var	0.576753
S.E. of regression	0.138361	Akaike info criterion	-1.090372
Sum squared resid	3.330993	Schwarz criterion	-1.001339
Log likelihood	102.5883	Hannan-Quinn criter.	-1.054270
F-statistic	729.7414	Durbin-Watson stat	1.211577
Prob(F-statistic)	0.000000		

Inverted AR Roots	.87

Figure 3.3 Statistical results based on the model in (3.8)

(b) The growth rate of *RS* for $t > 119$ is significantly lower than the growth rate of *RS* for $t \leq 119$, because the hypothesis H_0: $C(4) \geq 0$ is rejected. This is based on the t-test, with $t_0 = -4.43$ and a p-value of $0.0000/2 = 0.0000$.

(c) The growth rate of *RS* for $t \leq 119$ is significantly positive, because the null hypothesis H_0: $C(2) \leq 0$ is rejected; this is based on the t-test, with $t_0 = 6.33$ and a p-value of $0.0000/2 = 0.0000$.

(d) The small value of $DW = 1.21$ indicates that the model should be modified by using a higher-order autoregressive model. By using the trial-and-error methods, a fourth-order autoregressive growth model having $DW = 2.04$ is obtained, as presented in Figure 3.5, with its residual graph in Figure 3.6.

(e) In order to test the hypothesis on the growth rate of *RS* for $t > 119$, two kinds of hypotheses will be considered. The first hypothesis is a two-sided hypothesis:

$$H_0 : C(2) + C(4) = 0 \text{ versus } H_1 : C(2) + C(4) \neq 0 \tag{3.11}$$

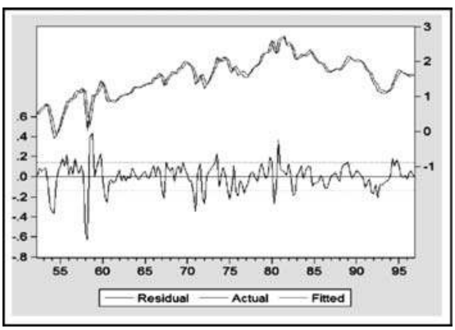


Figure 3.4 Residual graph of the regression in Figure 3.3

Dependent Variable: LOG(RS)
Method: Least Squares
Date: 10/15/07 Time: 11:36
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 5 iterations
LOG(RS)=C(1)+C(2)*T+C(3)*DRS2+C(4)*T*DRS2+[AR(1)=C(5),AR(2)=C(6),AR(3)=C(7),AR(4)=C(8)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.389068	0.112814	3.448753	0.0007
C(2)	0.016819	0.001486	11.31996	0.0000
C(3)	3.361288	0.517862	6.490708	0.0000
C(4)	-0.029844	0.004096	-7.286632	0.0000
C(5)	1.446201	0.073144	19.77188	0.0000
C(6)	-1.001877	0.122686	-8.166179	0.0000
C(7)	0.678876	0.122621	5.536386	0.0000
C(8)	-0.320660	0.073193	-4.381050	0.0000

R-squared	0.960597	Mean dependent var	1.558911
Adjusted R-squared	0.958956	S.D. dependent var	0.567913
S.E. of regression	0.115056	Akaike info criterion	-1.442412
Sum squared resid	2.223952	Schwarz criterion	-1.298299
Log likelihood	134.9322	Hannan-Quinn criter.	-1.383960
F-statistic	585.0975	Durbin-Watson stat	2.043661
Prob(F-statistic)	0.000000		

Inverted AR Roots	.77+.27i	.77-.27i	-.05-.69i	-.05+.69i

Figure 3.5 Statistical results based on an AR(4) growth model in (3.8)

and the second is a left-hand hypothesis:

$$H_0 : C(2) + C(4) \geq 0 \text{ versus } H_1 : C(2) + C(4) < 0 \qquad (3.12)$$

Both hypotheses can be tested using the Wald tests, by using the following processes:

(e.1) Click *View/Coefficient Tests/Wald-Coefficient Restriction ...*; then by entering the equation $C(2) + C(4) = 0$, the statistical results given in Figure 3.7 are obtained.

(e.2) Click *OK...*, which gives the needed statistical tests in Figure 3.7 on the right-hand side. This result shows that the null hypothesis in (3.11), $H_0 : C(2) + C(4) = 0$, is rejected based on the *F*- and *chi-square*-statistics with *p*-values = 0.0001. Hence, it can be concluded that *RS* has a significant growth rate for $t > 119$.

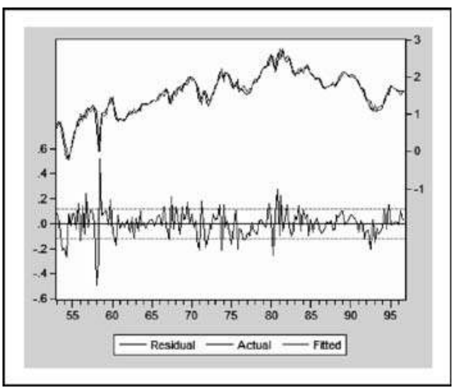
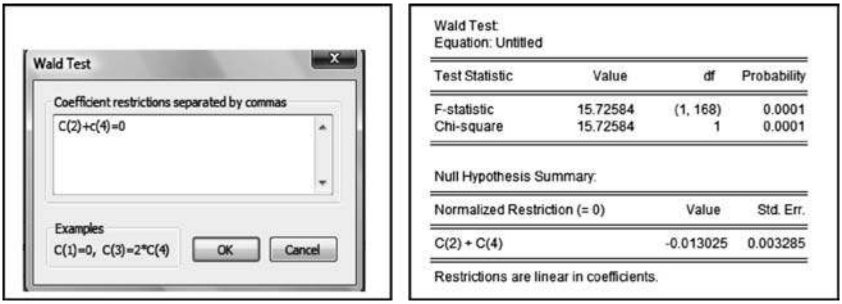


Figure 3.6 Residual graph of the regression in Figure 3.5



Wald Test
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	15.72584	(1, 168)	0.0001
Chi-square	15.72584	1	0.0001

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(2) + C(4)	-0.013025	0.003285

Restrictions are linear in coefficients.

Figure 3.7 The Wald test for the null hypothesis H_0: $C(2) + C(4) = 0$

(e.3) However, to test the left-hand hypothesis in (3.10), the t-statistic should be used. The observed value of the t-statistic can be easily computed by using the statistics in Figure 3.7, in the row of $C(2) + C(4)$, that is

$$t_0 = \text{Value}/\text{Std Err} = -\frac{0.013\,025}{0.003\,285} = -3.964\,992$$

Then, in making the conclusion of the testing hypothesis, two alternative methods are now presented, as follows:

(i) If the total number of observations is sufficiently large, that is $n > 20$ (Conover, 1980), the simplest method is to make the critical value of the t-test equal to $t_c = -2.0$ or -1.96; then the null hypothesis H_0: $C(2) + C(4) \geq 0$ is rejected, since the observed value of $t_0 = -3.964\,992 < t_c = -2$.

(ii) Note that the p-value of the F-statistic should be equal to the p-value of the two-sided t-test. Since $t_0 < 0$, then for testing the left-hand- sided hypothesis in (3.10), its p-value $= 0.0001/2 = 0.000\,05$.
Hence it can be concluded that, for $t > 119$, the time series RS has a significant negative growth rate.

(e.4) On the other hand, if a left-sided hypothesis that a growth rate of RS is less than -0.01 during the time period $t > 119$ needs to be tested, then the statistical hypothesis should be written as

$$H_0: 0.01 + C(2) + C(4) \geq 0 \quad \text{versus} \quad H_1: 0.01 + C(2) + C(4) < 0 \quad (3.13)$$

The statistical tests are obtained by entering $C(2) + C(4) = -0.01$ in the '*Coefficient restriction* …' window; then click *OK*, giving the results in Figure 3.8(a). Based on this result, the observed value of the t-statistic is found: $t_0 = -0.003\,025/0.003\,285 = -0.920\,852$. Since $t_0 > -2.0$, the null hypothesis is accepted.

(f) To test the effect of all independent variables t, $Drs2$ and $Drs2^{*}t$ on the dependent variable, the following hypothesis is used:

$$H_0: C(2) = C(3) = C(4) = 0 \quad \text{versus} \quad H_1: \text{If Otherwise} \qquad (3.14)$$

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	0.848272	(1, 168)	0.3584
Chi-square	0.848272	1	0.3570

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
0.01 + C(2) + C(4)	-0.003025	0.003285

Restrictions are linear in coefficients.

(a)

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	43.49318	(3, 168)	0.0000
Chi-square	130.4795	3	0.0000

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(2)	0.016819	0.001486
C(3)	3.361288	0.517862
C(4)	-0.029844	0.004096

Restrictions are linear in coefficients.

(b)

Figure 3.8 The Wald tests for testing (a) the hypothesis (3.13) and (b) the hypothesis (3.14)

The Wald test can be obtained by entering C(2) = C(3) = C(4) = 0, as presented in Figure 3.8(b). This result shows that the null hypothesis is rejected based on the *F*- and *chi-square*-statistics with *p*-values = 0.0000. □

Example 3.3. (Application of the model in (3.4)) Corresponding to the AR(4) growth model presented in the previous example, this example presents statistical results, as presented in Figure 3.9, based on the model without an intercept in (3.4), by entering the following equation specifications:

$$\begin{aligned} \log(rs) = {} & (C(1) + C(2)^* t)^* Drs1 + (C(3) + C(4)^* t)^* Drs2 \\ & + [ar(1) = c(5),\ ar(2) = c(6),\ ar(3) = c(7),\ ar(4) = c(8)] \end{aligned} \tag{3.15}$$

Based on this result, the *p*-values can be directly obtained for testing the one-sided hypothesis on the growth rates of *RS* during each time period. They are H_1: $C(2) > 0$

Dependent Variable: LOG(RS)
Method: Least Squares
Date: 10/15/07 Time: 11:53
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 5 iterations
LOG(RS)=(C(1)+C(2)*T)*DRS1 + (C(3)+C(4)*T)*DRS2+[AR(1)=C(5),AR(2)=C(6),AR(3)=C(7),AR(4)=C(8)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.389068	0.112814	3.448753	0.0007
C(2)	0.016819	0.001486	11.31996	0.0000
C(3)	3.750356	0.478703	7.834404	0.0000
C(4)	-0.013025	0.003285	-3.965582	0.0001
C(5)	1.446201	0.073144	19.77188	0.0000
C(6)	-1.001877	0.122686	-8.166179	0.0000
C(7)	0.678876	0.122621	5.536386	0.0000
C(8)	-0.320660	0.073193	-4.381050	0.0000

R-squared	0.960597	Mean dependent var	1.558911
Adjusted R-squared	0.958956	S.D. dependent var	0.567913
S.E. of regression	0.115056	Akaike info criterion	-1.442412
Sum squared resid	2.223952	Schwarz criterion	-1.298299
Log likelihood	134.9322	Hannan-Quinn criter.	-1.383960
Durbin-Watson stat	2.043661		

Inverted AR Roots	.77-.27i	.77+.27i	-.05-.69i	-.05+.69i

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	53.09500	(1, 168)	0.0000
Chi-square	53.09500	1	0.0000

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(2) - C(4)	0.029844	0.004096

Restrictions are linear in coefficients.

Figure 3.9 Statistical results and a Wald test based on the model in (3.15) and H_2: $C(4) < 0$. Note that the data support the hypothesis that RS has a significant positive growth rate of 0.0168 19 for $t \leq 119$ with a p-value = 0.0000/2 and a significant negative growth rate of −0.013 025 for $t > 119$ with a p-value = 0.0001/2. Furthermore, a test for the hypothesis that both growth rates of RS are equal to zero will be presented, with H_0: $C(2) = C(4) = 0$. Using the same process as presented above gives the result in Figure 3.9 on the right-hand side. Hence, the null hypothesis is rejected based on the F-test, with $F_0 = 64.137\,18$, $df = (2, 168)$ and a p-value = 0.0000.

On the other hand, a similar process may also be used to test the hypothesis that both growth rates of RS are equal, with $H_0 : C(2) = C(4)$. The null hypothesis is rejected based on the F-statistic of $F_0 = 53.095\,00$ with $df = (1, 168)$ and a p-value = 0.0000. □

3.3 Piecewise S-shape growth models

3.3.1 Two-piece linear growth models

Corresponding to the S-shape or bounded growth model in (2.16), a basic two-piece S-shape growth model should be defined as

$$\log\left(\frac{Y_t - L_1}{U_1 - Y_t}\right) = C(1) + C(2)^* t + \mu_t \quad \text{for} \quad t \leq t_1 \tag{3.16a}$$

$$\log\left(\frac{Y_t - L_2}{U_2 - Y_t}\right) = C(3) + C(4)^* t + \mu_t \quad \text{for} \quad t > t_1 \tag{3.16b}$$

where L_1 and U_1 are defined (subjectively selected) fixed values of lower and upper bounds of Y_t in the time period $t \leq t_1$ and L_2 and U_2 are defined values of lower and upper bounds of Y_t in the time period $t > t_1$. Note that, in some cases, $L_2 = U_1$.

In order to perform the data analysis, first a new variable or series should be generated, namely Lny, such as

$$\begin{aligned} Lny &= \log((Y - L_1)/(U_1 - Y)) \quad \text{for} \quad t \leq t_1 \\ Lny &= \log((Y - L_2)/(U_2 - Y)) \quad \text{for} \quad t > t_1 \end{aligned} \tag{3.17}$$

Then, following the growth models in (3.1) and (3.4), the S-shape growth models with intercepts can be written as

$$Lny = (C(11) + C(12)^* t) + (C(21) + C(22)^* t)^* D2 + \mu_t \tag{3.18}$$

or

$$Lny = (C(11) + C(12)^* t)^* D1 + (C(21) + C(22)^* t) + \mu_t \tag{3.19}$$

Note that $C(11)$ is the intercept of the model in (3.18) and for the model (3.19) the intercept is $C(21)$. Then the model can also be presented as a growth model without an intercept as follows:

$$Lny = (C(11) + C(12)^* t)^* D1 + (C(21) + C(22)^* t)^* D2 + \mu_t \tag{3.20}$$

Table 3.1 Parameters of the model in (3.20)

CV	*D*1	*D*2	Constant	*T*
1	1	0	*C*(11)	*C*(12)
2	0	1	*C*(21)	*C*(22)

It is recognized that it is easier or more convenient to apply the model in (3.20), especially for piecewise time series models with multivariate exogenous variables, which will be presented in Section 3.8. For this model it should be easy to construct the table of its parameters, as presented in Table 3.1. Note that the symbol *CV* in general indicates a defined *Categorical Variable*. In this case it is a dichotomous variable or time period, based on the time *t*.

In general, there may be an I x J table of model parameters, namely $C(ij)$ or $C(i,j)$, for $i = 1, 2 \ldots, I$ and $j = 1, 2, \ldots, J$. In some cases, there can be a regression with dummy variables, where the regressions within the defined time periods have unequal sets of independent variables, so that $j = 1,2,\ldots, J_i$. In this case, an incomplete table of the model parameters might exist. For example, Table 3.2 presents an incomplete 2×5 table by time period and independent variables, since the first regression does not have *X*1 as an independent variable and the second regression does not have *X*3 as an independent variable.

Table 3.2 Incomplete 2 × 5 table of the model parameters

CV	*D*1	*D*2	Constant (1)	*T* (2)	*X*1 (3)	*X*2 (4)	*X*3 (5)
1	1	0	*C*(11)	*C*(12)	—	*C*(14)	*C*(15)
2	0	1	*C*(21)	*C*(22)	*C*(23)	*C*(24)	—

Corresponding to this table, the two-piece growth model can be presented as

$$\begin{aligned} Lny = &(C(11) + C(12)^{*}t + C(14)^{*}X2 + C(15)^{*}X3)^{*}D1 \\ &+ (C(21) + C(22)^{*}t + C(23)^{*}X1 + C(24)^{*}X2)^{*}D2 + \mu_t \end{aligned} \tag{3.21}$$

where the subscripts of the parameters $C(ij)$ are not properly ordered, corresponding to the *j*th exogenous variable used as an independent variable of the regression in the *i*th time period. The general piecewise growth models will be presented in Section 3.8, which can easily be derived from the additive and interaction growth models presented in Chapter 2. For example, this situation could happen with the growth in productivity, gross domestic product, development, retail sales and macroeconomic indicators in general, because some produce factors such as internal and external environmental factors. The process of the data analysis is straightforward using the same steps presented above.

For illustration purposes, Figure 3.10 present two graphs of two-piece polynomial growth models, with $U_1 = L_2$. In practice, however, there may be either $U_1 < L_2$ or $U_1 > L_2$ if there is a breakpoint at $t = t_1$, e.g. for the social and economic indicators in a region or country before and after a critical event, such as a terrorist or a natural disaster. There may even be three or more pieces of growth model (refer to Figure 1.24).

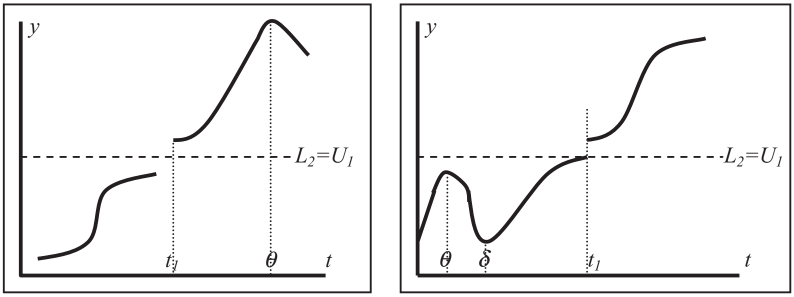


Figure 3.10 Illustrative two-piece bounded growth models

Example 3.4. (To generate the series *Lny*) For time series data, the process in generating a new variable, namely *Lny*, are as follows:

(1) After opening the Demo_Modified workfile, click *Sample* and then enter $D1 = 1$ in the *IF condition* window, as presented in Figure 3.11. Note that the dummy variables have been defined or constructed for the two time periods: 1952q1 up to 1975q4 and 1976q1 up to 1996q4.
(2) Click *OK*, which selects a subsample for the time period 1952q1 up to 1975q4 corresponding to the dummy variable $D1 = 1$.
(3) Then by clicking *Quick/Generate Series* ... the window in Figure 3.11(b) is produced, and the equation $Lny = \log((m1 - 100)/(320 - m1))$ can be entered; then click *OK*. The lower and upper bounds of $M1$ are defined by using personal judgment based on the minimum and maximum scores of $M1$ in the first time period, just for illustration purposes.
(4) The scores of *Lny* for $D1 = 0$ can be constructed by using the same process, such as click the '*Sample*' option to enter $D1 = 0$ in the '*IF condition*' window, then click *OK*, followed by selecting *Quick/Generate Series* ... to enter a new variable $Lny = \log((m1 - 320)/(1400 - m1))$.

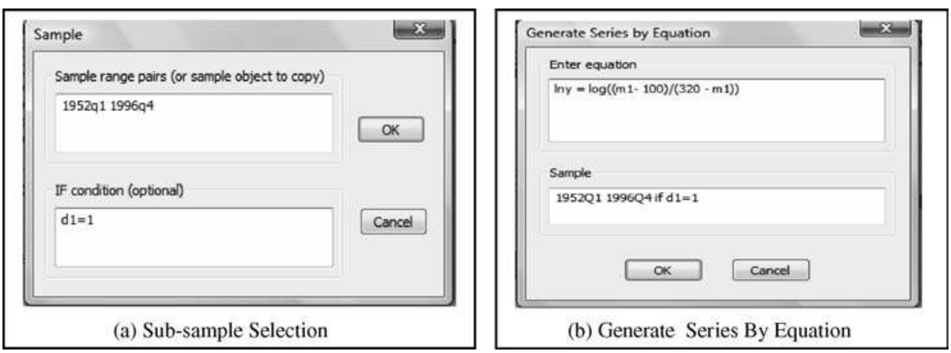


Figure 3.11 Two windows needed to generate a series in a subsample

(5) Here, $L_2 = U_1 = 320$ is used, which is a value between the observed values at 1975 : 4 and 1976 : 1. Any value of $U_1 > 320$ could be used, but for the value of L_2 it should be selected less than 320.
(6) In order to make sure that the correct scores of the variable *Lny* have been selected in both time periods, the scores should be presented on the screen together with the dummy variables *D*1 or *D*2. If they are acceptable or correct scores, then the whole data set or workfile can be saved.
(7) Note that, in some cases, there may be an '*Error Message*' to indicate that there is a logarithm of a nonpositive number. If you have difficulty in generating a new variable using EViews, go back to the Microsoft Excel file. □

Example 3.5. (Two-piece S-shape AR(1) growth model) By entering the equation

$$Lny = (C(11) + C(12)^{*}t) + (C(21) + C(22)^{*}t)^{*}D2 \tag{3.22}$$

where *Lny* has been generated in the previous example, and $D2 = 1$ for the second time period and $D2 = 0$ if otherwise, the results in Figure 3.12 are obtained, with the residual graph presented in Figure 3.13.

Based on these outputs, the following notes and conclusions can be made:

(1) Such a large value of *R*-squared = 0.870 757 indicates that the fitted and observed values are much closer over time. However, the model, in a statistical sense, is not a good model, specifically for doing statistical inference. Note that the residual plot shows that the sign (±) of the error terms have systematic changes over time and the DW-statistic is very small.
(2) Hence, an attempt is made to apply an AR(1) piecewise growth model, using the variable series

$$Lny\ c\ t\ D2\ D2^{*}t\ AR(1) \tag{3.23}$$

Dependent Variable: LNY
Method: Least Squares
Date: 10/15/07 Time: 13:36
Sample: 1952Q1 1996Q4
Included observations: 180
LNY=C(11)+C(12)*T + (C(21)+C(22)*T)*D2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-2.705246	0.117038	-23.11431	0.0000
C(12)	0.047611	0.002095	22.72330	0.0000
C(21)	-7.094401	0.378496	-18.74369	0.0000
C(22)	0.017821	0.003308	5.387178	0.0000

R-squared	0.870757	Mean dependent var	-0.555314
Adjusted R-squared	0.868554	S.D. dependent var	1.569117
S.E. of regression	0.568892	Akaike info criterion	1.731718
Sum squared resid	56.96024	Schwarz criterion	1.802673
Log likelihood	-151.8546	Hannan-Quinn criter.	1.760487
F-statistic	395.2574	Durbin-Watson stat	0.558506
Prob(F-statistic)	0.000000		

Figure 3.12 Statistical results based on the model in (3.22)

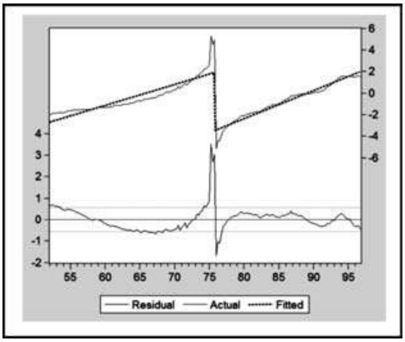


Figure 3.13 Residual graph of the regression in Figure 3.12

with the statistical results presented in Figure 3.14. Figure 3.15 presents its residual graphs. Based on this model the following notes and conclusions can be made:

- By comparing the residual graphs in Figures 3.13 and 3.15, it can be concluded that the AR(1) model in (3.22) is a better model, with the DW-statistic = 2.486.

Dependent Variable: LNY
Method: Least Squares
Date: 10/15/07 Time: 16:37
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 20 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-12.70242	27.73938	-0.457920	0.6476
T	0.168580	0.211416	0.797387	0.4263
D2	-3.303386	15.76675	-0.209516	0.8343
T*D2	-0.070577	0.163666	-0.431226	0.6668
AR(1)	0.980173	0.033342	29.39788	0.0000

R-squared	0.979050	Mean dependent var	-0.547318
Adjusted R-squared	0.978568	S.D. dependent var	1.569837
S.E. of regression	0.229818	Akaike info criterion	-0.075522
Sum squared resid	9.190042	Schwarz criterion	0.013511
Log likelihood	11.75925	Hannan-Quinn criter.	-0.039420
F-statistic	2032.851	Durbin-Watson stat	2.486261
Prob(F-statistic)	0.000000		
Inverted AR Roots	.98		

Figure 3.14 Statistical results based on the model in (3.23)

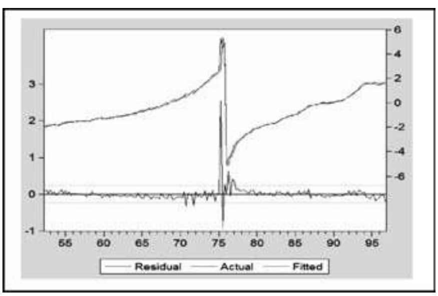


Figure 3.15 Residual graph of the regression in Figure 3.14

Dependent Variable: LNY
Method: Least Squares
Date: 11/19/07 Time: 16:25
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 23 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
D1	3.303517	15.76665	0.209526	0.8343
T*D1	0.070576	0.163665	0.431221	0.6668
C	-16.00569	12.53901	-1.276471	0.2035
T	0.098003	0.052982	1.849752	0.0660
AR(1)	0.980173	0.033342	29.39791	0.0000

R-squared	0.979050	Mean dependent var	-0.547318
Adjusted R-squared	0.978568	S.D. dependent var	1.569837
S.E. of regression	0.229818	Akaike info criterion	-0.075522
Sum squared resid	9.190042	Schwarz criterion	0.013511
Log likelihood	11.75925	Hannan-Quinn criter.	-0.039420
F-statistic	2032.851	Durbin-Watson stat	2.486261
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 3.16 Statistical results based on the model in (3.24)

- The joint effect of the independent variables, namely t, $D2$ and t^*D2, is significantly based on the F-statistic of 2032.852 with a p-value = 0.0000. However, each of these variables has an insignificant adjusted effect, so that a better model should be found in the statistical sense.

(3) For illustration purposes, Figures 3.16 and 3.17 present statistical results based on the following two equation specifications respectively:

$$Lny\ t\ t^*D1\ c\ t\ AR(1) \tag{3.24}$$

$$Lny\ D1\ t^*D1\ D2\ D2^*t\ AR(1) \tag{3.25}$$

(4) Considering the three AR(1) models in (3.23), (3.24) and (3.25), it is found that they are in fact the same two-piece regressions. Write the regression functions within each time period based on the three outputs as an exercise.

(5) However, the output in Figure 3.17 does not present the F-statistic, compared to the other outputs.

Dependent Variable: LNY
Method: Least Squares
Date: 11/19/07 Time: 16:21
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 17 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
D1	-12.70246	27.73638	-0.457971	0.6475
T*D1	0.168581	0.211399	0.797451	0.4263
D2	-16.00580	12.53713	-1.276672	0.2034
T*D2	0.098003	0.052974	1.850035	0.0660
AR(1)	0.980173	0.033341	29.39810	0.0000

R-squared	0.979050	Mean dependent var	-0.547318
Adjusted R-squared	0.978568	S.D. dependent var	1.569837
S.E. of regression	0.229818	Akaike info criterion	-0.075522
Sum squared resid	9.190042	Schwarz criterion	0.013511
Log likelihood	11.75925	Hannan-Quinn criter.	-0.039420
Durbin-Watson stat	2.486262		

Inverted AR Roots	.98

Figure 3.17 Statistical results based on the model in (3.25)

(6) Furthermore, note that a higher-order autoregressive growth model, as well as the lagged endogenous variables, could be used in many cases, as presented in the following example. □

Example 3.6. (Two-piece S-shape LV(1) and LVAR(1,1) growth models) For a further comparison, Figure 3.18 presents statistical results based on a two-piece S-shape LV(1)_GM in (3.26) and Figure 3.19 presents the statistical results based on the LVAR(1,1)_GM in (3.27):

$$Lny\ D1\ t^{*}D1\ D1^{*}Lny(-1)\ D2\ D2^{*}t\ D2^{*}Lny(-1) \tag{3.26}$$

$$Lny\ D1\ t^{*}D1\ D1^{*}Lny(-1)\ D2\ D2^{*}t\ D2^{*}Lny(-1)\ AR(1) \tag{3.27}$$

□

Dependent Variable: LNY
Method: Least Squares
Date: 11/19/07 Time: 16:52
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
D1	-0.011634	0.152454	-0.076311	0.9393
T*D1	0.001844	0.002612	0.705925	0.4812
D1*LNY(-1)	1.014025	0.051172	19.81611	0.0000
D2	-10.56107	0.373957	-28.24139	0.0000
T*D2	0.070497	0.002537	27.78569	0.0000
D2*LNY(-1)	-0.086151	0.035828	-2.404568	0.0172

R-squared	0.961014	Mean dependent var	-0.547318
Adjusted R-squared	0.959887	S.D. dependent var	1.569837
S.E. of regression	0.314409	Akaike info criterion	0.556702
Sum squared resid	17.10159	Schwarz criterion	0.663542
Log likelihood	-43.82484	Hannan-Quinn criter.	0.600025
Durbin-Watson stat	1.404370		

(a)

Dependent Variable: LNY
Method: Least Squares
Date: 11/19/07 Time: 17:11
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 22 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
D1	-22.77494	40.63730	-0.560444	0.5759
T*D1	0.256515	0.254349	1.008514	0.3146
D1*LNY(-1)	-0.321928	0.078684	-4.091400	0.0001
D2	-20.84526	26.97359	-0.772803	0.4407
T*D2	0.119716	0.110806	1.080406	0.2815
D2*LNY(-1)	-0.063229	0.022115	-2.859143	0.0048
AR(1)	0.984494	0.019690	49.99891	0.0000

R-squared	0.981741	Mean dependent var	-0.539463
Adjusted R-squared	0.981100	S.D. dependent var	1.570733
S.E. of regression	0.215941	Akaike info criterion	-0.189093
Sum squared resid	7.973804	Schwarz criterion	-0.063967
Log likelihood	23.82929	Hannan-Quinn criter.	-0.138351
Durbin-Watson stat	1.873644		
Inverted AR Roots	.98		

(b)

Figure 3.18 Statistical results based on two-piece growth models: (a) the LV(1)_GM in (3.26) and (b) the LVAR(1,1)_GM in (3.27)

Example 3.7. (Two-piece S-shape LVAR(p,q) growth models) By experimentation, it has been found that several good fit models can be applied to represent the adjusted effect of the time t within each defined time period. One of those models is presented in Figure 3.19(a) with its reduced model presented in Figure 3.19(b), which is considered to be the best model for an illustration. Based on the results in Figure 3.19(b) the following notes and conclusions may be made:

(1) In a statistical sense, this model is a good fit model, since all independent variables are significant with a sufficiently large value of the DW-statistic.
(2) The regression in the first time period has t and $Lny(-1)$ as independent variables, but the regression in the second time period has t and $Lny(-2)$ as independent variables. Therefore, this model can be considered as an unexpected two-piece growth model.
(3) These findings indicate that, based on time series data in various fields, a good fit piecewise S-shape LVAR(p,q) growth model could be found by using the

Dependent Variable: LNY
Method: Least Squares
Date: 11/19/07 Time: 17:17
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 11 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
D1	-0.826346	0.459024	-1.800223	0.0736
T*D1	0.016456	0.007716	2.132819	0.0344
D1*LNY(-1)	-0.048688	0.113585	-0.428649	0.6687
D1*LNY(-2)	0.778398	0.091334	8.522545	0.0000
D2	-10.60439	0.565115	-18.76499	0.0000
T*D2	0.070733	0.003866	18.29766	0.0000
D2*LNY(-1)	-0.108645	0.032138	-3.380522	0.0009
D2*LNY(-2)	-0.005314	0.030346	-0.175129	0.8612
AR(1)	1.008534	0.123770	8.148471	0.0000
AR(2)	-0.385414	0.104416	-3.691146	0.0003

R-squared	0.977949	Mean dependent var	-0.524148
Adjusted R-squared	0.976753	S.D. dependent var	1.573023
S.E. of regression	0.239837	Akaike info criterion	0.037422
Sum squared resid	9.548584	Schwarz criterion	0.217563
Log likelihood	6.706841	Hannan-Quinn criter.	0.110487
Durbin-Watson stat	1.915677		

Inverted AR Roots	.50-.36i	.50+.36i

(a)

Dependent Variable: LNY
Method: Least Squares
Date: 11/19/07 Time: 17:20
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
D1	-0.660424	0.253302	-2.607253	0.0099
T*D1	0.013691	0.004357	3.141966	0.0020
D1*LNY(-2)	0.790079	0.072235	10.93758	0.0000
D2	-10.50159	0.417864	-25.13159	0.0000
T*D2	0.070053	0.002926	23.94362	0.0000
D2*LNY(-1)	-0.100395	0.025498	-3.937323	0.0001
AR(1)	0.968195	0.076054	12.73039	0.0000
AR(2)	-0.364882	0.076477	-4.771124	0.0000

R-squared	0.977919	Mean dependent var	-0.524148
Adjusted R-squared	0.976999	S.D. dependent var	1.573023
S.E. of regression	0.238566	Akaike info criterion	0.016051
Sum squared resid	9.561545	Schwarz criterion	0.160164
Log likelihood	6.587472	Hannan-Quinn criter.	0.074503
Durbin-Watson stat	1.929941		

Inverted AR Roots	.48+.36i	.48-.36i

(b)

Figure 3.19 Statistical results based on two-piece S-shape growth models: (a) an LVAR(2,2)_GM and (b) its reduced model

trial-and-error methods in order to obtain relevant values of p and q. It is recognized that it is possible to have various sets of lagged endogenous variables within each time period, but only one set of autoregressive (AR) indicators should be used for all units of observations or both time periods, as presented in Figure 3.19, as well as in the previous piecewise AR models. □

3.4 Two-piece polynomial bounded growth models

For illustration purposes, only three special cases will be presented: (i) the quadratic growth model, (ii) the third-degree growth model and (iii) the generalized exponential growth model.

3.4.1 Two-piece quadratic growth models

Special cases of quadratic growth models are defined as

$$Lny = (c(11) + c(12)^{*}t) + (c(21) + c(22)^{*}(t-\theta)^{2})^{*}D2 + \mu_{t} \tag{3.28}$$

$$Lny = (c(11) + c(12)^{*}t) + (c(21) + c(22)^{*}(t-\theta)^{2})^{*}D1 + \mu_{t} \tag{3.29}$$

and

$$Lny = (C(11) + C(12)^{*}t)^{*}D1 + (C(21) + C(22)^{*}(t-\theta)^{2})^{*}D2 + \mu_{t} \tag{3.30}$$

where the dependent variable Lny is defined as in (3.17) and θ is a selected fixed number. Note that based on the model in (3.30), the following two regressions can be derived:

$$\begin{aligned} Lny &= C(11) + C(12)^{*}t + \mu_{t}, \quad t \leq t_{1}(D1 = 1, D2 = 0) \\ Lny &= C(21) + C(22)^{*}(t-\theta)^{2} + \mu_{t}, \quad t > t_{1}(D1 = 0, D2 = 1) \end{aligned} \tag{3.31}$$

Hence, the model in (3.30) represents a classical growth model in the first time period and a quadratic growth model in the second time period with a maximum or minimum value of $Lny = C(21)$ for $t = \theta$. This model can be generalized to the quadratic growth models in both time periods, with the following equation:

$$\begin{aligned} Lny &= (C(11) + C(12)^{*}t + C(13)^{*}t^{2})^{*}D1 \\ &\quad + (C(21) + C(22)^{*}t + C(22)^{*}t^{2})^{*}D2 + \mu_t \end{aligned} \tag{3.32}$$

3.4.2 Two-piece third-degree bounded growth model

A specific two-piece third-degree growth model is defined as

$$Lny = (C(11) + C(12)^{*}f(t))^{*}D1 + (C(21) + C(22)^{*}t)D2 + \mu_t \tag{3.33}$$

where the dependent variable *Ln(y)* is defined as in (3.17) and $f(t) = (t - \beta)^2(t - \delta)$, where β and δ, with $\beta < \delta$, are fixed selected values corresponding to estimated or predicted maximum and minimum observed values of $Ln(y) = (Y - L_1)/(U_1 - Y)$ during the first defined time period. This model represents the following two regressions:

$$Lny = (C(11) + C(12)^{*}(t-\beta)^2(t-\delta) + \mu_t \tag{3.34a}$$

$$Lny = (C(21) + C(22)^{*}t + \mu_t \tag{3.34b}$$

Based on the regression in (2.37), the first-order condition for the extreme values of *Lny* with respect to the time t is as follows:

$$\frac{\mathrm{d}Lny}{\mathrm{d}t} = C(12)\left[2(t-\beta)(t-\delta) + (t-\beta)^2\right] = 0 \tag{3.35}$$

Then $t_1^* = \beta$ and $t_2^* = (\beta + 2\delta)/3$, which can lead to a minimum or maximum value of *Lny* depending on the sign of the parameter $C(12)$, corresponding to the sign of the second-order condition

$$\frac{\mathrm{d}^2Lny}{\mathrm{d}t^2} = C(12)[2(t-\delta) + 4(t-\beta)] \tag{3.36}$$

and

$$\begin{aligned} Lny''(t_1^*) &= \frac{\mathrm{d}^2Lny}{\mathrm{d}t^2}(t_1^*) = 2C(12)(\beta-\delta) \\ Lny''(t_2^*) &= \frac{\mathrm{d}^2Lny}{\mathrm{d}t^2}(t_2^*) = -2C(12)(\beta-\delta) \end{aligned} \tag{3.37}$$

Therefore, for $C(12) < 0$ and selected $\beta < \delta$, the function has a minimum value of $Lny(t_1^*) = C(11)$ since $Lny''(t_1^*) > 0$ and a maximum value of $Lny(t_2^*) = C(11) + 4C(12)(\beta - \delta)^3/7$ because $Lny''(t_1^*) < 0$.

3.4.3 Two-piece generalized exponential growth model

As an extension of the bounded growth model in (3.33), Agung (1999a, 2007) presents a two-piece generalized exponential growth model or a third-degree polynomial growth model as follows:

$$\begin{aligned} Lny = \ & (C(11) + C(12)^* F1(t))^* D1 \\ & + (C(21) + C(22)^* F2(t))^* D2 + \mu_t \end{aligned} \tag{3.38}$$

where $F_1(t) = (t - a)^2(t - b)$ and $F_2(t) = (t - c)^2(t - d)$ with values of a, b, c and d selected as constant numbers. These numbers should be selected by taking into account the observed relative minimum and maximum values of the dependent variable *Lny*, as well as their predicted values, in the case of forecasting. For example, $F_1(t)$ is used for the estimation, so the values of a and b are selected based on the observed values, and $F_2(t)$ will be used in forecasting, so alternative values of c and d are subjectively selected.

3.5 Discontinuous translog linear AR(1) growth models

Corresponding to the two-piece growth models presented in the previous sections are the following possible translog (i.e. translogarithmic) linear AR(1) growth models:

$$\log(y_t) = C(1) + C(2)^*\log(t) + (C(3) + C(4)^*\log(t))^*D1 + [ar(1) = C(5)] \tag{3.39}$$

$$\log(y_t) = (C(1) + C(2)^*\log(t))^*D1 + (C(3) + C(4)^*\log(t))^*D2 + [ar(1) = C(5)] \tag{3.40}$$

$$\log(y_t) = C(1) + C(2)^*\log(t) + C(3)^*(\log(t) - \log(t1))^*D2 + [ar(1) = C(4)] \tag{3.41}$$

Furthermore, these translog growth models could easily be extended to the piecewise S-shape growth models, using dependent variables *Lny* as defined in (3.17).

3.6 Alternative discontinuous growth models

In fact, all two-piece growth models presented in the sections above are very closely related to the models presented in Chapter 2. Note that within each time interval, in fact, there are continuous growth models. Hence, a two-piece growth model can be considered as a linear combination of any two continuous growth models presented in Chapter 2.

Furthermore, the general discontinuous growth models should be easy to develop or derive using all of the continuous growth models presented in Chapter 2, including the advanced growth models having interaction factors as independent variables, the trigonometric models in Section 2.10.5 and the multivariate growth models. Hence, they will not be presented in detail again here. The data analysis using each of those discontinuous growth models are straightforward using the same process previously presented.

The following examples present additional illustrative graphical representations of discontinuous growth models without detailed results of data analysis and discussions.

Example 3.8. (Residual plots of Two-piece linear models) For a comparison, Figure 3.20 presents the residual graphs based on two models of two-piece linear models:

(1) The graph in Figure 3.20(a) is the residual graph of a two-piece linear model or multiple regressions of *RS* on four independent variables, namely *Drs*1, *Drs*1* *t*, *Drs*2 and *Drs*2* *t*, without an intercept. The equation specification of the model is as follows:

$$RS = C(1)^{*}Drs1 + C(2)^{*}Drs1^{*}t + C(2)^{*}Drs2 + C(4)^{*}Drs2^{*}t \qquad (3.42)$$

with a pair of simple regression functions

$$\begin{aligned} RS &= 0.4779 + 0.0724^{*}t \quad \text{for} \quad t < 119 \\ &= 22.0023 - 0.1029^{*}t \quad \text{for} \quad t > 119 \end{aligned} \qquad (3.43)$$

(2) The graph in Figure 3.20(b) is the residual graph of a two-piece AR(1) linear model with the following equation specification:

$$\log(rs)\ Drs1\ Drs1^{*}\log(t)\ Drs(2)\ Drs2^{*}\log(t)\ ar(1) \qquad (3.44)$$

with a pair of regression functions

$$\begin{aligned} \text{LOG}(RS) &= -2.4006 + 0.9628^{*}\text{LOG}(T) + [AR(1) = 0.8471] \quad \text{for} \quad t =< 119 \\ &= 8.8839 - 1.4223^{*}\text{LOG}(T) + [AR(1) = 0.8471] \quad \text{for} \quad t =< 119 \end{aligned} \qquad (3.45)$$

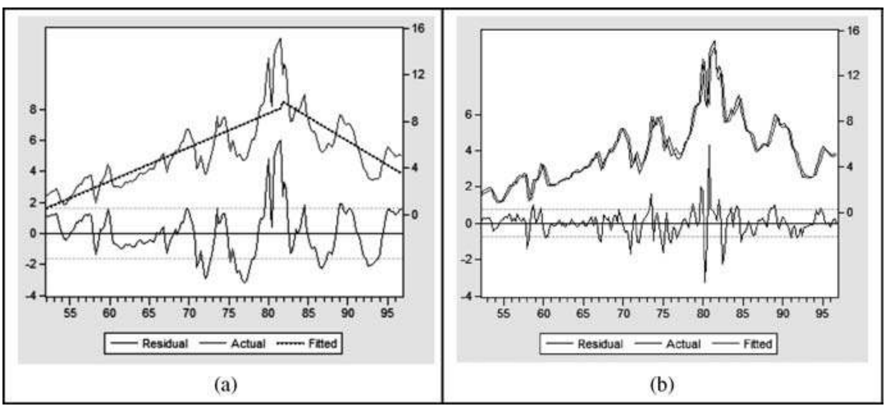


Figure 3.20 Comparison between residual plots of two-piece regressions: (a) simple linear model in (3.42) and (b) the AR(1) model in (3.44)

Note that, in Figure 3.20(b), the graphs of the two-piece regression functions could not be clearly identified because the indicator $AR(1)$ is used, so that the graph of the observed and fitted values are very close, corresponding to a high value of R-squared = 0.942 821. □

Example 3.9. (Residual graphs of discontinuous growth models) Figure 3.21(a) and (b) present the residual graphs of the two-pieces classical growth models, as presented in (2.3), of the variable $M1$ and its corresponding AR(1)_GM. The graph in Figure 3.21(a) is the residual graph of the growth model of $M1$ with a corner or discontinuity point at $Year = 80$, with the following equation specification:

$$\log(m1) \quad c \quad \text{Year}\, Dm1^{*}(\text{Year}-80) \tag{3.46}$$

where $Dm1$ is a dummy variable, generated by using

$$Dm1 = 1^{*}(\text{Year} =< 80) + 0^{*}(\text{Year} > 80) \tag{3.47}$$

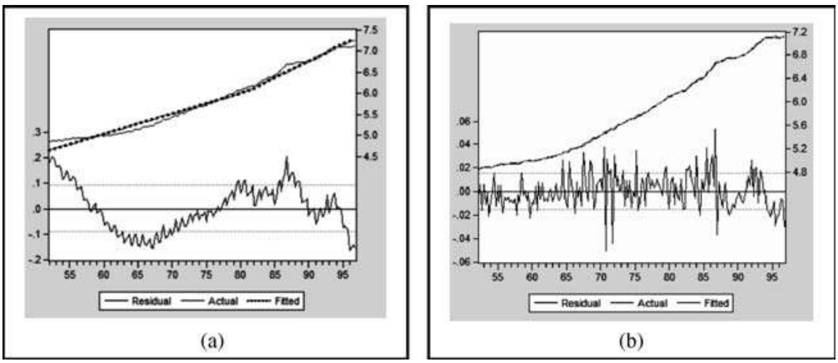


Figure 3.21 Comparison between residual plots of two-piece regressions: (a) classical growth models in (3.46) and (b) AR(1)_GM in (3.48)

For comparison, the graph in Figure 3.21(b) is the residual graph of a two-piece AR(1)_GM, with the following equation specification:

$$\log(m1) \quad c \quad \text{Year}\, Dm1^{*}(\text{Year}-80)\; ar(1) \tag{3.48}$$

Note that both graphs of the observed and fitted values are very close, corresponding to values of R-squared = 0.985 662 and 0.999 563 respectively. Therefore it is not possible to identify the positions of the corner points. However, their residual graphs are quite different. □

Example 3.10. (Step regression function having one breakpoint) Figure 3.22(a) and (b) presents the step growth curve of a hypothetical time series $Y3$ and a simple AR (1) model with trend respectively. The graph in Figure 3.22(b) is the residual graph

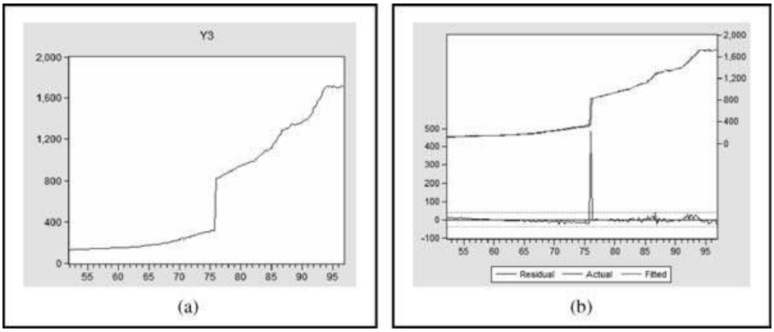


Figure 3.22 (a) A step growth curve of a time series Y3 and (b) its AR(1) model with trend in (3.49)

obtained by using the following equation specification:

$$Y3 = C(1) + C(2)^{*}t + [AR(1) = C(3)] \tag{3.49}$$

with R-squared = 0.997 07 and the DW-statistic = 2.022 607. Note that this model is not a classical growth model, because it has a dependent variable $Y3$ instead of log(y3). Furthermore, note that the AR(1) model in (3.49) does not have a dummy variable to identity the existence of a breakpoint. However, the breakpoint can be identified by a large value of the error terms at the time $Year = 80$ or by the residual plot having a long vertical line at $Year = 80$.

For comparison, if the simple step regression model is used,

$$Y3 = C(1) + C(2)^{*}t + (C(3) + C(4)^{*}t)^{*}D2 \tag{3.50}$$

gives the results in Figure 3.23, with its residual graph in Figure 3.24. Note that there are problems with a small value of the DW-statistic and the pattern of the residual graph, even though the R-squared value is much closer to one. Hence it is suggested that autoregressive or lagged variable growth models should be applied. As an illustration, Figure 3.25 presents the results of an AR(1) model, with its residual graph in Figure 3.26. □

Dependent Variable: Y3
Method: Least Squares
Date: 10/15/07 Time: 17:21
Sample: 1952Q1 1996Q4
Included observations: 180
Y3=C(1)+C(2)*T + (C(3)+C(4)*T)*D2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	94.00011	7.212811	13.03238	0.0000
C(2)	1.914927	0.129127	14.82984	0.0000
C(3)	-513.5706	23.32596	-22.01713	0.0000
C(4)	10.06764	0.203871	49.38233	0.0000

R-squared	0.996222	Mean dependent var	678.3398
Adjusted R-squared	0.996158	S.D. dependent var	565.6132
S.E. of regression	35.05971	Akaike info criterion	9.973954
Sum squared resid	216336.3	Schwarz criterion	10.04491
Log likelihood	-893.6559	Hannan-Quinn criter.	10.00272
F-statistic	15470.72	Durbin-Watson stat	0.073717
Prob(F-statistic)	0.000000		

Figure 3.23 Statistical results based on the model in (3.50)

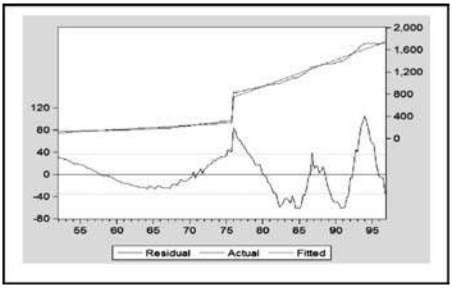


Figure 3.24 Residual graph of the regression in Figure 3.23

Dependent Variable: Y3
Method: Least Squares
Date: 10/15/07 Time: 17:23
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 25 iterations
Y3=C(1)+C(2)*T + (C(3)+C(4)*T)*D2 +[AR(1)=C(5)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-429.3192	724.7255	-0.592389	0.5544
C(2)	6.071529	4.005656	1.515739	0.1314
C(3)	-59.15594	219.8247	-0.269105	0.7882
C(4)	5.803191	2.278113	2.547367	0.0117
C(5)	0.986888	0.011051	89.30118	0.0000

R-squared	0.999762	Mean dependent var	681.4225
Adjusted R-squared	0.999756	S.D. dependent var	565.6814
S.E. of regression	8.831620	Akaike info criterion	7.222089
Sum squared resid	13571.57	Schwarz criterion	7.311123
Log likelihood	-641.3770	Hannan-Quinn criter.	7.258192
F-statistic	182523.8	Durbin-Watson stat	1.578776
Prob(F-statistic)	0.000000		

Inverted AR Roots	.99

Figure 3.25 Statistical results based on the AR(1) model of (3.50)

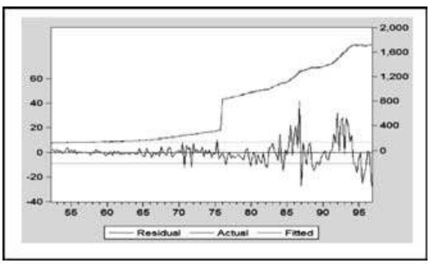


Figure 3.26 Residual graph of the regression in Figure 3.25

Example 3.11. (Step growth model with one breakpoint) Suppose the statistical results presented in Figure 3.27 are based on a growth model of *Y*, with its residual graph in Figure 3.28. Then an AR(1) classical growth model has been used as follows:

$$\begin{aligned} \log(y) &= \beta_0 + \beta_1{}^* t + \mu_t \\ \mu_t &= \rho_1 \mu_{t-1} + \varepsilon_t \end{aligned} \tag{3.51}$$

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/16/07 Time: 07:05
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	5.500584	3.340119	1.646823	0.1014
T	0.008028	0.016925	0.474359	0.6358
AR(1)	0.988798	0.017687	55.90527	0.0000

R-squared	0.993352	Mean dependent var	6.029911
Adjusted R-squared	0.993276	S.D. dependent var	0.923719
S.E. of regression	0.075744	Akaike info criterion	-2.305297
Sum squared resid	1.009739	Schwarz criterion	-2.252877
Log likelihood	209.4136	Hannan-Quinn criter.	-2.284636
F-statistic	13148.49	Durbin-Watson stat	1.988334
Prob(F-statistic)	0.000000		
Inverted AR Roots	.99		

Figure 3.27 Statistical results based on an AR(1)_GM in (3.51)

Without having the residual graph and observing only the values of R-squared, adjusted R-squared and the DW-statistic, it might be thought that this model is the best growth model for the series Y_t. Do you also think so?

However, the residual graph clearly shows that there is a breakpoint, corresponding to a long vertical line presented in the residual graph, as well as the two levels of the actual and fitted graphs. Therefore, a growth rate of $\hat{\beta}_1 = \hat{c}(2) = 0.6358$ cannot be presented for the series Y_t within the whole time period. Then what should be explored in order to obtain a better picture of the series Y_t? Observe the following example. □

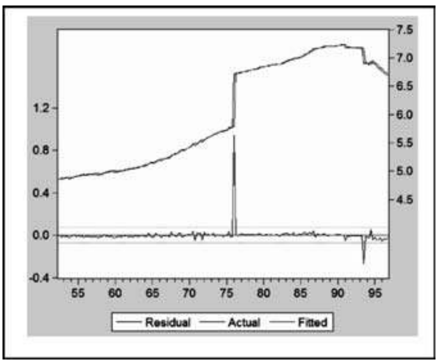


Figure 3.28 Residual graph of the regression in Figure 3.27

Example 3.12. (Growth model with two breakpoints) Figure 3.29(a) presents a scatter graph with a regression line of a hypothetical time series Y. For comparison, Figure 3.29(b) presents the residual graph of the simple linear regression of Y at the time t, with the regression function with the p-value of the t-statistic in $[\cdot]$, as follows:

$$Y = \underset{[0.00001]}{-125.\ 4393} + \underset{[0.0000]}{8.0958}{}^{*}t \tag{3.52}$$

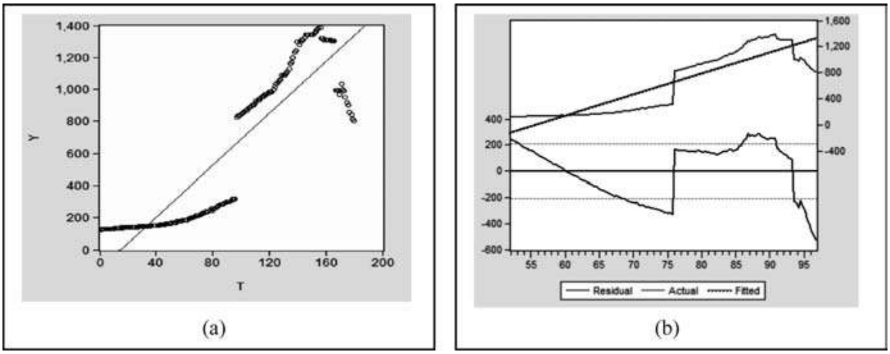


Figure 3.29 (a) Scatter graph with regression of Y on the time t and (b) the residual graph of the regression (3.52)

Even though the time t has a highly significant effect, both graphs in Figure 3.29 (the scatter graph with regression and the residual graph) show that the simple linear regression is not an appropriate model to be applied. Therefore, a three-piece growth model should be used, which will be presented in the following example. □

Example 3.13. (Three-piece classical growth model) Corresponding to the classical growth model in (2.3), Figure 3.30 presents the statistical results of a three-piece classical growth model as follows:

$$\log(Y) = (c(1) + c(2)^{*}t) + (c(3) + c(4)^{*}t)Dy2 + (c(5) + c(6)^{*}t)Dy3 \qquad (3.53)$$

with its residual graph in Figure 3.31. The growth model has two dummy variables $Dy2$ and $Dy3$, out of the three possible dummies, namely $Dy1$, $Dy2$ and $Dy3$, which can be defined for the three time periods observed in the previous example. Hence, this

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/15/07 Time: 21:30
Sample: 1952Q1 1996Q4
Included observations: 180
LOG(Y) = C(1)+C(2)*T + (C(3)+C(4)*T)*DY2 + (C(5)+C(6)*T)*DY3

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	4.714211	0.012752	369.6802	0.0000
C(2)	0.009788	0.000228	42.87459	0.0000
C(3)	1.211843	0.050422	24.03397	0.0000
C(4)	-0.001581	0.000432	-3.659846	0.0003
C(5)	5.301074	0.713316	7.431593	0.0000
C(6)	-0.028120	0.004115	-6.831951	0.0000

R-squared	0.995639	Mean dependent var	6.023304
Adjusted R-squared	0.995513	S.D. dependent var	0.925391
S.E. of regression	0.061985	Akaike info criterion	-2.691084
Sum squared resid	0.668532	Schwarz criterion	-2.584652
Log likelihood	248.1975	Hannan-Quinn criter.	-2.647930
F-statistic	7944.418	Durbin-Watson stat	0.085824
Prob(F-statistic)	0.000000		

Figure 3.30 Statistical results based on the model in (3.53)

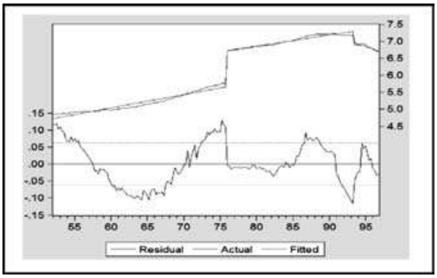


Figure 3.31 Residual graph of the regression in Figure 3.30

model in fact represents three classical growth models as follows:

$$\begin{aligned} \log(Y) &= C(1) + C(2)^{*}t \\ \log(Y) &= C(1) + C(3) + (C(2) + C(4))^{*}t \quad \text{and} \\ \log(Y) &= C(1) + C(5) + (C(2) + C(6))^{*}t \end{aligned} \tag{3.54}$$

within the first, second and third time intervals respectively.

Note that this model has a very small value of the DW-statistic and its residual graph indicates an autoregressive problem. Hence, it is suggested that an autoregressive or lagged-variable model be applied. For this reason and for comparison, three alternative models with their equations and statistical results are presented in Figures 3.32 to 3.34 together with their residual graphs in Figures 3.35 to 3.37 respectively. Based on these results, the following notes and conclusions are derived:

(1) Figure 3.32 presents statistical results and its residual graphs in Figure 3.35, based on a second-order lagged-variable three-piece growth model, namely the three-piece LV(2)_GM. Its residual graph as well as the small value of the DW-statistic show that the model should be modified. In other words, the model is not an acceptable time series model, in a statistical sense.

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/15/07 Time: 21:37
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
LOG(Y) = C(1)+C(2)*T + (C(3)+C(4)*T)*DY2 + (C(5)+C(6)*T)*DY3 +C(7)
*LOG(Y(-1))+C(8)*LOG(Y(-2))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.793921	0.185164	15.08889	0.0000
C(2)	0.005974	0.000426	14.03495	0.0000
C(3)	0.806653	0.056647	14.24011	0.0000
C(4)	-0.001657	0.000345	-4.800628	0.0000
C(5)	2.791905	0.605628	4.609930	0.0000
C(6)	-0.014864	0.003451	-4.307021	0.0000
C(7)	0.381386	0.060802	6.272603	0.0000
C(8)	0.024833	0.049928	0.497370	0.6196

R-squared	0.997391	Mean dependent var	6.036550
Adjusted R-squared	0.997283	S.D. dependent var	0.922032
S.E. of regression	0.048060	Akaike info criterion	-3.188842
Sum squared resid	0.392655	Schwarz criterion	-3.045840
Log likelihood	291.8069	Hannan-Quinn criter.	-3.130851
F-statistic	9282.598	Durbin-Watson stat	0.725185
Prob(F-statistic)	0.000000		

Figure 3.32 Statistical results based on a three-piece LV(2)_GM

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/15/07 Time: 21:40
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence not achieved after 500 iterations
LOG(Y) = C(1)+C(2)*T + (C(3)+C(4)*T)*DY2 + (C(5)+C(6)*T)*DY3 +C(7)
*LOG(Y(-1))+ [AR(1)=C(8)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	4.311204	0.178494	24.15322	0.0000
C(2)	0.015895	0.002171	7.320442	0.0000
C(3)	1.798183	0.292332	6.151161	0.0000
C(4)	-0.008852	0.003036	-2.915512	0.0040
C(5)	5.714174	0.676370	8.448296	0.0000
C(6)	-0.033938	0.004230	-8.022766	0.0000
C(7)	-0.009195	0.015290	-0.601357	0.5484
C(8)	0.979504	0.018745	52.25429	0.0000

R-squared	0.999747	Mean dependent var	6.036550
Adjusted R-squared	0.999737	S.D. dependent var	0.922032
S.E. of regression	0.014959	Akaike info criterion	-5.523142
Sum squared resid	0.038040	Schwarz criterion	-5.380140
Log likelihood	499.5596	Hannan-Quinn criter.	-5.465151
F-statistic	96043.53	Durbin-Watson stat	2.318880
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 3.33 Statistical results based on a three-piece LVAR(1,1)_GM

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/15/07 Time: 21:44
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence not achieved after 500 iterations
LOG(Y) = C(1)+C(2)*T + (C(3)+C(4)*T)*DY2 + (C(5)+C(6)*T)*DY3 +
[AR(1)=C(7)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	4.279238	0.415966	10.28746	0.0000
C(2)	0.015250	0.003902	3.908638	0.0001
C(3)	1.694669	0.385830	4.392267	0.0000
C(4)	-0.007774	0.004001	-1.943062	0.0536
C(5)	5.638084	0.712163	7.916850	0.0000
C(6)	-0.033033	0.005010	-6.593311	0.0000
C(7)	0.970835	0.017686	54.89301	0.0000

R-squared	0.999754	Mean dependent var	6.029911
Adjusted R-squared	0.999745	S.D. dependent var	0.923719
S.E. of regression	0.014752	Akaike info criterion	-5.556510
Sum squared resid	0.037433	Schwarz criterion	-5.431863
Log likelihood	504.3076	Hannan-Quinn criter.	-5.505967
F-statistic	116283.9	Durbin-Watson stat	2.358856
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97

Figure 3.34 Statistical results based on a three-piece AR(1)_GM

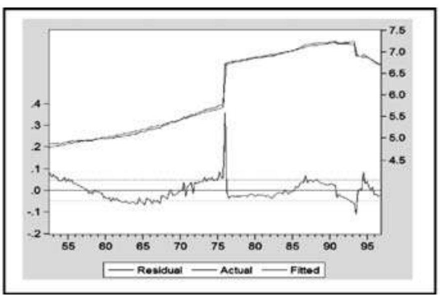


Figure 3.35 Residual graph of the regression in Figure 3.32

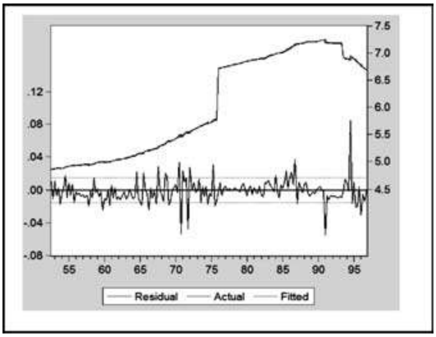


Figure 3.36 Residual graph of the regression in Figure 3.33

(2) Figure 3.33 presents statistical results based on a lagged-variable autoregressive three-piece growth model of the order (1,1), namely a three-piece LVAR(1,1)_GM. Compared to the first two models, this model is the best one, as presented by its residual graph in Figure 3.36. However, corresponding to the parameter $C(7)$, log $(Y(-1))$ is insignificant with a large p-value $= 0.5484$. On the other hand, this figure also presents a note 'Convergence not achieved after 500 iterations,' which indicates that the statistical results are not the optimal estimates. A decision was therefore made to produce a reduced model, as presented in Figure 3.34.

(3) The residual graphs in Figures 3.36 and 3.37 are very similar, since the reduced model is obtained by deleting an independent variable $\log(Y(-1))$ which has such a large p-value. However, the results in Figure 3.34 also present the note 'Convergence not achieved after 500 iterations.'

(4) Moreover, the residual graphs in Figures 3.36 and 3.37 also present the heteroskedasticity of their error terms. For these reasons an attempt is made to apply the White and the Newey–West estimation methods in the following examples. However, other forms of the model using the three defined dummy variables will also be presented. □

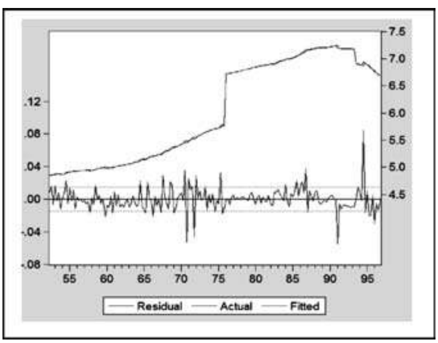


Figure 3.37 Residual graph of the regression in Figure 3.34

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/16/07 Time: 09:18
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 110 iterations
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
DY1	4.278934	0.408733	10.46877	0.0000
DY1*T	0.015253	0.004101	3.719438	0.0003
DY2	5.973873	0.173667	34.39851	0.0000
DY2*T	0.007476	0.001198	6.242908	0.0000
DY3	9.917263	1.043359	9.505129	0.0000
DY3*T	-0.017782	0.006214	-2.861630	0.0047
AR(1)	0.970848	0.016245	59.76246	0.0000

R-squared	0.999754	Mean dependent var	6.029911
Adjusted R-squared	0.999745	S.D. dependent var	0.923719
S.E. of regression	0.014752	Akaike info criterion	-5.556510
Sum squared resid	0.037433	Schwarz criterion	-5.431863
Log likelihood	504.3076	Hannan-Quinn criter.	-5.505967
Durbin-Watson stat	2.358887		

Inverted AR Roots	.97

Figure 3.38 The White estimates of the three-piece AR(1)_GM in (3.55)

Example 3.14. (The White and Newey–West estimation methods) As a modification of the three-piece growth model presented in the previous example, a three-piece AR(1)_GM is considered as follows:

$$\log(Y) \quad dy1\, dy1^{*}t \quad dy2\, dy2^{*}t \quad dy3\, dy3^{*}t \quad ar(1) \tag{3.55}$$

Figure 3.38 presents statistical results using the White estimation method with its residual graph in Figure 3.39. This figure shows that the convergence of the estimation process is achieved after 110 iterations, with a sufficiently large value of the DW-statistic. Therefore, it can be concluded that the model in (3.55) is an acceptable AR(1) model. Its estimation equation can easily be written based on the output or by selecting *Views/Representations* in Figure 3.38.

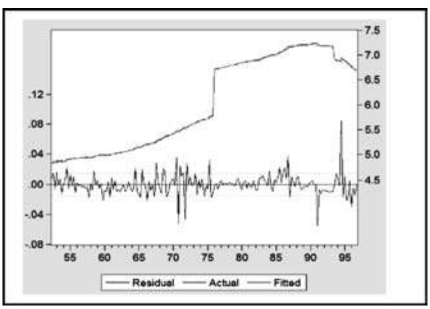


Figure 3.39 Residual graph of the regression in Figure 3.38

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/16/07 Time: 09:29
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 110 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

	Coefficient	Std. Error	t-Statistic	Prob.
DY1	4.278934	0.312875	13.67618	0.0000
DY1*T	0.015253	0.002875	5.305744	0.0000
DY2	5.973873	0.166976	35.77679	0.0000
DY2*T	0.007476	0.001320	5.662685	0.0000
DY3	9.917263	1.116058	8.885978	0.0000
DY3*T	-0.017782	0.006585	-2.700530	0.0076
AR(1)	0.970848	0.013884	69.92473	0.0000

R-squared	0.999754	Mean dependent var	6.029911
Adjusted R-squared	0.999745	S.D. dependent var	0.923719
S.E. of regression	0.014752	Akaike info criterion	-5.556510
Sum squared resid	0.037433	Schwarz criterion	-5.431863
Log likelihood	504.3076	Hannan-Quinn criter.	-5.505967
Durbin-Watson stat	2.358887		

Inverted AR Roots	.97

Figure 3.40 The Newey–West estimates of the three-piece AR(1)_GM in (3.55)

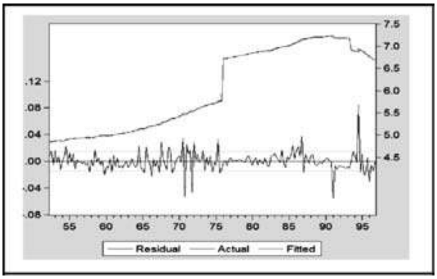


Figure 3.41 Residual graph of the regression in Figure 3.40

Furthermore, by using the Newey–West estimation method, the statistical results in Figure 3.40 are obtained, with its residual graph in Figure 3.41.

By looking at the statistical results based on the AR(1)_GM by using the OLS, White and Newey–West estimation methods, the following notes and conclusions are obtained:

(1) The OLS, White and Newey–West estimation methods will be exactly the same regression functions. Therefore, they will have exactly the same residual graphs, as well as the same values of the DW-statistic. As a result, their residual graphs cannot be used to differentiate the quality of their statistical results.

(2) The White and Newey–West estimation methods give different estimates of the standard error of the model parameters. As a result, they will give different values of the t-statistic, even though all reject the null hypothesis H_0: $C(k) = 0$ for each $k = 1, \ldots, 7$ and have such small p-values.

(3) Then a question can be asked: 'Which estimation method would you think is the best?' Since the White estimation method only takes into account the unknown heteroskedasticity, while the Newey–West method takes into account both the unknown autocorrelation and heteroskedasticity, then in general the Newey–West estimation method would be applied.

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/16/07 Time: 11:14
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence not achieved after 500 iterations
LOG(Y) = (C(1)+C(2)*T)*DY1 + (C(3)+C(4)*T)*DY2 + (C(5)+C(6)*T)*DY3 + [AR(1)=C(7)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	4.279390	0.416175	10.28266	0.0000
C(2)	0.015249	0.003903	3.906830	0.0001
C(3)	5.973924	0.172847	34.56192	0.0000
C(4)	0.007476	0.001174	6.368926	0.0000
C(5)	9.917351	0.671311	14.77310	0.0000
C(6)	-0.017783	0.003831	-4.642033	0.0000
C(7)	0.970829	0.017685	54.89623	0.0000

R-squared	0.999754	Mean dependent var	6.029911
Adjusted R-squared	0.999745	S.D. dependent var	0.923719
S.E. of regression	0.014752	Akaike info criterion	-5.556510
Sum squared resid	0.037433	Schwarz criterion	-5.431863
Log likelihood	504.3076	Hannan-Quinn criter.	-5.505967
Durbin-Watson stat	2.358840		
Inverted AR Roots	.97		

Figure 3.42 Statistical results using the equation specification in (3.56)

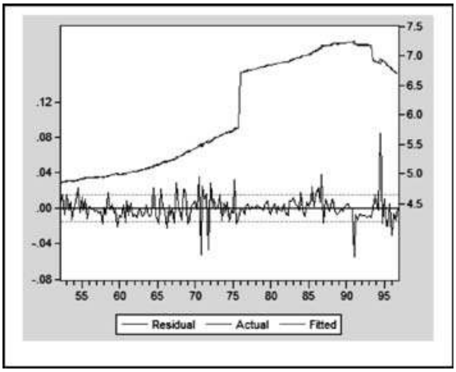


Figure 3.43 Residual graph of the regression in Figure 3.42

(4) Further experimentation has been done using the Newey–West estimation method, but with the equation specification given below (see Figure 3.42):

$$\begin{aligned}\text{Log}(Y) = {} & (C(1) + C(2)^{*}T)^{*}DY1 + (C(3) + C(4)T)^{*}DY2 \\ & + (C(5) + C(6)^{*}T)^{*}DY3 + [AR(1) = C(7)]\end{aligned} \tag{3.56}$$

However, the statistical results in Figure 3.42 present a note '*Convergence not achieved after* 500 *iterations*.' Note that this model and the model in (3.55) are in fact exactly the same regression, in theoretical statistics. It is surprising that Figure 3.43 presents the same estimates for the parameters but different t-statistics without a statement '*Newey–West* ...,' as presented in Figure 3.43. Based on these findings, it can be stated that:

(i) EViews should use different numerical processes in computing the estimates for the two equation specifications, namely (3.55) and (3.56).
(ii) The equation specification in (3.55) should be used in order to obtain the Newey–West estimates. □

Example 3.15. (A set of four growth models) Based on data in the Demo-Modified workfile, four dummy variables $dq1$, $dq2$, $dq3$ and $dq4$ are generated corresponding to the first, second, third and fourth quarterly time series data respectively. Hence, here a set of four growth models will be presented, one for each quarter. Two sets of regressions will be presented, such as (i) a set of four AR(1) growth models through the origin or without an intercept and (ii) a set of four AR(1) growth models with an intercept.

(i) *AR(1) Growth Model Through the Origin*

The equation specification entered is

$$\begin{aligned}\log(m1) = {} & (C(11)+C(12)^{*}t)^{*}dq1+(C(21)+C(22)^{*}t)^{*}dq2 \\ & +(C(31)+C(32)^{*}t)^{*}dq3+(C(41)+C(42)^{*}t)^{*}dq4+[ar(1)=C(1)]\end{aligned} \tag{3.57}$$

This equation is a representation of a set of four growth models, as follows:

$$\begin{aligned}\log(m1) &= (C(11)+C(12)^{*}t)+[ar(1)=C(1)] \text{ for } q=1 \\ \log(m1) &= (C(21)+C(22)^{*}t)+[ar(1)=C(1)] \text{ for } q=2 \\ \log(m1) &= (C(31)+C(32)^{*}t)+[ar(1)=C(1)] \text{ for } q=3 \\ \log(m1) &= (C(41)+C(42)^{*}t)+[ar(1)=C(1)] \text{ for } q=4\end{aligned} \tag{3.58}$$

One of the main objectives in using the equation specification in (3.57) is to present a special table for the model parameters as presented in Table 3.3. Corresponding to this table, the statistical results will directly present the growth rates (*GR*s) of the endogenous variable within each quarter, presented by the parameters $C(12)$, $C(22)$, $C(32)$ and $C(42)$.

Based on the statistical results in Figure 3.44, with its residual graph in Figure 3.45, the following findings and testing hypotheses are obtained:

(1) The slopes of the regressions for the growth rates of the money supply, $M1$, for the first, second, third and fourth quarters are $\hat{C}(12) = 16.639\%$, $\hat{C}(22) = 16.691\%$, $\hat{C}(32) = 16.607\%$ and $\hat{C}(42) = 16.652\%$.
(2) Each growth rate is significantly greater than zero, based on the t-statistic, with a p-value $= 0.0000$.

Table 3.3 The parameters of the models in (3.57) and (3.58)

	Quarter			
	$Q=1$	$Q=2$	$Q=3$	$Q=4$
Intercepts	$C(11)$	$C(21)$	$C(31)$	$C(41)$
Slopes/GR	$C(12)$	$C(22)$	$C(32)$	$C(42)$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 13:16
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 3 iterations
LOG(M1)=C(11)*DQ1+C(12)* DQ1*T+C(21)*DQ2 + C(22)*DQ2*T+C(31)
*DQ3+C(32)*DQ3*T+C(41)*DQ4+C(42)*DQ4*T+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.142506	0.203422	20.36407	0.0000
C(12)	0.016639	0.001182	14.08042	0.0000
C(21)	4.137352	0.203463	20.33471	0.0000
C(22)	0.016691	0.001182	14.12149	0.0000
C(31)	4.149541	0.203469	20.39396	0.0000
C(32)	0.016607	0.001182	14.05078	0.0000
C(41)	4.140260	0.203443	20.35093	0.0000
C(42)	0.016652	0.001181	14.09412	0.0000
C(1)	0.975144	0.008779	111.0828	0.0000

R-squared	0.999651	Mean dependent var	5.816642
Adjusted R-squared	0.999634	S.D. dependent var	0.753241
S.E. of regression	0.014409	Akaike info criterion	-5.592968
Sum squared resid	0.035295	Schwarz criterion	-5.432708
Log likelihood	509.5706	Hannan-Quinn criter.	-5.527984
Durbin-Watson stat	2.043570		

Inverted AR Roots .98

Figure 3.44 Statistical results based on the model in (3.57)

(3) For testing the null hypothesis of no growth rate differences between the four quarters, namely H_0: $C(12) = C(22) = C(32) = C(42)$, a chi-square-statistic is obtained: $\chi_0^2 = 6.929\,683$, with $df = 3$ and a p-value $= 0.0742$. Therefore, at a significant level of 5%, the four *growth rate parameters* (or the *subpopulation growth rates*) do not have significant differences.

(4) However, by doing pairwise comparisons, it is found that H_0: $C(22) = C(32)$ is rejected based on the F-statistic: $F_0 = 5.227\,54$ with $df = (1, 170)$ and a p-value $= 0.0211$, as well as the chi-square-statistic $\chi_0^2 = 5.227\,54$ with df 1 and a p-value $= 0.0199$. In fact, based on the t-statistic with a p-value $=$ $0.0211/2 = 0.010\,55$, it can be concluded that the growth rate in the second quarter is significantly greater than in the third quarter.

(5) Even though the regression function in Figure 3.44 represents a set of four regressions, note that their differences cannot be identified based on the actual and fitted graphs in Figure 3.45.

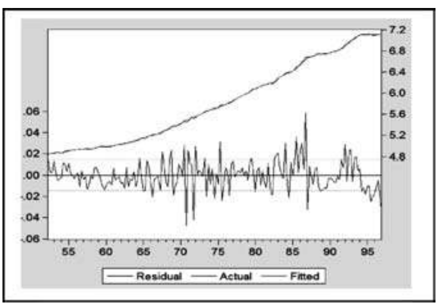


Figure 3.45 Residual graph of the regression in Figure 3.44

(6) Furthermore, in order to have the Newey–West estimates, the following equation specification has to be entered or used:

$$\log(m1)\ dq1\ dq1^{*}t \quad dq2\ dq2^{*}t \quad dq3\ dq3^{*}t \quad dq4\ dq4^{*}t \quad ar(1) \tag{3.59}$$

In this case, EViews will record or save the equation of the model in the following format, where the symbols $C(k), k=1,\ldots,9$ should be used for the testing hypothesis:

$$\begin{aligned}\log(m1) = {} & c(1)^{*}dq1 + c(2)^{*}dq1^{*}t + c(3)^{*}dq2 + c(4)^{*}dq2^{*}t \\ & + c(5)^{*}dq3 + c(6)^{*}dq3^{*}t + c(7)^{*}dq4 + c(8)^{*}dq4^{*}t \\ & + [ar(1) = c(9)]\end{aligned} \tag{3.60}$$

(ii) *AR(1) Growth Model With an Intercept*

The equation specification of the model is

$$\begin{aligned}\log(m1) = {} & (C(11)+C(12)^{*}t)+(C(21)+C(22)^{*}t)^{*}dq2 \\ & +(C(31)+C(32)^{*}t)^{*}dq3+(C(41)+C(42)^{*}t)^{*}dq4+[ar(1)=C(1)]\end{aligned} \tag{3.61}$$

This model presents the following four growth models:

$$\begin{aligned}\log(m1) &= (C(11)+C(12)^{*}t)+[ar(1)=C(1)] \text{ for } q=1 \\ \log(m1) &= (C(11)+C(21)+\{C12)+C(22)\}^{*}t)+[ar(1)=C(1)] \text{ for } q=2 \\ \log(m1) &= (C(11)+C(31)+\{C(12)+C(32)\}^{*}t)+[ar(1)=C(1)] \text{ for } q=3 \\ \log(m1) &= (C(11)+C(41)+\{C(12)+C(42)\}^{*}t)+[ar(1)=C(1)] \text{ for } q=4\end{aligned} \tag{3.62}$$

The parameters of these models and the growth rate differences can be summarized as shown in Table 3.4. Compare this table with Figure 3.44 based on the AR(1) growth model in (3.57).

The statistical results are presented in Figure 3.46, with its residual graph in Figure 3.47. Based on these results and the results in Table 3.4, the following notes and conclusions are obtained:

(1) Figure 3.46 shows that the growth model for the first quarter, namely $Q=1$, is selected as the reference group. $Q=2$, 3 or 4 could also be used as the reference group for alternative models.

Table 3.4 The parameters of the models in (3.61) and (3.62)

					Differences		
	$Q=1$	$Q=2$	$Q=3$	$Q=4$	$Q2$-$Q1$	$Q3$-$Q1$	$Q4$-$Q1$
Intercept	$C(11)$	$C(11)+C(21)$	$C(11)+C(31)$	$C(11)+C(41)$	$C(21)$	$C(31)$	$C(41)$
Slopes/GR	$C(12)$	$C(12)+C(22)$	$C(12)+C(32)$	$C(12)+C(42)$	$C(22)$	$C(32)$	$C(42)$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 17:09
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 2 iterations
LOG(M1)=C(11)+C(12)*T+C(21)*DQ2 + C(22)*DQ2*T+C(31)*DQ3+C(32)
*DQ3*T+C(41)*DQ4+C(42)*DQ4*T+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.142506	0.203422	20.36407	0.0000
C(12)	0.016639	0.001182	14.08042	0.0000
C(21)	-0.005154	0.003774	-1.365813	0.1738
C(22)	5.28E-05	3.64E-05	1.450179	0.1489
C(31)	0.007034	0.004403	1.597604	0.1120
C(32)	-3.19E-05	4.23E-05	-0.755617	0.4509
C(41)	-0.002246	0.003867	-0.580900	0.5621
C(42)	1.32E-05	3.72E-05	0.353659	0.7240
C(1)	0.975144	0.008779	111.0828	0.0000

R-squared	0.999651	Mean dependent var	5.816642
Adjusted R-squared	0.999634	S.D. dependent var	0.753241
S.E. of regression	0.014409	Akaike info criterion	-5.592968
Sum squared resid	0.035295	Schwarz criterion	-5.432708
Log likelihood	509.5706	Hannan-Quinn criter.	-5.527984
F-statistic	60783.25	Durbin-Watson stat	2.043570
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 3.46 Statistical results based on the model in (3.61)

(2) C(22), $C(32)$ and $C(42)$ provide the differences in the growth rates of the first quarter with the second, third and fourth quarters respectively.

(3) At a significant level of 10%, the null hypothesis H_0: $C(22) = 0$ is accepted, based on the t-test, with a p-value = 0.1489, and similarly for H_0: $C(32) = 0$ and H_0: $C(42) = 0$ with p-values of 0.4509 and 0.7240 respectively.

(4) However, for testing the right-hand side hypothesis H_0: $C(22) \leq 0$ versus H_1: $C(22) > 0$, at a significant level of 10%, the null hypothesis is rejected based on the t-test with a p-value = 0.1489/2 = 0.074 45 < 0.10. Hence, the growth rate of $M1$ in the second quarter is significantly greater than in the first quarter.

(5) Note that the four growth functions will present four heterogeneous regression lines in a two-dimensional coordinate system with log($m1$) and t axes.

(6) The values of R-squared, adjusted R-squared, the DW-statistic and other statistics should be exactly the same as those based on the first model, because both models in (3.59) and (3.61) present the same sets of four AR(1) growth models.

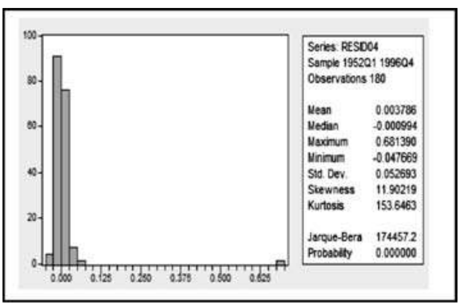


Figure 3.47 Residual histogram of the regression in Figure 3.46

(7) Finally, even though most of the independent variables are insignificant, either one of the variables cannot be deleted in order to have a reduced model, since each parameter has a specific characteristic or position. As an exercise, delete one of the independent variables and then construct a table of its model parameters.
(8) The residual histogram in Figure 3.47 presents an indication of outlier(s) with a very large positive value of skewness. The outlier(s) can also be identified by using the residual box plot, as presented in Section 1.4.2, and the outlier(s) should be treated by using the process suggested in Example 2.4. Do this as an exercise. □

3.7 Stability test

3.7.1 Chow's breakpoint test

Note that a breakpoint of any macroeconomic indicator or growth curve over time could be identified by an analyst or a researcher even before having or collecting the corresponding time series data. For example, it should already be known that any macroeconomic indicator in Indonesia had several breakpoints over the last five or ten years because of the first and second Bali bombings and other environmental factors. Hence, it is possible to apply the discontinuous growth models directly.

Considering that any growth curve might have a breakpoint, Chow presents a statistic that can be used to test the hypothesis that there is a break at a predetermined time point(s). As an illustration look at the growth curves of *GDP*, $M1$, *PR* and *RS* in Figure 3.48. Based on the graphs of *GDP*, $M1$ and *PR*, it is very difficult to identify or estimate whether they have a breakpoint or not and, if it does exist, where that breakpoint is. Hence, some critical events should be used in the corresponding region of observation (population) that can be identified as the cause of the breakpoint. On the other hand, based on the graph of *RS*, several breakpoints can easily be identified.

Example 3.16. (Identifying a breakpoint) The classical AR(1) growth model of $M1$can be seen by entering

$$\log(m1)\ c\ t\ ar(1) \tag{3.63}$$

in the '*Equation specification*' window, with the residual graph presented in Figure 3.49. Based on this graph, a guess can at least be made that there is a breakpoint, because the residual graph presents several high or long vertical lines.

In the first trial, two time points 1970 : 3 and 1987 : 1 are chosen corresponding to the two highest observed absolute values of the error term. Then, the Chow breakpoint test should be done as follows:

(1) Having the result on the screen, click *View* ... and then select *Stability Tests/ Chow Breakpoint Test .... click*. This will give the window of the Chow tests on the screen, as presented in Figure 3.50, together with its statistics.

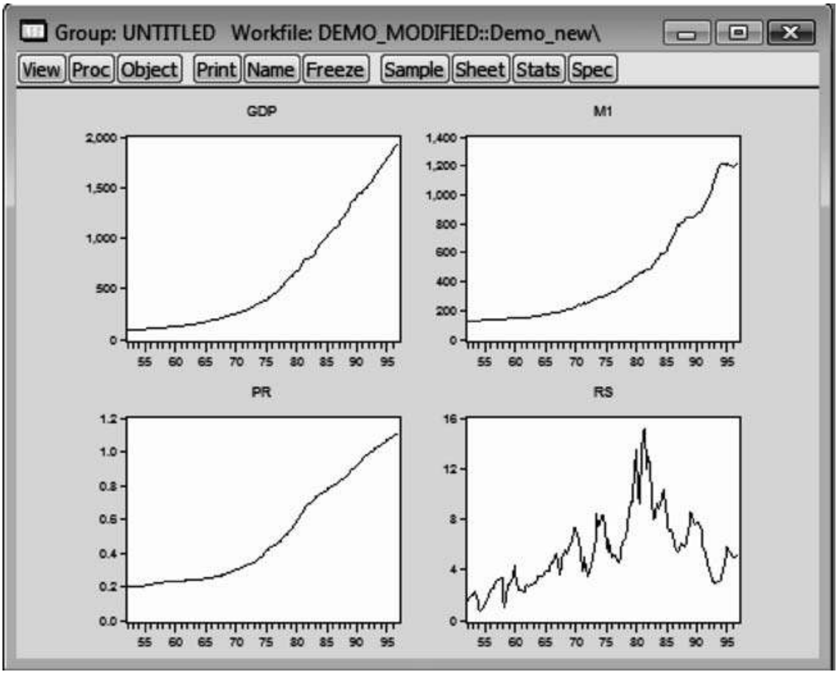


Figure 3.48 Growth curves of the variables *GDP*, *M*1, *PR* and *RS*

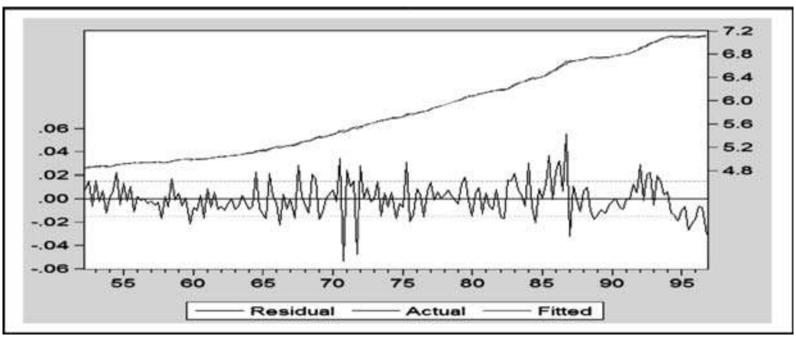


Figure 3.49 Residual graph of the model in (3.63)

(2) By entering *'1970 : 3 1987 : 1'* in the window shown on the screen and then clicking *OK*, the Chow Breakpoint Test will appear on the right-hand side in Figure 3.50.
(3) Hence, based on a p-value $= 0.0000$, the null hypothesis of no two breakpoints at time 1970 : 3 and 1987 : 1 is rejected.
(4) Note that the observed values at the two breakpoints could be outliers. If this is the case then a second data analysis may be done by using the alternative methods suggested in Example 2.4. □

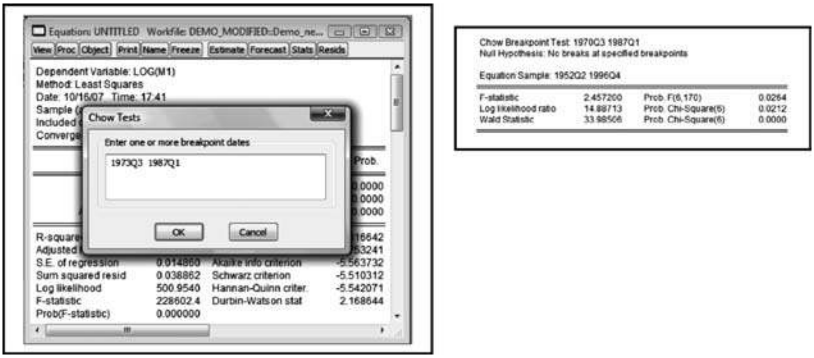


Figure 3.50 The Chow breakpoint tests for the variable $M1$

Example 3.17. (Effects of breakpoints or outliers) Corresponding to the two breakpoints or outliers of the observed values of $\log(m1)$, an attempt will now be made to present a method of how to explore the effects of those points on the growth model. In order to study the effect of the two breakpoints, two dummy variables should be defined, namely $Dbp1 = 1$ if $t = 1970:3 = 79$ and $Dbp1 = 0$ if otherwise; and $Dbp2 = 1$ if $t = 1980:1 = 105$ and $Dbp2 = 0$ if otherwise. Then the analysis can be done by using two alternative equation specifications as follows:

$$\begin{aligned}\log(m1) = {} & c(1) + c(2)^*t + c(3)^*Dbp1 + c(4)^*Dbp2 \\ & + c(5)^*t^*Dbp1 + c(6)^*t^*Dbp2 + [ar(1) = c(7)]\end{aligned} \tag{3.64}$$

$$\log(m1) = c(1) + c(2)^*t + c(3)^*Dbp1 + c(4)^*Dbp2 + [ar(1) = c(7)] \tag{3.65}$$

Table 3.5 presents the classical growth models in (3.64) and (3.65) by the two defined dummy variables. Based on this table, the following specific characteristics of each model can be seen:

(i) The model in (3.64) represents a set of four *heterogeneous regressions* (a set of regressions having different slopes), as presented in column (3), where the

Table 3.5 The classical growth models in (3.64) and (3.65) by the dummy variables $Dbp1$ and $Dbp2$

$Dbp1$ (1)	$Dbp2$ (2)	Growth models in (3.64) (3)	Growth models in (3.65) (4)
0	0	$c(1) + c(2)^*t$	$c(1) + c(2)^*t$
1	0	$c(1) + c(2)^*t + \{c(3) + c(5)^*t\}$	$c(1) + c(2)^*t + c(3)$
0	1	$c(1) + c(2)^*t + \{c(4) + c(6)^*t\}$	$c(1) + c(2)^*t + c(4)$
1	1	$c(1) + c(2)^*t + \{c(3) + c(4)\} + \{c(5) + c(6)\}^*t$	$c(1) + c(2)^*t + \{c(3) + c(4)\}$

regression for $Dbp1 = Dbp2 = 0$ is taken as the reference. Hence, this model will study or test the effects of the breakpoints or outliers on the growth rate of the variable $M1$, indicated by the parameters $c(5)$, $c(6)$ as well as $\{c(5) + c(6)\}$. Do this as an exercise.

(ii) The model in (3.65) should be considered as a reduced model of (3.64) under the assumption that $c(5) = c(6) = 0$. Therefore, this model represents a set of four *homogeneous regressions* (a set of regressions having equal slopes), as presented in column (4). This model can be considered as a covariance analysis model, which is used to study the intercept differences between the four homogeneous regressions. Those intercept differences are known as the adjusted effect differences of the time t on $\log(m1)$. □

Example 3.18. (Testing the breakpoints *Y* in Example 3.13) Example 3.13 presents a three-piece growth model of a hypothetical time series Y having two breakpoints at 1972Q1 and 1993Q4. The existence of these breakpoints will be tested using the Chow test. Figure 3.51 presents the statistical results of the test by entering the equation specification as follows:

$$Y\ c\ t\ ar(1) \tag{3.66}$$

□

Chow Breakpoint Test: 1973Q1 1993Q3
Null Hypothesis: No breaks at specified breakpoints

Equation Sample: 1952Q2 1996Q4

F-statistic	4.726251	Prob. F(6,170)	0.0002
Log likelihood ratio	27.61479	Prob. Chi-Square(6)	0.0001
Wald Statistic	19.83927	Prob. Chi-Square(6)	0.0030

Figure 3.51 The Chow breakpoint test for the variable *Y*, based on the model in (3.66)

3.7.2 Chow's forecast test

The processes of the Chow forecast test is first to estimate the model based on a subsample comprised of the first T_1 observations. Then this estimated model will be used to predict the values of the dependent variable in the remaining T_2 data points. This test will be used to test the hypothesis on the stability of the estimated relation over the two subsamples. The Chow forecast test can be used with least squares and two-stage least squares regressions.

EViews presents two statistics, the F-statistic and the log likelihood ratio (LR) statistic. The F-statistic follows an exact finite sample F-distribution if the errors are independent and, identically, normally distributed. The LR test statistic has an asymptotic chi-square distribution with $df = T_2$ under the null hypothesis of no structural change.

Example 3.19. (Chow's forecast test for log($m1$)) Considering the AR(1) growth model of the variable $M1$ in (3.63) for the whole data set from 1952 : 1 to 1996 : 4, Chow's forecast test will now be applied. Having the statistical results on the screen, click or select *View/Stability Tests/Chow Forecast Test* . . . .

Chow Forecast Test: Forecast from 1987Q1 to 1996Q4

F-statistic	1.294223	Prob. F(40,136)	0.1401
Log likelihood ratio	57.73776	Prob. Chi-Square(40)	0.0343

Test Equation:
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 18:00
Sample: 1952Q2 1986Q4
Included observations: 139
Convergence achieved after 349 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-150.8024	3200.228	-0.047122	0.9625
T	0.166698	1.639764	0.101660	0.9192
AR(1)	0.998939	0.011301	88.39501	0.0000

R-squared	0.999278	Mean dependent var	5.503644
Adjusted R-squared	0.999268	S.D. dependent var	0.531613
S.E. of regression	0.014386	Akaike info criterion	-5.623732
Sum squared resid	0.028147	Schwarz criterion	-5.560398
Log likelihood	393.8493	Hannan-Quinn criter.	-5.597994
F-statistic	94151.48	Durbin-Watson stat	2.473383
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00

(a)

Chow Forecast Test: Forecast from 1990Q1 to 1996Q4

F-statistic	1.056491	Prob. F(28,148)	0.3992
Log likelihood ratio	32.61717	Prob. Chi-Square(28)	0.2501

Test Equation:
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 18:03
Sample: 1952Q2 1989Q4
Included observations: 151
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.865441	0.502380	7.694250	0.0000
T	0.018711	0.002902	6.448442	0.0000
AR(1)	0.979738	0.009967	98.29419	0.0000

R-squared	0.999414	Mean dependent var	5.600104
Adjusted R-squared	0.999406	S.D. dependent var	0.607090
S.E. of regression	0.014793	Akaike info criterion	-5.569629
Sum squared resid	0.032388	Schwarz criterion	-5.509683
Log likelihood	423.5070	Hannan-Quinn criter.	-5.545276
F-statistic	126237.5	Durbin-Watson stat	2.442990
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

(b)

Figure 3.52 The Chow forecast tests using the growth model in (3.63): (a) for the period 1987 : 1 to 1996 : 4 and (b) for the period 1990 : 1 to 1996 : 4

Then by entering 1987 : 1 as the first observation in the forecast period, the results in Figure 3.52(a) are obtained. This figure shows that, at a significant level of 0.10, the null hypothesis of no structural change of log($m1$) before and after 1987 : 1 is accepted based on the F-statistic with a p-value = 0.1401, but it is rejected based on the LR-statistic with a p-value = 0.0343. This example illustrates the possibility that two tests may yield conflicting conclusions, but in a particular case, which one is more appropriate or suitable? By observing the growth curve of the time series $M1$, it could be said that there is a structural change of log($m1$) before and after 1987 : 1, showing that there is a significant structural change of log ($m1$) before and after 1987 : 1 based on the LR test. Hence, in the statistical sense, the model based on the subsample 1952 : 1 to 1987 : 1 is not an acceptable model to be used in forecasting.

On the other hand, by entering 1990 : 1 as the first observation in the forecast period the result in Figure 3.52(b) is obtained, which shows that both statistics could not reject the null hypothesis. Therefore, the AR(1)_GM based on the subsample 1952 : 1 to 1990 : 1 can be considered as an appropriate model to be used in forecasting. □

3.8 Generalized discontinuous models with trend

Based on the previous examples in this chapter and all of the continuous growth models in Chapter 2, it is easy to derive many discontinuous growth models or generalized discontinuous models with trend, by using one or several dummy variables as an additional independent variable(s). For illustration purposes, the following sections will present two-piece growth models that are derived from the selected continuous growth models in Chapter 2.

3.8.1 General two-piece univariate models with trend

Corresponding to the general growth model (2.37) or (2.45), there is a general two-piece model with trend and multivariate exogenous variables, as follows:

$$\begin{aligned} g(y_t) = {} & (c(11) + c(12)^* t + f_1(x_1, x_2, \ldots, x_K))^* D1 \\ & + (c(21) + c(22)^* t + f_2(x_1, x_2, \ldots, x_K))^* D2 + \mu_t \end{aligned} \tag{3.67a}$$

where x_k, $k = 1,2, \ldots, x_K$, are exogenous variables, which could be the main factors, together with their interaction factors, of pure exogenous variables. The lagged variables of the endogenous or exogenous variables, $f_1(^*)$ and $f_2(^*)$ are functions having a finite number of unknown parameters and $g(y_t)$ is a defined function without parameters.

The dummy variables $D1$ and $D2$ are defined for the two time periods considered, as presented in the previous examples. An alternative of the model in (3.67a) is

$$\begin{aligned} g(y_t) = {} & (c(11) + c(12)^* t + f_1(x_1, x_2, \ldots, x_K))^* D1 \\ & + (c(21) + c(22)^* t + f_2(x_1, x_2, \ldots, x_K)) + \mu_t \end{aligned} \tag{3.67b}$$

Table 3.6 presents a summary of the models in (3.67a) and (3.67b), using the defined time periods or dummy variables in modified forms. Please note that $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are multivariate exogenous variables, which could be equal, and $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are unequal vectors of the model parameters. This table clearly shows the differential meanings of the parameters between the two models.

Table 3.6 A summary of the models in (3.67a) and (3.67b) by time periods

$D1$	$D2$	Model (3.67a)	Model (3.67b)
1	0	$c(11) + c(12)t + f_1(x_1,\theta_1)$	$\{c(11) + c(12)t + f_1(x_1,\theta_1)\} + \{c(21) + c(22)t + f_2(x_2,\theta_2)\}$
0	1	$c(21) + c(22)t + f_2(x_2,\theta_2)$	$c(21) + c(22)t + f_2(x_2,\theta_2)$

Example 3.20. (Two-piece models with trend) In this example two alternative models will be presented, an additive model with trend and an interaction model with trend, as follows:

(1) Two-Piece Additive Model with Trend

Corresponding to the LVAR(1,1) growth model in Example 2.17, this is a two-piece lagged-variable autoregressive model with trend for $M1$. In order to obtain

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 18:27
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 12 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(GDP)+C(14)*LOG(M1(-1)))*D1 +
(C(21)+C(22)*T+C(23)*LOG(GDP)+C(24)*LOG(M1(-1)))*D2 +
[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.029850	0.111481	-0.267761	0.7892
C(12)	-0.000960	0.000782	-1.227013	0.2215
C(13)	0.141251	0.083760	1.686384	0.0936
C(14)	0.876062	0.066563	13.16135	0.0000
C(21)	-0.084545	0.146529	-0.576980	0.5647
C(22)	-0.000594	0.000603	-0.985416	0.3258
C(23)	0.033864	0.026917	1.258053	0.2101
C(24)	0.991896	0.036959	26.83758	0.0000
C(1)	-0.097293	0.083673	-1.162780	0.2466

R-squared	0.999641	Mean dependent var	5.822083
Adjusted R-squared	0.999624	S.D. dependent var	0.751831
S.E. of regression	0.014571	Akaike info criterion	-5.570381
Sum squared resid	0.035879	Schwarz criterion	-5.409504
Log likelihood	504.7639	Hannan-Quinn criter.	-5.505141
Durbin-Watson stat	1.961834		

Inverted AR Roots -.10

Figure 3.53 Statistical results based on the two-piece model in (3.68)

the statistical results in Figure 3.53, the following equation specification should be used:

$$\begin{aligned}\log(m1) = {} & (c(11)+c(12)^{*}t+c(13)^{*}\log(gdp)+c(14)^{*}\log(m1(-1)))^{*}D1 \\ & +(c(21)+c(22)^{*}t+c(23)^{*}\log(gdp)+c(24)^{*}\log(m1(-1)))^{*}D2 \\ & +[ar(1)=c(1)]\end{aligned} \tag{3.68}$$

In fact, this equation represents the two following regressions in the first and second time periods defined by the dummy variables ($D1=1$, $D2=0$) and ($D=0$, $D2=1$) respectively:

$$\begin{aligned}\log(m1) &= (c(11)+c(12)^{*}t+c(13)^{*}\log(gdp)+c(14)^{*}\log(m1(-1))) \\ \log(m1) &= (c(21)+c(22)^{*}t+c(23)^{*}\log(gdp)+c(24)^{*}\log(m1(-1))) \\ \mu_t &= c(1)^{*}\mu_{t-1}+\varepsilon_t\end{aligned} \tag{3.69}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 18:52
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(GDP)+C(14)*LOG(M1(-1)))*D1 +
(C(21)+C(22)*T+C(23)*LOG(GDP)+C(24)*LOG(M1(-1)))*D2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.061053	0.119244	-0.512001	0.6093
C(12)	-0.001440	0.000795	-1.812423	0.0717
C(13)	0.196423	0.083875	2.341866	0.0203
C(14)	0.831705	0.066159	12.57126	0.0000
C(21)	-0.061883	0.157883	-0.391953	0.6956
C(22)	-0.000500	0.000649	-0.771261	0.4416
C(23)	0.035951	0.029226	1.230109	0.2203
C(24)	0.984191	0.039362	25.00388	0.0000

R-squared	0.999642	Mean dependent var	5.816642
Adjusted R-squared	0.999627	S.D. dependent var	0.753241
S.E. of regression	0.014541	Akaike info criterion	-5.580034
Sum squared resid	0.036156	Schwarz criterion	-5.437581
Log likelihood	507.4131	Hannan-Quinn criter.	-5.522271
Durbin-Watson stat	2.113841		

(a)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 18:33
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 7 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(GDP)+C(14)*LOG(M1(-1)))*D1 +
(C(21)+C(22)*T+C(23)*LOG(GDP)+C(24)*LOG(M1(-1)))*D2
+[AR(2)=C(2)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.003495	0.143127	-0.024417	0.9805
C(12)	-0.001492	0.000871	-1.713958	0.0884
C(13)	0.221253	0.090040	2.457284	0.0150
C(14)	0.796285	0.071877	11.07847	0.0000
C(21)	0.008418	0.188016	0.044771	0.9643
C(22)	-0.000226	0.000770	-0.293563	0.7695
C(23)	0.036027	0.034176	1.054156	0.2933
C(24)	0.967481	0.047293	20.45699	0.0000
C(2)	0.173401	0.082150	2.110794	0.0363

R-squared	0.999647	Mean dependent var	5.827503
Adjusted R-squared	0.999630	S.D. dependent var	0.750468
S.E. of regression	0.014432	Akaike info criterion	-5.589164
Sum squared resid	0.034994	Schwarz criterion	-5.427665
Log likelihood	503.6410	Hannan-Quinn criter.	-5.523667
Durbin-Watson stat	2.051683		

Inverted AR Roots	.42	-.42

(b)

Figure 3.54 Statistical results based on (a) a reduced model and (b) an unexpected modified model of the model in (3.68)

Since the null hypothesis of no first-order autocorrelation, namely H_0: $C(1) = 0$, is accepted with a p-value = 0.2394, as presented in Figure 3.53, a reduced model is produced, as presented in Figure 3.54(a) without an indicator AR(1). By using the trial-and-error methods an unexpected model is obtained in Figure 3.54(b), with its residual histogram in Figure 3.55, since the model has an indicator AR(2) without the indicator AR(1). Readers may find other acceptable or unexpected models by using the higher-order lagged endogenous variable or indicator AR(p).

Based on the statistics in Figure 3.54(b), the following notes and conclusions can be obtained:

(a) Table 3.7 presents the model parameters by the time periods and exogenous variables, which can be used to write hypotheses on differences between the first and second time periods. Then the tests can be conducted using the Wald test.

(b) Since each of the exogenous variables t and log(gdp) is insignificant, a reduced model may result. Hence, a model will be produced where the

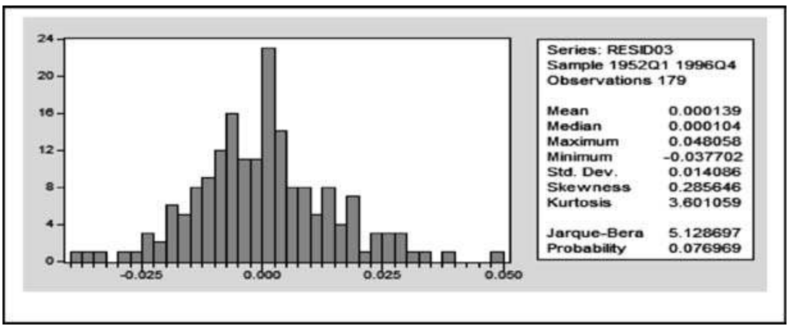


Figure 3.55 Residual histogram of the regression in Figure 3.54(b)

Table 3.7 The parameters of the model in Figure 3.54(b) by time periods and exogenous variables

$D1$	$D2$	Constant	t	log(gdp)	log($M1(-1)$)	$AR(2)$
1	0	$C(11)$	$C(12)$	$C(13)$	$C(14)$	$C(2)$
0	1	$C(21)$	$C(22)$	$C(23)$	$C(24)$	$C(2)$

regressions in the first and second time periods have a different set of exogenous variables.

(c) Finally, the residual histogram in Figure 3.55, including the statistics of the residual, can be used to evaluate the limitation of the model. For example, in a theoretical sense, the mean should be equal to zero and the kurtosis should be equal to 3.0, based on the assumptions of the basic regression.

(2) Two-Piece Interaction Model with Trend

Corresponding to the interaction growth model in (2.59), a two-piece interaction model is now considered with the following equation specification:

$$\begin{aligned}\log(m1) = {} & (c(11) + c(12)^* t + c(13)^* \log(gdp) + c(14)^* \log(pr) \\ & + c(15)^* \log(gdp)^* \log(pr))^* D1 \\ & + (c(21) + c(22)^* t + c(23)^* \log(gdp) + c(24)^* \log(pr) \\ & + c(25)^* \log(gdp)^* \log(pr))^* D2 \\ & + [ar(1) = c(1), ar(2) = c(2)]\end{aligned} \tag{3.70}$$

However, EViews presents the '*Near singular matrix*' error message. An experimentation should now be performed. At the first stage, by deleting the indicator AR(2), the error message will still be received and at the second stage an attempt is made to delete the indicator AR(1). This produces the statistical results given in Figure 3.56 based on a two-piece growth model with exogenous

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 18:58
Sample: 1952Q1 1996Q4
Included observations: 180
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(GDP)+C(14)*LOG(PR) + C(15)
*LOG(GDP)*LOG(PR))*D1 + (C(21)+C(22)*T+C(23)*LOG(GDP)
+C(24)*LOG(PR) + C(25)*LOG(GDP)*LOG(PR))*D2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.270936	1.159501	0.233666	0.8155
C(12)	-0.007245	0.002450	-2.956674	0.0036
C(13)	1.060017	0.162590	6.519576	0.0000
C(14)	-0.186011	0.902192	-0.206176	0.8369
C(15)	0.064675	0.141040	0.458555	0.6471
C(21)	1.638560	1.320863	1.240522	0.2165
C(22)	0.021593	0.005203	4.150372	0.0001
C(23)	0.241082	0.280972	0.858029	0.3921
C(24)	2.370935	1.143398	2.073586	0.0396
C(25)	-0.493729	0.189871	-2.600337	0.0101

R-squared	0.998274	Mean dependent var	5.811220
Adjusted R-squared	0.998183	S.D. dependent var	0.754650
S.E. of regression	0.032169	Akaike info criterion	-3.981651
Sum squared resid	0.175928	Schwarz criterion	-3.804265
Log likelihood	368.3486	Hannan-Quinn criter.	-3.909729
Durbin-Watson stat	0.320685		

Figure 3.56 Statistical results based on a two-piece interaction growth model

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 19:03
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(GDP)+C(14)*LOG(PR) + C(15)
*LOG(GDP)*LOG(PR))*D1 + (C(21)+C(22)*T+C(23)*LOG(GDP)
+C(24)*LOG(PR) + C(25)*LOG(GDP)*LOG(PR))*D2 + C(1)*LOG(M1(
-1)) + C(2)*LOG(M1(-2))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.374631	0.541873	-0.691364	0.4903
C(12)	2.95E-05	0.001156	0.025514	0.9797
C(13)	0.068107	0.085449	0.797051	0.4266
C(14)	-0.151620	0.415739	-0.364700	0.7158
C(15)	0.015246	0.064900	0.234913	0.8146
C(21)	0.884625	0.604146	1.464257	0.1450
C(22)	0.001661	0.002491	0.666751	0.5059
C(23)	-0.142924	0.129848	-1.100707	0.2726
C(24)	0.521528	0.526112	0.991286	0.3230
C(25)	-0.068821	0.087997	-0.782076	0.4353
C(1)	0.816101	0.077488	10.53197	0.0000
C(2)	0.171030	0.082246	2.079488	0.0391

R-squared	0.999645	Mean dependent var	5.822083
Adjusted R-squared	0.999621	S.D. dependent var	0.751831
S.E. of regression	0.014630	Akaike info criterion	-5.546467
Sum squared resid	0.035530	Schwarz criterion	-5.331965
Log likelihood	505.6355	Hannan-Quinn criter.	-5.459480
Durbin-Watson stat	1.951058		

Figure 3.57 Statistical results based on a two-piece LV(2)_GM

variables. Since this model does not take into account the autocorrelation of the error terms, the regression presents a very small value of the DW-statistic.

Hence, the lagged endogenous variables should be used in order to take into account the autocorrelation of the error terms. Then we obtain the LV(2)_GM is obtained with interaction exogenous variable(s), as presented in Figure 3.57. Based on this model the following notes and conclusions are given:

(a) Compared with the lagged endogenous variables, each of the other exogenous variables is insignificant. Even though this model can be considered as an acceptable time series model, in a statistical sense, since it has $DW = 1.951\,058$, it is sufficient to declare that the data and the model used support the basic assumptions of the error terms.

(b) Then this model can be used as a base or full model to construct a reduced model by deleting one or two exogenous variables. However, the trial-and-error methods should be used to delete an exogenous variable, but the model is not mainly based on a variable with a large or the largest p-value. Refer to Section 2.14 and do this as an exercise. □

Example 3.21. (Unexpected models) In the previous examples, three variables, $M1$, GDP and PR, were used having the same pattern of growth curves over time. In this example, the relationship will be considered between variables $M1$ and RS having different patterns of growth curves, as presented in Figure 3.48, while Figure 3.58 presents the scatter graph with a regression line of log($M1$) on RS. This figure clearly shows that the simple linear regression of log($M1$) on RS is not an appropriate model.

The data show that RS and log($M1$) have a positive correlation based on the subsample from $t = 1952:1$ up to $t = 1981:3$ and they have a negative correlation based on the other subsample from $t = 1981:4$ up to $t = 1996:4$. In fact, RS has a

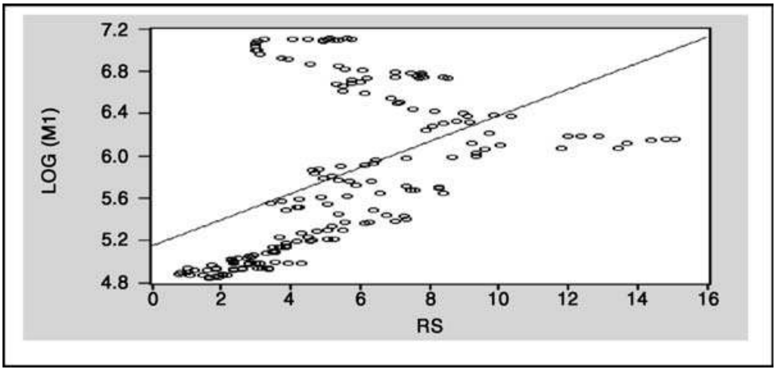


Figure 3.58 Scatter graph with regression line of log($M1$) on RS

maximum observed value of 15.087 33 at $t = 1981:4$. Corresponding to these subsamples, two dummy variables have been generated, namely $Drs1$ and $Drs2$, which should be used to present any two-piece models having RS as one of the independent, exogenous or source variables. Figure 3.59 presents two scatter graphs with regressions of log($M1$) on RS with the first and second time periods.

The simplest two-piece model with trend considered is

$$\begin{aligned}\log(m1) = & (c(11) + c(12)^*t + c(13)^*\log(m1(-1)) + c(14)^*rs)^*Drs1 \\ & + (c(21) + c(22)^*t + c(23)^*\log(m1(-1)) + c(24)^*rs)^*Drs2\end{aligned} \quad (3.71)$$

Note that the first lag log($m1(-1)$) should be used in both time periods in order to take into account the differential autocorrelation in the two time periods. Figure 3.60 presents its statistical results with a large value of $DW = 2.713\,615$. Using an additional indicator AR(1) in the model gives the statistical results in Figure 3.61 with a value of $DW = 2.074\,438$.

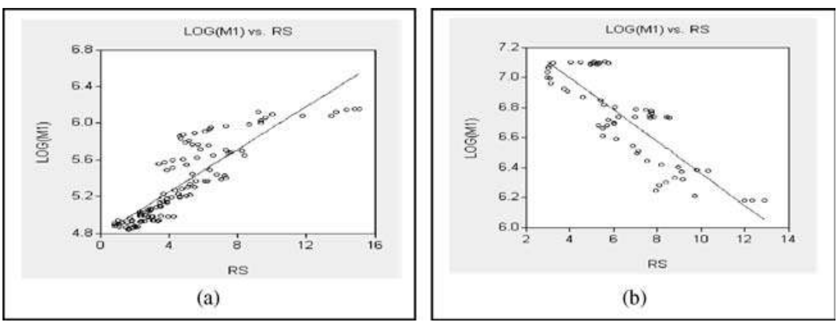


Figure 3.59 Scatter graphs with regression lines of log($M1$) on RS, based on (a) subsample 1952 : 1 to 1981.3 and (b) subsample 1984 : 4 to 1996 : 4

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 19:47
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(M1(-1))+C(14)*RS)*DRS1 + (C(21)
+ C(22)*T+C(23)*LOG(M1(-1))+C(24)*RS)*DRS2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.033884	0.066304	0.511049	0.6100
C(12)	0.000324	0.000155	2.096883	0.0375
C(13)	0.993482	0.014303	69.45954	0.0000
C(14)	-0.001558	0.000786	-1.981650	0.0491
C(21)	0.842727	0.179264	4.701047	0.0000
C(22)	0.001112	0.000589	1.885859	0.0610
C(23)	0.860517	0.038436	22.38848	0.0000
C(24)	-0.008865	0.001332	-6.654238	0.0000

R-squared	0.999714	Mean dependent var	5.816642
Adjusted R-squared	0.999702	S.D. dependent var	0.753241
S.E. of regression	0.012994	Akaike info criterion	-5.804942
Sum squared resid	0.028874	Schwarz criterion	-5.662489
Log likelihood	527.5424	Hannan-Quinn criter.	-5.747179
Durbin-Watson stat	2.713615		

Figure 3.60 Statistical results based on the model in (3.71)

By using further trial-and-error methods, statistical results based on two models are obtained, which are statistically good models, namely an LVAR(1,3)_GM (i.e. a first lagged-variable–third-order autoregressive two-piece growth model) and an LV (2)_GM, as presented in Figure 3.62. Note that both models have sufficient values of the DW-statistics and each of the independent variables are significant.

Finally, an attempt is made to develop an AR(*p*)_GM without using the lagged endogenous variable. In this case, the first lagged variable of *RS*, namely *RS*(−1), is tried

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 11/23/07 Time: 10:22
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 4 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(M1(-1))+C(14)*RS)*DRS1+(C(21)
+C(22)*T+C(23)*LOG(M1(-1))+C(24)*RS)*DRS2+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.016376	0.045314	0.361401	0.7183
C(12)	0.000268	0.000106	2.523603	0.0125
C(13)	0.997182	0.009783	101.9259	0.0000
C(14)	-0.001323	0.000549	-2.409503	0.0170
C(21)	0.754042	0.124742	6.044808	0.0000
C(22)	0.000863	0.000409	2.109328	0.0364
C(23)	0.878905	0.026719	32.89490	0.0000
C(24)	-0.008457	0.000929	-9.107243	0.0000
C(1)	-0.382883	0.071974	-5.319731	0.0000

R-squared	0.999753	Mean dependent var	5.822083
Adjusted R-squared	0.999741	S.D. dependent var	0.751831
S.E. of regression	0.012098	Akaike info criterion	-5.942232
Sum squared resid	0.024737	Schwarz criterion	-5.781356
Log likelihood	537.8587	Hannan-Quinn criter.	-5.876993
Durbin-Watson stat	2.074438		

Inverted AR Roots -.38

Figure 3.61 Statistical results based on the AR(1) of the model in (3.71)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 11/23/07 Time: 10:30
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 4 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(M1(-1))+C(14)*RS)*DRS1+(C(21)
+C(22)*T+C(23)*LOG(M1(-1))+C(24)*RS)*DRS2+[AR(1)=C(1),AR(2)=
C(2),AR(3)=C(3)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.018955	0.034004	0.557422	0.5780
C(12)	0.000261	8.08E-05	3.224104	0.0015
C(13)	0.996536	0.007355	135.4916	0.0000
C(14)	-0.001059	0.000429	-2.468861	0.0146
C(21)	0.731797	0.096986	7.545399	0.0000
C(22)	0.000803	0.000316	2.543593	0.0119
C(23)	0.883487	0.020704	42.67310	0.0000
C(24)	-0.008378	0.000737	-11.36373	0.0000
C(1)	-0.461685	0.077364	-5.967669	0.0000
C(2)	-0.211223	0.083839	-2.519396	0.0127
C(3)	-0.187872	0.077068	-2.437749	0.0158

R-squared	0.999763	Mean dependent var	5.833023
Adjusted R-squared	0.999749	S.D. dependent var	0.748997
S.E. of regression	0.011872	Akaike info criterion	-5.968742
Sum squared resid	0.023257	Schwarz criterion	-5.770587
Log likelihood	536.2493	Hannan-Quinn criter.	-5.888371
Durbin-Watson stat	1.914538		

Inverted AR Roots	.08+.55i	.08-.55i	-.61

(a)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 11/23/07 Time: 10:36
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(RS)+C(14)*LOG(M1(-1))+C(15)
*LOG(M1(-2)))*DRS1+(C(21) +C(22)*T +C(23)*RS+C(24)*LOG(M1(
-1))+C(25)*LOG(M1(-2)))*DRS2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.095611	0.058177	1.643447	0.1022
C(12)	0.000556	0.000164	3.382898	0.0009
C(13)	-0.009393	0.003970	-2.365705	0.0191
C(14)	0.629687	0.084303	7.469360	0.0000
C(15)	0.351371	0.083871	4.189443	0.0000
C(21)	0.909830	0.169778	5.358925	0.0000
C(22)	0.000878	0.000559	1.571394	0.1180
C(23)	-0.010774	0.001434	-7.510705	0.0000
C(24)	0.547975	0.121222	4.520431	0.0000
C(25)	0.310349	0.114938	2.700151	0.0076

R-squared	0.999751	Mean dependent var	5.822083
Adjusted R-squared	0.999738	S.D. dependent var	0.751831
S.E. of regression	0.012174	Akaike info criterion	-5.924452
Sum squared resid	0.024900	Schwarz criterion	-5.745700
Log likelihood	537.2762	Hannan-Quinn criter.	-5.851963
Durbin-Watson stat	2.137470		

(b)

Figure 3.62 Statistical results based on two-piece growth models: (a) an LVAR(1,3)_GM and (b) an LV(2)_GM

as an independent variable in both time periods, giving the results in Figure 3.63(a), which presents a note '*Convergence not achieved after* 500 *iterations*.' This indicates that the estimates are not optimal estimates. For this reason the model needs to be modified.

By using the trial-and-error methods when selecting *RS*, *RS*(−1), as well as *RS*(−2), as the possible independent variable(s) of a model, a statistically optimal estimate was found in Figure 3.63(b), where the '*Convergence achieved after* 132

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 20:22
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence not achieved after 500 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*RS(-1))*DRS1 + (C(21) + C(22)*T+C(23)
*RS(-1))*DRS2 +[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	3.896288	0.332490	11.71851	0.0000
C(12)	0.019220	0.002926	6.568000	0.0000
C(13)	-0.003405	0.001733	-1.964543	0.0511
C(21)	4.563595	0.220596	20.68760	0.0000
C(22)	0.014375	0.001409	10.20205	0.0000
C(23)	-0.008610	0.002581	-3.335563	0.0010
C(1)	0.982494	0.014055	69.90328	0.0000

R-squared	0.999639	Mean dependent var	5.822083
Adjusted R-squared	0.999626	S.D. dependent var	0.751831
S.E. of regression	0.014534	Akaike info criterion	-5.586154
Sum squared resid	0.036120	Schwarz criterion	-5.461028
Log likelihood	504.1677	Hannan-Quinn criter.	-5.535412
Durbin-Watson stat	2.342531		

Inverted AR Roots	.98

(a)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/07 Time: 20:25
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 132 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*RS)*DRS1 + (C(21) + C(22)*T+C(23)
*RS(-1))*DRS2 +[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	4.022703	0.552144	7.285608	0.0000
C(12)	0.017788	0.004066	4.375314	0.0000
C(13)	-0.001102	0.001739	-0.633642	0.5272
C(21)	4.517560	0.262580	17.20453	0.0000
C(22)	0.014648	0.001500	9.768581	0.0000
C(23)	-0.008417	0.002600	-3.237327	0.0014
C(1)	0.976160	0.013954	69.95345	0.0000

R-squared	0.999637	Mean dependent var	5.822083
Adjusted R-squared	0.999625	S.D. dependent var	0.751831
S.E. of regression	0.014564	Akaike info criterion	-5.582013
Sum squared resid	0.036270	Schwarz criterion	-5.456887
Log likelihood	503.7991	Hannan-Quinn criter.	-5.531271
Durbin-Watson stat	2.328585		

Inverted AR Roots	.98

(b)

Figure 3.63 Statistical results based on two alternative AR(1)_GMs: (a) convergence not achieved after 500 iterations and (b) convergence achieved after 132 iterations

iterations' and the regression in the first time period show *RS* as an independent variable and the regression in the second time period shows *RS*(−1) as an independent variable. It is really an unexpected model. Readers may find other alternative models; do this as an exercise.

By observing all models presented above, their differences were easy to identify. However, to select the best one is not an easy task. The author considers that the two models in Figure 3.62 are the best compared to the others, since these models present or indicate unequal autocorrelations in the two time periods by having lagged endogenous variables. Then, based on the values of AIC and SC, the model in Figure 3.62(a) will be chosen, namely LVAR(1,3)_GM, as the best model, since it has smaller values of AIC as well as SC. □

3.8.2 Special notes and comments

It is recognized that many students and young researchers have been applying time series models without taking into account the time *t* as an independent variable of their models. The special case in the previous examples, as well as in Chapter 2, shows that the time *t* has to be used, at least the dummy variables of time periods, as an independent variable of a time series model.

By referring to both scatter graphs with regressions in Figure 3.59, the following notes and conclusions have been derived:

(1) The observed values of *RS* are within the interval [0,16] in the first time period and within the interval [2,14] in the second time period, which is a subset or subinterval of [0,16]. As a result, the scatter plots of (*RS*, log(*M*1)), based on the time periods, will be mixed and overlaid in a region between the line $RS = 0$ and $RS = 16$. Hence, in general, it was not possible to differentiate between the two subsamples.
(2) Note again that log(*M*1) and *RS* have a positive correlation within the first interval, but they have a negative correlation in the second interval, as presented in Figure 3.59.
(3) Based on points (1) and (2), if at least one of the two dummy variables in the model is not being used, namely *Drs*1 and *Drs*2, to present the relationship between the two variables *RS* and log(*M*1), the conclusion is most likely to be misleading. As an illustration, refer to the following example, which presents a simple linear regression of log(*M*1) on *RS*, without taking into account the time *t* and the dummy variables.

Example 3.22. (A simple linear regression of log(*M*1) on *RS*) Figure 3.64 presents statistical results based on a simple linear regression (SLR) of log(*M*1) on *RS*, which shows that *RS* has a significant positive effect on log(*M*1), based on the *t*-statistic of 7.199 409. It has been recognized, in some or many cases, that students would be happy with this finding, since they can prove that *RS* has a

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 11/24/07 Time: 07:22
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	5.144359	0.105090	48.95194	0.0000
RS	0.123198	0.017112	7.199409	0.0000

R-squared	0.225520	Mean dependent var	5.811220
Adjusted R-squared	0.221168	S.D. dependent var	0.754650
S.E. of regression	0.665989	Akaike info criterion	2.035962
Sum squared resid	78.95039	Schwarz criterion	2.071439
Log likelihood	-181.2366	Hannan-Quinn criter.	2.050347
F-statistic	51.83148	Durbin-Watson stat	0.022864
Prob(F-statistic)	0.000000		

Figure 3.64 Simple linear regression of log(M1) on RS

significant positive *linear effect* on log(M1). Figure 3.58 clearly shows that the linear regression of log(M1) on RS is not an appropriate model.

For illustration purposes, Figure 3.65 presents the statistical results based on the SLR of RS on log(M1), which also gives the t-statistic of 7.199 409, and Figure 3.66 presents the moment product correlation of RS and log(M1) of 0.474 889 with the same value of the t-statistic of 7.199 409.

Based on these findings, the following general conclusions can be derived:

(1) The causal relationship between a pair of numerical variables cannot be proven using the simple linear regression (SLR) as well as the moment product correlation, but it should be defined when supported on a relevant and strong theoretical basis.
(2) Either the SLR or the moment product correlation only provide a quantitative measure of their relationship, which is highly dependent on the data set that happens to be available for a researcher.
(3) The testing hypothesis on the linear causal relationship between numerical variables X and Y can be done by using either the SLR or the moment product

Dependent Variable: RS
Method: Least Squares
Date: 11/24/07 Time: 07:24
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.224793	1.489921	-3.506759	0.0006
LOG(M1)	1.830549	0.254264	7.199409	0.0000

R-squared	0.225520	Mean dependent var	5.412928
Adjusted R-squared	0.221168	S.D. dependent var	2.908939
S.E. of regression	2.567180	Akaike info criterion	4.734542
Sum squared resid	1173.094	Schwarz criterion	4.770020
Log likelihood	-424.1088	Hannan-Quinn criter.	4.748927
F-statistic	51.83148	Durbin-Watson stat	0.094339
Prob(F-statistic)	0.000000		

Figure 3.65 Simple linear regression of RS on log(M1)

Covariance Analysis: Ordinary
Date: 11/24/07 Time: 07:30
Sample: 1952Q1 1996Q4
Included observations: 180

Correlation t-Statistic Probability Cases	RS	LOG(M1)
RS	1.000000 ----- ----- 180	
LOG(M1)	0.474889 7.199409 0.0000 180	1.000000 ----- ----- 180

Figure 3.66 Correlation between the variables *RS* and log(*M*1)

correlation, under a precondition that the variables have a causal relationship, in a theoretical sense. However, it should always be remembered that the conclusion is also highly dependent on the data set used and cannot be used to prove or disprove the causal relationship.

(4) The moment product correlation $\rho(X,Y)$ also can be used to present or test the linear effect of X on Y, as well as the effect of Y on X.

(5) Furthermore, in order to develop an *empirical association model* based on a set of variables, it is suggested that the following points should be considered, under the assumption that the data set is valid and reliable:
- Even before collecting the data, the best possible judgment should have been used to evaluate whether there are at least two time periods where the growth curves of any variable would be different over time (refer to Figures 3.48, 3.58 and 3.59).
- Scatter plot(s) need to be used between each independent variable and the corresponding dependent variable with their regression or kernel density as a guide, in order to develop or define an empirical model (refer to Section 1.4).
- Use the corresponding correlation matrix of all numerical variables as basic information to evaluate the limitation of a defined model having multivariate independent variables (refer to Section 1.4.5, as well as Section 2.14.2). For example, the correlation matrices have been presented in the dissertation of the author's students, as well as in international journals, such as those of Hamsal (2006), Hamzal and Agung (2007), Billett, King and Maucer (2007) and Chapers, Koh and Stapledon (2006).
- Note that by having a statistical result based on a time series model, it does not directly mean that the result is a good and acceptable result, in a statistical sense. Refer to the alternative statistical results or models presented in Chapter 2, the previous examples, as well as the following examples. Furthermore, refer to the discussions on the true population model presented in Section 2.14.1.
- Since the effect of an exogenous variable on an endogenous variable is most likely to be dependent on other variables, it is suggested that an interaction

model should be defined or proposed, such as the model with a time-related effect and other interaction factors, as presented in Chapter 2. Moreover, the interaction is between the dummy and the numerical variables (Agung, 2006; Neter and Wasserman, 1974). On the other hand, there may be a set of heterogeneous linear regressions, which is known as the Johnson–Neyman technique (1936, quoted by Huitema, 1980, p. 270). Note that the effect of an interaction factor, namely $X_1{}^*X_2$, on a dependent variable Y will indicate that the effect of X_1 on Y is dependent on X_2, or the effect of X_2 on Y is dependent on X_1. This type of association or hypothesis should be easy to define, even before the data collection, by relevant theoretical and substantive bases. Then the statistical result will show whether the data supports the hypothesis or not.

- Furthermore, the statistical results of this experimentation, based on the Demo workfile, as well as the hypothetical data set, support the use of two-way or three-way interactions as independent variable(s) of the time series models. Many papers in international journals and the scientific papers of the author's students have used interaction models (i.e. models with two-way or three-way interactions as independent variables). For example, the three-way interaction models have been presented in Agung (2006), Bertrand, Schoar and Thesmar (2007), Hamzal and Agung (2007) and Harford and Li (2007).
- Finally, if a good model or a set of regressions has been obtained, it is wise to learn the limitation of each regression, by doing a residual analysis. □

3.8.3 General two-piece multivariate models with trend

The models presented in this subsection will refer to the multivariate continuous models in Section 2.15. Corresponding to the univariate model with trend in (3.67) and the symbols presented in Table 3.6, the following general multivariate model or system of univariate model is found:

$$\begin{aligned} g_h(y_{h,t}) = & (c(h11) + c(h12)^*t + f_{h,1}(x_{h,1}, \theta_{h,1}))^*D1 \\ & + (c(h21) + c(h22)^*t + f_{h,2}(x_{h,2}, \theta_{h,2}))^*D2 + \mu_{h,t} \end{aligned} \tag{3.72}$$

where $\boldsymbol{x}_{h,1}$ and $\boldsymbol{x}_{h,2}$ are multivariate exogenous variables, which can be equal for each or all $h = 1, 2, \ldots, H$, and $\theta_{h,1}$ and $\theta_{h,2}$ are unequal vectors of model parameters for all $h = 1, 2, \ldots, H$.

Example 3.23. (A model in (3.72) with endogenous variables *M*1 and *GDP*) Figure 3.67 presents statistical results based on a translog (translogarithmic) model with endogenous variables *M*1 and *GDP* by taking into account the first-order autocorrelation of their error terms. Note that both regressions have sufficient values of the DW-statistics, and their error terms have significant first-order autocorrelations. Since some of the independent variables have insignificant adjusted effects, the model can be reduced. Do this as an exercise.

Estimation Method: Iterative Least Squares
Date: 11/06/07 Time: 06:38
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.103706	0.321239	0.322831	0.7470
C(12)	0.000217	0.000217	0.999538	0.3182
C(13)	0.984364	0.044551	22.09530	0.0000
C(14)	0.017141	0.071452	0.239898	0.8106
C(21)	0.156165	0.146470	1.066191	0.2871
C(22)	-0.000605	0.000553	-1.093147	0.2751
C(23)	0.993421	0.032101	30.94702	0.0000
C(24)	0.053723	0.020973	2.561549	0.0109
C(1)	-0.163879	0.079380	-2.064476	0.0397
C(31)	0.524390	0.356387	1.471405	0.1421
C(32)	0.001003	0.000629	1.596198	0.1114
C(33)	0.912778	0.059234	15.40963	0.0000
C(34)	0.080167	0.066179	1.211360	0.2266
C(41)	0.639739	0.425104	1.504898	0.1333
C(42)	0.000553	0.000501	1.102226	0.2711
C(43)	0.902583	0.066956	13.48024	0.0000
C(44)	0.090312	0.073030	1.236651	0.2171
C(2)	0.338972	0.095570	3.546826	0.0004

Determinant residual covariance 1.63E-08

Equation: LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(M1(-1))+C(14)*LOG(PR))
*D1+(C(21)+C(22)*T+C(23)*LOG(M1(-1))+C(24)*LOG(PR))*D2
+[AR(1)=C(1)]
Observations: 178

R-squared	0.999640	Mean dependent var	5.822083
Adjusted R-squared	0.999623	S.D. dependent var	0.751831
S.E. of regression	0.014595	Sum squared resid	0.036000
Durbin-Watson stat	1.956318		

Equation: LOG(GDP)=(C(31)+C(32)*T+C(33)*LOG(GDP(-1))+C(34)
*LOG(PR))*D1+(C(41)+C(42)*T+C(43)*LOG(GDP(-1))+C(44)
*LOG(PR))*D2+[AR(1)=C(2)]
Observations: 178

R-squared	0.999916	Mean dependent var	6.008518
Adjusted R-squared	0.999912	S.D. dependent var	0.995104
S.E. of regression	0.009344	Sum squared resid	0.014756
Durbin-Watson stat	2.004899		

Figure 3.67 Statistical results based on a translog linear model with trend

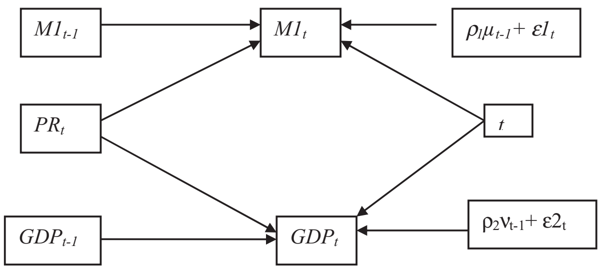


Figure 3.68 The path diagram of the model in Figure 3.67

For both time periods, the bivariate model can easily be written based on the output, which can be obtained by clicking *View/Representations*. Furthermore, each regression in the first and second time periods can be presented in the form of the path diagram in Figure 3.68. Note that this figure clearly shows that $M1_t$ and GDP_t are the downstream (endogenous or dependent) variables and the variables t, $M1_{t-1}$, GDP_{t-1} and PR_t are the source (exogenous or independent) variables. Furthermore, note that the relationships between the exogenous variables, $M1_{t-1}$, GDP_{t-1} and PR_t, as well as the time t, are not identified or presented, and likewise between the error terms $\varepsilon 1_t$ and $\varepsilon 2_t$.

In addition to testing an hypothesis by using the t-statistics presented in Figure 3.67, other univariate and multivariate hypotheses can be tested by using the Wald test, as presented in the previous examples. On the other hand, the residual analysis can also be done in order to study the limitation of the model. □

Example 3.24. (A simultaneous piecewise causal effect model) As an extension of the model in Figure 3.67 and its path diagram in Figure 3.68, in this example an

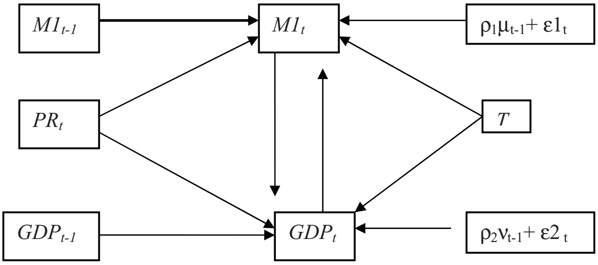


Figure 3.69 A path diagram of a simultaneous causal effects model, as a modification of the path diagram in Figure 3.68

hypothesis is proposed that the endogenous $M1_t$ and GD_t have a simultaneous causal effect. In this case, in fact, the path diagram presented in Figure 3.69 is considered.

The equation of the regressions and their parameter estimates are presented in Figure 3.70. Based on these results, the following notes and conclusions are obtained:

(1) Based on the first regression, $\log(GDP_t)$ has a significant adjusted effect on $\log(M1_t)$ in the first time period with a p-value = 0.0382, but in the second time period it is insignificant with a p-value = 0.4311.
(2) Similarly, based on the second regression, $\log(M1_t)$ has a significant positive adjusted effect on $\log(GDP_t)$ in the first time period, but not in the second time period.
(3) As a final conclusion, it can be said that the data supports the hypothesis that the variables $\log(M1_t)$ and $\log(GDP_t)$ have simultaneous causal effects.

Estimation Method: Iterative Least Squares
Date: 11/06/07 Time: 06:34
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.140647	0.337628	0.416575	0.6773
C(12)	-0.001597	0.000901	-1.771723	0.0774
C(13)	0.820023	0.090811	9.029986	0.0000
C(14)	0.188330	0.090524	2.080434	0.0382
C(15)	0.067167	0.078211	0.858798	0.3911
C(21)	0.433446	0.362680	1.195120	0.2329
C(22)	-0.000437	0.000597	-0.731497	0.4650
C(23)	1.002039	0.037490	26.72808	0.0000
C(24)	-0.049458	0.062744	-0.788246	0.4311
C(25)	0.104091	0.066707	1.560405	0.1196
C(1)	-0.098624	0.087221	-1.130740	0.2590
C(31)	-0.138353	0.334151	-0.414044	0.6791
C(32)	0.002995	0.000754	3.970822	0.0001
C(33)	0.690089	0.076943	8.968793	0.0000
C(34)	0.283533	0.059370	4.775677	0.0000
C(35)	-0.095547	0.067926	-1.406630	0.1605
C(41)	0.701772	0.425491	1.649321	0.1000
C(42)	0.000232	0.000573	0.403951	0.6865
C(43)	0.868664	0.073291	11.85230	0.0000
C(44)	0.034570	0.033198	1.041305	0.2985
C(45)	0.121338	0.076915	1.577556	0.1156
C(2)	0.363794	0.101993	3.566871	0.0004
Determinant residual covariance		1.38E-08		

Equation: LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(M1(-1))+C(14)*LOG(GDP)
+C(15)*LOG(PR))*D1+(C(21)+C(22)*T+C(23)*LOG(M1(-1))+C(24)
*LOG(GDP)+C(25)*LOG(PR))*D2+[AR(1)=C(1)]
Observations: 178

R-squared	0.999650	Mean dependent var	5.822083
Adjusted R-squared	0.999630	S.D. dependent var	0.751831
S.E. of regression	0.014471	Sum squared resid	0.034971
Durbin-Watson stat	1.963459		

Equation: LOG(GDP)=(C(31)+C(32)*T+C(33)*LOG(GDP(-1))+C(34)
*LOG(M1)+C(35)*LOG(PR))*D1+(C(41)+C(42)*T+C(43)*LOG(GDP(-1))
+C(44)*LOG(M1)+C(45)*LOG(PR))*D2+[AR(1)=C(2)]
Observations: 178

R-squared	0.999928	Mean dependent var	6.008518
Adjusted R-squared	0.999924	S.D. dependent var	0.995104
S.E. of regression	0.008688	Sum squared resid	0.012605
Durbin-Watson stat	1.959680		

Figure 3.70 Statistical results of a two-piece simultaneous causal model, with the path diagram presented in Figure 3.69

(4) Note that it is common to have one or two independent variables having insignificant adjusted effect(s), if the model has multivariate independent variables. Refer to the special notes and comments in Section 2.14. □

3.9 General two-piece models with time-related effects

Based on the general two-piece multivariate model with trend in (3.72), the equation of a general two-piece model with time-related effects can easily be derived as follows:

$$\begin{aligned} g_h(y_{ht,}) &= (c(h11) + c(h12)^* t + f_{h,1}(x_{h,1}, \theta_{h,1}) + t^* f_{h,1}(x_{h,1}, \theta^1_{h,1}))^* D1 \\ &\quad + (c(h21) + c(h22)^* t + f_{h,2}(x_{h,2}, \theta_{h,2}) + t^* f_{h,2}(x_{h,2}, \theta^2_{h,2}))^* D2 + \mu_{h,t} \\ &= F_{h,1}(t, x_{h,1}, \theta_{h,1})^* D1 + F_{h,2}(t, x_{h,2}, \theta_{h.2})^* D2 + \mu_{h,t} \end{aligned} \tag{3.73}$$

Note that for $h = 1$ there is a univariate two-piece model with time-related effects; if $\theta^1_{h,1} = \theta^2_{h,2} = 0$ then there is a multivariate model with trend, as in (3.73). For example, the univariate time-related effect models have been applied by Delong and Deyoung (2007) and Bansal (2005).

Furthermore, note that in order to have a specific or explicit model of this type the functions $F_{h,i}(t,^*,^*)$ can be substituted by the right-hand side of any models presented in Chapter 2. Refer to the following example as an illustration. Other types of models can easily be applied, since the process of data analysis is a straightforward process using EViews.

Example 3.25. (An extension of Example 3.21) As an extension of the additive regressions presented in the Example 3.21, here a simple two-piece AR(1) interaction model is considered as follows:

$$\begin{aligned} \log(m1) &= (c(11) + c(12)^* t + c(13)^* rs + c(14)^* t^* rs)^* Drs1 \\ &\quad + (c(21) + c(22)^* t + c(23)^* rs + c(24)^* t^* rs)^* Drs2 + [ar(1) = c(1)] \end{aligned} \tag{3.74}$$

Note that this model, in fact, has two three-way interactions as exogenous variables, namely t^*rs^*Drs1 and t^*rs^*Drs2, and it represents two first-order autoregressive regressions having two-way interaction, namely t^*rs, as follows:

$$\begin{aligned} \log(m1) &= (c(11) + c(12)^* t + c(13)^* rs \\ &\quad + c(14)^* t^* rs) + [ar(1) = c(1)], \quad \text{and} \\ \log(m1) &= (c(21) + c(22)^* t + c(23)^* rs \\ &\quad + c(24)^* t^* rs) + [ar(1) = c(1)] \end{aligned} \tag{3.75}$$

However, for this model the '*Near singular matrix*' error message is obtained. Corresponding to this interaction model, an experimentation should be performed

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/17/07 Time: 07:58
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence not achieved after 500 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*RS+C(14)*T*RS)*DRS1+(C(21)+C(22)
*T+C(24)*T*RS(-1))*DRS2+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	3.932263	0.227036	17.31996	0.0000
C(12)	0.019279	0.002421	7.964223	0.0000
C(13)	0.005186	0.005879	0.882132	0.3790
C(14)	-6.87E-05	5.97E-05	-1.149713	0.2519
C(21)	4.632054	0.256786	18.03861	0.0000
C(22)	0.014172	0.001575	8.995434	0.0000
C(24)	-6.97E-05	2.00E-05	-3.487899	0.0006
C(1)	0.985979	0.014330	68.80659	0.0000

R-squared	0.999622	Mean dependent var	5.822083
Adjusted R-squared	0.999607	S.D. dependent var	0.751831
S.E. of regression	0.014913	Akaike info criterion	-5.529240
Sum squared resid	0.037809	Schwarz criterion	-5.386238
Log likelihood	500.1024	Hannan-Quinn criter.	-5.471249
Durbin-Watson stat	2.244655		

Inverted AR Roots	.99

Figure 3.71 Statistical results with a note 'convergence not achieved after 500 iterations'

using the trial-and-error methods, similar to the process of obtaining the models based on the three variables $M1$, RS and t in Example 3.21. For illustration purposes, the following alternative statistical results are presented:

(1) Corresponding to the model in Figure 3.63(b), a model with additional independent variables is considered whose interactions are t^*RS and $t^*RS(-1)$ in the first and second time periods respectively. The equation of the model and its statistical results are presented in Figure 3.71, but with a note that the convergence is not achieved after 500 iterations.
(2) Corresponding to the two-way interaction models in Figures 3.71 and 3.72, statistical results are presented based on a reduced model, where the 'Convergence achieved after 34 iterations' is given but with a note or an error message '*Estimated AR process is nonstationary.*' Therefore, other interaction models have to be found.
(3) Figure 3.73(a) and (b) presents two interaction models that should be considered as acceptable models in a statistical sense, by using the Newey–West estimation method. However, only the first statistical results in (a) present the statement '*Newey–West HAC . . .*' while the second statistical results do not, even though the same option has been used. Furthermore, note that these models are three-way interaction models, since they have $t^*RS(-1)^*Drs1$ and $t^*RS(-1)^*Drs2$ as independent variables. It is a certainty that other two-piece acceptable models could be constructed based on the three variables $M1$, RS and the time t, such as the models presented in the following examples.

The AR(1) model with time-related effects in Figure 3.51(b), within each time period, can be presented as the path diagram in Figure 3.74. This graph shows that an

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 11/24/07 Time: 09:37
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 62 iterations
LOG(M1)=(C(11)+C(13)*RS+C(14)*T*RS)*DRS1+(C(21)+C(22)*T+C(23)
*RS(-1)+C(24)*T*RS(-1))*DRS2+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2.431028	2.176392	1.116999	0.2656
C(13)	0.003433	0.005784	0.593474	0.5537
C(14)	-4.56E-05	5.87E-05	-0.776695	0.4384
C(21)	2.551129	2.749131	0.927977	0.3547
C(22)	-0.000524	0.005658	-0.092611	0.9263
C(23)	0.030456	0.020771	1.466287	0.1444
C(24)	-0.000297	0.000156	-1.899167	0.0592
C(1)	1.003877	0.002871	349.6514	0.0000

R-squared	0.999623	Mean dependent var	5.822083
Adjusted R-squared	0.999608	S.D. dependent var	0.751831
S.E. of regression	0.014892	Akaike info criterion	-5.532100
Sum squared resid	0.037701	Schwarz criterion	-5.389099
Log likelihood	500.3569	Hannan-Quinn criter.	-5.474110
Durbin-Watson stat	2.279977		

Inverted AR Roots 1.00
Estimated AR process is nonstationary

Figure 3.72 Statistical results with a note 'estimated AR process is nonstationary'

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/17/07 Time: 13:36
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)
LOG(M1)=(C(11)+C(12)*T+C(13)*RS(-1)+C(14)*RS(-1)*T+C(15)*LOG(M1(
-1)))*DRS1+(C(21)+C(22)*T+C(24)*RS(-1)*T+C(25)*LOG(M1(-1)))
*DRS2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.184165	0.087794	2.097704	0.0374
C(12)	0.000544	0.000172	3.164356	0.0018
C(13)	-0.005746	0.002359	-2.435422	0.0159
C(14)	4.32E-05	2.11E-05	2.047052	0.0422
C(15)	0.963533	0.017957	53.65823	0.0000
C(21)	0.802388	0.201666	3.978806	0.0001
C(22)	0.001643	0.000692	2.375608	0.0186
C(24)	-5.14E-05	9.33E-06	-5.512775	0.0000
C(25)	0.853443	0.043960	19.41400	0.0000

R-squared	0.999691	Mean dependent var	5.816642
Adjusted R-squared	0.999676	S.D. dependent var	0.753241
S.E. of regression	0.013556	Akaike info criterion	-5.715019
Sum squared resid	0.031240	Schwarz criterion	-5.554760
Log likelihood	520.4942	Hannan-Quinn criter.	-5.650035
Durbin-Watson stat	2.597103		

(a)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/17/07 Time: 13:38
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 3 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*RS(-1)+C(14)*RS(-1)*T+C(15)*LOG(M1(
-1)))*DRS1+(C(21)+C(22)*T+C(24)*RS(-1)*T+C(25)*LOG(M1(-1)))
*DRS2+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.119966	0.079021	1.518162	0.1309
C(12)	0.000429	0.000150	2.863883	0.0047
C(13)	-0.004411	0.002052	-2.150203	0.0330
C(14)	3.03E-05	1.94E-05	1.566814	0.1190
C(15)	0.976575	0.016214	60.23133	0.0000
C(21)	0.739429	0.147785	5.003419	0.0000
C(22)	0.001422	0.000500	2.842068	0.0050
C(24)	-5.15E-05	7.49E-06	-6.874167	0.0000
C(25)	0.867764	0.032190	26.95727	0.0000
C(1)	-0.328418	0.074150	-4.429094	0.0000

R-squared	0.999720	Mean dependent var	5.822083
Adjusted R-squared	0.999705	S.D. dependent var	0.751831
S.E. of regression	0.012903	Akaike info criterion	-5.808170
Sum squared resid	0.027970	Schwarz criterion	-5.629418
Log likelihood	526.9271	Hannan-Quinn criter.	-5.735681
Durbin-Watson stat	2.021606		

Inverted AR Roots -.33

(b)

Figure 3.73 Statistical results of two-piece three-way interaction models: (a) an LV(1) model and (b) an LVAR(1,1) model

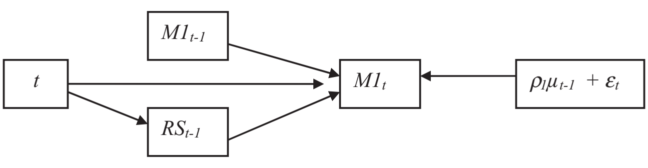


Figure 3.74 The path diagram of the AR(1) model in Figure 3.73(b)

arrow from the time t to the first lagged variable RS_{t-1} represents the interaction t^*RS_{t-1}, as already mentioned in Chapter 2, which also indicates that the effect of RS_{t-1} depends on the time t. □

Example 3.26. (Modified interaction models) Corresponding to the models in Figure 3.62(a) and (b), Figure 3.75(a) and (b) presents statistical results based on three-way interaction models, which should be considered as the extension of the two-way interaction models in Figure 3.62. Based on the results in this table,the following notes and conclusions are made:

(1) Even though both models in Figure 3.75 are acceptable, in a statistical sense, the LVAR(1,3) in Figure 3.75(a) should be considered as a good model, in a statistical sense, since almost all independent variables of the LV(2) model are insignificant.
(2) The LVAR(1,3) model, as well as its regression function, can easily be written based on the output or obtained by selecting *View/Representations*. Therefore, a pair of regressions exists in the first and second time periods, namely for ($t \leq 119$) and ($t > 119$), respectively, as follows:

$$\begin{aligned}\log(m1)_1 = {} & c(11) + c(12)^*t + c(13)^*\log(m1(-1)) + c(14)^*rs \\ & + c(15)^*t^*\log(m1(-1)) + c(16)^*t^*rs) \\ & + [ar(1) = c(1), ar(2) = c(2), ar(3) = c(3)]\end{aligned} \tag{3.76a}$$

$$\begin{aligned}\log(m1)_2 = {} & c(21) + c(22)^*t + c(23)^*\log(m1(-1)) + c(24)^*rs \\ & + c(25)^*t^*\log(m1(-1)) + c(26)^*t^*rs) \\ & + [ar(1) = c(1), ar(2) = c(2), ar(3) = c(3)]\end{aligned} \tag{3.76b}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 11/24/07 Time: 09:57
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 5 iterations
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(M1(-1))+C(14)*RS+C(15)*T
*LOG(M1(-1))+C(16)*T*RS)*DRS1+(C(21)+C(22)*T+C(23)*LOG(M1(
-1))+C(24)*RS+C(25)*T*LOG(M1(-1))+C(26)*T*RS)*DRS2
+[AR(1)=C(1),AR(2)=C(2),AR(3)=C(3)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.165842	0.135669	-1.222404	0.2233
C(12)	0.001784	0.000874	2.040759	0.0429
C(13)	1.035646	0.028359	36.51937	0.0000
C(14)	-0.003855	0.001718	-2.243995	0.0262
C(15)	-0.000340	0.000200	-1.704742	0.0902
C(16)	3.16E-05	1.78E-05	1.773293	0.0781
C(21)	0.141619	0.336617	0.420713	0.6745
C(22)	0.005951	0.002517	2.364237	0.0193
C(23)	0.957253	0.045188	21.18392	0.0000
C(24)	-0.003893	0.005696	-0.683530	0.4953
C(25)	-0.000671	0.000323	-2.077543	0.0393
C(26)	-2.94E-05	3.76E-05	-0.780142	0.4365
C(1)	-0.511883	0.078061	-6.557490	0.0000
C(2)	-0.277012	0.085755	-3.230281	0.0015
C(3)	-0.232396	0.077999	-2.979487	0.0033

R-squared	0.999775	Mean dependent var	5.833023
Adjusted R-squared	0.999755	S.D. dependent var	0.748997
S.E. of regression	0.011714	Akaike info criterion	-5.974646
Sum squared resid	0.022093	Schwarz criterion	-5.704435
Log likelihood	540.7689	Hannan-Quinn criter.	-5.865050
Durbin-Watson stat	1.916781		

(a)

Method: Least Squares
Date: 11/24/07 Time: 10:13
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
LOG(M1)=(C(11)+C(12)*T+C(13)*LOG(M1(-1))+C(14)*LOG(M1(-2))+C(15)
*RS+C(16)*T*LOG(M1(-1))+C(17)*T*LOG(M1(-2))+C(18)*T*RS)
*DRS1+(C(21)+C(22)*T+C(23)*LOG(M1(-1))+C(24)*LOG(M1(-2))
+C(25)*RS+C(26)*T*LOG(M1(-1))+C(27)*T*LOG(M1(-2))+C(28)*T
*RS)*DRS2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.044988	0.280281	0.160512	0.8727
C(12)	0.000626	0.001769	0.353671	0.7240
C(13)	0.648306	0.248244	2.611564	0.0099
C(14)	0.343576	0.252108	1.362813	0.1748
C(15)	-0.003369	0.003151	-1.069037	0.2866
C(16)	-0.000229	0.003330	-0.068856	0.9452
C(17)	0.000186	0.003381	0.055005	0.9562
C(18)	1.71E-05	3.27E-05	0.522121	0.6023
C(21)	0.229880	0.798892	0.287749	0.7739
C(22)	0.006803	0.006115	1.112466	0.2676
C(23)	0.428040	1.257864	0.340291	0.7341
C(24)	0.514627	1.192792	0.431448	0.6667
C(25)	-0.004594	0.012719	-0.361193	0.7184
C(26)	0.000662	0.008811	0.075180	0.9402
C(27)	-0.001430	0.008327	-0.171761	0.8638
C(28)	-4.05E-05	8.83E-05	-0.458126	0.6475

R-squared	0.999754	Mean dependent var	5.822083
Adjusted R-squared	0.999731	S.D. dependent var	0.751831
S.E. of regression	0.012324	Akaike info criterion	-5.868985
Sum squared resid	0.024604	Schwarz criterion	-5.582982
Log likelihood	538.3397	Hannan-Quinn criter.	-5.753003
Durbin-Watson stat	2.145088		

(b)

Figure 3.75 Statistical results based on two-piece interaction models, as the extension of the models in Figure 3.62, namely (a) the LVAR(1,3) and (b) the LV(2) interaction models

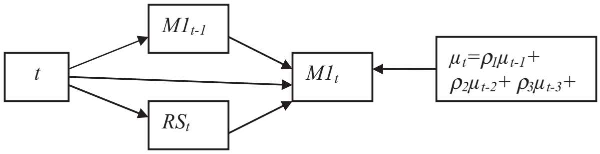


Figure 3.76 The path diagram of the AR(1) model in Figure 3.75(a)

(3) Based on the model in (3.76), it is easy to write hypotheses on the differences between the regressions in the two defined time periods and use the Wald test to conduct the testing. Do this as an exercise.

(4) The LVAR(1,3) model, within each time period, can be presented as the path diagram in Figure 3.76. This path diagram shows that the effects of $M1_{t-1}$ and RS_t on $M1_t$ are dependent on the time *t*. On the other hand, this model does not take into account the possible causal effect of $M1_{t-1}$ on RS_t. Refer to the path diagrams in Figures 2.66 and 2.85 to modify this path diagram, and then write or define possible univariate as well as multivariate models with interaction exogenous variables.

(5) For further illustration, the regression in (3.76a) can be written as follows:

$$\begin{aligned}\log(m1)_1 = &\{c(11)+c(13)^*\log(m1(-1))+c(14)^*rs\}\\ &+\{c(12)+c(15)^*\log(m1(-1))+c(16)^*rs\}^*t\\ &+[ar(1)=c(1),ar(2)=c(2),ar(3)=c(3)]\end{aligned} \quad (3.77)$$

This model shows that the effect of the time *t* on $\log(m1)_1$ is dependent on the function $\{c(12) + c(15)^*\log(m1(-1)) + c(16)^*rs\}$, which is significant based on the chi-square-statistic of 15.579 48 with $df = 3$ and a *p*-value = 0.0014. It can also be said that the joint effect of $\log(m1(-1))$ and *RS* depends on the time *t*. Corresponding to this statement, it might be considered useful to present the path diagram in Figure 3.77 for the model in Figure 3.76(a). What do you think?

(6) On the other hand, the model in Figure 3.76(b) has so many insignificant independent variables that an attempt should be made to try to obtain a reduced model, which has a better estimate. Do this as an exercise. However, note that

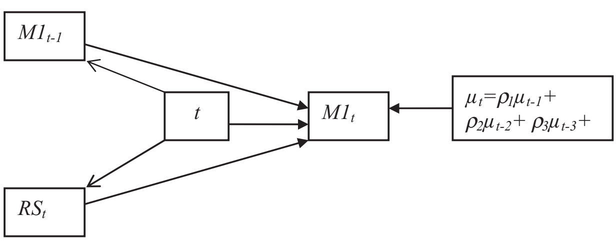


Figure 3.77 The path diagram of the AR(1) model in Figure 3.75(a)

these insignificant effects do not directly indicate that the model is a bad model, since good estimates may be based on other data sets. Refer to the special notes on unpredictable effects or impacts of multicollinearity presented in Section 2.14.2. □

Example 3.27. (An advanced Two-piece interaction model) Corresponding to the model with time-related effects in Example 2.45, a two-piece AR(p) model time-related effects model may be considered as follows:

$$\begin{aligned}\log(m1) = {} & (c(11)+c(12)\log(gdp)+c(13)\log(pr)+c(14)\log(gdp)^{*}\log(pr)+c(15)t \\ & +c(16)t^{*}\log(gdp)+c(17)t^{*}\log(pr)+c(18)t^{*}\log(gdp)^{*}\log(pr))^{*}D1 \\ & +(c(21)+c(22)\log(gdp)+c(23)\log(pr)+c(24)\log(gdp)^{*}\log(pr)+c(25)t \\ & +c(26)t^{*}\log(gdp)+c(27)t^{*}\log(pr)+c(28)t^{*}\log(gdp)^{*}\log(pr))^{*}D2 \\ & +[ar(1)=c(1),ar(2)=c(2),\ldots,ar(p)=c(p)]+\epsilon_t\end{aligned} \tag{3.78}$$

For $p=2$, the statistical results in Figure 3.78 can be obtained by using the procedure or estimation method '*System*.' However, the estimates present so many independent variables that they have an insignificant adjusted effect on $\log(m1)$. Hence, an attempt should be made to try to obtain a reduced or modified model. Do this as an exercise.

However, based on these estimates, at a significant level of $\alpha = 0{,}10$, the four-way interaction $t^{*}\log(gdp)^{*}\log(pr)^{*}D1$ is insignificant, but $t^{*}\log(gdp)^{*}\log(pr)^{*}D2$ is significant with a p-value $= 0.0070$.

Furthermore, note that the DW-statistic of 1.974 179 is sufficient to conclude that the model is a good model in controlling the autocorrelation of the error terms. By

System: UNTITLED
Estimation Method: Weighted Least Squares
Date: 10/17/07 Time: 14:34
Sample: 1952Q3 1996Q4
Included observations: 180
Total system (balanced) observations 178
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 52 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	3.077135	12.64899	0.243271	0.8081
C(12)	0.430109	2.669594	0.161114	0.8722
C(13)	-1.063358	8.075071	-0.131684	0.8954
C(14)	0.246077	1.709634	0.143935	0.8857
C(15)	0.037246	0.087452	0.425897	0.6708
C(16)	-0.006203	0.019093	-0.324885	0.7457
C(17)	0.047267	0.045793	1.032202	0.3035
C(18)	-0.008774	0.010197	-0.860497	0.3908
C(21)	6.444461	10.69832	0.602381	0.5478
C(22)	-0.803903	1.583807	-0.507576	0.6124
C(23)	-10.09237	2.575592	-3.918465	0.0001
C(24)	1.117324	0.392463	2.846955	0.0050
C(25)	0.036476	0.069532	0.524594	0.6006
C(26)	0.000396	0.010041	0.039402	0.9686
C(27)	0.179818	0.068662	2.618887	0.0097
C(28)	-0.024919	0.009124	-2.731067	0.0070
C(1)	0.722831	0.077371	9.342434	0.0000
C(2)	0.121266	0.079808	1.519462	0.1306
Determinant residual covariance		0.000170		

Equation: LOG(M1) = C(11)*D1+C(12)*LOG(GDP)*D1+C(13)*LOG(PR)*D1
+C(14)*LOG(GDP)*LOG(PR)*D1+C(15)*T*D1+ C(16)*T*LOG(GDP)*D1
+C(17)*T*LOG(PR)*D1+C(18)*T*LOG(GDP)*LOG(PR)*D1+ C(21)*D2
+C(22)*LOG(GDP)*D2+C(23)*LOG(PR)*D2+C(24)*LOG(GDP)
*LOG(PR)*D2+C(25)*T*D2 + C(26)*T*LOG(GDP)*D2+C(27)*T
*LOG(PR)*D2+C(28)*T*LOG(GDP)*LOG(PR)*D2+ [AR(1)=C(1),
AR(2)=C(2)]
Observations: 178

R-squared	0.999697	Mean dependent var	5.822083
Adjusted R-squared	0.999665	S.D. dependent var	0.751831
S.E. of regression	0.013770	Sum squared resid	0.030340
Durbin-Watson stat	1.974179		

Figure 3.78 Statistical results based on the AR(2) model in (3.78), using the WLS estimation method

using the WLS estimation method, the heteroskedasticity of the error terms has also been taken into account. □

3.10 Multivariate models by states and time periods

For illustration purposes, here only two states and two time periods are considered. As a result, there will be four pieces of growth models or four regressions with trend and time-related effects. The general equation of the model can easily be derived from the two-piece model in (3.73) as follows:

$$\begin{aligned} g_h(y_{h,t}) &= F_{h,1}(t, x_{h,1}, \theta_{h,1})^* D1 + F_{h,2}(t, x_{h,2}, \theta_{h.2})^* D2 \\ &\quad + F_{h,3}(t, x_{h,3}, \theta_{h,3})^* D3 + F_{h,4}(t, x_{h,4}, \theta_{h.4})^* D4 + \mu_{h,t} \end{aligned} \tag{3.79}$$

where $D1, D2, D3$ and $D4$ are the four dummy variables of the four cells by '*States* and *Time-Periods*' and the functions $F_{h,k}(t, x_{h,k}, \theta_{h,k})$, $k = 1$, 2, 3 and 4, are defined functions having a finite number of parameters for all h.

If there is a relevant data set, then all models presented in Chapter 2 can be used to represent the four functions $F_{h,k}(t, x_{h,k}, \theta_{h,k})$ in order to develop an explicit model by states and time periods. Then the data analysis can be done by using a similar process, as presented in the previous examples.

Example 3.28. (Univariate model by states and times) The following equation presents a simple AR(p) interaction model by state and time:

$$\begin{aligned} y_t &= (c(11) + c(12)^* t + c(13)^* x + c(14)^* t^* x)^* D1 \\ &\quad + (c(21) + c(22)^* t + c(23)^* x + c(24)^* t^* x)^* D2 \\ &\quad + (c(31) + c(32)^* t + c(33)^* x + c(34)^* t^* x)^* D3 \\ &\quad + (c(41) + c(42)^* t + c(43)^* x + c(44)^* t^* x)^* D4 + \mu_t \\ \mu_t &= \rho_1 \mu_{t-1} + \cdots + \rho_p \mu_{t-p} + \varepsilon_t \end{aligned} \tag{3.80}$$

Note that this model is a three-way interaction model, since it has the independent variables $t^* x^* Di$, $i = 1, 2, 3$ and 4. This model represents four interaction regressions or models having exactly the same set of independent or exogenous variables, namely t, x and $t^* x$. In general, however, there could be different sets of variables, such as in the previous examples.

Any statistical hypothesis based on this model can easily be defined by using the summary of the model parameters presented in Table 3.8. For illustrative purposes, refer to the following specific hypotheses:

(1) *Conditional hypotheses:*
   - Specific for State = 1, the adjusted effects differences of $t^* x$ on the endogenous variable y between the two time periods can be tested by entering $c(14) = c(24)$ for the Wald test.

Table 3.8 Parameters of the model in (3.80), for $p = 2$, by states and time periods

Cells		Dummy of the cells				Exogenous variables					
State	Time	$D1$	$D2$	$D3$	$D4$	Constant	T	x	t^*x	AR(1)	AR(2)
1	1	1	0	0	0	$C(11)$	$C(12)$	$C(13)$	$C(14)$	$c(1)$	$c(2)$
1	2	0	1	0	0	$C(21)$	$C(22)$	$C(23)$	$C(24)$	$c(1)$	$c(2)$
2	1	0	0	1	0	$C(31)$	$C(32)$	$C(33)$	$C(34)$	$c(1)$	$c(2)$
2	2	0	0	0	1	$C(41)$	$C(42)$	$C(43)$	$C(44)$	$c(1)$	$c(2)$

- Specific for Time = 1, the adjusted effects differences of t^*x on the endogenous variable y between the two states can be tested by entering $c(14) = c(34)$ for the Wald test.
- Note that the statistical results for testing these hypotheses can also be used to test a one-sided hypothesis, such as H_1: $c(14) > c(24)$, and H_2: $c(14) < c(34)$.

(2) *Unconditional hypothesis:*

- The effects differences of t^*x on the endogenous variable y between the four cells (by states and time periods) can be tested by entering $c(14) = c(24) = c(34) = c(44)$. □

Example 3.29. (Alternative model of the model in Example 3.28) Note that the model in (3.80) has a single error term for both states. This could be considered a limitation of the model, since in practice the two states are most likely to have different association models, as well as different error terms. As a result, an alternative model should be considered.

In order to have two error terms, one for each state, a different type of datafile should be developed. For an illustration, suppose there are two states with three variables, namely t, X and Y; then the datafile should be developed to have six variables, namely $t1, X1$ and $Y1$ for the first state and $t2, X2$ and $Y2$ for the second state. Then, corresponding to the model in (3.80), the following AR(p) bivariate model is given:

$$\begin{aligned} y1_t &= (c(111) + c(112)^*t1 + c(113)^*x1 + c(114)^*t1^*x1)^*Dt1 \\ &\quad + (c(121) + c(122)^*t1 + c(123)^*x1 + c(124)^*t1^*x1)^*Dt2 + \mu_t \\ \mu_t &= \rho_1\mu_{t-1} + \cdots + \rho_p\mu_{t-p} + \varepsilon 1_t \\ y2_t &= (c(211) + c(212)^*t2 + c(213)^*x2 + c(214)^*t2^*x2)^*Dt1 \\ &\quad + (c(221) + c(222)^*t2 + c(223)^*x2 + c(224)^*t2^*x2)^*Dt2 + \nu_t \\ \nu_t &= \rho_1\nu_{t-1} + \cdots + \rho_p\nu_{t-p} + \varepsilon 2_t \end{aligned} \tag{3.81}$$

where $Dt1$ and $Dt2$ are the two dummies of the two defined time periods. In general, there may be an AR(p) model for the endogenous variable $y1_t$, and an AR(q) model for the endogenous variable $y2_t$.

Table 3.9 Parameters of the bivariate model in (3.81) by time periods

Dependent variable	Time periods	$Dt1$	$Dt2$	Exogenous variables Constant	T	x	t^*x	AR(1)	AR(2)
$y1$	1	1	0	$C(111)$	$C(112)$	$C(113)$	$C(114)$	$c(11)$	$c(12)$
	2	0	1	$C(121)$	$C(122)$	$C(123)$	$C(124)$	$c(11)$	$c(12)$
$y2$	1	1	0	$C(211)$	$C(212)$	$C(213)$	$C(214)$	$c(21)$	$c(22)$
	2	0	1	$C(221)$	$C(222)$	$C(223)$	$C(224)$	$c(21)$	$c(22)$

Compared to the model in (3.80), for this model there is the model parameter summary presented in Table 3.9. Note that the symbol $C(ijk)$ is used as the model parameter for the ith endogenous variable, the jth time period and the kth parameter of the intercept or exogenous variables, compared to $C(ij)$ for the model in (3.80).

Based on this model, the univariate and multivariate hypotheses are as follows:

(1) *Univariate hypotheses:*
- The adjusted effects differences of t^*x (or $t1^*x1$) on $y1$ between the two time periods can be tested by entering $C(114) = C(124)$ for the Wald test. Refer to the first conditional hypothesis based on the model in (2.80).
- Specific for the *Time-period* = 1 or ($Dt1 = 1, Dt2 = 0$), the difference between the effect of t^*x (or $t1^*x1$) on $y1$ and the effect of t^*x (or $t2^*x2$) on $y2$ can be tested by entering $C(114) = C(214)$. Compare this with the second conditional hypothesis based on the model in (2.80). Furthermore, note that $t1 = t2$ and $x1$ and $x2$ are the same variables, as well as the variables $y1$ and $y2$.
- Specific for the *Time-period* = 1, the joint effects of all exogenous variables on $y2$ can be tested by entering the equation $C(212) = C(213) = C(214) = 0$.

(2) *Multivariate hypotheses:*
- Specific for the *Time-period* = 1, the adjusted effects of t^*x on the bivariate exogenous variables ($y1$, $y2$) can be tested by entering $C(114) = C(214) = 0$. Note that $t1^*x1$ and $t2^*x2$ are the same variables, that is t^*x.
- The adjusted effects of t^*x on the bivariate exogenous variables ($y1, y2$) in both time periods can be tested by entering $C(114) = C(124) = C(214) = C(224) = 0$.
- Specific for the *Time-period* = 1, the joint effects of all exogenous variables t, x and t^*x on ($y1$, $y2$) can be tested by entering $C(112) = C(113) = C(114) = 0$, $C(212) = C(213) = C(214) = 0$.
- To test the first-order partial autocorrelations differences, the equation $c(11) = c(21)$ should be used, as well as the pair of equations $c(11) = c(21)$, $c(12) = c(22)$ in order to test both partial autocorrelation differences. □

3.10.1 Alternative models

Note that the model in (3.79) uses four dummy variables of the four cells or categories by states and time periods. However, only three out of the four dummy variables may be used, such as:

$$g_h(y_{h,}) = F_{h,1}(t, x_{h,1}, \theta_{h,1})^* D1 + F_{h,2}(t, x_{h,2}, \theta_{h.2})^* D2 \\ + F_{h,3}(t, x_{h,3}, \theta_{h,3})^* D3 + F_{h,4}(t, x_{h,4}, \theta_{h.4}) + \mu_{h,t} \tag{3.82}$$

Corresponding to the model in (3.80), the following univariate AR(p) model is given. For a comparison, construct its model parameters, which will show their differences with the parameters in Table 3.8:

$$\begin{aligned} y_t &= (c(11) + c(12)^* t + c(13)^* x + c(14)^* t^* x)^* D1 \\ &\quad + (c(21) + c(22)^* t + c(23)^* x + c(24)^* t^* x)^* D2 \\ &\quad + (c(31) + c(32)^* t + c(33)^* x + c(34)^* t^* x)^* D3 \\ &\quad + (c(41) + c(42)^* t + c(43)^* x + c(44)^* t^* x) + \mu_t \\ \mu_t &= \rho_1 \mu_{t-1} + \cdots + \rho_p \mu_{t-p} + \varepsilon_t \end{aligned} \tag{3.83}$$

Then corresponding to the bivariate model in (3.81), the following AR(p) model in the first state and AR(q) model in the second state are given:

$$\begin{aligned} y1_t &= (c(111) + c(112)^* t1 + c(113)^* x1 + c(114)^* t1^* x1)^* Dt1 \\ &\quad + (c(121) + c(122)^* t1 + c(123)^* x1 + c(124)^* t1^* x1) + \mu_t \\ \mu_t &= \rho_1 \mu_{t-1} + \cdots + \rho_p \mu_{t-p} + \varepsilon 1_t \\ y2_t &= (c(211) + c(212)^* t2 + c(213)^* x2 + c(214)^* t2^* x2)^* Dt1 \\ &\quad + (c(221) + c(222)^* t2 + c(223)^* x2 + c(224)^* t2^* x2) + \nu_t \\ \nu_t &= \rho_1 \nu_{t-1} + \cdots + \rho_q \nu_{t-q} + \varepsilon 2_t \end{aligned} \tag{3.84}$$

3.10.2 Not recommended models

It is recognized that most students and some researchers have been applying the following multivariate model, instead of the model in (3.79):

$$g_h(y_{h,t}) = F_h(t, x_h, \theta_h) + c(1h)^* Dt1 + c(2h)^* Ds1 + \mu_{h,t} \tag{3.85}$$

where $Dt1$ is a dummy variable of the dichotomous time variable and $Ds1$ is a dummy variable of the two states, and $F_h(t_h, x_h, \theta_h)$ are additive functions having a finite number of parameters, for all h. For $h = 1$, this gives a univariate additive model. To study this model in detail, the four regressions of the model in (3.85) need to be investigated by states ($Ds1$) and time periods ($Dt1$), as presented in Table 3.10.

Table 3.10 The regressions in the model (3.85) by states and time periods

	$Dt1 = 1$	$Dt1 = 0$	Difference
$Ds1 = 1$	$F_h(t_h, x_h, \theta_h) + c(1h) + c(2h)$	$F_h(t_h, x_h, \theta_h) + c(1h)$	$\mathbf{c(2h)}$
$Ds1 = 0$	$F_h(t_h, x_h, \theta_h) + c(2h)$	$F_h(t_h, x_h, \theta_h)$	$\mathbf{c(2h)}$
Difference of the intercept	$\mathbf{c(1h)}$	$\mathbf{c(1h)}$	

This table clearly shows:

(1) The differences of the regressions, specifically their intercepts, between the two states is equal to $c(1h)$ for both time periods, while $c(2h)$ indicates the differences of the regressions between the two time periods in both states, for each h.
(2) The effects of all exogenous variables on the corresponding endogenous variables will be exactly the same within the four cells, which is presented by the function $F_h(t_h,x_h,\theta_h)$. For illustration purposes, construct a similar table based on the following univariate additive model:

$$\begin{aligned} y_t = &\ c(1) + c(2)^*t + c(3)^*x_t + c(4)^*z_{t-1} \\ &c(11)^*Dt1 + {}^*c(21)Ds1 + \mu_t \end{aligned} \tag{3.86}$$

(3) Based on these limitations of the models in (3.85), as well as the model in (3.86), it can be said that these types of models are unacceptable models. In other words, these models are not recommended models. Moreover, for the general model with dummy variables,

$$y_t = F(t, \boldsymbol{x}_t, \theta) + \sum_i c(1i)^*Dt, i + \sum_j c(2j)^*Ds, j + \mu_t \tag{3.87}$$

where Dt,i is a zero–one indicator of the ith time period and Ds,j is an indicator of the jth state, and $F(t,\boldsymbol{x}_t,\theta)$ is a function of the time t, a multidimensional exogenous variable $\boldsymbol{x}_t$ with the vector parameters θ.

4

Seemingly causal models

4.1 Introduction

Chapters 2 and 3 presented time series models having the time t as an exogenous variable. However, it is recognized that many time series models have been presented or applied without using the time t as an independent or exogenous variable. In this chapter, selected illustrative time series models will be presented without using the time t as an exogenous variable. As a result, the models will look like causal models between the exogenous variables and the corresponding endogenous variable(s).

Note that it has been well known that the causal relationship between variables should be defined on a theoretical and substantive basis. However, in some cases, it was found that each independent variable of a model had been incorrectly stated as a pure cause factor of the corresponding dependent variable. This problem arises because the statement '*the effect of the X-variable on the Y-variable*' had been used.

Considering the growth models, the time t should not be stated as a pure cause factor of the corresponding dependent variables. This is also the case for the effects of some X_t-variables on the Y_t-variables in time series data. In such cases, the X_t-variables should be considered as predictor, explanatory or source variables. For this reason, the term 'seemingly causal model (SCM)' or 'explanatory model (EM)' is used instead of '*growth model*' if and only if the model does not have the time t as an independent variable.

Furthermore, note that a pair of dated variables (X_t, Y_t) could have a significant correlation coefficient, either positive or negative, but they do not have any causal relationships. For example, even though RS_t (reason sale) and M_t (money supply) have a significant positive correlation with a p-value = 0.0002, it is known that RS cannot be a cause factor of $M1$. The main reason is that they do not have the same pattern of growth curves over time (refer to Figure 1.24). In addition, X_{t-p}, $p > 0$, does not directly imply that it is a cause factor of Y_t, even though they are observed or measured in sequence.

Based on any growth model, either continuous or discontinuous, that has been presented in Chapters 2 and 3, it is easy to derive seemingly causal models or explanatory models just by replacing the t-variable with a relevant X-variable, or by deleting the t-variable from the models. For this reason, this chapter will only present

Time Series Data Analysis Using EViews IGN Agung

some selected SCMs or EMs, starting with the simplest model based on a single time series, namely $\{Y_t\}$, for $t = 1, 2, \ldots, T$, and then a bivariate time series (X_t, Y_t).

Without using the time t as an independent variable of a time series model, there could be some problems in developing an empirical model. The following section will present illustrative case problems that have been found in developing or defining a seemingly causal model.

4.2 Statistical analysis based on a single time series

In this section, alternative models based on a single time series are considered, namely $\{Y_t\}$, for $t = 1, 2, \ldots, T$, without using the discrete time t as an exogenous (independent) variable.

4.2.1 The means model

The means model can be considered as the simplest model for the time series $\{Y_t\}$, $t = 1,2,\ldots, T$, which is presented (in EViews) as

$$Y_t = c(1) + \mu_t \tag{4.1}$$

For this model the estimated mean $\hat{Y} = \hat{c}(1) = \bar{Y} = \sum_{t=1}^{T} Y_t/T$ and $R^2 = 0$. Refer to Example 2.1 for the computational formula of R^2. Since R^2 is always equal to zero, then, based on a time series, a good fit model could never be achieved.

For illustration purposes, Figures 4.1 and 4.2 present the residual graphs of the log(M1) and RS means models respectively. These graphs clearly show the autocorrelations of the error terms of the models, which are related to low values of the DW-statistics.

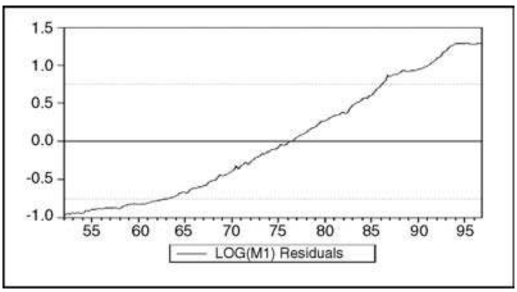


Figure 4.1 Residual graph of the log(M1) mean model

The residual graph of the log(M1) means model shows that the time t has a positive correlation with log(m1), which can also be proven by using correlation or regression analysis. Therefore, in order to have a better model, the time t should be used as an independent variable, or another variable that has a high positive correlation with the time t. However, in this chapter, consideration will only be taken using other variables.

4.2.2 The cell-means models

Even though the means model is considered as the worst fit model, in many cases the means differences between defined time intervals should be studied. For example, if there

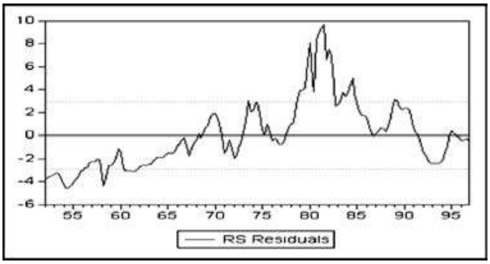


Figure 4.2 Residual graph of the *RS* mean model

is a monthly or weekly time series $\{Y_t\}$, for $t = 1, 2, \ldots, T$, its means differences might be studied between seasons, namely Summer, Fall, Winter and Spring, or between quarters, Q1, Q2, Q3 and Q4. Moreover, if a time series is in days or even in 15-minute intervals, the means differences between smaller time intervals could be studied.

For illustration purposes, Table 4.1 presents the means of the time series $\{Y_t\}$ by years and quarters. Corresponding to this table is a *cell-means model* as follows:

$$\begin{aligned} Y_t = \; & c(11)^*D11 + c(12)^*D12 + c(13)^*D13 + c(14)^*D14 \\ & + c(21)^*D21 + c(22) + c(23)^*D23 + c(24)^*D24 + \mu_t \end{aligned} \tag{4.2}$$

where *Dij* is the zero–one indicator or dummy variable of the cell (*i*, *j*), i.e. the *i*th year and the *j*th quarter. In this case, $i = 1$, 2 and $j = 1$, 2, 3 and 4.

Table 4.1 The means of the variable Y_t by years and quarters

	Quarters			
	1	2	3	4
Year = 1	*C*(11)	*C*(12)	*C*(13)	*C*(14)
Year = 2	*C*(21)	*C*(22)	*C*(23)	*C*(24)

The main objectives of this model are:

(1) To estimate and test the hypothesis on the means differences of the time series $\{Y_t\}$ between the *Years* for all *Quarters*, which can easily be tested by entering the equations $c(11) = c(21)$, $c(12) = c(22)$, $c(13) = c(23)$ and $c(14) = c(24)$ for the Wald test.
(2) To estimate and test a one-sided hypothesis on the means differences of the time series $\{Y_t\}$ between the *Year's* levels, for each *Quarter*. For example, for the first quarter, a right-hand hypothesis H_0: $c(11)–c(21) \leq 0$ versus H_1: $c(11)–c(21) > 0$ could be found. The statistical result can be obtained by entering the equation $c(11) = c(21)$ for the Wald test.
(3) Similarly, the hypothesis can be estimated or tested on the means differences of the time series $\{Y_t\}$ between quarters, for all years or each year's level.

This model can be presented in several other forms by using the dummy variables of the year's levels and the quarter's levels. By defining or generating *dy*1 and *dy*2 as the dummy variables of the two year's levels and $dq1$, $dq2$, $dq3$ and $dq4$ as the dummy variables of the quarter's levels, the following alternative models can be found:

(1) *Cell-Means Model I*

$$\begin{aligned} Y_t = {} & (c(11)*dq1 + c(12)*dq2 + c(13)*dq3 + c(14)*dq4)*dy1 \\ & + (c(21)*dq1 + c(22)*dq2 + c(23)*dq3 + c(24)*dq4) + dy2 + \mu_t \end{aligned} \tag{4.3}$$

Note that the cell-means table of this model is exactly the same as Table 4.1.

Example 4.1. (A 2 × 4 cell-means model) In order to apply the model in (4.3), the time series *POLI*_1 in the BASICS workfile is selected for the first two years, 1959 and 1960. The stages of analysis are as follows:

(1) By using Excel, it is easy to generate or develop the dummy variables or zero–one indicators of the year's levels, namely $dy1$ and $dy2$, and the quarter's levels, namely $dq1$, $dq2$, $dq3$ and $dq4$.
(2) The dummy variables data in Excel can be inserted in the original BASICS workfile by using the process presented in Chapter 1. If EViews 5 or 6 is used, the Excel datafile can be directly opened as a workfile. Refer to Sections 1.2 and 1.3.
(3) Then the statistical results in Figure 4.3 can easily be obtained, with the residual graph presented in Figure 4.4. Based on these results, the following notes and comments can be made:
    (i) The DW-statistic of 2.0275, as well as the following BG serial correlation LM test, indicates that the null hypothesis of no first-order autocorrelation of the error terms is accepted (Table 4.2).

Dependent Variable: POLI_1
Method: Least Squares
Date: 10/17/07 Time: 18:14
Sample (adjusted): 1959M01 1960M12
Included observations: 21 after adjustments
POLI_1=(C(11)*DQ1+C(12)*DQ2+C(13)*DQ3+C(14)*DQ4)*DY1+(C(21)
*DQ1+C(22)*DQ2+C(23)*DQ3+C(24)*DQ4)*DY2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.227716	0.005496	41.43044	0.0000
C(12)	0.205223	0.004488	45.72963	0.0000
C(13)	0.222319	0.004488	49.53915	0.0000
C(14)	0.220671	0.004488	49.17181	0.0000
C(21)	0.196404	0.004488	43.76448	0.0000
C(22)	0.203636	0.005496	37.04922	0.0000
C(23)	0.210533	0.005496	38.30408	0.0000
C(24)	0.222271	0.004488	49.52826	0.0000

R-squared	0.746835	Mean dependent var	0.213544
Adjusted R-squared	0.610515	S.D. dependent var	0.012455
S.E. of regression	0.007773	Akaike info criterion	-6.593986
Sum squared resid	0.000785	Schwarz criterion	-6.196073
Log likelihood	77.23685	Hannan-Quinn criter.	-6.507628
Durbin-Watson stat	2.027516		

Figure 4.3 Statistical results based on the cell-means model in (4.3)

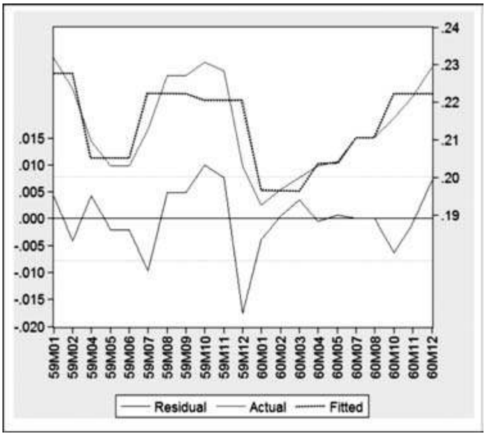


Figure 4.4 Residual graph of the regression in Figure 4.3

(ii) This example demonstrates a special case of a basic regression based on a time series, which does not have a first-order autoregressive problem. On the other hand, the basic regression based on a cross-sectional data set could have autoregressive problems.

(iii) Based on the *t*-statistic presented in Figure 4.3, it can be concluded that each of the cell-means is significantly greater than zero. Other hypotheses can easily be tested by using the Wald tests.

(2) *Cell-Means Model II*

$$\begin{aligned} Y_t = \ & (c(11)^*dq1 + c(12)^*dq2 + c(13)^*dq3 + c(14)^*dq4)^*dy1 \\ & + (c(21)^*dq1 + c(22)^*dq2 + c(23)^*dq3 + c(24)^*dq4) + \mu_t \end{aligned} \tag{4.4a}$$

with the cell-means as presented in Table 4.3. This table clearly shows that the parameters *C*(11), *C*(12), *C*(13) and *C*(14) are the means differences between the

Table 4.2 BG serial correlation test of the regression in Figure 4.3

Breusch–Godfrey serial correlation LM test			
F-statistic	0.042 642	Prob. *F*(1,12)	0.8399
Obs*R-squared	0.074 359	Prob. chi-squared (1)	0.7851

Table 4.3 The cell-means based on the model in (4.4a)

	Quarters			
	1	2	3	4
Year = 1	*C*(11) + *C*(21)	*C*(12) + *C*(22)	*C*(13) + *C*(23)	*C*(14) + *C*(22)
Year = 2	*C*(21)	*C*(22)	*C*(23)	*C*(24)
Differences	*C*(11)	*C*(12)	*C*(13)	*C*(14)

levels Year = 1 and Year = 2, for each of the four quarters. Hence by applying this model, the t-statistic presented in the printout can directly be used to test two-sided and one-sided hypotheses on each of these parameters.

(3) *Cell-Means Model III*

$$\begin{aligned} Y_t = &\ (c(11)^*dy1 + c(21)^*dy2)^*dq1 + (c(12)^*dy1 + c(22)^*dy2)^*dy2 \\ &+ (c(13)^*dy1 + c(23)^*dy2)^*dq3 + (c(14)^*dy1 + c(24)^*dy2) + \mu_t \end{aligned} \tag{4.4b}$$

where $C(ij)$ indicates the model parameter in the ith row and jth column or cell (i, j). For this model, the cell-means is as presented in Table 4.4. This table shows that the fourth quarter is taken as a reference group. The model can easily be modified in order to have a different reference group if it is needed.

Table 4.4 The cell-means based on the model in (4.4b)

	Quarters				Differences		
Years	1	2	3	4	1–4	2–4	3–4
1	$C(11) + C(14)$	$C(12) + C(14)$	$C(13) + C(14)$	$C(14)$	$C(11)$	$C(12)$	$C(13)$
2	$C(21) + C(24)$	$C(22) + C(24)$	$C(23) + C(24)$	$C(24)$	$C(12)$	$C(22)$	$C(23)$

Note that these models can easily be extended to $I \times 4$ cell-means models with $I > 2$. Furthermore, if there is a weekly time series, then the cell-means may be considered by *Years*, *Quarters* and *Months*, namely the $I \times 4 \times 12$ cell-means models (see Agung, 2006), which can be considered as a '*multilevel cell-means model*' or '*multifactorial cell-means models.*' Moreover, if a daily or a 15-minute time series exists, they could easily be developed into multilevel cell-means models. One of the author's students, Ekaputra (2003), has been considering the 15-minute time intervals in order to study the mean differences of the intra-day stocks by days and 15-minute time intervals.

(4) *Not Recommended Cell-Means Model*

In many cases, corresponding to the 2×4 cell-means table above, it has been recognized that an analyst presents an additive model having dummy variables as independent variables, besides several selected numerical exogenous or source variables, as follows:

$$Y_t = c(1) + c(2)dy1 + c(3)dq1 + c(4)dq2 + c(5)dq3 + \sum_{k=1}^{K} \beta_k Xk_t + \mu_t \tag{4.5}$$

where all Xk's are numerical exogenous variables.

Note that this model only has five parameters, $c(1)$ to $c(5)$, which indicate the intercepts of the corresponding set of homogeneous regressions or surfaces within the eight (=2 × 4) cells by years and quarters. Therefore, these intercepts are not sufficient to represent the eight homogeneous regressions. The parameters $c(1)$ to $c(5)$ represent the intercepts of the eight homogeneous regressions with endogenous variables Y_t and numerical independent variables Xk. The eight intercepts can be summarized as in Table 4.5.

Table 4.5 The intercepts of the homogenous regressions in (4.5)

	Quarters				Differences		
Years	1	2	3	4	1–4	2–4	3–4
1	$C(1) + C(2) + C(3)$	$C(1) + C(2) + C(4)$	$C(1) + C(2) + C(5)$	$C(1) + C(2)$	C(3)	C(4)	C(5)
2	$C(1) + C(3)$	$C(1) + C(4)$	$C(1) + C(5)$	$C(1)$	C(3)	C(4)	C(5)
Diff.	$C(2)$	$C(2)$	$C(2)$	$C(2)$			

This table shows a specific pattern of the intercept differences, which are considered to be an unrealistic pattern. Hence, these types of models are considered to be poor or worst models, which will be stated as not recommended models. If all $\beta_k' s = 0$, then the worst cell-means model is obtained.

Other additive models with dummy variables, which are also considered as poor models, are as follows:

(a) *A Model Through the Origin*

$$Y_t = c(1)dy1 + c(2)dy2 + c(3)dq1 + c(4)dq2 + c(5)dq3 + \sum_{k=1}^{K} \beta_k Xk_t + \mu_t \tag{4.6}$$

(b) *Another Model Through the Origin*

$$Y_t = c(1)dy1 + c(2)dq1 + c(3)dq2 + c(4)dq3 + c(5)dq4 + \sum_{k=1}^{K} \beta_k Xk_t + \mu_t \tag{4.7}$$

Note that these last two models also have five intercept parameters $c(1)$ to $c(5)$, which are considered as the models through the origin, since they only have one set of independent variables (i.e. the dummy variables and the numerical exogenous variables). The model (4.6) uses both dummies of the years and three out of the four dummy variables of the quarters, while the model (4.7) uses one out of the two dummies of the years and all dummies of the quarters. In fact, these models represent the same set of regressions as the model in (4.5), but they have different forms of intercepts. Construct those tables as an exercise and for comparison.

On the other hand, if the following model having six intercept parameters is used, a singular design matrix is formed, since $c(1) = c(2) + c(3)$, and the '*Near Singular Matrix*' error message would be obtained:

$$Y_t = c(1) + c(2)dy1 + c(3)dy2 + c(4)dq1 + c(5)dq2 + c(6)dq3 + \sum_{k=1}^{K} \beta_k Xk_t + \mu_t \tag{4.8}$$

Likewise, if the following model is used, a singular design matrix is also formed, since $c(1) = c(3) + c(4) + c(4) + c(5)$:

$$Y_t = c(1) + c(2)dy1 + c(3)dq1 + c(4)dq2 + c(5)dq3 + c(6)dq4 + \sum_{k=1}^{K} \beta_k Xk_t + \mu_t \tag{4.9}$$

□

4.2.3 The lagged-variable models

The general lagged (endogenous)-variable model, namely LV(p), is defined as

$$Y_t = c(1) + \sum_{i=1}^{p} c(1+i)^*Y_{t-i} + \mu_t \tag{4.10}$$

For specific time series, at the first stage of data analysis, the following simple models may be considered:

(1) For a yearly time series, the first lagged-variable model, LV(1), is as follows:

$$Y_t = c(1) + c(2)^*Y_{t-1} + \mu_t \tag{4.11}$$

(2) For a quarterly time series, in order to match the quarters between a recent year with the previous year, the fourth lagged-variable simple model is as follows:

$$Y_t = c(1) + c(2)^*Y_{t-4} + \mu_t \tag{4.12}$$

(3) For a monthly time series, in order to match the months in a recent year with the previous year, the twelfth lagged-variable simple model is as follows:

$$Y_t = c(1) + c(2)^*Y_{t-12} + \mu_t \tag{4.13}$$

Note that these models are in fact a simple linear regression of Y_t on each of the independent variables Y_{t-1}, Y_{t-4} and Y_{t-12} respectively. Hence each of these models can be presented in the form of a scatter plot/graph with a simple linear regression, as presented in the following example. Furthermore, each model can be considered as a model based on the bivariate (X_i,Y_i), with $X_i \le X_{i+1} \le \max(Y_{t-p})$ for all i, $p = 1$, 4 or 12.

Example 4.2. (LV(p) Models based on the variable *RS*) Figure 4.5 presents the growth curve of the variable *RS* and Figure 4.6 presents the scatter graph of (RS_{t-1}, RS_t) with the regression line in (4.10). Since $\max(RS_t) = 15.087\,33$ for $t = 119$, then based on the scatter graph of RS_t on RS_{t-1}, the following notes and

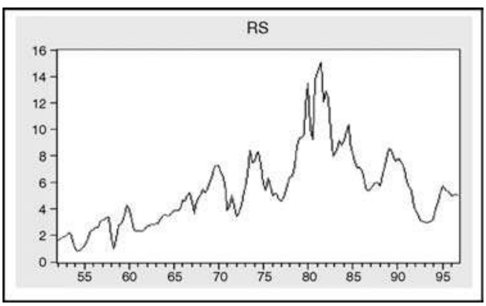


Figure 4.5 Growth curve of the variable RS_t

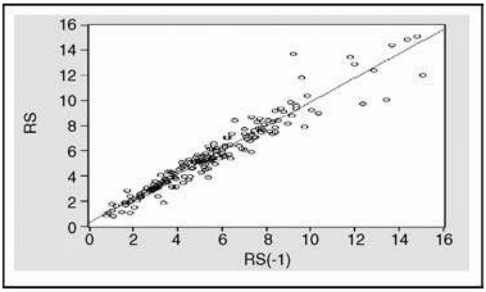


Figure 4.6 Scatter graph and regression line of RS_t on RS_{t-1}

comments can be made:

(1) The scatter graph of (RS_{t-1}, RS_t) is an overlay of two scatter graphs of (RS_{t-1}, RS_t) for $t \leq 119$ and $t > 119$. This scatter graph could be considered as a scatter graph of cross-section data, namely (X_i, RS_i) with $X_i \leq X_{i+1} \leq \max(RS_t)$, for all $i = 2, \ldots, N$.
(2) The scatter graph with the regression of RS_t on RS_{t-1} shows the heterogeneity of the error terms, since the absolute values of the error term increase with increasing values of RS_t.
(3) The graph also shows that RS_t and RS_{t-1} have positive correlation, and RS_{t-1} has a significant '*effect*' on RS_t, based on the t-test: $t_0 = 48.07336$, with $df =$ and p-value $= 0.000$. Considering the use of the word '*effect*,' would you declare that RS_{t-1} is a (pure) cause factor of RS_t? It would be better to say or conclude that RS_t and RS_{t-1} have a *significant positive correlation*, rather than RS_{t-1} has a significant positive effect on RS_t, since there might be other variables that are the real cause factors of RS_t.
(4) The LV(1) model of RS has $R^2 = 0.926$ and $DW = 1.564$.
(5) For a comparison, since there is a quarterly time series, it is suggested that an LV(4) model should be used, in order to match the quarters between two consecutive years, giving the statistical results presented in Figure 4.7, with its residual graph in

Dependent Variable: RS
Method: Least Squares
Date: 10/17/07 Time: 18:36
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	1.050145	0.252584	4.157612	0.0001
RS(-4)	0.819961	0.040976	20.01097	0.0000

R-squared	0.697096	Mean dependent var	5.495778
Adjusted R-squared	0.695355	S.D. dependent var	2.888665
S.E. of regression	1.594389	Akaike info criterion	3.782157
Sum squared resid	442.3212	Schwarz criterion	3.818185
Log likelihood	-330.8298	Hannan-Quinn criter.	3.796769
F-statistic	400.4391	Durbin-Watson stat	0.375546
Prob(F-statistic)	0.000000		

Figure 4.7 Statistical results based on a simple model in (4.12) for RS_t

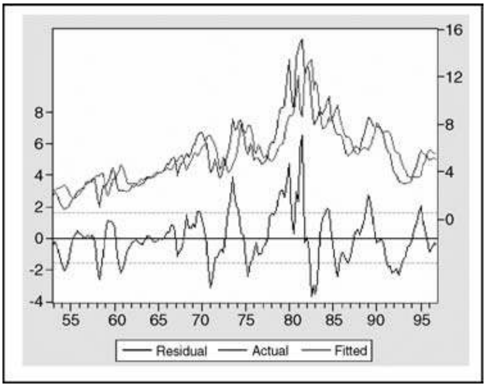


Figure 4.8 Residual graph of the regression in Figure 4.7

Figure 4.8. As this model has a very small DW-statistic, it is a worst or poor model compared to the LV(1) model, with respect to the autocorrelation problem.

(6) After conducting further experimentation, the statistical results based on an LV(5) model are obtained, which are presented in Figure 4.9. However, since $RS(-5)$ has an insignificant effect with a p-value $= 0.4805$, then the LV(4) model, given in Figure 4.10, was found to be an acceptable model, in a statistical sense, with $R^2 = 0.939\,720$ and $DW = 1.971\,842$. Do you think this LV(4) model is the best model for RS? The limitation of this model can be explored by doing residual tests. The results of these residual tests are presented in Figure 4.11, which shows that the null hypothesis of no serial correlation, as well as its homogeneity and normal distribution, for the error terms is rejected. This model could therefore be thought to be the worst model with relation to its error terms. Corresponding to the residual tests, refer to the special notes presented in Section 2.14.3 with the topic '*To Test or Not*' *the Assumptions of the Error Terms*. However, for illustration

Dependent Variable: RS
Method: Least Squares
Date: 10/17/07 Time: 18:54
Sample (adjusted): 1953Q2 1996Q4
Included observations: 175 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.250178	0.120037	2.084184	0.0386
RS(-1)	1.317640	0.076815	17.15349	0.0000
RS(-2)	-0.703540	0.124638	-5.644683	0.0000
RS(-3)	0.616993	0.127330	4.845609	0.0000
RS(-4)	-0.328420	0.124680	-2.634097	0.0092
RS(-5)	0.054077	0.076488	0.706997	0.4805

R-squared	0.939403	Mean dependent var	5.515484
Adjusted R-squared	0.937610	S.D. dependent var	2.885066
S.E. of regression	0.720630	Akaike info criterion	2.216301
Sum squared resid	87.76291	Schwarz criterion	2.324808
Log likelihood	-187.9264	Hannan-Quinn criter.	2.260315
F-statistic	523.9847	Durbin-Watson stat	1.975689
Prob(F-statistic)	0.000000		

Figure 4.9 Statistical results based on an LV(5) model for RS_t

Dependent Variable: RS
Method: Least Squares
Date: 10/17/07 Time: 18:56
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.259687	0.117737	2.205651	0.0287
RS(-1)	1.303669	0.073884	17.64474	0.0000
RS(-2)	-0.672059	0.115885	-5.799357	0.0000
RS(-3)	0.580549	0.115884	5.009726	0.0000
RS(-4)	-0.257354	0.073532	-3.499865	0.0006

R-squared	0.939720	Mean dependent var	5.495778
Adjusted R-squared	0.938310	S.D. dependent var	2.888665
S.E. of regression	0.717469	Akaike info criterion	2.201825
Sum squared resid	88.02437	Schwarz criterion	2.291896
Log likelihood	-188.7606	Hannan-Quinn criter.	2.238357
F-statistic	666.4450	Durbin-Watson stat	1.971842
Prob(F-statistic)	0.000000		

Figure 4.10 Statistical results based on an LV(4) model for RS_t

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	4.670127	Prob. F(2,169)	0.0106
Obs*R-squared	9.217686	Prob. Chi-Square(2)	0.0100

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	14.22009	Prob. F(4,171)	0.0000
Obs*R-squared	43.93071	Prob. Chi-Square(4)	0.0000
Scaled explained SS	161.4095	Prob. Chi-Square(4)	0.0000

Figure 4.11 BG serial correlation LM test for the model in Figure 4.10

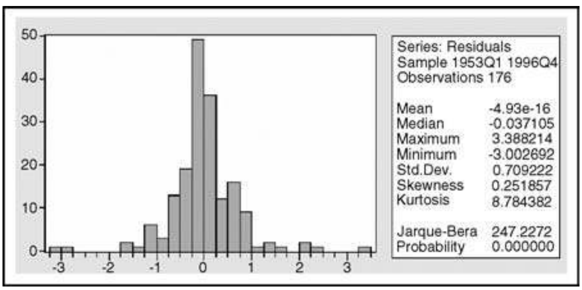
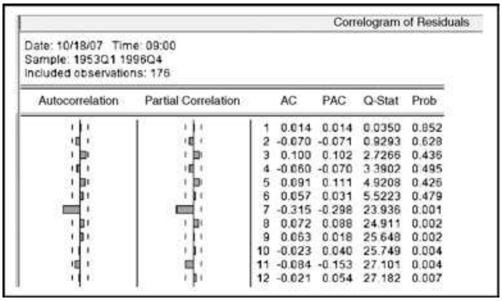


Figure 4.12 Residual histogram of the regression in Figure 4.10.

purposes, Figures 4.12 to 4.14 present three alternative residual graphs to evaluate visually the characteristics of the error terms of the model.

The correlogram of residuals shows that the autocorrelation is significant at level $k = 7$, as well as its PAC (i.e. partial correlation). On the other hand, the correlogram of residuals squared shows that the partial correlations are significant at the levels $k = 2$ and $k = 9$. Based on these findings, it could be said that there is always a final problem or question: ‘What should we do in order to obtain a better model, and

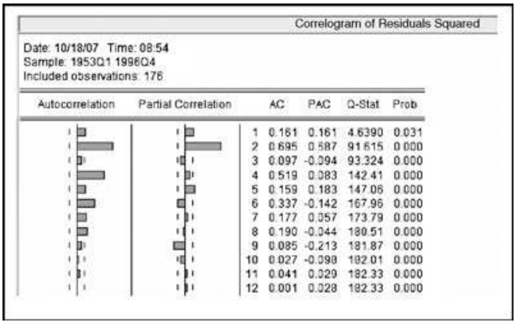


Correlogram of Residuals

Date: 10/18/07 Time: 09:00
Sample: 1953Q1 1996Q4
Included observations: 176

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.014	0.014	0.0350	0.852
		2	-0.070	-0.071	0.9293	0.628
		3	0.100	0.102	2.7266	0.436
		4	-0.060	-0.070	3.3902	0.495
		5	0.091	0.111	4.9208	0.426
		6	0.057	0.031	5.5223	0.479
		7	-0.315	-0.298	23.936	0.001
		8	0.072	0.088	24.911	0.002
		9	0.063	0.018	25.648	0.002
		10	-0.023	0.040	25.749	0.004
		11	-0.084	-0.153	27.101	0.004
		12	-0.021	0.054	27.182	0.007

Figure 4.13 Residual correlogram of the regression in Figure 4.10.



Correlogram of Residuals Squared

Date: 10/18/07 Time: 08:54
Sample: 1953Q1 1996Q4
Included observations: 176

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.161	0.161	4.6390	0.031
		2	0.695	0.587	91.615	0.000
		3	0.097	-0.094	93.324	0.000
		4	0.519	0.083	142.41	0.000
		5	0.159	0.183	147.06	0.000
		6	0.337	-0.142	167.96	0.000
		7	0.177	0.057	173.79	0.000
		8	0.190	-0.044	180.51	0.000
		9	0.085	-0.213	181.87	0.000
		10	0.027	-0.098	182.01	0.000
		11	0.041	0.029	182.33	0.000
		12	0.001	0.028	182.33	0.000

Figure 4.14 Residual squared correlogram of the regression in Figure 4.10.

moreover the best possible model?' The answer to this question could be derived from the special notes on the true population model in Section 2.14.1. □

Example 4.3. (Special models based on the variable *POLI_1*) Figure 4.15 presents the growth curve of the variable *POLI_1*, in the BASICS workfile, and the scatter graph of ($POLI\_1_t$, $POLI\_1_{t-1}$) with their simple linear regression is presented in Figure 4.16. This graphical presentation is considered as a special case, corresponding to the growth curve of $POLI\_1_t$, which is quite different from RS_t in the previous example, but the scatter graph also shows that $POLI\_1_t$ and $POLI\_1_{t-1}$ have a positive correlation.

After doing additional data analysis, the statistical results in Figure 4.17 are obtained based on an LV(4) model, with $DW = 1.927\,228$. However, the independent variables *POLI_1*(−2) and *POLI_1*(−3) are insignificant. Hence, the following modified models and notes are presented:

(1) The residual graph in Figure 4.18 clearly shows that the data do not support the error terms, which are homogeneous. For this reason, it is suggested that other estimation methods should be tried, such as the White or Newey–West estimation methods. Do this as an exercise.

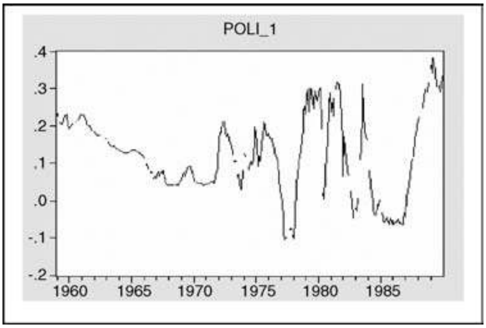


Figure 4.15 Growth curve of the variable $POLI\_1_t$

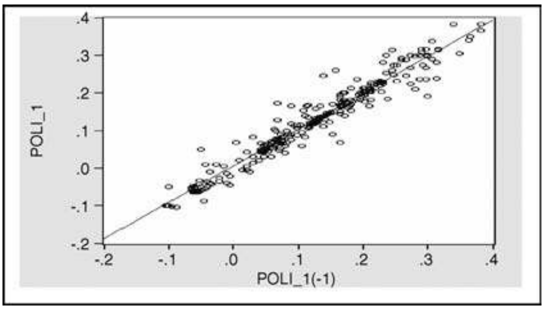


Figure 4.16 Scatter graph with the regression line of $POLI\_1_t$ on $POLI\_1_{t-1}$

Dependent Variable: POLI_1
Method: Least Squares
Date: 10/18/07 Time: 09:43
Sample (adjusted): 1959M08 1989M07
Included observations: 235 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007012	0.002393	2.930207	0.0037
POLI_1(-1)	1.126509	0.067206	16.76203	0.0000
POLI_1(-2)	-0.165640	0.103305	-1.603399	0.1102
POLI_1(-3)	0.112824	0.099955	1.128746	0.2602
POLI_1(-4)	-0.141684	0.063851	-2.218974	0.0275

R-squared	0.943323	Mean dependent var	0.115543
Adjusted R-squared	0.942337	S.D. dependent var	0.099948
S.E. of regression	0.024001	Akaike info criterion	-4.600420
Sum squared resid	0.132488	Schwarz criterion	-4.526812
Log likelihood	545.5493	Hannan-Quinn criter.	-4.570744
F-statistic	957.0158	Durbin-Watson stat	1.927228
Prob(F-statistic)	0.000000		

Figure 4.17 Statistical results based on an LV(4) model of the variable $POLI\_1$

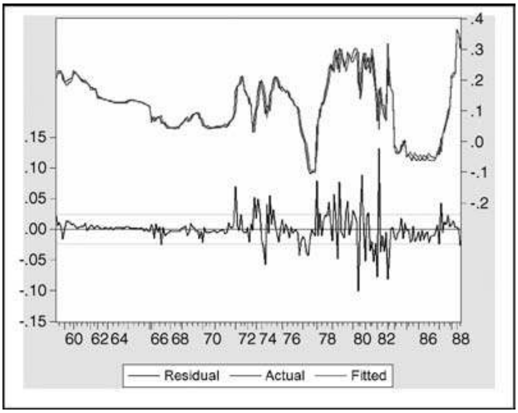


Figure 4.18 Residual graph of the regression in Figure 4.17

(2) For illustration purposes, since the variable *POLI*_1(−4) is significant, two alternative reduced models are presented in Figure 4.19, which are considered as unexpected models, since they have unordered lagged dependent variables.

(3) Compared to the models in Figure 4.19, Figure 4.20 presents statistical results using the White and the Newey–West estimation methods based on an LV(2) model of *POLI*_1, which is a common model and an acceptable one, since $DW = 1.961\,506$ and each of the independent variables is significant. In order to know the limitation of this model, Figure 4.21(a) presents the residual histogram together with its descriptive statistics and Figure 4.21(b) presents the residual box plot. Both graphs clearly show several outliers. Refer to Example 2.4 for descriptions of how outliers are treated. □

Example 4.4. (An application of the model in (4.13)) Since the BASIC work file contains monthly time series, the model in (4.13) may be applied as an alternative model of all models presented in Example 4.3. The equation of the model is

$$POLI\_1_t = c(1) + c(2)^* POLI\_1_{t-12} + \mu_t \tag{4.14}$$

Dependent Variable: POLI_1
Method: Least Squares
Date: 10/18/07 Time: 09:49
Sample (adjusted): 1959M06 1989M11
Included observations: 258 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006846	0.002352	2.910664	0.0039
POLI_1(-1)	1.127190	0.066608	16.92262	0.0000
POLI_1(-2)	-0.097403	0.083135	-1.171621	0.2424
POLI_1(-4)	-0.087848	0.039603	-2.218227	0.0274

R-squared	0.942462	Mean dependent var	0.116658
Adjusted R-squared	0.941782	S.D. dependent var	0.102435
S.E. of regression	0.024716	Akaike info criterion	-4.547361
Sum squared resid	0.155162	Schwarz criterion	-4.492277
Log likelihood	590.6096	Hannan-Quinn criter.	-4.525211
F-statistic	1386.814	Durbin-Watson stat	1.813018
Prob(F-statistic)	0.000000		

Dependent Variable: POLI_1
Method: Least Squares
Date: 10/18/07 Time: 09:51
Sample (adjusted): 1959M05 1989M11
Included observations: 282 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006434	0.002237	2.875951	0.0043
POLI_1(-1)	1.063401	0.027338	38.89798	0.0000
POLI_1(-4)	-0.118377	0.027568	-4.293977	0.0000

R-squared	0.945043	Mean dependent var	0.119092
Adjusted R-squared	0.944649	S.D. dependent var	0.103435
S.E. of regression	0.024335	Akaike info criterion	-4.583211
Sum squared resid	0.165223	Schwarz criterion	-4.544468
Log likelihood	649.2328	Hannan-Quinn criter.	-4.567675
F-statistic	2398.830	Durbin-Watson stat	1.775322
Prob(F-statistic)	0.000000		

Figure 4.19 Unexpected reduced models of the LV(4) model in Figure 4.17

Dependent Variable: POLI_1
Method: Least Squares
Date: 11/26/07 Time: 10:44
Sample (adjusted): 1959M06 1989M12
Included observations: 281 after adjustments
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005999	0.001991	3.013167	0.0028
POLI_1(-1)	1.231517	0.114598	10.74638	0.0000
POLI_1(-2)	-0.275331	0.116664	-2.360042	0.0190

R-squared	0.945469	Mean dependent var	0.118761
Adjusted R-squared	0.945077	S.D. dependent var	0.104966
S.E. of regression	0.024599	Akaike info criterion	-4.561563
Sum squared resid	0.168228	Schwarz criterion	-4.522719
Log likelihood	643.8995	Hannan-Quinn criter.	-4.545984
F-statistic	2410.027	Durbin-Watson stat	1.961506
Prob(F-statistic)	0.000000		

(a)

Dependent Variable: POLI_1
Method: Least Squares
Date: 11/26/07 Time: 10:46
Sample (adjusted): 1959M06 1989M12
Included observations: 281 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005999	0.002648	2.265571	0.0242
POLI_1(-1)	1.231517	0.100510	12.25272	0.0000
POLI_1(-2)	-0.275331	0.100919	-2.728248	0.0068

R-squared	0.945469	Mean dependent var	0.118761
Adjusted R-squared	0.945077	S.D. dependent var	0.104966
S.E. of regression	0.024599	Akaike info criterion	-4.561563
Sum squared resid	0.168228	Schwarz criterion	-4.522719
Log likelihood	643.8995	Hannan-Quinn criter.	-4.545984
F-statistic	2410.027	Durbin-Watson stat	1.961506
Prob(F-statistic)	0.000000		

(b)

Figure 4.20 Statistical results using (a) the White and (b) the Newey–West estimation methods, based on an LV(2) model of the variable $POLI\_1_t$

Figure 4.22 presents its statistical results using the LS estimation method and Figure 4.23 presents its residual graph.

Compared to the models in the previous example, this model is a nonnested model; hence the AIC and SC statistics could be applied in selecting a better model. Since this model has larger values of AIC and SC statistics, it will be considered as a worst model, compared to the previous models.

Considering the very small value of the DW-statistic, an autoregressive model should be used, which will be presented in the following subsection. □

4.2.3.1 Special notes and comments

(1) The regression function of $\hat{Y}_t = \hat{c}(1) + \hat{c}(2)Y_{t-k}$ for a selected k will present a straight line in a two-dimensional coordinate system, with the intercept $\hat{c}(1)$ and slope $\hat{c}(2)$.

(2) The regression function of $\hat{Y}_t = \hat{c}(1) + \hat{c}(2)Y_{t-m} + \hat{c}(3)Y_{t-k}$ for a selected (m,k), such as the regression $POLI\_1_t = 0.006\,434 + 1.634\,01 POLI\_1_{t-1} - 0.118\,37 POLI\_1_{t-4}$ presented in Example 4.3, will present a plane in a

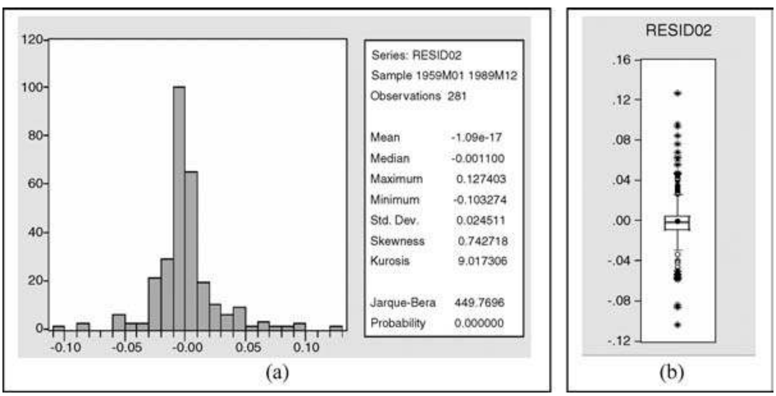


Figure 4.21 (a) The residual histogram and (b) the residual box plot of the regression in Figure 4.20

Dependent Variable: POLI_1
Method: Least Squares
Date: 10/18/07 Time: 09:55
Sample (adjusted): 1960M01 1989M11
Included observations: 301 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.075768	0.008661	8.748017	0.0000
POLI_1(-12)	0.347724	0.057962	5.999159	0.0000

R-squared	0.107436	Mean dependent var	0.114178
Adjusted R-squared	0.104451	S.D. dependent var	0.106936
S.E. of regression	0.101197	Akaike info criterion	-1.736865
Sum squared resid	3.062035	Schwarz criterion	-1.712233
Log likelihood	263.3981	Hannan-Quinn criter.	-1.727008
F-statistic	35.98990	Durbin-Watson stat	0.073291
Prob(F-statistic)	0.000000		

Figure 4.22 Statistical results based on the model in (4.14)

three-dimensional coordinate system, with the intercept $\hat{c}(1)$ and partial derivatives $\partial\hat{Y}_t/\partial Y_{t-m} = \hat{c}(2)$ and $\partial\hat{Y}_t/\partial Y_{t-k} = \hat{c}(3)$, which indicate that each of the lagged variables Y_{t-m} and Y_{t-k} has a constant partial (adjusted) effect on Y_t.

In general, the regression function of the LV(p) model in (4.10) will present an hyperplane in a (p + 1)-dimensional coordinate system, with each of the lagged variables having a constant adjusted effect on the recent time series Y_t.

(3) Corresponding to the use of two-way and three-way interactions in a time series model, as presented in Chapter 2, a lagged endogenous variables regression may be applied with interaction exogenous variables, such as follows as an illustration:

$$\hat{Y}_t = \hat{c}(1) + \hat{c}(2)Y_{t-m} + \hat{c}(3)Y_{t-k} + \hat{c}(4)Y_{t-m}Y_{t-k} \qquad (4.15)$$

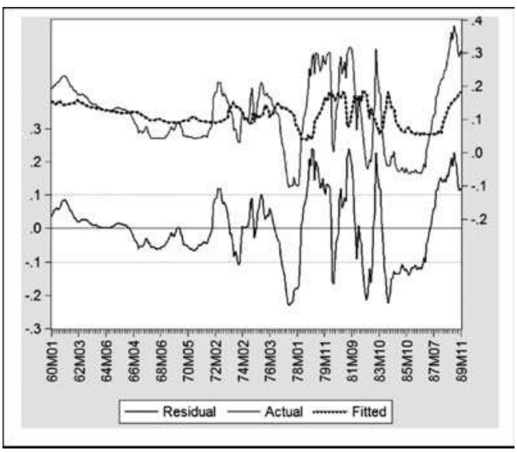


Figure 4.23 Residual graph of the regression in Figure 4.22

with $m < k$. This regression shows that the effect of Y_{t-m} on Y_t is dependent on Y_{t-k}, which can be presented as the partial derivative.

$$\frac{\partial \hat{Y}_t}{\partial Y_{t-m}} = \hat{c}(2) + \hat{c}(4) Y_{t-k} \tag{4.16}$$

4.2.4 Autoregressive models

A general autoregressive model, namely the AR(q) model, will be presented as

$$Y_t = c(1) + \sum_{i=1}^{q} c(1+i)^* \mu_{t-i} + \varepsilon_t \tag{4.17}$$

As presented in the previous chapter, in order to apply this model, the following alternative 'equation specification' should be used or entered:

$$Y = c(1) + [ar(1) = c(2), ar(2) = c(3), \ldots, ar(p) = c(p+1)] \tag{4.18}$$

or

$$y\ c\ ar(1)\ ar(2) \cdots ar(p) \tag{4.19}$$

4.2.5 Lagged-variable autoregressive models

The general lagged (endogenous)-variable autoregressive model, namely LVR(p, q), is presented as

$$\begin{aligned} Y_t &= c(1) + \sum_{i=1}^{p} c(1+i)^* Y_{t-i} + \mu_t \\ \mu_t &= \sum_{j=1}^{q} \rho_j \mu_{t-j} + \varepsilon_t \end{aligned} \tag{4.20}$$

The process of data analysis based on this model, for all possible values of p and q, can be done easily. However, by using the trial-and-error methods, unexpected models may be obtained, as presented in the following example.

In order to apply this model, it is suggested that the following equation specification should be used:

$$y\ c\ y(-1)\ y(-2) \cdots y(p)\ ar(1)\ ar(2) \cdots ar(q) \tag{4.21}$$

so that the printout will directly show the variables in the model. Then the equation can be saved in the workfile by clicking the option '*Names*,' to be recalled later if there is a need to modify the model.

Dependent Variable: POLI_1
Method: Least Squares
Date: 10/18/07 Time: 10:04
Sample (adjusted): 1960M12 1989M07
Included observations: 209 after adjustments
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.132814	0.039356	3.374702	0.0009
POLI_1(-12)	0.000125	0.077613	0.001607	0.9987
AR(1)	1.176395	0.070514	16.68307	0.0000
AR(2)	-0.225150	0.069525	-3.238430	0.0014

R-squared	0.942272	Mean dependent var	0.107391
Adjusted R-squared	0.941427	S.D. dependent var	0.105099
S.E. of regression	0.025436	Akaike info criterion	-4.486369
Sum squared resid	0.132630	Schwarz criterion	-4.422401
Log likelihood	472.8256	Hannan-Quinn criter.	-4.460507
F-statistic	1115.380	Durbin-Watson stat	1.982543
Prob(F-statistic)	0.000000		

Inverted AR Roots	.94	.24

Figure 4.24 Statistical results based on the model in (4.22)

Example 4.5. (AR(q) Models of the model in (4.14)) Corresponding to a very low value of $DW = 0.073291$ of the model in (4.14), as presented in Figure 4.22, the model should be improved by using autoregressive models in order to find a sufficient value of the DW-statistic. By using the trial-and-error methods, the statistical results in Figures 4.24 and 4.25 are obtained with sufficient values of the DW-statistics, based on the following models:

$$Poli\_1_t = c(1) + c(2)*Poli\_1_{t-12} + [ar(1) = c(3), ar(2) = c(4)] \quad (4.22)$$

$$Poli\_1_t = c(1) + c(2)*Poli\_1_{t-12} + [ar(1) = c(3), ar(3) = c(4)] \quad (4.23)$$

Dependent Variable: POLI_1
Method: Least Squares
Date: 10/18/07 Time: 10:07
Sample (adjusted): 1961M01 1988M03
Included observations: 177 after adjustments
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.112180	0.031290	3.585164	0.0004
POLI_1(-12)	0.043604	0.083765	0.520553	0.6033
AR(1)	1.096587	0.048609	22.55940	0.0000
AR(3)	-0.162530	0.047479	-3.423204	0.0008

R-squared	0.939975	Mean dependent var	0.106270
Adjusted R-squared	0.938934	S.D. dependent var	0.102920
S.E. of regression	0.025433	Akaike info criterion	-4.483198
Sum squared resid	0.111903	Schwarz criterion	-4.411420
Log likelihood	400.7630	Hannan-Quinn criter.	-4.454087
F-statistic	903.0491	Durbin-Watson stat	2.096463
Prob(F-statistic)	0.000000		

Inverted AR Roots	.89	.54	-.34

Figure 4.25 Statistical results based on the model in (4.23)

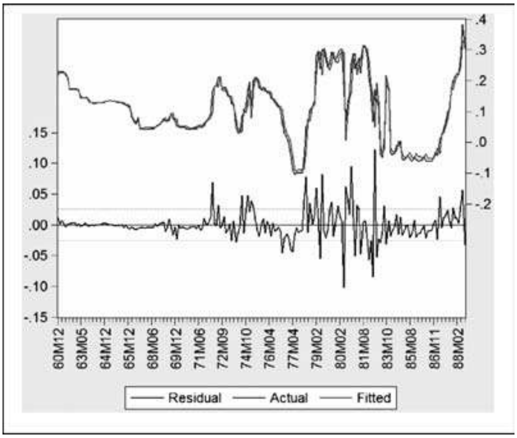


Figure 4.26 Residual graph of the regression in Figure 4.24

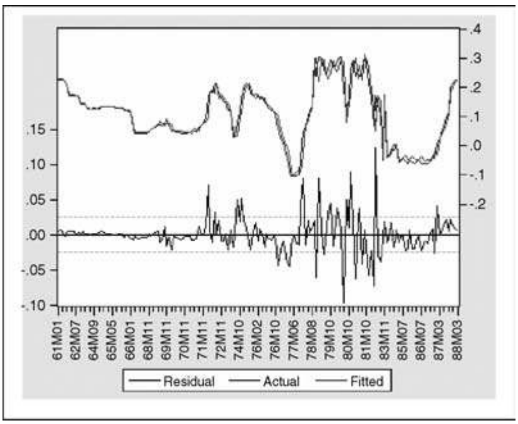


Figure 4.27 Residual graph of the regression in Figure 4.25

However, the model in (4.23) should be considered as an unexpected or uncommon model, since it has two indicators AR(1) and AR(3), without the indicator AR(2).

Compared to the model in (4.14), these models are much better and are based on the values of the DW-statistic, as well as their residual, actual and fitted value graphs, as presented in Figures 4.26 and 4.27. However, both residual graphs are very similar, which show the heterogeneity of the error terms. Therefore, it is suggested that the White or Newey–West estimation methods should be used. Do this as an exercise. □

4.3 Bivariate seemingly causal models

Based on a bivariate time series, namely (X_t, Y_t), $t = 1, 2, \ldots, T$, the following alternative *SCMs* (*seemingly causal models*) may be obtained.

4.3.1 The simplest seemingly causal models

Based on a bivariate (X_t, Y_t), the two simplest SCMs are as follows:

$$Y_t = c(1) + c(2)X_t + \mu_t \tag{4.24}$$

$$Y_t = c(1) + c(2)X_{t-k} + \mu_t \tag{4.25}$$

Note that the model (4.24) shows that X_t looks like a cause factor of the endogenous variable Y_t, but it could only be a predictor, an explanatory or a source variable, since both variables X_t and Y_t are observed or measured at the same time point. On the other hand, X_{t-k}, for a specific value of $k \geq 1$, could be a cause factor of Y_t, since they are observed or measured in a sequence. However, note that this condition is not a sufficient condition for X_{t-k} to be declared or named as a cause factor of Y_t, but is a necessary condition.

Based on a sample survey, the causal effect of X_t (or X_{t-k}), either direct or indirect, on Y_t is very highly dependent on expert judgment, which can be very subjective, and is similar for the simultaneous effects between both variables. Refer to alternative models, with their path diagrams, presented in Chapters 2 and 3. In this subsection, however, only some of the problems in applying the models in (4.24) or (4.25) are considered, as presented in the following examples.

Example 4.6. (The relationship between log(M1) and RS) By looking at the scatter plot with the regression line of log(M1) on RS in the previous examples, as presented again in Figure 4.28, it can be concluded that RS_t is not an appropriate variable to be used as an explanatory variable for the endogenous variable $M1_t$ or log($M1_t$), even though RS_t has a significant linear effect on log($M1_t$) with a p-value = 0.0000, based on the standard t-test.

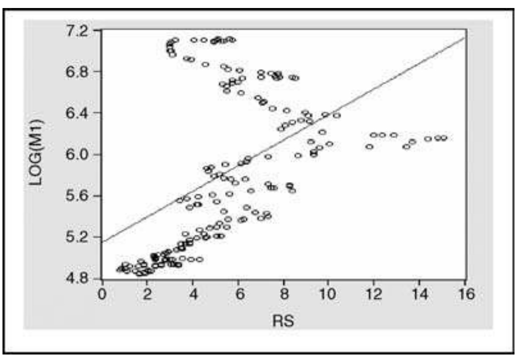


Figure 4.28 Scatter graph with a regression line of log(M1) on RS

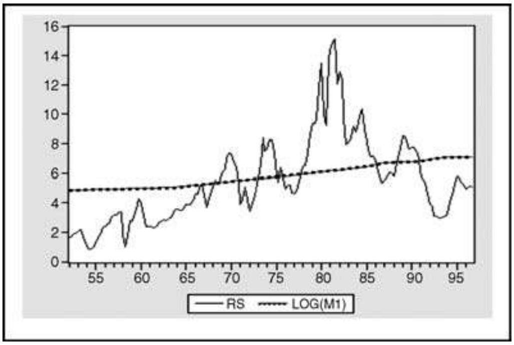


Figure 4.29 Overlay growth curves of the variables *RS* and log(*M*1)

For further discussion, overlay growth curves of the time series RS_t and $\log(M1_t)$ are also presented in Figure 4.29. By comparing both graphs, the following notes and conclusions can be derived:

(1) The scatter plot or graph of $(\log(M1_t), RS_t)$, $t = 1, 2, \ldots, T$, is, in fact, exactly the same as the scatter plot of $(\log(M1_i), RS_i)$, with $RS_i \le RS_{i+1}$ for all $i = 1, 2, \ldots, n = T$, as presented in Figure 4.28. Hence, the time series data analysis or a model based on only the dated variables $\log(M1_t)$ and RS_t, without using the time t, can be considered as a cross-sectional data analysis, based on the variables $\log(M1_i)$ and RS_i. As a result, for further analysis, the subscript i can be used instead of t.

(2) Note that the maximum observed values of $RS_i = 15.08\,733$ occur in both graphs. Corresponding to the graph in Figure 4.29, the maximum value of RS_i is achieved for $t = 119$. For $t \le 119$, $\log(M1_t)$ and RS_t have a positive correlation, but they have a negative correlation for $t > 119$. As a result, the model based on $\log(M1_i)$ and RS_i should have at least two branches, with the following alternative equations:

$$\log(M1_i) = (c(11) + c(12)^*RS_i)^*Drs1 + (c(21) + C(22)^*RS_i)^*Drs2 + \mu_t \tag{4.26}$$

$$\log(M1_i) = (c(11) + c(12)^*RS_i) + (c(21) + C(22)^*RS_i)^*Drs2 + \mu_t \tag{4.27}$$

where $Drs1$ and $Drs2$ are the previously defined dummy variables of the two time periods. Based on the model in (4.26), the following results are obtained:

$$\begin{aligned} &\text{LOG}(M1) = (4.780 + 0.117^*RS)^*DRS1 + (7.422 - 0.106^*RS)^*DRS2 \\ &\quad \text{with } R^2 = 0.934\,147 \quad \text{and} \quad DW = 0.215\,959 \end{aligned} \tag{4.28}$$

Compare this with all models with additional independent variables, including the time t, as presented in Examples 3.21 and 3.22. This result clearly shows that

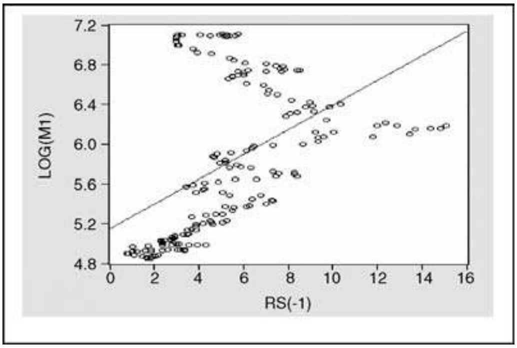


Figure 4.30 Scatter graph with a regression line of log($M1$) on $RS(-1)$

there are two lines with different intercepts and slopes in a two-dimensional coordinate system with axes log($M1$) and RS, with the following equations:

$$\begin{aligned} \text{LOG}(M1) &= 4.780 + 0.117{*}RS \\ \text{LOG}(M1) &= 7.422 - 0.106{*}RS \end{aligned} \tag{4.29}$$

Since the value of DW is very small, the standard t-test cannot be applied and therefore further analysis should be done by taking into account the autocorrelation of the error terms. However, here, no further analysis will take place, since this has been demonstrated in previous examples. Do it as an exercise by using the lagged dependent variables log($m1(-1)$) and log($m1(-2)$) or the AR indicators $ar(1)$ and $ar(2)$, or a combination of both.

(3) Similar statistical results will be obtained by using the first lagged RS_{t-1} as an independent variable, instead of RS_t. Figure 4.30 presents an illustrative scatter graph with linear regression of log($m1_t$) on $RS(-1) = RS_{t-1}$ and Figure 4.31 presents the scatter graph with a nearest neighbor fit as a nonparametric estimation method, which will be discussed in Chapter 11. These graphical representations also show that RS_{t-1} should not be used as a linear predictor of $M1_t$ or log($M1_t$).

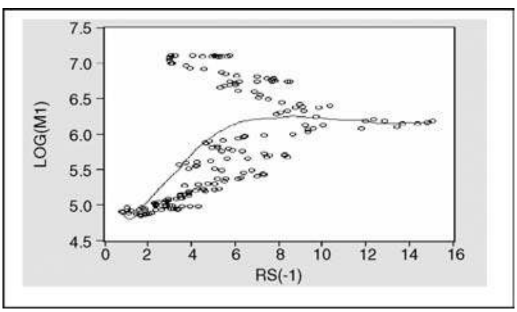


Figure 4.31 Scatter graph with a nearest neighbor fit of log($M1$) on $RS(-1)$

(4) These illustrations show that by not using the time t as an independent variable of a model, there could be an unexpected association or correlation, as well as a multiple correlation, between a set of dated variables. Hence, without using the time t as an independent variable, it may be concluded that bivariate scatter plots or graphs between each of the independent variables and the dependent variable are important to be used as a guide to decide or select whether or not a variable is a good or an appropriate independent variable. Compare this with the following examples. □

Example 4.7. (Not recommended model) Considering the relationship between $\log(m1)$ with RS and the dummy variables $Drs1$ and $Drs2$, the following alternative additive models may be presented:

$$\log(m1_t) = c(1) + c(2)^*Drs1 + c(3)^*RS_t + \mu_t \qquad (4.30)$$

which can be presented as a pair of homogeneous simple regressions or a set of regressions having equal slopes, $c(3)$, as follows:

$$\begin{aligned} \log(m1_t) &= c(1) + c(2) + c(3)^*RS_t + \mu_t \\ \log(m1_t) &= c(2) + c(3)^*RS_t + \mu_t \end{aligned} \qquad (4.31)$$

Based on this model, the following regression functions are obtained, with the p-value of the t-statistics in [·]:

$$\log(m1)\_head = 6{\cdot}305 - 1{\cdot}270 \underset{[0{\cdot}0000]}{\text{Drs1}} + 0{\cdot}063\,970^* \underset{[0{\cdot}0000]}{\text{RS}} \qquad (4.32)$$

This equation in fact represents two parallel lines (homogeneous regressions) with a slope = 0.063 970, which can be presented as in Figure 4.32. This graph presents overly scatter graphs with regression lines of observed values of $\log(M1)$ and the fitted values of the model in (4.30), namely $\log(m1)$_*head* in (4.32), on RS, which clearly shows that the model, i.e. the set of homogeneous regressions, is not an appropriate empirical model.

Furthermore, even though RS has a significant effect on $\log(m1)$ with a p-value 0.0000 based on the standard t-test, it is suggested that these results, namely the model in (4.30) as well as the conclusion of the t-test, should not be used as research findings. In other words, this model is not a recommended model. □

Example 4.8. (The relationship between log(*M*1) and log(*GDP*) or *PR*) Compared to the two-piece model of $\log(M1)$ on RS in (4.26) and (4.30), Figures 4.33 and 4.34 present scatter graphs of $\log(M1)$ on each of the variables $\log(GDP)$ and PR respectively. Compared to the variable RS, the data show that both variables, $\log(GDP)$ and PR, are better linear predictors than RS.

Note that the graphs presented in Figures 4.33 and 4.34 are in fact based on the variables $\log(M1_i)$, $\log(GDP_i)$ with $\log(GDP_i) \le \log(GDP_{i+1})$ and PR_i with $PR_i \le PR_{i-1}$, for all $i = 1, 2, \ldots, n = T$, as mentioned in the previous examples.

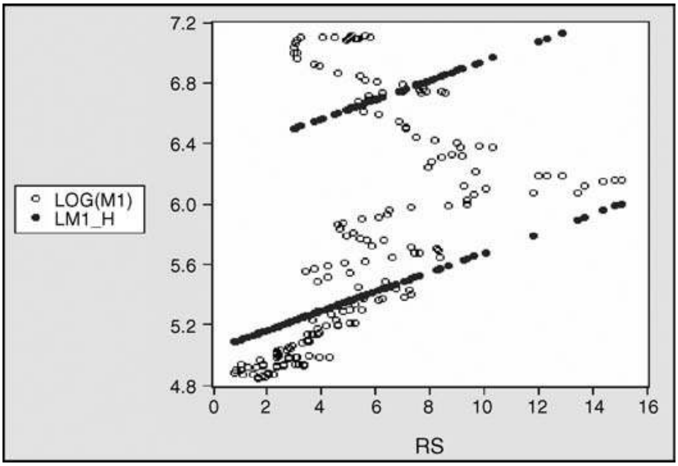


Figure 4.32 Overlay scatter graphs with regressions lines of log($M1$) on RS and the fitted values of the model in (4.32)

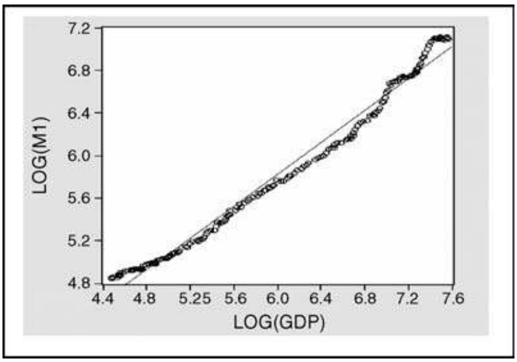


Figure 4.33 Scatter graph with a regression line of log($M1$) on log(GDP)

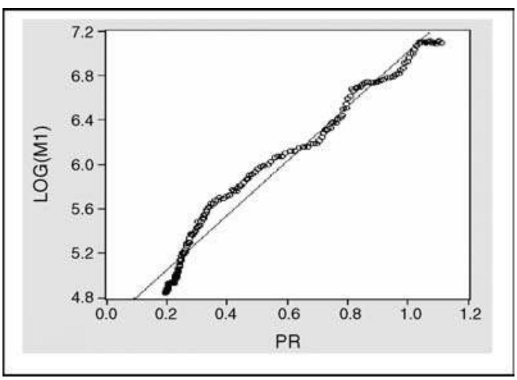


Figure 4.34 Scatter graph with a regression line of log($M1$) on PR

The simple linear regressions (SLR) of log(M1) on each of $l$og(GDP) and PR have R-squared of 0.984 292 and 0.972 889 respectively. However, these SLRs have very small values of DW-statistics of 0.027 162 and 0.017 985, so are not acceptable time series models. □

Example 4.9. (An extreme case based on data in the BASICS workfile) Referring back to the illustrative graphs presented in Section 1.4 based on a set of variables in BASICS.wf1, Figure 4.35 presents scatter plots of a bivariate time series (X_t,Y_t), with (d) its simple linear regression and (e) its nearest neighbor fit.

Based on these graphs, it could be said that Y_t cannot be predicted by using X_t. In other words, X_t cannot be a good predictor of Y_t, since its simple linear regression has a very small $R^2 = 0.000\,213$ and X_t has an insignificant effect with a p-value = 0.7885, as presented in Figure 4.36.

Even though its $DW = 1.956\,719$ is sufficient to indicate that the null hypothesis of no first-order autocorrelation is accepted, the model cannot be considered as a good fit model. By observing the scatter graphs, as presented in Figure 4.37, it may be

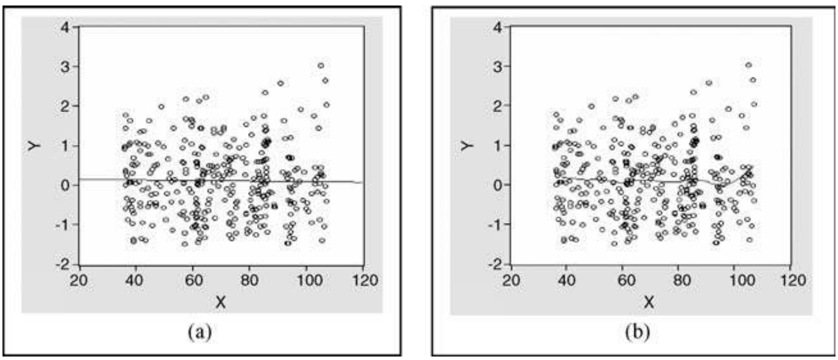


Figure 4.35 Scatter graphs (X_t, Y_t) in BASICS.wf1 with (a) a simple linear regression and (b) a nearest neighbor fit

Dependent Variable: Y
Method: Least Squares
Date: 10/18/07 Time: 19:57
Sample: 1959M01 1989M12
Included observations: 340

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.154206	0.176299	0.874683	0.3824
X	-0.000645	0.002403	-0.268515	0.7885

R-squared	0.000213	Mean dependent var	0.108655
Adjusted R-squared	-0.002745	S.D. dependent var	0.883724
S.E. of regression	0.884936	Akaike info criterion	2.599263
Sum squared resid	264.6919	Schwarz criterion	2.621786
Log likelihood	-439.8746	Hannan-Quinn criter.	2.608237
F-statistic	0.072100	Durbin-Watson stat	1.965179
Prob(F-statistic)	0.788467		

Figure 4.36 Statistical results based on a simple regression of Y_t on X_t

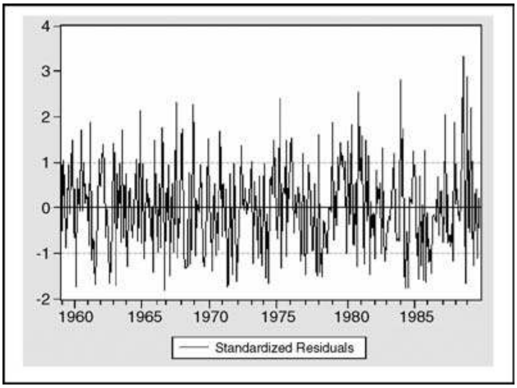


Figure 4.37 Residual graph of the regression in Figure 4.36

concluded that any defined models will have very small values of R^2. In other words, it is impossible to find a regression that can have a sufficiently large value of R^2. Therefore, in this case, a model having a small value of R^2 should be accepted. □

Example 4.10. (A special case based on data in BASICS.wf1) Referring to the growth patterns of Y_t and $POLI\_1_t$ over time, as presented in Figure 4.38(a), it could be said that $POLI\_1_t$ looks as though it has an insignificant linear effect on Y_t. However, a regression function has been obtained as follows:

$$\hat{Y} = -0{\cdot}097\,031 + 1{\cdot}704\,116 * \underset{(3{\cdot}860437)}{POLI\_1} \tag{4.33}$$

with a small value of $R^2 = 0.042\,230$ and a sufficient value of $DW = 2.034$; $POLI\_1$ has a significant effect on Y with a p-value $= 0.0001$. Figure 4.38(b) presents the scatter graph with a regression line of Y on $POLI\_1$.

It is surprising that based on the Breusch–Godfrey serial correlation LM test, the null hypothesis of no first-order autocorrelation is accepted with a p-value =

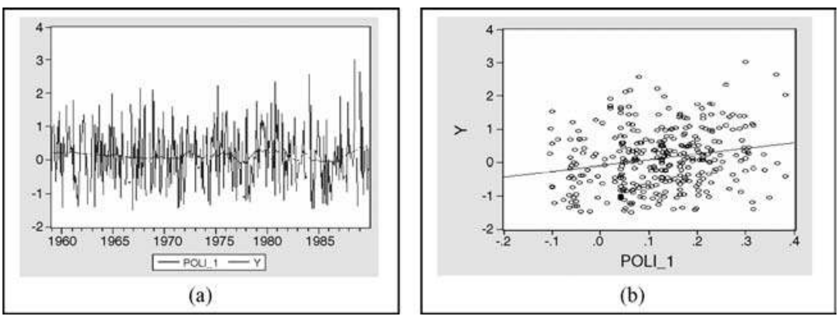


Figure 4.38 Graphs of the bivariate (*POLI_1*, *Y*): (a) overlay growth curves and (b) a scatter graph with a regression line

0.413 401. This result also shows that the time series ($POLI\_1_t$, Y_t), $t = 1, 2, \ldots T$; can be analyzed as cross-sectional data ($POLI\_1_i$, Y_i), $i = 1, 2, \ldots, n = T$.

Even though the R-squared value is very small, these findings show that the simple linear model is an acceptable model in a statistical sense, since it is certain that there can never be a model with a large value of R-squared. Furthermore, even $POLI\_1$ has a significant effect on Y, which means that $POLI\_1$ is not a good linear predictor for Y, since the R-squared value is very small. For a comparison, do the analysis using the variable $POLI\_3$. □

Example 4.11. (Another special case based on data in BASICS.wf1) First refer to the overlay growth patterns of Y_t and $URATE_t$ over time, as presented in Figure 4.39(a), which are quite different from the growth patterns of Y_t and $POLI\_1_t$ in Figure 4.38(a). Figure 4.39(b) presents the scatter graph with a regression line of Y_t on $URATE_t$ with the following regression function with the t-statistic in [.]:

$$\hat{Y} = 0{\cdot}128\,493 - 0{\cdot}003\,263 * \underset{[-0{\cdot}106796]}{URATE} \tag{4.34}$$

with $R^2 = 0.000\,034$, $DW = 1.964\,872$, and $URATE$ has insignificant effect on Y with a p-value $= 0.9150$. This value of the DW-statistic also indicates that the series ($URATE_t$, Y_t), $t = 1, 2, \ldots T$, can be analyzed as cross-sectional data ($URATE_i$, Y_i), $i = 1, 2, \ldots, n = T$. However, the simple linear regression may not be an appropriate model. Corresponding to this type of scatter graph it is suggested that a nonparametric regression should be applied, which will be presented in Chapter 11. □

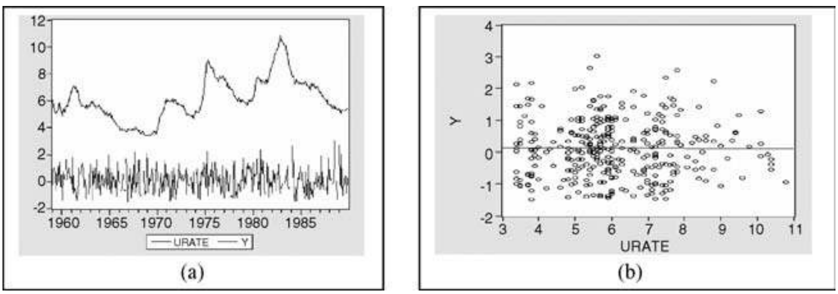


Figure 4.39 Graphs of the bivariate ($URATE$, Y): (a) overlay growth curves and (b) a scatter graph with a regression line

4.3.2 Simplest models in three-dimensional space

Two simple (additive) models in a three-dimensional space or a coordinate system, based on the bivariate (X_t, Y_t), can have the following alternative models:

$$Y_t = c(1) + c(2)*X_t + c(3)*X_{t-m} + \mu_t \tag{4.35a}$$

$$Y_t = c(1) + c(2)*Y_{t-k} + c(3)*X_{t-m} + \mu_t \tag{4.35b}$$

for selected values of (k, m). However, in general, $k = m = 1$ may be used.

In a special case, for $k = 1$ and $m = 0$, there is a simple model as follows:

$$Y_t = c(1) + c(2)^*Y_{t-1} + c(3)^*X_t + \mu_t \tag{4.36}$$

where X_t can be an *environmental* or *instrumental* (exogenous) variable (Gourierroux and Manfort, 1997).

Note that the corresponding regression function of this additive model can be presented as a plane in a three-dimensional coordinate system. For example, the regression function of (2.35) will represent a plane in the three-dimensional rectangular coordinate with Y_t, Y_{t-k} and X_{t-m} axes. Hence, this type of model is considered as the simplest model in a three-dimensional space.

Furthermore, note that the X and Y variables used in the model can be the original variables or their transformation, such as $\log(Y)$, $\log(Y-L)/(U-Y)$, $\log(X)$ or X^α, which have been presented in Chapter 2. If $0 < Y_t < 1$ for all t, then there will be a logistic model with $\log(Y_t/(1-Y_t))$ as the dependent variable, and if $0 < Y_t < 100$ for all t, then there will be a modified logistic model with $\log(Y_t/(100-Y_t))$ as the dependent variable.

4.3.3 General univariate LVAR(p,q) seemingly causal model

Besides the simple models presented in the previous subsections, based on a bivariate time series (X_t, Y_t), the following general univariate LVAR(p, q) seemingly causal model, namely LVAR(p, q)_SCM, may be considered:

$$\begin{aligned} Y_t &= c(10) + c(11)Y_{t-1} + \cdots + c(1p)Y_{t-p} + c(20)X_t + \cdots + c(2k)X_{t-k} + \mu_t \\ \mu_t &= c(31)\mu_{t-1} + \cdots + c(3q)\mu_{t-q} + \varepsilon_t \end{aligned} \tag{4.37}$$

Note that different symbols of parameters are used, such as $c(10)$, $c(1p)$, $c(2k)$ and $c(3q)$, to indicate their positions corresponding to the intercept, lagged dependent variables, the exogenous variable and its lags, and the autoregressive indicators. By using these specific symbols, an independent variable can easily be added or deleted while performing data analysis based on a series of alternative models.

Furthermore, note that this general model is a modification or derivation of a multivariate macroeconomic model, presented in Gourierroux and Manfort (1997, p. 356).

Corresponding to the model in (4.37), Enders (2004, p.7) presents another form of the lagged-variable model, as follows:

$$Y_t = a_0 + \sum_{i=1}^{p} a_i Y_{t-i} + X_t \tag{4.38}$$

where the various parameters a_i are functions of economic variables, but do not depend on any of the values Y_t or X_t. The term X_t is called the *forcing process*, and can be any function of time, current and lagged values of other variables and/or stochastic disturbance.

Example 4.12. (An additive seemingly causal model) Corresponding to the LVAR (1,1) growth model in Example 2.17, after doing some exercises, an additive SCM is obtained, namely a LVAR(2,1) with exogenous variables $\log(GDP_t)$ and $\log(GDP_{t-1})$, without the time t, as follows:

$$\begin{aligned} \log(m1) = {} & c(10) + c(11)^*\log(m1(-1)) + c(12)^*\log(m1(-2)) \\ & + c(20)^*\log(gdp) + c(21)^*\log(gdp(-1)) + [ar(1) = c(1)] \end{aligned} \tag{4.39}$$

Figure 4.40 presents the statistical results based on the model in (4.39) with $R^2 = 0.999\,646$ and $DW = 1.978\,124$, and its residual graph presented in Figure 4.41.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/18/07 Time: 20:42
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 8 iterations
LOG(M1)=C(10)+C(11)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(20)
*LOG(GDP)+C(21)*LOG(GDP(-1))+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(10)	0.064997	0.027230	2.386960	0.0181
C(11)	0.560353	0.117836	4.755356	0.0000
C(13)	0.381471	0.108780	3.506822	0.0006
C(20)	0.224623	0.116404	1.929684	0.0553
C(21)	-0.176830	0.121774	-1.452124	0.1483
C(1)	0.295773	0.123823	2.388668	0.0180

R-squared	0.999646	Mean dependent var	5.827503
Adjusted R-squared	0.999636	S.D. dependent var	0.750468
S.E. of regression	0.014317	Akaike info criterion	-5.621380
Sum squared resid	0.035053	Schwarz criterion	-5.513714
Log likelihood	503.4921	Hannan-Quinn criter.	-5.577715
F-statistic	96678.28	Durbin-Watson stat	1.978124
Prob(F-statistic)	0.000000		

Inverted AR Roots .30

Figure 4.40 Statistical results based on the LVAR(2,1) model in (4.39)

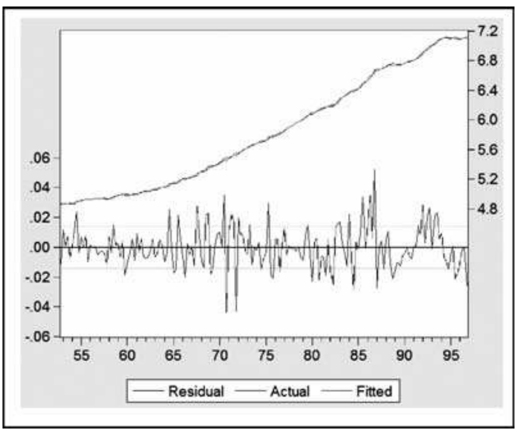


Figure 4.41 Residual graph of the regression in Figure 4.40

Therefore, it can be concluded that this model is an acceptable model, in a statistical sense. Note that this model is an LVAR(2,1) model with two exogenous variables, namely log(*gdp*) and log(*gdp*(−1)). □

Example 4.13. (LVAR(2,1) model with three exogenous variables) Figure 4.42 presents the statistical result based on an LVAR(2,1) model with three exogenous variables, which has a '*forcing process*' X_t (Enders, 2004) that is a linear combination of log(*gdp*), log(*gdp*(−1)), *rs* and *ar*(1), using the following model:

$$\log(m1_t) = c(1) + c(2)\log(m1_{t-1}) + c(3)\log(m1_{t-2}) + c(4)\log(gdp_t) + c(5)\log(gdp_{t-1}) + c(6)RS + c(7)\mu_{t-1} + \varepsilon t \quad (4.40)$$

However, the statistical results in Figure 4.42 are obtained by entering the following equation specification:

$$\log(m1) = c\,\log(m1(-1))\log(m1(-2))\log(gdp)\log(gdp(-1))rs\;ar(1) \quad (4.41)$$

and by clicking *View/Actual/Fitted/Residual Table*, the residual plot in Figure 4.43 is obtained. This plot shows two of the error terms out of the confident interval, namely at 1953Q1 ($t = 5$) and 1954Q3 ($t = 11$).

Based on this model, the hypotheses could easily be tested using the Wald test, besides using the *t*-statistic and *F*-statistic presented in the printout. Note that log(*gdp*) has a significant effect on log(*m*1), but log(*gdp*(−1)) has an insignificant effect on log(*m*1), based on the *t*-statistic with a *p*-value = 0.1975.

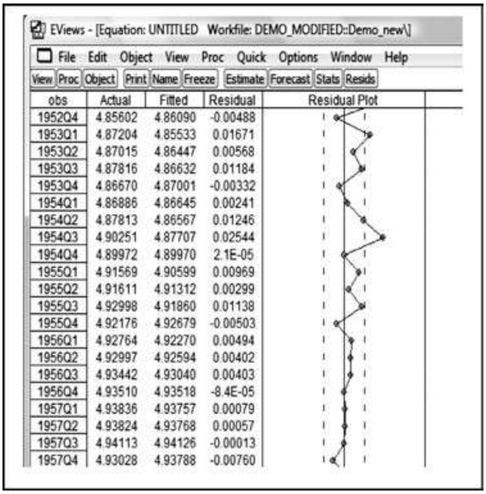
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/18/07 Time: 20:55
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.155308	0.034313	4.526212	0.0000
LOG(M1(-1))	0.507067	0.107531	4.715532	0.0000
LOG(M1(-2))	0.352879	0.092534	3.813508	0.0002
LOG(GDP)	0.258039	0.105186	2.453184	0.0152
LOG(GDP(-1))	-0.142149	0.109880	-1.293679	0.1975
RS	-0.004333	0.000953	-4.545744	0.0000
AR(1)	0.280385	0.122887	2.281654	0.0237

R-squared	0.999696	Mean dependent var	5.827503
Adjusted R-squared	0.999686	S.D. dependent var	0.750468
S.E. of regression	0.013303	Akaike info criterion	-5.762865
Sum squared resid	0.030086	Schwarz criterion	-5.637255
Log likelihood	517.0136	Hannan-Quinn criter.	-5.711923
F-statistic	93320.25	Durbin-Watson stat	1.994235
Prob(F-statistic)	0.000000		

Inverted AR Roots	.28

Figure 4.42 Statistical results based on the LV(2,1) model in (4.40)


EViews - [Equation: UNTITLED Workfile: DEMO_MODIFIED::Demo_new\]

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obs	Actual	Fitted	Residual	Residual Plot
1952Q4	4.85602	4.86090	-0.00488	
1953Q1	4.87204	4.85533	0.01671	
1953Q2	4.87015	4.86447	0.00568	
1953Q3	4.87816	4.86632	0.01184	
1953Q4	4.86670	4.87001	-0.00332	
1954Q1	4.86886	4.86645	0.00241	
1954Q2	4.87813	4.86567	0.01246	
1954Q3	4.90251	4.87707	0.02544	
1954Q4	4.89972	4.89970	2.1E-05	
1955Q1	4.91569	4.90599	0.00969	
1955Q2	4.91611	4.91312	0.00299	
1955Q3	4.92998	4.91860	0.01138	
1955Q4	4.92176	4.92679	-0.00503	
1956Q1	4.92764	4.92270	0.00494	
1956Q2	4.92997	4.92594	0.00402	
1956Q3	4.93442	4.93040	0.00403	
1956Q4	4.93510	4.93518	-8.4E-05	
1957Q1	4.93836	4.93757	0.00079	
1957Q2	4.93824	4.93768	0.00057	
1957Q3	4.94113	4.94126	-0.00013	
1957Q4	4.93028	4.93788	-0.00760	

Figure 4.43 Residual plot of the regression in Figure 4.42

For illustration purposes, the joint effects of log(gdp) and log($gdp(-1)$) are tested using a null hypothesis H_0: $C(4) = C(5) = 0$. Based on the Wald test, the null hypothesis is rejected either based on the F-statistic = 18.546 21, df = (2, 170) or on the chi-squared-statistic = 327.092 42, df = 2, with a p-value = 0.000.

Then, based on this conclusion, if a reduced model is required, either one of these independent variables can be deleted. For a further discussion, see the following example. However, both variables may be kept in the model, since at a significant level 0.10, log($gdp(-1)$) has a significant negative effect on log($m1$) with a p-value = 0.1975/2 = 0.098 75 < 0.10. □

Example 4.14. (**Possible reduced models of the model in (4.40)**) Figure 4.44 presents the statistical results based on a reduced model of (4.40) by deleting log($gdp(-1)$) as an independent variable and using the OLS and the Newey–West estimation methods respectively.

On the other hand, Figure 4.45 presents the statistical results based on another reduced model by deleting log(gdp) as an independent variable, even though it has a significant adjusted effect on log($m1$), by using the OLS and the Newey–West estimation methods respectively.

Based on these results, the following notes and conclusions are presented:

(1) It has been found that log(gdp) and log($gdp(-1)$) have a high or significant coefficient of correlation. This can be easily tested by using a simple linear regression.
(2) There is a general conclusion or rule that if a pair of independent variables has a high or significant coefficient of correlation, then either one of those variables could be used to develop a reduced model. However, which one should be used is a matter of judgment.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 09:50
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.169590	0.032858	5.161339	0.0000
LOG(M1(-1))	0.483312	0.103043	4.690413	0.0000
LOG(M1(-2))	0.366763	0.089210	4.111210	0.0001
LOG(GDP)	0.123609	0.021485	5.753177	0.0000
RS	-0.004459	0.000969	-4.601538	0.0000
AR(1)	0.294591	0.119305	2.469233	0.0145

R-squared	0.999693	Mean dependent var	5.827503
Adjusted R-squared	0.999684	S.D. dependent var	0.750468
S.E. of regression	0.013331	Akaike info criterion	-5.764197
Sum squared resid	0.030388	Schwarz criterion	-5.656531
Log likelihood	516.1314	Hannan-Quinn criter.	-5.720532
F-statistic	111525.5	Durbin-Watson stat	2.003639
Prob(F-statistic)	0.000000		

Inverted AR Roots .29

(a)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 09:52
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 7 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.169590	0.038285	4.429690	0.0000
LOG(M1(-1))	0.483312	0.101054	4.782729	0.0000
LOG(M1(-2))	0.366763	0.082513	4.444910	0.0000
LOG(GDP)	0.123609	0.025481	4.851116	0.0000
RS	-0.004459	0.001122	-3.973940	0.0001
AR(1)	0.294591	0.147863	1.992326	0.0479

R-squared	0.999693	Mean dependent var	5.827503
Adjusted R-squared	0.999684	S.D. dependent var	0.750468
S.E. of regression	0.013331	Akaike info criterion	-5.764197
Sum squared resid	0.030388	Schwarz criterion	-5.656531
Log likelihood	516.1314	Hannan-Quinn criter.	-5.720532
F-statistic	111525.5	Durbin-Watson stat	2.003639
Prob(F-statistic)	0.000000		

Inverted AR Roots .29

(b)

Figure 4.44 Statistical results based on a reduced model of the model in (4.40), by deleting log(GDP(−1)) using (a) the LS and (b) the Newey–West estimation methods

(3) A researcher could have difficulty in selecting one out of the four statistical results that might be considered as the best fit model, as well as choosing the best estimation method to use. Note also that there are several or many other possible lagged-variable autoregressive models having the endogenous variable $\log(m1)$. See the following examples.
(4) However, if a reduced model needs to be presented, then the model with $\log(gdp(-1))$ can be selected as an independent variable by using the Newey–West estimation method, because $\log(gdp(-1)) = \log(gdp_{t-1})$ is more appropriate to use than $\log(gdp_t)$ as a cause or an explanatory factor of $\log(m1_t)$.

The Newey–West estimation method takes into account the unknown autocorrelation, as well as the heteroskedasticity, of the error terms. On the other hand, the WLS or the White estimation methods may be used, since the model has been using the AR(1) indicator. □

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 09:58
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.167401	0.033865	4.943232	0.0000
LOG(M1(-1))	0.492885	0.111110	4.436022	0.0000
LOG(M1(-2))	0.361209	0.096201	3.754742	0.0002
LOG(GDP(-1))	0.120189	0.021983	5.467301	0.0000
RS	-0.004197	0.000967	-4.339421	0.0000
AR(1)	0.274451	0.124730	2.200357	0.0291

R-squared	0.999686	Mean dependent var	5.827503
Adjusted R-squared	0.999677	S.D. dependent var	0.750468
S.E. of regression	0.013497	Akaike info criterion	-5.739342
Sum squared resid	0.031152	Schwarz criterion	-5.631676
Log likelihood	513.9317	Hannan-Quinn criter.	-5.695677
F-statistic	108786.8	Durbin-Watson stat	2.008954
Prob(F-statistic)	0.000000		

Inverted AR Roots 27

(a)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 09:56
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 7 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.167401	0.041031	4.079907	0.0001
LOG(M1(-1))	0.492885	0.113991	4.323880	0.0000
LOG(M1(-2))	0.361209	0.092853	3.890120	0.0001
LOG(GDP(-1))	0.120189	0.026898	4.468308	0.0000
RS	-0.004197	0.001154	-3.636269	0.0004
AR(1)	0.274451	0.158326	1.733458	0.0848

R-squared	0.999686	Mean dependent var	5.827503
Adjusted R-squared	0.999677	S.D. dependent var	0.750468
S.E. of regression	0.013497	Akaike info criterion	-5.739342
Sum squared resid	0.031152	Schwarz criterion	-5.631676
Log likelihood	513.9317	Hannan-Quinn criter.	-5.695677
F-statistic	108786.8	Durbin-Watson stat	2.008954
Prob(F-statistic)	0.000000		

Inverted AR Roots 27

(b)

Figure 4.45 Statistical results based on a reduced model of the model in (4.40), by deleting log(*GDP*) using (a) the LS and (b) the Newey–West estimation methods

Covariance Analysis: Ordinary
Date: 10/19/07 Time: 10:24
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Balanced sample (listwise missing value deletion)

Correlation Probability	RESID01	LOG(M1(-1))	LOG(M1(-2))	LOG(GDP...	RS	LOG(PR)	T
RESID01	1.000000 -----						
LOG(M1(-1))	-0.014512 0.8475	1.000000 -----					
LOG(M1(-2))	-0.014664 0.8460	0.999798 0.0000	1.000000 -----				
LOG(GDP(-1))	-0.016986 0.8219	0.992092 0.0000	0.991490 0.0000	1.000000 -----			
RS	-0.018908 0.8022	0.464353 0.0000	0.462707 0.0000	0.547873 0.0000	1.000000 -----		
LOG(PR)	-0.004050 0.9572	0.993178 0.0000	0.992880 0.0000	0.994637 0.0000	0.527411 0.0000	1.000000 -----	
T	-0.035135 0.6415	0.986827 0.0000	0.986228 0.0000	0.995617 0.0000	0.525782 0.0000	0.983267 0.0000	1.000000 -----

Figure 4.46 A correlation matrix of selected variables with their probabilities

4.3.3.1 A specific residual analysis

Figure 4.46 presents a correlation matrix of the error term, namely *Resid*01, of the model in Figure 4.45 with selected variables either in or out of the model. The steps for obtaining a correlation matrix have been presented in Section 1.3.5.

Based on this correlation matrix the following notes are produced:

(1) The correlation matrix should be used to study the correlations between each of the independent variables with the error term. Since they are insignificant it can be concluded that the model is an appropriate model, in a statistical sense, specifically the linear forms of the independent variables. If at least one of them has a significant correlation, it is suggested that an instrumental model should be applied, which will be presented in Chapter 7.

(2) The correlation can be used to study whether a variable outside the model should be used to improve the quality of the model or to modify it. In this case, since the variables $\log(pr)$ and the time t have insignificant correlations with the error terms, it can be concluded that the model does not have to use these variables in order to improve or modify the model.

(3) Note that each of the independent variables $\log(m1(-1))$, $\log(m1(-2)$, $\log(gdp(-1))$ and *RS* has a significant effect on $\log(m1)$, even though they have significant bivariate correlations. These statistical results show the unpredictable impact(s) of the multicollinearity or correlations between the independent variables, which have been presented in Section 2.14, since in general there would be insignificant adjusted effect(s) whenever the independent variables are significantly or highly correlated.

(4) Furthermore, it is suggested that the scatter graphs between the *Resid*01 and each independent variable should be observed in order to explore the possibility of a nonlinear relationship. Do this as an exercise.

Example 4.15. (Advanced additive models for log(*m*1)) By experimentation or using the '*trial and error methods*,' other lagged-variable autoregressive models have been found for log($m1$) that can be considered as acceptable models, in a statistical sense. The statistical results presented in Figure 4.47 are based on the following model:

$$\begin{aligned} \log(m_t) &= c(1) + c(2)^*\log(gdp_t) + c(3)^*\log(gdp_{t-1}) + c(4)^*\log(rs_t) \\ &\quad + c(5)^*\log(rs_{t-1}) + \mu t \\ \mu_t &= \rho_1\mu_{t-1} + \rho_{2-1}\mu_t\varepsilon t \end{aligned} \tag{4.42}$$

The statistical results show that the model is an acceptable model.

For illustration purposes, Figure 4.48 presents a correlation matrix of the error terms, namely *Resid*02, and three variables out of the model, namely log($m1(-1)$),

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 11:11
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 34 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.922054	0.296720	3.107485	0.0022
LOG(GDP)	0.499182	0.101856	4.900846	0.0000
LOG(GDP(-1))	0.324398	0.102674	3.159490	0.0019
LOG(RS)	-0.019863	0.007938	-2.502162	0.0133
LOG(RS(-1))	-0.038562	0.008126	-4.745713	0.0000
AR(1)	0.822029	0.076344	10.76747	0.0000
AR(2)	0.145247	0.075732	1.917910	0.0568

R-squared	0.999661	Mean dependent var	5.827503
Adjusted R-squared	0.999649	S.D. dependent var	0.750468
S.E. of regression	0.014058	Akaike info criterion	-5.652555
Sum squared resid	0.033595	Schwarz criterion	-5.526945
Log likelihood	507.2511	Hannan-Quinn criter.	-5.601612
F-statistic	83570.55	Durbin-Watson stat	1.921839
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97	-.15

Figure 4.47 Statistical results based on the model in (4.42)

Covariance Analysis: Ordinary
Date: 10/19/07 Time: 11:14
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Balanced sample (listwise missing value deletion)

Correlation Probability	RESID02	LOG(M1(-1))	LOG(PR)	T
RESID02	1.000000 -----			
LOG(M1(-1))	-0.094696 0.2073	1.000000 -----		
LOG(PR)	-0.099108 0.1869	0.993240 0.0000	1.000000 -----	
T	-0.142578 0.0569	0.986459 0.0000	0.983020 0.0000	1.000000 -----

Figure 4.48 Correlation matrix of the error terms with variables outside the regression in Figure 4.47

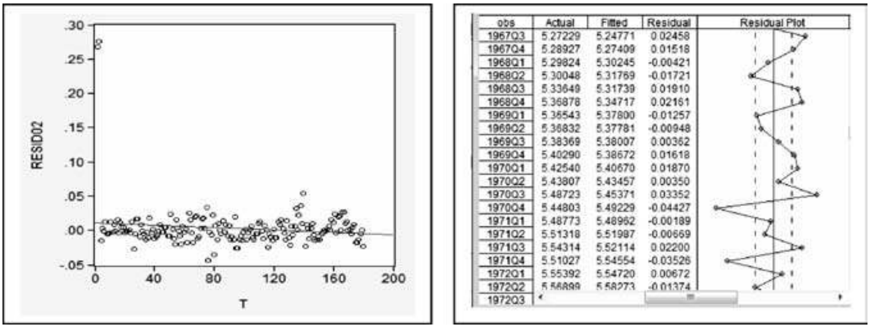


Figure 4.49 (a) Residual graph and (b) residual plot of the regression in Figure 4.47

log(pr) and the time t. At a significant level of 0.10, the $Resid02$ and the time t have a significant correlation with a p-value $= 0.0649$. On the other hand, log(pr) also a has positive correlation with $Resid02$ with a p-value $= 0.1869/2 = 0.093\,45 < 0.10$. These indicate that the model can be improved or modified by using an additional variable, either t or log(pr), or both.

Another limitation of this model can be identified by observing the residual graphs in Figure 4.49(a) and (b). Figure 4.49(a) clearly presents two points that are a very long way from the others, which should be considered as outliers, and Figure 4.49(b) presents several points that are outside the confidence interval. These findings indicate that the analysis is being done based on data that does not include the outliers or by transforming the outliers to the means of their neighbors, as has been suggested in previous examples. As this process is not being presented here, there is no need to do it as an exercise. □

4.3.3.2 Special comments

(1) The last two examples present models having RS_t or $\log(RS_t)$ and $\log(RS_{t-1})$ as exogenous variables and can exert significant adjusted effects on $\log(M1_t)$.
(2) On the other hand, Examples 4.6 and 4.7 show that RS_t is not an appropriate linear predictor or explanatory variable for $\log(M1_t)$. This example also shows problems with its error terms.
(3) Based on these contradictory conclusions, the acceptability of any continuous regression of $\log(M1_t)$ on RS_t or $\log(RS_t)$ may be argued. From the present point of view, it could be said that RS_t and $\log(RS_t)$ are not appropriate linear explanatory variables for $M1$ or $\log(M1)$. Hence, it is recommended not to use RS_t or $\log(RS_t)$ as predictors of $M1$ or $\log(M1)$, based on any models without dummy variable(s) of the RS_t. However, note that this recommendation cannot be generalized to all pairs of variables X_t and Y_t, but needs to be observed and evaluated case by case.
(4) Instead of having a conclusion that RS_t has a significant adjusted effect on $\log(M1)$, it is wise or recommended to explore external information in order to be able to present an explanation of why the values of RS_t decrease after $t = 119$.

4.4 Trivariate seemingly causal models

4.4.1 Simple models in three-dimensional space

Based on a trivariate time series, namely (X_t, Y_t, Z_t), $t = 1, 2, \ldots, T$, there may be additive and interaction seemingly causal models (SCMs) or explanatory models (EMs), as follows.

4.4.1.1 Simple additive models

An additive model is considered as the simplest model in three-dimensional space, with the following general form:

$$Y_t = c(1) + c(2)X_{t-k} + c(3)Z_{t-m} + \mu_t \tag{4.43}$$

for selected values of (k, m), which are highly dependent on the time intervals. Larger values of k and m will correspond to smaller time intervals, such as days and intra-day intervals. However, in general, $k = m = 0$, $k = m = 1$ or $k = 1$ and $m = 0$ are used, especially for yearly time series. For a special case, where $m = 0$, a model with the *environmental* or *instrumental* variable Z_t is found.

Note that the corresponding regression of this model can be presented as a plane in a three-dimensional coordinate system, with Y_t, X_{t-k} and Z_{t-k} as the axes.

4.4.1.2 Two-way interaction models

Referring to the two-way analysis of variance models based on two treatments, experimental or classification factors A and B, Agung (2006) presents four alternative two-way interaction ANOVA models, which are presented as designs $A\,B\,A^*B$, $A\,A^*B$, $B\,A^*B$ and A^*B. Corresponding to these ANOVA models, Agung also presents similar models based on numerical variables. Now, based on the trivariate time series (X_t, Y_t, Z_t), the following two-way interaction models may be considered:

$$Y_t = c(1) + c(2)X_{t-k} + c(3)Z_{t-m} + c(4)X_{t-k}Z_{t-m} + \mu_t \tag{4.44}$$

for fixed values k and m, which should be selected based on *expert judgment*. This model is considered as *an hierarchical model* if $c(2) \neq 0$, $c(3) \neq 0$ and $c(4) \neq 0$. On the other hand, there will be three possible nonhierarchical models as follows:

(i) If $c(2) = 0$, $c(3) \neq 0$ and $c(4) \neq 0$, the nonhierarchical model is

$$Y_t = c(1) + c(3)Z_{t-m} + c(4)X_{t-k}Z_{t-m} + \mu_t \tag{4.45}$$

(ii) If $c(2) \neq 0$, $c(3) = 0$ and $c(4) \neq 0$, the nonhierarchical model is

$$Y_t = c(1) + c(2)X_{t-k} + c(4)X_{t-k}Z_{t-m} + \mu_t \tag{4.46}$$

(iii) If $c(2) = c(3) = 0$ and $c(4) \neq 0$, the nonhierarchical model is

$$Y_t = c(1) + c(4)X_{t-k}Z_{t-m} + \mu_t \tag{4.47}$$

Example 4.16. (An additive model, compared to Example 2.16) Corresponding to the additive model with trend or the growth model in the Example 2.16, after doing experimentation based on the time series $M1_t$, GDP_t and PR_t, the following acceptable additive LVAR(2,1) model is obtained:

$$\begin{aligned}\log(m1) = {} & c(10) + c(11)^*\log(gdp) + c(12)\log(gdp(-1)) + c(21)^*\log(m1(-1)) \\ & + c(22)^*\log(m1(-2)) + c(31)^*\log(pr) + c(32)^*\log(pr(-1)) \\ & + [ar(1) = c(41)]\end{aligned} \tag{4.48}$$

with the following regression function, with the t-statistic in $(\cdot)$:

$$\begin{aligned}\log(m1) = {} & \underset{(0.397)}{0.118} + \underset{(2.258)}{0.267}^*\log(gdp) - \underset{(-1.677)}{0.207}^*\log(gdp(-1)) + \underset{(4.512)}{0.552}^*\log(m1(-1)) \\ & + \underset{(3.338)}{0.370}^*\log(m1(-2)) - \underset{(-1.723)}{0.508}^*\log(pr) + \underset{(1.768)}{0.515}^*\log(pr(-1)) + [ar(1) = \underset{(2.194)}{0.282}]\end{aligned} \tag{4.49}$$

with $R^2 = 0.999\,653$ and $DW = 1.977\,283$. □

Example 4.17. (Two-way interaction models with endogenous variable log(*m*1)) After experimentation, the following acceptable AR(3) two-way interaction SCM was found:

$$\begin{aligned}\log(m1_t) &= c(1) + c(2)\log(gdp_{t-1}) + c(3)\log(rs_{t-1}) \\ &\quad + c(4)\log(gdp_{t-1})^*\log(rs_{t-1}) + \mu_t \\ \mu_t &= \rho_1\mu_{t-1} + \rho_2\mu_{t-2} + \rho_3\mu_{t-3} + \varepsilon_t\end{aligned} \tag{4.50}$$

The results in Figure 4.50 are obtained by using the following equation specification:

$$\begin{aligned}&\log(m1)\ c\ \log(gdp(-1))\ \log(rs(-1))\ \log(gdp(-1))^*\log(rs(-1)) \\ &ar(1)\ ar(2)\ ar(3)\end{aligned} \tag{4.51}$$

However, by using the equation specification

$$\begin{aligned}\log(m1_t) = {} & c(1) + c(2)\log(gdp_{t-1}) + c(3)\log(rs_{t-1}) + c(4)\log(gdp_{t-1})^*\log(rs_{t-1}) \\ & + [ar(1) = c(11), ar(2) = c(12), ar(3) = c(13)]\end{aligned} \tag{4.52}$$

the results in Figure 4.51 are obtained. Note that the symbols $c(i)$ and $c(1j)$ are used to identify the differential status or meaning of the model parameters. The background to using the first lagged exogenous variables lies in the fact that recent observations or events should be explained by the events in the previous time period(s). This model could easily be extended by using lagged endogenous variables, as well as higher lagged variables.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 12:37
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 50 iterations
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.906683	0.206625	4.388067	0.0000
LOG(GDP(-1))	0.835088	0.032428	25.75190	0.0000
LOG(RS(-1))	0.080081	0.033316	2.403672	0.0173
LOG(GDP(-1))*LOG(RS(-1))	-0.024886	0.006387	-3.896373	0.0001
AR(1)	0.704820	0.089626	7.863989	0.0000
AR(2)	0.440100	0.091336	4.818486	0.0000
AR(3)	-0.183439	0.075505	-2.429494	0.0162

R-squared	0.999651	Mean dependent var	5.833023
Adjusted R-squared	0.999639	S.D. dependent var	0.748997
S.E. of regression	0.014241	Akaike info criterion	-5.626487
Sum squared resid	0.034272	Schwarz criterion	-5.500388
Log likelihood	502.1309	Hannan-Quinn criter.	-5.575342
F-statistic	80657.12	Durbin-Watson stat	2.004839
Prob(F-statistic)	0.000000		

Inverted AR Roots	.96	.33	-.58

Figure 4.50 Statistical results using the equation specification in (4.51)

Note that by using the equation specification in (4.51) EViews saves or records the model as follows:

$$\log(m1_t) = c(1) + c(2)\log(gdp_{t-1}) + c(3)\log(rs_{t-1}) + c(4)\log(gdp_{t-1})^*\log(rs_{t-1})$$
$$[ar(1) = c(5), ar(2) = c(6), ar(3) = c(8)]. \qquad (4.53)$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 12:45
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 50 iterations
LOG(M1)= C(1)+C(2)*LOG(GDP(-1))+C(3)*LOG(RS(-1))+C(4)*LOG(GDP(
-1))*LOG(RS(-1))+[AR(1)=C(11), AR(2)=C(12), AR(3)=C(13)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.906686	0.217720	4.164468	0.0000
C(2)	0.835088	0.033469	24.95099	0.0000
C(3)	0.080081	0.039369	2.034105	0.0435
C(4)	-0.024886	0.007195	-3.458759	0.0007
C(11)	0.704820	0.077878	9.050270	0.0000
C(12)	0.440100	0.086687	5.076885	0.0000
C(13)	-0.183439	0.076064	-2.411629	0.0170

R-squared	0.999651	Mean dependent var	5.833023
Adjusted R-squared	0.999639	S.D. dependent var	0.748997
S.E. of regression	0.014241	Akaike info criterion	-5.626487
Sum squared resid	0.034272	Schwarz criterion	-5.500388
Log likelihood	502.1309	Hannan-Quinn criter.	-5.575342
F-statistic	80657.12	Durbin-Watson stat	2.004838
Prob(F-statistic)	0.000000		

Inverted AR Roots	.96	.33	-.58

Figure 4.51 Statistical results using the equation specification in (4.52)

where the parameters $c(5)$, $c(6)$ and $c(7)$ represent the autocorrelation parameters ρ_1, ρ_2 and ρ_3 respectively. □

Example 4.18. (Another interaction model, compared to Example 2.18) Corresponding to the interaction growth model in Example 2.18, an interaction model will be considered, without the time t, as follows:

$$\begin{aligned}\log(m1) \;=\; & c(1) + c(2)*\log(gdp) + c(3)*\log(pr) \\ & + c(4)*\log(gdp)*\log(pr) + [ar(1) = c(5)]\end{aligned} \tag{4.54}$$

The following regression is obtained, with the t-statistic in $(\cdot)$:

$$\begin{aligned}\log(m1) = \underset{(2.630)}{2.237} & + \underset{(5.441)}{0.632}*\log(gdp) - \underset{(-1.109)}{0.465}*\log(pr) \\ & + \underset{(2.442)}{0.136}*\log(gdp)*\log(pr) + [ar(1) = \underset{(41.779)}{0.958}]\end{aligned} \tag{4.55}$$

with $R^2 = 0.999\,603$ and $DW = 2.013\,201$.

Note that this model can easily be extended using the lagged variables, as presented in the previous example. □

4.4.2 General LVAR(p,q) with exogenous variables

Based on a trivariate time series, namely (X_t, Y_t, Z_t), $t = 1, 2, \ldots, T$, we may have the following general additive *SCMs* or *EMs*, namely the LVAR(*p,q*) with exogenous variables X_t and Z_t, as follows:

$$\begin{aligned}Y_t \;=\; & c(10) + c(11)Y_{t-1} + \cdots + c(1p)Y_{t-p} + c(20)X_t + \cdots + c(2k)X_{t-k} \\ & + c(30)Z_t + \cdots + c(3m)Z_{t-m} + \mu_t \\ \mu_t \;=\; & c(41)\mu_{t-1} + \cdots + c(4q)\mu_{t-q} + \varepsilon_t\end{aligned} \tag{4.56}$$

This is an additive model, which can be extended to present two-way or three-way interaction models, but only specific selected two-way or three-way interaction factors are used from such a large number of possible interactions. Refer to the additive and interaction models presented in the previous chapters.

On the other hand, judgment should also be used to select appropriate values of k, m, p and q. However, in most cases, trial-and-error methods will be used or experimentation as demonstrated in the previous examples will be performed.

For illustration purposes, some simple models, such as additive and interaction models, will be presented as follows.

4.4.2.1 Additive models

For illustration purposes, the following three additive seemingly causal or explanatory models will be presented:

$$Y_t = c(1) + c(2)Y_{t-1} + c(3)X_t + c(4)Z_t + \mu_t \tag{4.57a}$$

$$Y_t = c(1) + c(2)Y_{t-1} + c(3)X_t + c(4)X_{t-1} + c(5)Z_t + \mu_t \quad (4.57b)$$

$$Y_t = c(1) + c(2)Y_{t-1} + c(3)Y_{t-2} + c(4)X_t + c(5)X_{t-1} + c(6)Z_t + \mu_t \quad (4.57c)$$

Note that the corresponding regressions of these models represent hyperplanes in four-, five- and six-dimensional coordinate systems respectively, where Z_t is considered as an environmental or instrumental variable (Gourierroux and Manfort, 1997). These models show that the partial effect or adjusted effect of Z_t on Y_t are $c(4)$, $c(5)$ and $c(6)$ respectively, which are the partial derivatives $\partial Y_t/\partial Z_t$. Compare these to the following interaction models.

4.4.2.2 Two-way interaction models

For example, corresponding to the additive SCMs (4.57a), there is a complete two-way interaction SCM, which is an hierarchical model, as follows:

$$Y_t = c(1) + c(2)Y_{t-1} + c(3)X_t + c(4)Z_t + c(23)X_tY_{t-1} + c(24)Y_{t-1}Z_t + c(34)X_tZ_t + \mu_t \quad (4.58)$$

This model can be presented as

$$\begin{aligned} Y_t &= \{c(1) + c(2)Y_{t-1} + c(3)X_t + c(23)X_tY_{t-1}\} + \{c(4) + c(24)Y_{t-1} + c(34)X_t\}Z_t + \mu_t \\ \text{with} \quad \frac{\partial Y_t}{\partial Z_t} &= c(4) + c(24)Y_{t-1} + c(34)X_t \end{aligned} \quad (4.59)$$

which shows that the effect of the environmental variable Z_t is dependent on the exogenous variables X_t and $Y_{t-1.}$

However, if a model with environmental variables and *environment-related effects* is considered, then the following two-way interaction model is needed:

$$Y_t = c(1) + c(2)Y_{t-1} + c(3)X_t + c(4)Z_t + c(24)Y_{t-1}Z_t + c(34)X_tZ_t + \mu_t \quad (4.60)$$

which has been found by deleting the two-way interaction X_tY_{t-1} from the model in (4.58). Compare this model with the model with trend and the *time-related effects* presented in Chapter 2.

This model is also considered as an hierarchical model. On the other hand, if at least one of the parameters $c(2)$, $c(3)$ and $c(4)$ is equal to zero, then it is a nonhierarchical SCM.

Note a two-way interaction should be used as an independent variable, since the effect of a factor on the corresponding dependent variable is dependent on the second factor. Other two-way interaction models, including the model with environmental variables and environment-related effects, can easily be derived from the models in (4.57b) and (4.57c).

4.4.2.3 Three-way interaction models

As an extension of the model in (4.58), a complete three-way interaction model, which is a full hierarchical model, is presented as follows:

$$\begin{aligned} Y_t = {} & c(1) + c(2)Y_{t-1} + c(3)X_t + c(4)Z_t + c(23)X_tY_{t-1} + c(24)Y_{t-1}Z_t + c(34)X_tZ_t \\ & + c(234)X_tY_{t-1}Z_t + \mu_t \end{aligned} \tag{4.61}$$

This gives the partial derivative

$$\frac{\partial Y_t}{\partial Z_t} = c(4) + c(24)Y_{t-1} + c(34)X_t + c(234)X_tY_{t-1} \tag{4.62}$$

which shows that the effect of Z_t on Y_t is dependent on a nonlinear function of Y_{t-1} and X_t.

If at least one of the parameters $c(2)$, $c(3)$, $c(4)$, $c(23)$, $c(24)$ and $c(34)$ is equal to zero, but $c(234) \neq 0$, then a nonhierarchical model is produced. As usual, the exogenous variables Y_{t-1}, X_t and Z_t are called the main factors, X_tY_{t-1}, $Y_{t-1}Z_t$ and X_tZ_t are the two-way interaction factors and $X_tY_{t-1}Z_t$ is the three-way interaction factor. Therefore, a three-way interaction factor is used as an independent variable, under the assumption that their main factors have a complete association. Refer to the three-way interactions presented in Chapter 2.

4.4.2.4 Higher-interaction models

In the case of a model having more than three main factors, such as the models in (4.57b) and (4.57c), a four-way or higher interaction of numerical variables or factors will never be used as an independent variable, since it is very difficult to judge whether a set of four variables or factors has a complete association. In fact, even for the three-way interactions, only one or two three-way interactions should be selected out of all possible three-way interactions of the numerical variables.

For comparison, however, in a multifactorial analysis of variance (ANOVA or MANOVA), an analysis of covariance (ANCOVA or MANCOVA) and heterogeneous regressions, four-way or higher-interaction factors may be used between the categorical independent variables or between the categorical and numerical variables. For example, based on three treatment or classification factors, namely A, B and C, and a numerical exogenous variable, X_t, the following four-way interaction nonhierarchical model could be produced (see Agung, 2006, pp. 301–307), as follows:

$$\begin{gathered} Y_t = (ABC)_{ijk} + X_t + (ABC)_{ijk}X_t + \mu_t, \\ \text{with} \quad \sum\nolimits_{ijk}(ABC)_{ijk} = 0 \end{gathered} \tag{4.63}$$

This model represents a set of heterogeneous regressions or a model with the three factors A, B and C. Corresponding to the weekly or monthly time series data, A is a factor of time periods, such as before and after the monetary crises in Indonesia (before and after 1997), B is a factor of the years and C is a factor of the semesters or a semi-annual factor. The main objective of this model is to study the differences in the linear effects of X_t on Y_t between all time intervals by semesters, years and time periods.

In order to do the data analysis using EViews, a regression should be used with dummy variables of all cells defined by the three factors A, B and C. Then there will be a regression with eight (=$2 \times 2 \times 2$) possible dummies, namely $D1$ up to $D8$, with the following equation:

$$Y_t = \sum_{k=1}^{8} c(1k)^*Dk + \sum_{k=1}^{8} c(2k)^*Dk^*X_t + \mu_t \tag{4.64a}$$

or

$$Y_t = \sum_{k=1}^{8} c(1k)^*Dk + c(20)^*X_t + \sum_{k=1}^{7} c(2k)^*Dk^*X_t + \mu_t \tag{4.64b}$$

Furthermore, a more advanced model with a five-way interaction as an independent variable will follow if there is an additional quarterly factor Q. In this case, a nonhierarchical model will be presented, as follows:

$$\begin{aligned} Y_t = (ABCQ)_{ijkl} + X_t + (ABC)_{ijk}X_t + (ABCQ)_{ijkl}X_t + \mu_t, \\ \text{with} \quad \sum\nolimits_{ijk}(ABC)_{ijk} = 0, \forall l \quad \text{and} \quad \sum\nolimits_{ijkl}(ABCQ)_{ijkl} = 0 \end{aligned} \tag{4.65}$$

The main objective of this model is to study the differences of the linear effects of X_t on Y_t between the four quarters, for all and for each time interval, by semesters, years and time periods. For the data analysis using EViews, a regression with 32 (=$2 \times 2 \times 2 \times 4$) dummy variables should be used, giving an equation similar to either model in (4.64a) or (4.64b).

Then the hypotheses on the slope differences can easily be tested by using the Wald tests.

Furthermore, refer to the notes and comments on the true population model, multicollinearity problem and the '*near singular matrix*' error message, presented in Section 2.14, corresponding to the application of an SCM having many exogenous variables, particularly numerical variables.

4.5 System equations based on trivariate time series

Based on a trivariate time series, namely (X_t, Y_t, Z_t), $t = 1, 2, \ldots, T$, a general system equation or multivariate additive LVAR(p,q) with exogenous variables may be produced as follows:

$$\begin{aligned} Y_t &= c(10) + c(11)Y_{t-1} + \cdots + c(1p)Y_{t-p} + c(20)X_t + \cdots + c(2m)X_{t-m} \\ &\quad + c(30)Z_t + \cdots + c(3k)Z_{t-k} + \mu_t \\ \mu_t &= c(41)\mu_{t-1} + \cdots + c(4p)\mu_{t-p} + \varepsilon 1_t \\ X_t &= c(50) + c(51)Y_{t-1} + \cdots + c(5p)Y_{t-k} + c(60)X_t + \cdots + c(6m)X_{t-m} \\ &\quad + c(70)Z_t + \cdots + c(7k)Z_{t-k} + \nu_t \\ \nu_t &= c(81)\nu_{t-1} + \cdots + c(8q)\nu_{t-q} + \varepsilon 2_t \end{aligned} \tag{4.66}$$

In time series data analysis, the multivariate autoregressive model, namely the MAR model, is known as a vector autoregressive model (VAR model). However, since EViews uses the VAR function to present a special case of the multivariate autoregressive models, then there is a preference to use the acronyms MAR model.

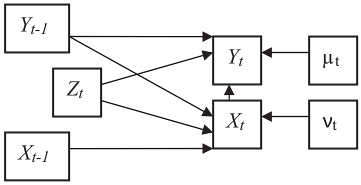


Figure 4.52 Path diagram of the model in (4.66)

For example, a special case of this model is an additive model as follows:

$$\begin{aligned} Y_t &= c(11) + c(12)Y_{t-1} + c(13)X_t + c(14)X_{t-1} + c(15)Z_t + \mu_t \\ X_t &= c(21) + c(22)Y_{t-1} + c(23)X_{t-1} + c(24)Z_t + \nu_t \end{aligned} \tag{4.67}$$

The seemingly causal effects between the variables in this model can be presented as a path diagram in Figure 4.52. Compare this to the path diagrams presented in the previous chapters.

This path diagram shows that the three exogenous variables X_{t-1}, Y_{t-1} and Z_t have direct effects on both X_t and Y_t, as well as indirect effects on Y_t through X_t.

Example 4.19. (Extension of the model in (4.48)) Corresponding to the univariate model in (4.48), the following additive bivariate model can be produced:

$$\begin{aligned} \log(m1) &= c(1) + c(11)^*\log(gdp) + c(12)^*\log(gdp(-1)) \\ &\quad + c(21)^*\log(m1(-1)) + c(22)^*\log(m1(-2)) \\ &\quad + c(31)^*\log(pr) + c(32)^*\log(pr(-1)) + [ar(1) = c(41)] \\ \log(gdp) &= c(2) + c(51)^*\log(gdp(-1)) \\ &\quad + c(61)^*\log(m1(-1)) + c(62)^*\log(m1(-2)) \\ &\quad + c(71)^*\log(pr) + c(72)^*\log(pr(-1)) + [ar(1) = c(81)] \end{aligned} \tag{4.68}$$

The statistical results are obtained by using the 'system equation,' as presented in the previous chapters. Regressions having DW-statistics of both 1.977 and 1.963 will be obtained, making it sufficient to conclude that the null hypothesis of no first-order autocorrelation is accepted. As a result, it can be declared that this bivariate model is an acceptable model, in a statistical sense.

The associations between the variables in this LV(2,1) additive bivariate model can be presented as path diagrams in Figure 4.53. Compare these with the path diagram of a simultaneous causal model as presented in Figure 2.82.

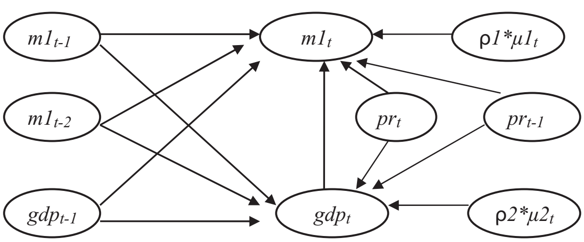


Figure 4.53 Path diagram of the endogenous and exogenous variables of the model in (4.68)

Corresponding to all multivariate growth models, as well as the path diagrams presented in Chapter 2, this bivariate SCM could easily be extended to multivariate seemingly causal models with two-way and three-way interaction factors as independent variables, such as the model in (2.82) and the simultaneous causal models in (2.81). □

4.6 General system of equations

Based on a multivariate time series, namely ($X1$, $X2$, $X3$, $Y1$, $Y2$), a path diagram may be produced as presented in Figure 4.54. This path diagram is derived from the path diagram in Figure 2.89, by deleting the time t-variable.

Corresponding to this path diagram, multivariate additive, two-way interaction and three-way interaction models may be produced, which can easily be written based on the models presented in Section 2.13, specifically on the models in (2.83) to (2.85) by deleting the time t from the models.

Furthermore, in addition to those models many other multivariate models can easily be developed, by using the five recent time series considered, namely $X1_t, X2_t, X3_t, Y1_t$ and $Y2_t$, their possible lagged variables and some of the autoregressive indicators, which could be unexpected or unpredictable empirical models. For illustration purposes, by assuming that $Y1_t$ and $Y2_t$ have simultaneous causal effects, the following examples present selected bivariate SCMs.

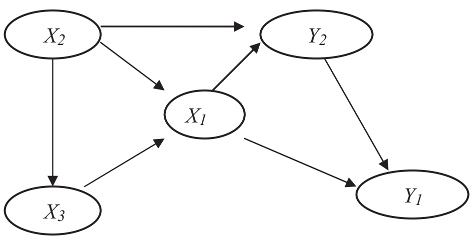


Figure 4.54 Path diagram based on Figure 2.89

Example 4.20. (Simultaneous seemingly causal models) In this example two alternative simultaneous seemingly causal models will be presented, namely an additive and an interaction model as follows:

(1) *Simultaneous Additive SCM*

Among a lot of possible alternative models, suppose the following multivariate additive simultaneous SCMs are given:

$$\begin{aligned} Y1_t &= c(11) + c(12)Y2_t + c(13)Y1_{t-1} + c(14)Y2_{t-1} + c(15)X1_t + \mu_t \\ Y2_t &= c(21) + c(22)Y1_t + c(23)Y1_{t-1} + c(24)Y2_{t-1} + c(25)X1_t + \nu_t \\ X1_t &= c(31) + (32)Y1_{t-1} + c(33)Y2_{t-1} + c(34)X2_t + c(35)X3_t + \theta_t \end{aligned} \quad (4.69)$$

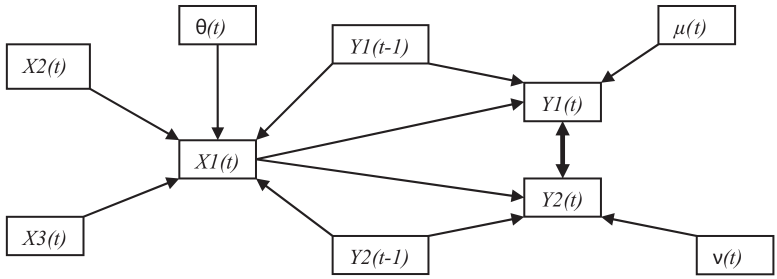


Figure 4.55 Path diagram of the model in (4.69)

Note that this seemingly causal model can be presented as a path diagram in Figure 4.55. The double arrows between $Y1(t)$ and $Y2(t)$ indicate the simultaneous causal effects of these endogenous variables.

(2) *Interaction Simultaneous Seemingly Causal Model*
The path diagram in Figure 4.55 shows that $Y1(_{t-1})$ and $Y2(_{t-1})$ have indirect effects on the endogenous variables $Y1(t)$ and $Y2(t)$ through $X1(t)$. Then a two-way interaction model could be obtained as follows:

$$\begin{aligned} Y1_t &= c(11)+c(12)Y2_t+c(13)Y1_{t-1}+c(14)Y2_{t-1}+c(15)X1_t \\ &\quad +c(16)X1_tY1_{t-1}+c(17)X1_tY2_{t-1}+\mu_t \\ Y2_t &= c(21)+c(22)Y1_t+c(23)Y1_{t-1}+c(24)Y2_{t-1}+c(25)X1_t \\ &\quad +c(26)X1_tY1_{t-1}+c(27)X1_tY2_{t-1}+\nu_t \\ X1_t &= c(31)+(32)Y1_{t-1}+c(33)Y2_{t-1}+c(34)X2_t+c(35)X3_t+\theta_t \end{aligned} \quad (4.70)$$

Furthermore, by considering that the $X2(t)$ and $X3(t)$ also have indirect effects on $Y1(t)$ and $Y2(t)$, a more advanced two-way interaction model could be produced as follows:

$$\begin{aligned} Y1_t &= c(11)+c(12)Y2_t+c(13)Y1_{t-1}+c(14)Y2_{t-1}+c(15)X1_t \\ &\quad +c(16)X1_tY1_{t-1}+c(17)X1_tY2_{t-1}+c(18)X1_tX2_t+c(19)X1_tX3_t+\mu_t \\ Y2_t &= c(21)+c(22)Y1_t+c(23)Y1_{t-1}+c(24)Y2_{t-1}+c(25)X1_t \\ &\quad +c(26)X1_tY1_{t-1}+c(27)X1_tY2_{t-1}+c(28)X1_tX2_t+c(29)X1_tX3_t+\nu_t \\ X1_t &= c(31)+(32)Y1_{t-1}+c(33)Y2_{t-1}+c(34)X2_t+c(35)X3_t+\theta_t \end{aligned} \quad (4.71)$$

Based on this model the partial derivatives are as follows:

$$\begin{aligned} \frac{\partial Y1_t}{\partial X1_t} &= c(15)+c(16)Y1_{t-1}+c(17)Y2_{t-1}+c(18)X2_t+c(19)X3_t \\ \frac{\partial Y2_t}{\partial X1_t} &= c(25)+c(26)Y1_{t-1}+c(27)Y2_{t-1}+c(28)X2_t+c(29)X3_t \end{aligned} \quad (4.72)$$

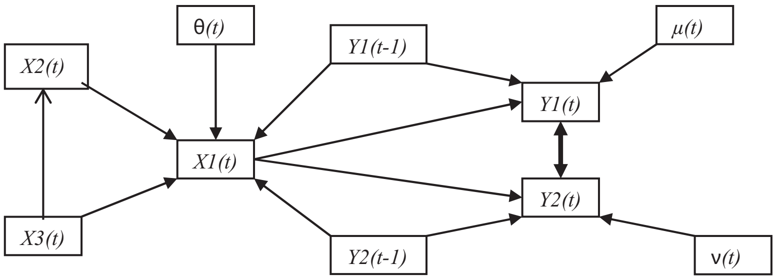


Figure 4.56 Path diagram of the model in (4.71)

which indicates that the effect of $X1_t$ on the bivariate $(Y1, Y2)_t$ is dependent on the first lagged variables $(Y1, Y2)_{t-1}$, as well as the exogenous variables $X2_t$ and $X3_t$ (Figure 4.56).

Finally, note that there is a possibility that the three variables $X1(t)$, $Y1(t)$ and $Y1(t-1)$ have a complete association. If this is the case, then the three-way interaction $X1(t)^*Y1(t)^*Y1(t-1)$ can be a source or cause factor of $Y2(t)$. Similarly, for the three-way interaction $X1(t)^*Y2(t)^*Y2(t-1)$ is a source or cause factor of $Y1(t)$. For this reason, there may be a three-way interaction SCM as follows:

$$
\begin{aligned}
Y1_t = {} & c(11) + c(12)Y2_t + c(13)Y1_{t-1} + c(14)Y2_{t-1} + c(15)X1_t + c(16)X1_tY1_{t-1} \\
& + c(17)X1_tY2_{t-1} + c(18)X1_tX2_t + c(19)X1_tX3_t + c(110)X1_tY2_tY2_{t-1} + \mu_t \\
Y2_t = {} & c(21) + c(22)Y1_t + c(23)Y1_{t-1} + c(24)Y2_{t-1} + c(25)X1_t + c(26)X1_tY1_{t-1} \\
& + c(27)X1_tY2_{t-1} + c(28)X1_tX2_t + c(29)X1_tX3_t + c(210)X1_tY1_tY1_{t-1} + \nu_t \\
X1_t = {} & c(31) + (32)Y1_{t-1} + c(33)Y2_{t-1} + c(34)X2_t + c(35)X3_t + \theta_t
\end{aligned}
\tag{4.73}
$$

For illustration purposes, Figure 4.57 presents a modified path diagram of the path diagram in Figure 4.55. This path diagram shows that there should be four dependent or downstream variables, namely $Y1(t)$, $Y2(t)$, $X1(t)$ and $X2(t)$. Similar to the models in (4.70), (4.71) and (4.73), as well as the models presented in the previous chapters, based on this path diagram, it should be easy to define or write additive, two-way and three-way interaction models.

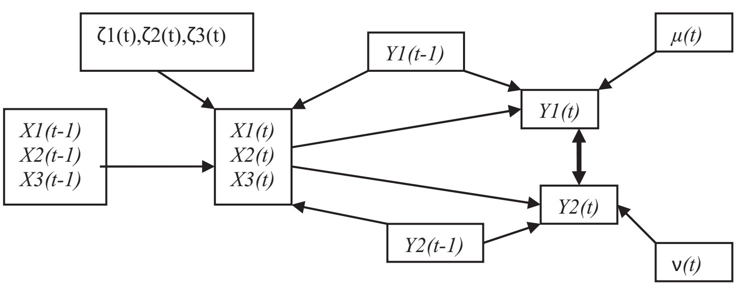


Figure 4.57 Simple path diagram of the model in (4.73)

Additional alternative models can be developed by using higher-order lagged endogenous variable autoregressive models, as well as lagged exogenous variables, and by introducing an *environmental* or *instrumental variable*, namely $Z(t)$. □

Example 4.21. (Higher-dimensional multivariate models) Based on the multivariate time series ($X1$, $X2$, $X3$, $Y1$, $Y2$), there might be a higher- dimensional or level additive SCM, as follows:

$$\begin{aligned}
Y1_t &= c(11) + c(12)Y2_t + c(13)Y1_{t-1} + c(14)Y2_{t-1} + c(15)X1_t \\
&\quad + c(16)X2_t + c(17)X3_t + \mu_t \\
Y2_t &= c(21) + c(22)Y1_t + c(23)Y1_{t-1} + c(24)Y2_{t-1} + c(25)X1_t \\
&\quad + c(26)X2_t + c(27)X3_t + \nu_t \\
X1_t &= c(31) + (32)Y1_{t-1} + c(33)Y2_{t-1} + c(34)X1_{t-1} + c(35)X2_{t-1} + c(36)X3_{t-1} + \theta_t \\
X2_t &= c(41) + (42)Y1_{t-1} + c(43)Y2_{t-1} + c(44)X1_{t-1} + c(45)X2_{t-1} + c(46)X3_{t-1} + \vartheta_t \\
X3_t &= c(51) + (52)Y1_{t-1} + c(53)Y2_{t-1} + c(54)X1_{t-1} + c(55)X2_{t-1} + c(56)X3_{t-1} + \varsigma_t
\end{aligned} \tag{4.74}$$

Note that this model is derived from the model in (2.98) by deleting the time t. Therefore, the path diagram of this model can also be presented as the path diagram in Figure 2.108 by deleting the time t-variable. However, here a simpler path diagram is presented, as in Figure 4.57.

Similar to the models in the previous example, this model can easily be modified in order to develop many other alternative models, such as the higher-order lagged-variable autoregressive SCMs, either additive or interaction models. On the other hand, use may not be made of the three exogenous variables $X1(t)$, $X2(t)$ and $X3(t)$, or the first lagged endogenous variables, to define alternative models.

By having an environmental variable $Z(t)$, the path diagram presented in Figure 4.58 may be obtained. Corresponding to this path diagram, several alternative interaction models could be presented, including a model with *environmental-related effects*, by using the main factor $Z(t)$ and the two-way interactions $X1(t)^*Z(t)$, $X2(t)^*Z(t)$ and $X3(t)^*Z(t)$ as additional independent variables of the first two regressions in (4.74).

Even though the data analysis is a straightforward method, the trial-and-error methods should be used, since the good fit model(s) could be unexpected model(s), which is(are) highly dependent on the data set that happens to be selected or available.

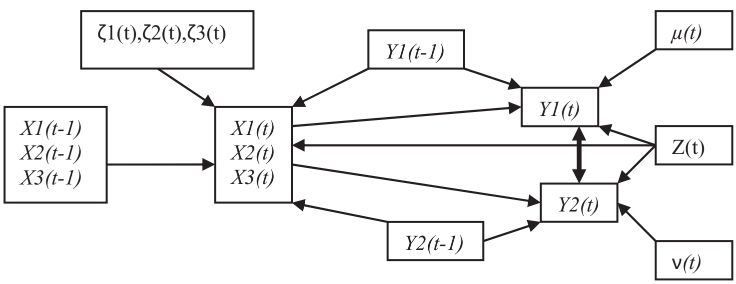


Figure 4.58 Path diagram of the model in (4.74) with an additional environment independent variable $Z(t)$

Refer to the special notes and comments in Section 2.14. So far, several unexpected models have been presented that are acceptable in a statistical sense. For a comparison, the following example presents an unexpected model from another source. □

Example 4.22. (An unexpected model of Yaffee and McGee (2000, p. 45)) Yaffee and McGee present the following autoregressive growth model:

$$\begin{aligned} Y_t &= 9.844 + 0.505 time + e_t \\ e_t &= 0.295 e_{t-1} - 0.21 e_{t-5} + 0.237 e_{t-6} + v_t \end{aligned} \tag{4.75}$$

This model could be considered as an unexpected model, corresponding to the model of the error term e_t. The question could be asked: 'Why are e_{t-1}, e_{t-5} and e_{t-6} used instead of the other lags of the error term?' It is certain that this result is highly dependent on the data, which cannot be generalized. □

4.7 Seemingly causal models with dummy variables

In the previous sections, as well as in Chapter 3, models with dummy variables have been presented, which could be defined or constructed based on the time t-variable, both exogenous as well as endogenous variables. Based on the same reasons, there could also be various lagged-variable autoregressive seemingly causal or explanatory models (SCMs or EMs) with dummy variables. Find the following time series models with dummy variables.

4.7.1 Homogeneous time series models

Based on a bivariate time series $\{X_{t-m}, Y_t\}$, $t = 1, 2, \ldots, T$ and a selected $m \geq 1$, the following general additive model with dummy variables may be considered:

$$Y_t = \sum_{k=1}^{K-1} c(k)Dk + c(K) + c(11)X_{t-m} + \mu_t \tag{4.76a}$$

where $Dk = D(k)$ is a zero–one indicator of the k-th category of a defined categorical variable having K categories, and is either constructed based on the time variable, one or more endogenous or exogenous variables, or other external variables (such as regional or environmental variables). This model should be considered as an analysis of covariance (ANCOVA) time series model, with a covariate X_{t-m}, for a selected value of $m \geq 1$. Note that this regression model is a model with intercept, $c(K)$, or a model not through the origin, since the other terms on the right-hand side are independent variables, i.e. dummy variables and a numerical variable.

In fact, this model represents a set of homogeneous regressions (Agung, 2006), with a covariate X_{t-m} or a set of parallel lines in a two-dimensional coordinate system with

X_{t-m} and Y_t axes. This model can be presented or written as a set of simple homogeneous linear regressions having the same slopes, namely $c(11)$, as follows:

$$\begin{aligned}
Y_t &= c(1)+c(K)+c(11)X_{t-m}+\mu_t \\
Y_t &= c(2)+c(K)+c(11)X_{t-m}+\mu_t \\
&\cdots\cdots\cdot \\
Y_t &= c(K-1)+c(K)+c(11)X_{t-m}+\mu_t \\
Y_t &= c(K)+c(11)X_{t-m}+\mu_t
\end{aligned} \tag{4.76b}$$

Note that the selected value of $m \geq 1$ is highly dependent on the researchers' judgment, which can be very subjective. The main objective of this model is to study and to test the hypotheses on the *adjusted mean differences* of Y_t between the Kth defined categories, under the assumption that X_{t-m} has equal effects on Y_t within all categories considered. However, note that this condition is almost never ovserved in reality with a large value of K. Refer to the not recommended model presented in Figure 4.32.

For this reason, a more general model should be considered, i.e. a set of heterogeneous regressions, which is known as the Johnson–Neyman technique or approach (1936, in Huitema, 1980, p.270), presented in the following subsection.

4.7.2 *Heterogeneous time series models*

Corresponding to the homogeneous regressions in (4.76), the equation of a set of heterogeneous regressions will be presented as follows:

$$Y_t = \sum_{k=1}^{K-1} c(k)Dk + c(K) + \sum_{k=1}^{K} c(1k)^*Dk^*X_{t-m} + \mu_t \tag{4.77}$$

This regression can be considered as a model with an intercept $c(K)$. An alternative general model is a regression through the origin, as follows:

$$Y_t = \sum_{k=1}^{K} c(k)Dk + \sum_{k=1}^{K} c(1k)^*Dk^*X_{t-m} + \mu_t \tag{4.78}$$

This model represents a set of K regression lines, as follows:

$$\begin{gathered}
Y_t = c(k) + c(1k)^*X_{t-m} + \mu_t \\
\text{for} \quad k = 1, 2, \ldots, K
\end{gathered} \tag{4.79}$$

The main objectives of these last two models in (4.77) and (4.78) are to study and to test the hypotheses on (i) the linear effect of X_{t-m} on Y_t within each category and (ii) the differences of the linear effects of X_{t-m} on Y_t between pairs of the Kth defined categories, which can easily be done by using the Wald tests.

Furthermore, this model can easily be extended to lagged (endogenous)-variable autoregressive SCMs, either with a single exogenous variable, X_{t-m}, or multivariate exogenous variables. For multivariate numerical exogenous variables, the following general equation is presented:

$$Y_t = \sum_{k=1}^{K} \sum_{g=0}^{G} (c(gk)^* X_{g,t})^* Dk + \mu_t \tag{4.80}$$

where X_g, for $g = 0, 1, 2, \ldots, G$, and $X_0 = 1$ are numerical exogenous variables, as well as other endogenous variables, their lags and interactions between selected main factors. The following example presents illustrative statistical results. □

Example 4.23. (SCM with a dichotomous independent variable) By using the two dummy variables, $D1$ and $D2$, based on the model in (4.80), an LVAR(2,1) model may be applied as follows:

$$\begin{aligned}\log(m1_t) = \; & [c(11) + c(12)^*\log(m1_{t-1}) + c(13)^*\log(m1_{t-2}) + c(14)^*\log(gdp_t) \\ & + c(15)^*\log(pr_{t-1})]^*D1 + [c(21) + c(22)^*\log(m1_{t-1}) \\ & + c(23)^*\log(m1_{t-2}) + c(24)^*\log(gdp_t) \\ & + c(25)^*\log(pr_{t-1}))]^*D2 + [ar(1) = c(1)] + \varepsilon t\end{aligned} \tag{4.81}$$

with the statistical results presented in Figure 4.59.

In order to test hypotheses further using the Wald tests, based on the model in (4.81), it is suggested that the model parameters presented in Table 4.6 should be used or referred to. For example, since each of the variables log($m1(-2)$, log(gdp) and log($pr(-1)$ is insignificant, their joint effect on log($m1$) needs to be tested. The test can be done by entering the equation $c(13) = c(14) = c(15) = 0$. Do this as an exercise. □

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 12/01/07 Time: 06:50
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 10 iterations
LOG(M1)=(C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(14)
*LOG(GDP)+C(15)*LOG(PR(-1)))*D1+(C(21)+C(22)*LOG(M1(-1))
+C(23)*LOG(M1(-2))+C(24)*LOG(GDP)+C(25)*LOG(PR(-1)))*D2
+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.102942	0.264340	-0.389431	0.6975
C(12)	0.892952	0.181348	4.923957	0.0000
C(13)	0.093798	0.185214	0.506427	0.6132
C(14)	0.029563	0.018639	1.586089	0.1146
C(15)	-0.022055	0.057476	-0.383727	0.7017
C(21)	0.399577	0.255789	1.562134	0.1202
C(22)	1.379688	0.135359	10.19284	0.0000
C(23)	-0.397088	0.135666	-2.926952	0.0039
C(24)	-0.036861	0.048342	-0.762497	0.4468
C(25)	0.077174	0.052940	1.457759	0.1468
C(1)	-0.358776	0.135600	-2.645835	0.0089

R-squared	0.999669	Mean dependent var	5.827503
Adjusted R-squared	0.999650	S.D. dependent var	0.750468
S.E. of regression	0.014049	Akaike info criterion	-5.632341
Sum squared resid	0.032766	Schwarz criterion	-5.434954
Log likelihood	509.4622	Hannan-Quinn criter.	-5.552289
Durbin-Watson stat	2.055062		
Inverted AR Roots	-.36		

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 12/01/07 Time: 06:53
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 4 iterations
LOG(M1)=(C(11)+C(12)*LOG(M1(-1))+C(14)*LOG(GDP))*D1+(C(21)
+C(22)*LOG(M1(-1))+C(23)*LOG(M1(-2))+C(25)*LOG(PR(-1)))*D2
+[AR(1)=C(1)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.000716	0.061098	0.011726	0.9907
C(12)	0.975681	0.027662	35.27132	0.0000
C(14)	0.026235	0.016436	1.596206	0.1123
C(21)	0.211718	0.079371	2.667459	0.0084
C(22)	1.417482	0.106543	13.30430	0.0000
C(23)	-0.447098	0.103511	-4.319312	0.0000
C(25)	0.038437	0.016442	2.337741	0.0206
C(1)	-0.404259	0.083276	-4.854441	0.0000

R-squared	0.999668	Mean dependent var	5.827503
Adjusted R-squared	0.999654	S.D. dependent var	0.750468
S.E. of regression	0.013964	Akaike info criterion	-5.660561
Sum squared resid	0.032953	Schwarz criterion	-5.517007
Log likelihood	508.9597	Hannan-Quinn criter.	-5.602341
Durbin-Watson stat	2.071103		
Inverted AR Roots	-.40		

Figure 4.59 Statistical results based on the model in (4.81) and a reduced model

Table 4.6 Parameters of the model in (4.81) by the dummy and exogenous variables

CV	*D*1	*D*2	Constant	$\log(m1_{t-1})$	$\log(m1_{t-2})$	$\log(gdp_t)$	$\log(pr_{t-1})$
1	1	0	*C*(11)	*C*(12)	*C*(13)	*C*(14)	*C*(15)
2	0	1	*C*(21)	*C*(22)	*C*(23)	*C*(24)	*C*(25)

Example 4.24. (Unexpected models) This example will demonstrate that there can be statistically acceptable causal models based on a pair of variables that are not related at all. For this illustration, the hypothetical variable *Y* presented in Example 3.12 is considered as an endogenous variable. Corresponding to the three-piece AR(1) growth model presented in Example 3.13, a three-piece AR(1) SCM is proposed, as follows:

$$\begin{aligned}\log(Y) = {} & (C(11) + C(12)^*\log(y(-1) + c(13)^*\log(gdp))^*Dy1 \\ & + (C(21) + c(22)^*\log(y(-1)) + C(23)^*\log(gdp))^*Dy2 \\ & + (C(31) + C(32)^*\log(y(-1)) + C(33)^*\log(gdp))^*Dy3 \\ & + [AR(1) = C(1)]\end{aligned} \tag{4.82}$$

Note that, even though the endogenous variable is an hypothetical data, any original observed variables in Demo.wf1 can be used as independent variables of a model for illustration purposes.

The statistical results in Figure 4.60 and its reduced model in Figure 4.61 are based on the model in (4.82). Then based on this reduced model the following notes and conclusions are presented:

(1) This model represents the following three models within the first, second and third time periods respectively:

$$\begin{aligned}&\log(y) = c(11) + c(13)^*\log(gdp) + [ar(1) = c(1)] \\ &\log(y) = (c(21) + c(22)^*\log(y(-1)) + c(23)^*\log(gdp) + [ar(1) = c(1)] \\ &\log(y) = c(31) + c(33)^*\log(gdp) + [a(1) = c(1)]\end{aligned} \tag{4.83}$$

Note that each of the independent variables has a significant adjusted effect on ln(*y*), but the first and third regressions only has log(*gdp*) as an independent variable.

(2) Even though the variable *GDP* does not have any relationship with the endogenous variables *Y*, this statistical result shows that log(*gdp*) has a significant effect on log(*y*). This example shows that a regression analysis can be used to show a significant causal relationship between a pair of unrelated variables. Note the following additional illustrations.

(3) For another illustration, Figure 4.62 presents statistical results based on the following AR(3) three-piece model of log(*y*) on log(*pr*):

$$\begin{aligned}\log(Y) = {} & (C(11) + 12)^*\log(pr))^*Dy1 + (C(21) + c(22)^*\log(pr)^*Dy2 \\ & + (C(31) + C(32)^*\log(pr))^*Dy3 \\ & + [AR(1) = C(1),\ AR(2) = C(2),\ AR(3) = C(3)]\end{aligned} \tag{4.84}$$

Estimation Method: Iterative Least Squares
Date: 12/01/07 Time: 17:50
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 178
Convergence achieved after 11 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	7.437121	3.091717	2.405498	0.0172
C(12)	-0.034966	0.428974	-0.081510	0.9351
C(13)	0.071295	0.032630	2.184936	0.0303
C(21)	4.601597	0.225918	20.36847	0.0000
C(22)	0.076253	0.022401	3.404028	0.0008
C(23)	0.353786	0.025011	14.14545	0.0000
C(31)	1.465125	0.200067	7.323185	0.0000
C(32)	-0.006066	0.010549	-0.574988	0.5661
C(33)	0.810459	0.026776	30.26808	0.0000
C(1)	0.895685	0.045542	19.66731	0.0000

Determinant residual covariance 5.04E-05

Equation: LOG(Y)=(C(11)+C(12)*LOG(Y(-1))+C(13)*LOG(GDP))*DY1
+(C(21)+C(22)*LOG(Y(-1))+C(23)*LOG(GDP))*DY2+(C(31)+C(32)
*LOG(Y(-1))+C(33)*LOG(GDP))*DY3+[AR(1)=C(1)]
Observations: 178

R-squared	0.998399	Mean dependent var	7.420576
Adjusted R-squared	0.998313	S.D. dependent var	0.177909
S.E. of regression	0.007307	Sum squared resid	0.008969
Durbin-Watson stat	2.010891		

Figure 4.60 Statistical results based on the model in (4.82)

For the final illustration, Figure 4.63 presents statistical results based on a model as follows:

$$\log(y_t) = c(1) + c(2)^*\log(y_{t-1}) + c(3)^*\log(gdp_t) + \varepsilon_t \tag{4.85}$$

This figure shows that log(*gdp*) has an insignificant effect on log(*y*). In fact, log(*gdp*) and log(*y*) have a significant negative correlation of −0.514 190, based

Estimation Method: Iterative Least Squares
Date: 12/01/07 Time: 17:54
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 178
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	7.185406	0.088355	81.32451	0.0000
C(13)	0.068979	0.016732	4.122567	0.0001
C(21)	4.588063	0.222294	20.63965	0.0000
C(22)	0.076606	0.022255	3.442249	0.0007
C(23)	0.355651	0.024165	14.71732	0.0000
C(31)	1.430477	0.189918	7.532074	0.0000
C(33)	0.809087	0.026726	30.27383	0.0000
C(1)	0.895414	0.045489	19.68404	0.0000

Determinant residual covariance 5.05E-05

Equation: LOG(Y)=(C(11)+C(13)*LOG(GDP))*DY1+(C(21)+C(22)*LOG(Y(
-1))+C(23)*LOG(GDP))*DY2+(C(31)+C(33)*LOG(GDP))*DY3
+[AR(1)=C(1)]
Observations: 178

R-squared	0.998396	Mean dependent var	7.420576
Adjusted R-squared	0.998330	S.D. dependent var	0.177909
S.E. of regression	0.007271	Sum squared resid	0.008987
Durbin-Watson stat	2.026027		

Figure 4.61 Statistical results based on a modified model in (4.82)

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/19/07 Time: 17:31
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence not achieved after 500 iterations
LOG(Y)=(C(11)+C(12)*LOG(PR))*DY1+(C(21)+C(22)*LOG(PR))*DY2
+(C(31)+C(32)*LOG(PR))*DY3+[AR(1)=C(1),AR(2)=C(2),AR(3)=C(3)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	7.766919	0.074441	104.3372	0.0000
C(12)	0.167287	0.055814	2.997214	0.0031
C(21)	7.722881	0.034453	224.1542	0.0000
C(22)	0.443776	0.049438	8.976418	0.0000
C(31)	7.387342	0.016226	455.2842	0.0000
C(32)	1.490991	0.070192	21.24161	0.0000
C(1)	1.024440	0.076490	13.39310	0.0000
C(2)	0.116215	0.110243	1.054170	0.2933
C(3)	-0.181715	0.078515	-2.314414	0.0219

R-squared	0.998240	Mean dependent var	7.420061
Adjusted R-squared	0.998156	S.D. dependent var	0.178280
S.E. of regression	0.007655	Akaike info criterion	-6.857295
Sum squared resid	0.009846	Schwarz criterion	-6.695796
Log likelihood	615.8706	Hannan-Quinn criter.	-6.791797
Durbin-Watson stat	1.899225		

Inverted AR Roots	.94	.48	-.40

Figure 4.62 Statistical results based on the model in (4.84)

on the t-statistic of $-7.998\,523$ and a p-value $= 0.0000$. Therefore, this model and the correlation analysis gives contradictory conclusions.

(4) These illustrations have demonstrated that statistically acceptable models can be constructed based on an unrelated pair of variables. It is certain that statistically acceptable multivariate models can also be constructed based on a set of unrelated variables. For this reason, based on any empirical data sets, best judgment has to be used to select relevant endogenous and exogenous variables before a model is defined or proposed.

(5) In order to obtain a better picture of the relationship between log(y) and log(gp), Figure 4.64 presents the growth curve of log(y) and Figure 4.65 presents the scatter graph with the regression line of log(y) on log(gdp). This scatter graph clearly shows that the simple model in (4.85) should be considered as an

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/19/07 Time: 17:12
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.522281	0.230335	2.267488	0.0246
LOG(Y(-1))	0.933157	0.028674	32.54348	0.0000
LOG(GDP)	-0.004363	0.005096	-0.856326	0.3930

R-squared	0.895090	Mean dependent var	7.421080
Adjusted R-squared	0.893898	S.D. dependent var	0.177536
S.E. of regression	0.057829	Akaike info criterion	-2.846018
Sum squared resid	0.588588	Schwarz criterion	-2.792598
Log likelihood	257.7186	Hannan-Quinn criter.	-2.824357
F-statistic	750.8126	Durbin-Watson stat	1.910907
Prob(F-statistic)	0.000000		

Figure 4.63 Statistical results based on the model in (4.85)

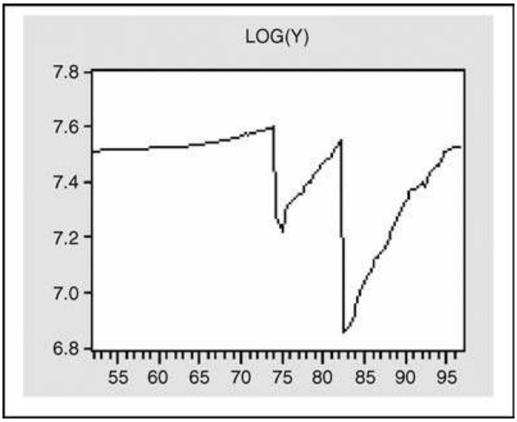


Figure 4.64 Growth curve of log(y) based on hypothetical data

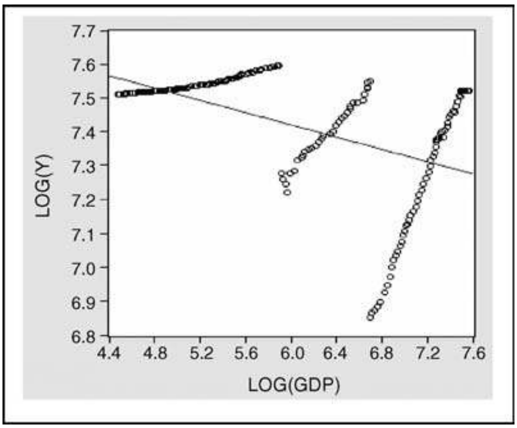


Figure 4.65 Scatter graph with regression of log(y) on log(gdp)

unacceptable model, in a theoretical as well as a statistical sense. As noted in the previous chapters, this illustration again proves or shows the importance of a scatter graph in statistical model development. □

4.8 General discontinuous seemingly causal models

Note that based on the three-piece univariate linear model presented in the previous example, a general equation of a three-piece autoregressive seemingly causal model, namely an AR(p)_SCM, can easily be obtained as follows:

$$Y_t = \sum_{i=0}^{I} c(1i)*X1_i*D1 + \sum_{j=0}^{J} c(2j)*X2_j*D2 + \sum_{k=0}^{K} c(3k)*X3_k*D3 \\ + [ar(1) = c(1), \ldots, ar(p) = c(p)] + \varepsilon_t \qquad (4.86)$$

where Y_t can be any type of endogenous variable and $\{X1_i\}$, $\{X2_j\}$ and $\{X3_k\}$ are sets of exogenous variables that can be equal or unequal sets of variables, with $X1_0 = X2_0 = X3_0 = 1$. Note that the exogenous variables could be other endogenous variable(s), pure exogenous variables, lagged endogenous and exogenous variables, dummy variables and interaction between selected exogenous variables.

Hence, there could be many alternative linear models, including the lagged (endogenous)-variables autoregressive models. Those models could easily be extended to multivariate or vector autoregressive models with dummy variables. Refer to all models presented in the previous subsection and Chapter 3.

Furthermore, note that by using many exogenous variables, there is a great possibility of producing an error message such as '*Near Singular Matrix*' or '*Overflow*' for the estimation methods using the iterative process. Refer to the notes and comments presented in Section 2.14.

Example 4.25. (A three-piece AR(2) additive model) The following equation presents a three-piece AR(2) additive model with three exogenous variables, X_1, X_2 and X_3:

$$\begin{aligned} Yt = {} & (C(11) + C(12)*Xl + C(13)*X2 + C(14)*X3)*D1 \\ & + (C(21) + C(22)*X1 + C(23)*X2 + C(24)*X3)*D2 \\ & + (C(31) + C(32)*X1 + C(32)*X2 + C(34)*X3)*D3 \\ & + [AR(1) = C(1), AR(2) = C(2)] + \varepsilon_t \end{aligned} \tag{4.87}$$

Note that the three dummy variables, $D1, D2$ and $D3$, could be defined based on a selected numerical variable, either the exogenous, endogenous, the time variables or the variables out of the model, called CV (i.e. categorical variable). Then for testing various hypotheses based on this model, the model parameters in Table 4.7 should be considered.

For examples, the following hypothesis can be considered:

(1) The adjusted effect of $X1$ on Y, within $CV = 1$, could be tested using the t-test presented in the printout, by looking at the parameter $C(12)$ or the coefficient of $X1$.
(2) The joint effects of the three independent variables on Y, within $CV = 1$, can be tested by entering the equation $C(12) = C(13) = C(14) = 0$.
(3) The differential adjusted effects of $X1$ on Y, between $CV = 1$, $CV = 2$ and $CV = 3$, can be tested by entering the equation $C(12) = C(22) = C(32)$.
(4) The differential joint effects of $X1$, $X2$ and $X3$ on Y, between $CV = 1$ and $CV = 3$, can be tested by entering the equation $C(12) = C(32)$, $C(13) = C(33)$ and $C(14) = C(34)$. □

Table 4.7 Parameters of the model in (4.87) by the dummy and exogenous variables

C V	*D*1	*D*2	*D*3	Constant	*X*1	*X*2	*X*3
1	1	0	0	*C*(11)	*C*(12)	*C*(13)	*C*(14)
2	0	1	0	*C*(21)	*C*(22)	*C*(23)	*C*(24)
3	0	0	1	*C*(31)	*C*(32)	*C*(33)	*C*(34)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 18:12
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 29 iterations
LOG(M1)=(C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(14)
*LOG(GDP(-1)))*D1+(C(21)+C(22)*LOG(M1(-1))+C(23)*LOG(M1(-2))
+C(24)*LOG(GDP(-1)))*D2+[AR(1)=C(1),AR(2)=C(2)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.033297	0.048588	-0.685294	0.4941
C(12)	1.059372	0.172581	6.138409	0.0000
C(13)	-0.066025	0.173191	-0.381229	0.7035
C(14)	0.015027	0.012911	1.163932	0.2461
C(21)	0.020933	0.012211	1.714224	0.0884
C(22)	1.633470	0.104270	15.66583	0.0000
C(23)	-0.662499	0.101187	-6.547264	0.0000
C(24)	0.024989	0.011732	2.129936	0.0347
C(1)	-0.605649	0.128323	-4.719727	0.0000
C(2)	-0.230329	0.104091	-2.212768	0.0283

R-squared	0.999670	Mean dependent var	5.833023
Adjusted R-squared	0.999652	S.D. dependent var	0.748997
S.E. of regression	0.013962	Akaike info criterion	-5.649755
Sum squared resid	0.032362	Schwarz criterion	-5.469614
Log likelihood	507.1785	Hannan-Quinn criter.	-5.576691
Durbin-Watson stat	2.050006		

Inverted AR Roots	-.30+.37i	-.30-.37i

Figure 4.66 Statistical results based on an AR(2) three-piece model

Example 4.26. (A two-piece translog LVAR(2,2)_SCM) Figure 4.66 presents statistical results of a two-pieces LVAR(2,2)_SCM, with dummy variables *D*1 and *D*2 corresponding to a defined dichotomous time variable in Demo.wf1. Since

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 18:15
Sample (adjusted): 1953Q2 1996Q4
Included observations: 175 after adjustments
Convergence achieved after 10 iterations
LOG(M1)=(C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(14)
*LOG(GDP(-1)))*D1+(C(21)+C(22)*LOG(M1(-1))+C(23)*LOG(M1(-2))
+C(24)*LOG(GDP(-1)))*D2+[AR(1)=C(1),AR(2)=C(2),AR(3)=C(3)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.021251	0.034455	-0.616764	0.5382
C(12)	1.352081	0.161997	8.346329	0.0000
C(13)	-0.357848	0.162943	-2.196156	0.0295
C(14)	0.011213	0.008970	1.249992	0.2131
C(21)	0.014437	0.008328	1.733665	0.0849
C(22)	1.775375	0.069669	25.48305	0.0000
C(23)	-0.798970	0.067655	-11.80943	0.0000
C(24)	0.020516	0.008082	2.538497	0.0121
C(1)	-0.833130	0.107170	-7.773887	0.0000
C(2)	-0.521236	0.127597	-4.085004	0.0001
C(3)	-0.284722	0.092591	-3.075062	0.0025

R-squared	0.999685	Mean dependent var	5.838514
Adjusted R-squared	0.999665	S.D. dependent var	0.747585
S.E. of regression	0.013678	Akaike info criterion	-5.685317
Sum squared resid	0.030681	Schwarz criterion	-5.486387
Log likelihood	508.4652	Hannan-Quinn criter.	-5.604625
Durbin-Watson stat	2.021756		

Inverted AR Roots	-.08+.64i	-.08-.64i	-.68

Figure 4.67 Statistical results based on an AR(3) three-piece model

log(m1(−2)) has an insignificant adjusted effect within the first time period (p-value = 0.5673), then a reduced model might be obtained.

However, the reduced model is not presented here, but the result of a two-piece LVAR(2,3)_SCM is presented in Figure 4.67 as an illustration. Based on these results, the following notes and conclusions are presented:

(1) By adding the indicator AR(3) to the model, a model is produced where log(m1(−2)) has a significant adjusted effect in both time periods. Hence, it can be said that unexpected results may be produced by inserting an additional AR (3) indicator in the model.
(2) In fact, in general there might also be unexpected statistical result(s) by adding or deleting an exogenous variable. Refer to the following example.
(3) By observing the residual graphs of both models, as well as their DW-statistics, it can be concluded that both models are acceptable models, in a statistical sense. □

Example 4.27. (Another two-piece translog LVAR(2,3)_SCM) Since *Drs*1 and *Drs*2 are zero–one indicators of the variable *RS* with a breakpoint $RS = \max(RS_t) = 15.078\,33$ *at* $t = 119$, then by using *RS* as an additional variable in the model in the previous example, a piecewise LVAR(2,3)_SCM would be obtained. By using the trial-and-error methods, a good model is obtained, as presented in Figure 4.68. Since the model has *RS*(−1) as an additional independent variable, then this model can be considered as a mixed translog model.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/19/07 Time: 18:24
Sample (adjusted): 1953Q2 1996Q4
Included observations: 175 after adjustments
Convergence achieved after 21 iterations
LOG(M1)=(C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(14)
*LOG(GDP(-1))+C(15)*RS(-1))*DRS1+(C(21)+C(22)*LOG(M1(-1))
+C(23)*LOG(M1(-2))+C(24)*LOG(GDP(-1))+C(25)*RS(-1))*DRS2
+[AR(1)=C(1),AR(2)=C(2),AR(3)=C(3)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.023650	0.080121	0.295173	0.7682
C(12)	0.174450	0.084610	2.061808	0.0408
C(13)	0.756995	0.078601	9.630876	0.0000
C(14)	0.070327	0.027269	2.579004	0.0108
C(15)	-0.003631	0.001011	-3.591938	0.0004
C(21)	0.619661	0.102504	6.045270	0.0000
C(22)	0.205569	0.129589	1.586310	0.1146
C(23)	0.529711	0.111348	4.757259	0.0000
C(24)	0.175812	0.061220	2.871815	0.0046
C(25)	-0.011365	0.001628	-6.979073	0.0000
C(1)	0.335893	0.104451	3.215793	0.0016
C(2)	-0.352503	0.091219	-3.864358	0.0002
C(3)	0.239602	0.093094	2.573763	0.0110

R-squared	0.999746	Mean dependent var	5.838514
Adjusted R-squared	0.999727	S.D. dependent var	0.747585
S.E. of regression	0.012355	Akaike info criterion	-5.878172
Sum squared resid	0.024728	Schwarz criterion	-5.643074
Log likelihood	527.3401	Hannan-Quinn criter.	-5.782810
Durbin-Watson stat	2.013857		

Inverted AR Roots	.53	-.10+.67i	-.10-.67i

Figure 4.68 Statistical results based on a piecewise LVAR(2,3) model

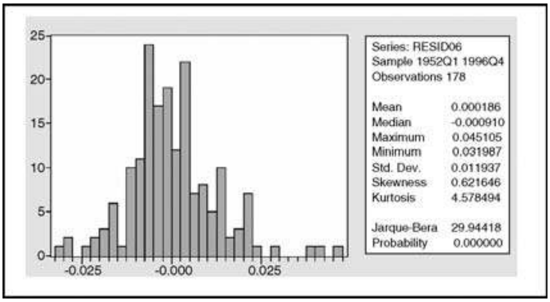


Figure 4.69 Residual histogram of the regression in Figure 4.70

Correlation Probability	RESID06	T	LOG(PR)
RESID06	1.000000		

T	-0.022629	1.000000	
	0.7643	-----	
LOG(PR)	-0.026929	0.983267	1.000000
	0.7212	0.0000	-----

Figure 4.70 Correlation matrix of *Resid*06, time t and log(PR)

Note that both log($m1(-1)$) and log($m1(-2)$) have a significant adjusted effect on log($m1$) in both intervals of the values of RS (i.e. $RS \leq 15.078\,33$ and $RS > 15.078\,33$), as well as other explanatory variables and the three AR indicators. Since, based on the model in the previous example, log($m1(-1)$) has an insignificant effect in the first interval, then this model can be considered as an unexpected model.

Furthermore, the residual histogram in Figure 4.69 and the correlation matrix in Figure 4.70 are presented to study the limitations of the model considered. □

Example 4.28. (Three-piece AR(3) interaction model) The following equation presents a three-piece AR(2) interaction model with two exogenous variables, X_1 and X_2. For illustration purposes, an interaction translog linear SCM is considered as follows:

$$\begin{aligned}\log(Y) = &(C(11)+C(12)*\log(X1)+C(13)*\log(X2)+C(14)*\log(X1)*\log(X2))*D1\\ &+(C(21)+C(22)*\log(X1)+C(23)*\log(X2)+C(24)*\log(X1)*\log(X2))*D2\\ &+(C(31)+C(32)*\log(X1)+C(32)*\log(X2)+C(34)*\log(X1)*\log(X2))*D3\\ &+[AR(1)=C(1),AR(2)=C(2)]+\varepsilon t\end{aligned} \tag{4.88}$$

Note that the interaction log(X_1)*log(X_2) should be used as an independent variable, since it is defined or well known that the effect of X_1 on Y depends on X_2 or that the

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/20/07 Time: 09:13
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 13 iterations
LOG(Y)=(C(11)+C(12)*LOG(X1)+C(13)*LOG(X2)+C(14)*LOG(X1)
*LOG(X2))*DY1+(C(21)+C(22)*LOG(X1)+C(23)*LOG(X2)+C(24)
*LOG(X1)*LOG(X2))*DY2+(C(31)+C(32)*LOG(X1)+C(33)*LOG(X2)
+C(34)*LOG(X1)*LOG(X2))*DY3+[AR(1)=C(1),AR(2)=C(2)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	6.770895	0.472575	14.32765	0.0000
C(12)	0.141610	0.081137	1.745316	0.0828
C(13)	-0.139643	0.288784	-0.483555	0.6293
C(14)	0.022064	0.054104	0.407807	0.6839
C(21)	3.470734	0.859639	4.037430	0.0001
C(22)	0.656476	0.136589	4.806230	0.0000
C(23)	-0.717246	0.721874	-0.993588	0.3219
C(24)	0.115651	0.116830	0.989904	0.3237
C(31)	7.547519	0.323706	23.31599	0.0000
C(32)	-0.018334	0.046479	-0.394464	0.6938
C(33)	5.934364	0.539395	11.00189	0.0000
C(34)	-0.677184	0.074742	-9.060314	0.0000
C(1)	0.910123	0.076852	11.84256	0.0000
C(2)	-0.239687	0.078723	-3.044670	0.0027

R-squared	0.998724	Mean dependent var	7.420576
Adjusted R-squared	0.998623	S.D. dependent var	0.177909
S.E. of regression	0.006602	Akaike info criterion	-7.127534
Sum squared resid	0.007148	Schwarz criterion	-6.877281
Log likelihood	648.3505	Hannan-Quinn criter.	-7.026050
Durbin-Watson stat	2.092318		

Inverted AR Roots	.46+.18i	.46-.18i

Figure 4.71 Statistical results based on an AR(2) interaction model

effect of X_2 on Y depends on X_1. Its statistical results are presented in Figure 4.71. Based on these results, the following notes and conclusions are presented:

(1) Since it is defined that the effect of $X1$ on Y is dependent on $X2$, then in order to construct an acceptable reduced model, no attempt should be made to delete or omit an interaction factor from the regression. By using the trial-and-error methods, a reduced model is obtained, as shown in Figure 4.72, p. 244, which is considered to be the best among all possible reduced models.
(2) Based on the statistical results of the reduced model, it can be concluded that the interaction factor $\log(x1)^*\log(x2)$ has a significant effect on $\log(y)$ within the three defined time periods. In other words, the data support the hypothesis that the effect of $X1$ on Y is dependent on $X2$, based on a translog linear model.
(3) Note again that the exogenous variables $X1$ and $X2$ should be selected based on best judgment, so that they are good predictors (source or cause factors) of the endogenous variable Y. □

4.9 Additional selected seemingly causal models

This section presents three types of simple SCMs, namely the polynomial model, the Cobb–Douglass model and the CES (i.e. *constant elasticity of substitution*) model, which could easily be extended to the lagged-variable autoregressive models, either univariate or multivariate, with multivariate exogenous variables.

Dependent Variable: LOG(Y)
Method: Least Squares
Date: 10/20/07 Time: 09:31
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 16 iterations
LOG(Y)=(C(11)+C(13)*LOG(X2)+C(14)*LOG(X1)*LOG(X2))*DY1+(C(21)
+C(22)*LOG(X2)+C(24)*LOG(X1)*LOG(X2))*DY2+(C(31)+C(32)
*LOG(X1)+C(33)*LOG(X2)+C(34)*LOG(X1)*LOG(X2))*DY3
+[AR(1)=C(1),AR(2)=C(2)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	7.477784	0.080312	93.10947	0.0000
C(13)	0.426178	0.110841	3.844934	0.0002
C(14)	-0.091396	0.032277	-2.831639	0.0052
C(21)	7.575733	0.035292	214.6605	0.0000
C(22)	2.507240	0.533495	4.699654	0.0000
C(24)	-0.380757	0.098753	-3.855664	0.0002
C(31)	7.563318	0.385064	19.64174	0.0000
C(32)	-0.020640	0.055286	-0.373333	0.7094
C(33)	6.084282	0.702958	8.655263	0.0000
C(34)	-0.698796	0.098643	-7.084081	0.0000
C(1)	0.989931	0.079667	12.42592	0.0000
C(2)	-0.235043	0.077226	-3.043578	0.0027

R-squared	0.998551	Mean dependent var	7.420576
Adjusted R-squared	0.998455	S.D. dependent var	0.177909
S.E. of regression	0.006994	Akaike info criterion	-7.022587
Sum squared resid	0.008119	Schwarz criterion	-6.808084
Log likelihood	637.0102	Hannan-Quinn criter.	-6.935600
Durbin-Watson stat	2.109222		

Inverted AR Roots	.59	.40

Figure 4.72 Statistical results based on a reduced model in Figure 4.71

4.9.1 A Third-degree polynomial function

Griffiths and Wall (1996, p. 573) presented a third-degree polynomial cost function, which can be generalized as follows:

$$TC = C(1) + C(2)*Q + C(3)*Q^2 + C(4)*Q^3 + u_t \tag{4.89}$$

where TC is the total cost and Q is the output of the firm. Therefore, the marginal cost (MC) will be a quadratic function as follows:

$$MC = \frac{\mathrm{d}(TC)}{\mathrm{d}Q} = C(2) + 2C(3)*Q + 3C(4)*Q^2 \tag{4.90}$$

4.9.2 A Three-dimensional bounded semilog linear model

A bounded time series model can be defined using a semilog linear model as follows:

$$\log\left(\frac{Y_t - L}{U - Y_t}\right) = C(1) + C(2)*X_1 + C(3)*X_2 + u_t \tag{4.91}$$

where L and U are the lower and upper bounds of all possible values of the variable Y_t or values of Y_t in the corresponding population. Note that in the three-dimensional coordinate system with X_1, X_2 and $\log[(Y-L)/(U-Y)]$ axes, the corresponding regression function will present a plane.

Special cases of this model are as follows.

4.9.2.1 Logistic seemingly causal model

If $0 < Y_t < 1$ for all t, then a logistic SCM is as follows:

$$\log\left(\frac{Y_t}{1-Y_t}\right) = C(1) + C(2)^*X_1 + C(3)^*X_2 + u_t \tag{4.92}$$

4.9.2.2 Modified logistic SCM

If Y_t is a variable of percentages and $0 < Y_t < 100$ for all t, then a modified logistic SCM is as follows:

$$\log\left(\frac{Y_t}{100-Y_t}\right) = C(1) + C(2)^*X_1 + C(3)^*X_2 + u_t \tag{4.93}$$

4.9.3 Time series Cobb–Douglas models

For illustration purposes, in three-dimensional space, the basic Cobb–Douglas (CD) model can be presented as a translog (i.e. translogarithmic) linear model as follows:

$$\log(Y_t) = C(1) + C(2)^*\log(X_1) + C(3)^*\log(X_2) + \mu_t \tag{4.94}$$

with a *constant partial elasticity* of $\log(\hat{Y})$ with respect to X_1, which is computed as

$$\eta = \frac{\partial \hat{Y}}{\partial X_1} \cdot \frac{\hat{X}_1}{\hat{Y}} = \hat{C}(2) \tag{4.95}$$

This basic CD model can be extended to a bounded CD model as follows:

$$\log\left(\frac{Y_t-L}{U-Y_t}\right) = C(1) + C(2)^*\log(X_1) + C(3)^*\log(X_2) + u_t \tag{4.96}$$

Furthermore, since this is time series data that is being investigated, then the model may be a lagged-variables autoregressive CD model. Refer to all possible models based on the trivariate time series (X_t, Y_t, Z_t), as presented in Section 4.4. Furthermore, note the following example.

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 09:52
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 20 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.279737	0.761798	4.305255	0.0000
LOG(GDP)	0.485966	0.103300	4.704397	0.0000
LOG(PR)	0.474420	0.169475	2.799350	0.0057
AR(1)	0.978000	0.014786	66.14546	0.0000

R-squared	0.999594	Mean dependent var	5.816642
Adjusted R-squared	0.999587	S.D. dependent var	0.753241
S.E. of regression	0.015300	Akaike info criterion	-5.499823
Sum squared resid	0.040966	Schwarz criterion	-5.428597
Log likelihood	496.2342	Hannan-Quinn criter.	-5.470942
F-statistic	143748.4	Durbin-Watson stat	1.984622
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 4.73 Statistical results based on the AR(1) model in (4.97)

Example 4.29. (An AR(1) Cobb–Douglas model) The printout in Figure 4.73 (with its residual graph in Figure 4.74) presents the results based on an AR(1) Cobb–Douglas model, by entering the following equation specification:

$$\log(m1)\ C \log(gdp) \log(pr)\ AR(1) \tag{4.97}$$

For a comparison, Figure 4.75 (with its residual graph in Figure 4.76) and Figure 4.77 present the statistical results by entering the following equation specifications respectively:

$$\log(m1)\ C \log(gdp) \log(pr) \log(m1(-1))AR(1) \tag{4.98}$$

$$\log(m1)\ C \log(gdp) \log(pr) \log(m1(-1) \log(gdp(-1)) \log(pr(-1))AR(1) \tag{4.99}$$

which are the LVAR(1,1) models.

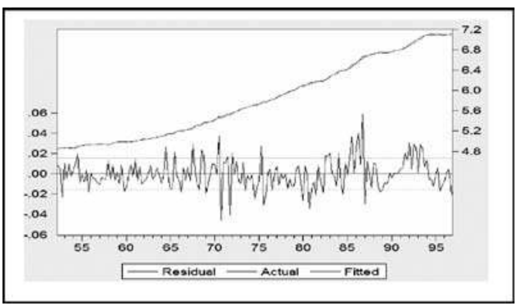


Figure 4.74 Residual graph of the regression in Figure 4.73

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 09:52
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 20 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.279737	0.761798	4.305255	0.0000
LOG(GDP)	0.485966	0.103300	4.704397	0.0000
LOG(PR)	0.474420	0.169475	2.799350	0.0057
AR(1)	0.978000	0.014786	66.14546	0.0000

R-squared	0.999594	Mean dependent var	5.816642
Adjusted R-squared	0.999587	S.D. dependent var	0.753241
S.E. of regression	0.015300	Akaike info criterion	-5.499823
Sum squared resid	0.040966	Schwarz criterion	-5.428597
Log likelihood	496.2342	Hannan-Quinn criter.	-5.470942
F-statistic	143748.4	Durbin-Watson stat	1.984622
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 4.75 Statistical results based on the model in (4.98)

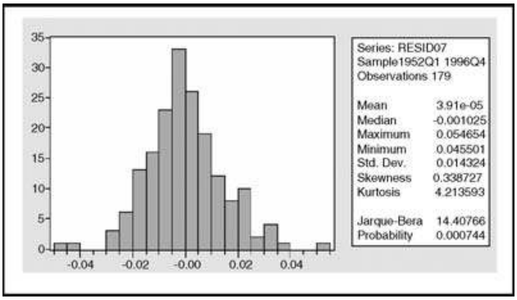


Figure 4.76 Residual histogram of the regression in Figure 4.75

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 09:59
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.087038	0.088790	0.980275	0.3283
LOG(GDP)	0.241630	0.106006	2.279386	0.0239
LOG(PR)	-0.362399	0.214893	-1.686415	0.0935
LOG(M1(-1))	0.948906	0.014052	67.52622	0.0000
LOG(GDP(-1))	-0.203718	0.108858	-1.871405	0.0630
LOG(PR(-1))	0.368825	0.212399	1.736470	0.0843
AR(1)	-0.121704	0.077636	-1.567629	0.1188

R-squared	0.999647	Mean dependent var	5.822083
Adjusted R-squared	0.999635	S.D. dependent var	0.751831
S.E. of regression	0.014368	Akaike info criterion	-5.609087
Sum squared resid	0.035301	Schwarz criterion	-5.483961
Log likelihood	506.2088	Hannan-Quinn criter.	-5.558345
F-statistic	80744.45	Durbin-Watson stat	1.952668
Prob(F-statistic)	0.000000		

Inverted AR Roots	-.12

Figure 4.77 Statistical results based on the model in (4.99)

Covariance Analysis: Ordinary
Date: 10/20/07 Time: 10:04
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Balanced sample (listwise missing value deletion)

Correlation Probability	RESID01	T	RS
RESID01	1.000000 -----		
T	-0.009056 0.9042	1.000000 -----	
RS	-0.180449 0.0156	0.531466 0.0000	1.000000 -----

Figure 4.78 Correlation matrix of the residual of the model in Figure 4.77 with time *t* and *RS*

Based on these three statistical results, the following notes and conclusions are presented:

(1) The model in (4.98) has the largest value of the *DW*-statistic, even though it is less than two, and this model is the simplest model. For these reasons, this model could be said to be the best of the three.
(2) The *R*-squared value of this model is the lowest, that is 0.999 594, because it has the least number of numerical independent variables or this is a nested model of the others.
(3) At the significant level $\alpha = 0.10$, log(*pr*) and *AR*(1) in the model in (4.99) have insignificant adjusted effects. However, in this model only the indicator *AR*(1) is insignificant.
(4) Corresponding to the model in (4.99), special notes are given as follows:
   - Since the model has the first lagged endogenous variable as an independent variable and AR(1) is insignificant with a negative estimate, then a reduced model may be obtained by deleting AR(1). Therefore, an LV(1) model would be used with the following equation specification:

$$\log(m1)C\ \log(gdp)\log(pr)\log(m1(-1)\log(gdp)(-1))\log(pr(-1)) \tag{4.100}$$

   However, the results are not presented.
   - The matrix correlation in Figure 4.78 shows that its residual, namely *Resid*01, and *RS* have a significant correlation with a *p*-value = 0.0156, which indicates that *RS* may be used as an additional independent variable. However, the scatter graphs (log(*M*1), *RS*) in Figures 4.28 and 4.30 show that *RS* is not a relevant linear predictor of log(*M*1). For this reason, *RS* should not be used as an additional variable of the model, except when using a model with a dummy variable(s). □

Example 4.30. (A two-piece Cobb–Douglas model) A two-piece CD model with two input variables can be presented as

$$\begin{aligned}\log(y_t) = {} & (C(11)) + C(12)^*\log(X_1) + C(13)^*\log(X_2)) \\ & (C(21) + C(22)^*\log X_1 + C(23)^*\log(X_2))^*D2 + \mu_t\end{aligned} \tag{4.101}$$

or

$$\log(y_t) = (C(11)) + C(12)^*\log(X_1) + C(13)^*\log(X_2))^*D1 \\ (C(21) + C(22)^*\log X_1 + C(23)^*\log(X_2))^*D2 + \mu_t \qquad (4.102)$$

where $D1$ and $D2$ are two dummy variables, which can be defined:

(a) using or based on the time t-variable as previously presented;
(b) using either X_1 or X_2 independent variables;
(c) using the endogenous variable;
(d) based on any variables outside the model.

For example, by using the median of X_1, say m, $D1 = 1$ if $X_1 \leq m$ and $D1 = 0$ if otherwise; and $D2 = 1$ if $D1 = 0$ and $D2 = 0$ if otherwise. □

4.9.4 Time series CES models

It is recognized that the constant elasticity of substitution (CES) model for time series data can be estimated by using its Taylor approximation as a translog quadratic model as follows:

$$\log(Y_t) = C(1) + C(2)^*\log(X_1) + C(3)^*\log(X_2) + C(4)^*\log(X_1)^2 \\ + C(5)^*\log(X_1)^*\log(X_2) + C(6)^*\log(X_2)^2 + \mu_t \qquad (4.103)$$

Agung and Pasay dan Sugiharso (1994, p. 53) proposed a modified translog quadratic model as follows:

$$\log(Y_t) = C(1) + C(2)\log(X_1) + C(2)\log(X_2) \\ + C(4)^*(\log(X_1) - \log(X_2))^2 + \mu_t \qquad (4.104)$$

Note that, under the null hypothesis H_0: $C(4) = 0$, this model becomes the translog linear model (CD model) in (4.94).

Furthermore, the models in (4.103) and (4.104) can easily be extended to lagged (endogenous)-variable autoregressive CES models, either univariate or multivariate or vector CES models, with dummy variables. Refer to the special notes presented in Section 2.14 corresponding to the model(s) having a multivariate exogenous variable(s).

Example 4.31. (An AR(1) CES model) The printout in Figure 4.79 presents the results based on an AR(1) CES model:

$$\log(m1_t) = C(1) + C(2)^*\log(gdp) + C(3)^*\log(pr) + C(4)^*\log(gdp)^2 \\ + C(5)^*\log(gdp)^*\log(pr) + C(6)^*\log(pr)^2 + [AR(1) = C(7)] + \varepsilon t \qquad (4.105)$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 18:12
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 17 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	37.92139	23.92269	1.585164	0.1148
LOG(GDP)	-9.018341	6.413244	-1.406206	0.1615
LOG(PR)	15.71778	11.17867	1.406050	0.1615
LOG(GDP)^2	0.652513	0.430393	1.516086	0.1313
LOG(GDP)*LOG(PR)	-2.050343	1.493562	-1.372788	0.1716
LOG(PR)^2	1.836160	1.318401	1.392717	0.1655
AR(1)	0.955167	0.025583	37.33584	0.0000

R-squared	0.999609	Mean dependent var	5.816642
Adjusted R-squared	0.999595	S.D. dependent var	0.753241
S.E. of regression	0.015159	Akaike info criterion	-5.502095
Sum squared resid	0.039526	Schwarz criterion	-5.377449
Log likelihood	499.4375	Hannan-Quinn criter.	-5.451552
F-statistic	73217.26	Durbin-Watson stat	1.981494
Prob(F-statistic)	0.000000		

Inverted AR Roots	.96

Figure 4.79 Statistical results based on the model in (4.105)

Since the results show that each of the independent variables has an insignificant adjusted effect, it might be preferable to find a reduced model. However, the following notes are given:

(1) Figure 4.80 shows that the null hypothesis H_0: $C(4) = C(5) = C(6) = 0$ is rejected based on the F-and chi-squared-statistics. Therefore, it can be concluded that the quadratic exogenous variables, namely $\log(gdp)^2$, $\log(gdp)^*\log(pr)$ and $\log(pr)^2$, have a significant joint effect on $\log(m1)$. This significant joint effect indicates that all of these variables cannot be deleted from the model if a reduced model is required.

(2) On the other hand, at the level of significance of $\alpha = 0.10$, each of these quadratic variables and the interaction factor have a significant adjusted effect based the one-sided hypothesis. For examples, each of $\log(gdp)^2$ and $\log(pr)^2$ has a significant positive effect on $\log(m1)$ with p-values of $0.1313/2 = 0.065\,65$ and $0.1655/2 = 0.082\,75$ respectively, and $\log(gdp)^*\log(pr)$ has a significant negative

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	2.999888	(3, 172)	0.0321
Chi-square	8.999665	3	0.0293

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(4)	0.652513	0.430393
C(5)	-2.050343	1.493562
C(6)	1.836160	1.318401

Restrictions are linear in coefficients.

Figure 4.80 The Wald test for H_0: $C(4) = C(5) = C(6) = 0$, based on the model in (4.105)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 18:24
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 10 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.236624	0.850438	2.629966	0.0093
LOG(GDP)	0.632320	0.116207	5.441323	0.0000
LOG(PR)	-0.464532	0.418983	-1.108713	0.2691
LOG(GDP)*LOG(PR)	0.135924	0.055659	2.442097	0.0156
AR(1)	0.957798	0.022925	41.77984	0.0000

R-squared	0.999603	Mean dependent var	5.816642
Adjusted R-squared	0.999594	S.D. dependent var	0.753241
S.E. of regression	0.015175	Akaike info criterion	-5.510766
Sum squared resid	0.040070	Schwarz criterion	-5.421733
Log likelihood	498.2136	Hannan-Quinn criter.	-5.474664
F-statistic	109593.4	Durbin-Watson stat	2.013206
Prob(F-statistic)	0.000000		

Inverted AR Roots	.96

Figure 4.81 Statistical results based on a reduced model in Figure 4.79

effect on log($m1$) with a p-value = 0.1716/2 = 0.0858. For these reasons, the model does not need to be reduced.

(3) However, for further illustration purposes, the following data analysis has been based on selected models.

(4) Since $M1$, GDP and PR have the same growth patterns, then the squared independent variables are deleted, one by one, giving the results in Figure 4.81 (with its residual graph in Figure 4.82), which shows that log(gdp)*log(pr) is significant, but log(pr) is insignificant with a p-value = 0.2691.

(5) Now there is another choice as to whether there should be an additional reduced model or not. If one is required, then a choice needs to be made as to which variable should be deleted from the model. Under the assumption that the effect of GDP on $M1$ is highly dependent on PR (or the effect of PR on $M1$ is dependent on GDP) then either one of the main factors should be deleted, even though log(gdp)* log(pr) might have an insignificant effect. Therefore, there could be two possible reduced models, as presented in Figures 4.83 and 4.84. In many cases, it has been recognized that an independent variable would be deleted from a model based on

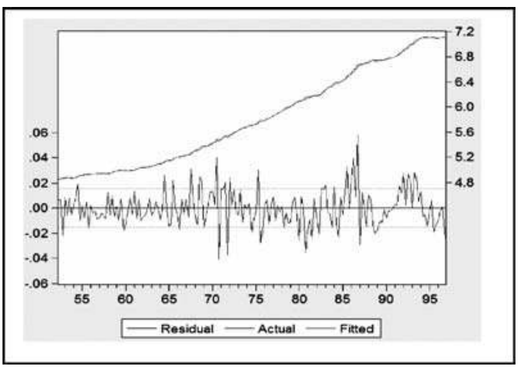


Figure 4.82 Residual graph of the regression in Figure 4.81

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 18:35
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.956211	0.557136	5.306080	0.0000
LOG(GDP)	0.532985	0.075742	7.036889	0.0000
LOG(GDP)*LOG(PR)	0.076805	0.022135	3.469794	0.0007
AR(1)	0.968098	0.018594	52.06641	0.0000

R-squared	0.999601	Mean dependent var	5.816642
Adjusted R-squared	0.999594	S.D. dependent var	0.753241
S.E. of regression	0.015176	Akaike info criterion	-5.516150
Sum squared resid	0.040303	Schwarz criterion	-5.444923
Log likelihood	497.6954	Hannan-Quinn criter.	-5.487268
F-statistic	146115.5	Durbin-Watson stat	2.008681
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97

Figure 4.83 Statistical results based on a reduced model in Figure 4.81 by deleting log(*pr*)

the largest *p*-value. However, it is suggested that judgment should be used to delete a variable that is less important on a theoretical basis (refer to the special notes in Section 2.14).

(6) Based on these last two reduced models, the model in Figure 4.83 will always be chosen as the best model, which is highly dependent on the data set used. In other words, the data supports the model in Figure 4.83 as the best model.

(7) Now, suppose in other cases that both statistical results show that each independent variable has a significant effect, *the interaction factor in particular*. Which model would be your choice? If the main objective is to study the effect of *GDP* on *M*1 dependent on *PR*, then the two models should be written or presented as follows:

$$\log(m1) = c(11) + \{c(12) + c(13)*\log(pr)\}*\log(gdp) + [ar(1) = c(1)]$$
$$\log(m1) = \{c(21) + c(22)*\log(pr)\} + \{c(23)*\log(pr)\}*\log(gdp) + [ar(1) = c(2)] \quad (4.106)$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 18:36
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 132 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	397.7811	57970.20	0.006862	0.9945
LOG(PR)	0.416114	0.545464	0.762863	0.4466
LOG(GDP)*LOG(PR)	-0.007680	0.089731	-0.085589	0.9319
AR(1)	0.999977	0.003372	296.5333	0.0000

R-squared	0.999591	Mean dependent var	5.816642
Adjusted R-squared	0.999584	S.D. dependent var	0.753241
S.E. of regression	0.015368	Akaike info criterion	-5.490996
Sum squared resid	0.041329	Schwarz criterion	-5.419770
Log likelihood	495.4442	Hannan-Quinn criter.	-5.462114
F-statistic	142484.6	Durbin-Watson stat	2.085484
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00

Figure 4.84 Statistical results based on a reduced model in Figure 4.81 by deleting log(*gdp*)

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 19:29
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.146275	0.079106	1.849111	0.0661
LOG(M1(-1))	0.980597	0.011504	85.24011	0.0000
LOG(PR)	0.065065	0.015226	4.273279	0.0000
LOG(GDP)*LOG(PR)	-0.007526	0.002110	-3.566823	0.0005
AR(1)	-0.144729	0.076583	-1.889838	0.0605

R-squared	0.999635	Mean dependent var	5.822083
Adjusted R-squared	0.999627	S.D. dependent var	0.751831
S.E. of regression	0.014524	Akaike info criterion	-5.598381
Sum squared resid	0.036492	Schwarz criterion	-5.509005
Log likelihood	503.2559	Hannan-Quinn criter.	-5.562137
F-statistic	118533.1	Durbin-Watson stat	1.951140
Prob(F-statistic)	0.000000		

Inverted AR Roots	-.14

Figure 4.85 Statistical results based on an LVAR(1,1)_SCM

In the two-dimensional coordinate system with log(m1) and log(gdp) as the coordinate axes, the first regression (i) represents a set of straight lines with *a single intercept*, namely $c(11)$, and *various slopes* that are dependent on PR, namely $\{c(12) + c(13)^*\log(pr)\}$. On the other hand, the second regression (ii) represents a set of straight lines with various intercepts, namely $\{c(21) + c(22)^*\log(pr)\}$, as well as various slopes, namely $\{c(23)^*\log(pr)\}$. From the present point of view, if each independent variable is significant, then the second regression would be chosen as the best model, since a set of regressions with a single intercept is an impossible model in reality or practice.

(8) By doing further experimentation, an alternative acceptable model is obtained, which is an LVAR(1,1) SCM, with the statistical results in Figure 4.85. Compared to the results in Figure 4.84, note that Figure 4.85 shows that each of the variables log(pr) and log(gdp)* log(pr) has a significant adjusted effect, but the model in Figure 4.84 shows that both variables have insignificant effects. On the other hand, the correlation matrix in Figure 4.86 shows that the independent variables

Covariance Analysis: Ordinary
Date: 10/20/07 Time: 20:34
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Balanced sample (listwise missing value deletion)

Correlation Probability	RESID02	LOG(M1(-1))	LOG(PR)	LOG(GDP...
RESID02	1.000000			

LOG(M1(-1))	-0.003629	1.000000		
	0.9615	-----		
LOG(PR)	-0.002573	0.993240	1.000000	
	0.9727	0.0000	-----	
LOG(GDP)*LOG(PR)	-0.002965	0.984926	0.986358	1.000000
	0.9686	0.0000	0.0000	-----

Figure 4.86 Correlation matrix of the residual of regression in Figure 4.85 and its independent variables

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 20:41
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.198574	0.528117	6.056562	0.0000
LOG(GDP)	0.500815	0.071485	7.005903	0.0000
LOG(GDP)*LOG(PR)	0.087278	0.022463	3.885442	0.0001
AR(1)	0.937183	0.075533	12.40753	0.0000
AR(2)	0.173505	0.096081	1.805823	0.0727
AR(4)	-0.152973	0.056439	-2.710431	0.0074

R-squared	0.999612	Mean dependent var	5.833023
Adjusted R-squared	0.999600	S.D. dependent var	0.748997
S.E. of regression	0.014971	Akaike info criterion	-5.531848
Sum squared resid	0.038105	Schwarz criterion	-5.423763
Log likelihood	492.8026	Hannan-Quinn criter.	-5.488009
F-statistic	87565.43	Durbin-Watson stat	2.059988
Prob(F-statistic)	0.000000		

Inverted AR Roots	.94	.65	-.33+.38i	-.33-.38i

Figure 4.87 Statistical results based on an unexpected or interaction model

have significant bivariate correlations with the p-values $= 0.0000$. These results indicate that the impact of correlations or multicollinearity between independent variables on the parameter estimates is unpredictable, so that the statistical results can also be considered as unexpected results, which are highly dependent on the data set used. Refer to the special notes and comments on multicollinearity problems in Section 2.14.

(9) Furthermore, corresponding to the model in Figure 4.84, two modified models have been found with the statistical results presented in Figures 4.87 and 4.88, which show that the interaction factor $\log(gdp)^*\log(pr)$ has a significant negative effect on $\log(m1)$, based on the p-values of $0.0001/2 = 0.000\,05$ and $0.0814/2 = 0.0407$ respectively. The model in Figure 4.88 should be considered as an unexpected model, since it has the indicators AR(1), AR(2) and AR(4) without AR (3). □

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 12/02/07 Time: 10:48
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.104819	0.056544	-1.853756	0.0655
LOG(M1(-1))	1.017390	0.008140	124.9866	0.0000
LOG(GDP)*LOG(PR)	-0.003714	0.002119	-1.752801	0.0814
AR(1)	-0.094221	0.076638	-1.229421	0.2206

R-squared	0.999597	Mean dependent var	5.822083
Adjusted R-squared	0.999590	S.D. dependent var	0.751831
S.E. of regression	0.015219	Akaike info criterion	-5.510380
Sum squared resid	0.040299	Schwarz criterion	-5.438879
Log likelihood	494.4238	Hannan-Quinn criter.	-5.481384
F-statistic	143935.2	Durbin-Watson stat	1.944800
Prob(F-statistic)	0.000000		

Inverted AR Roots	-.09

Figure 4.88 Statistical results based on an LVAR(1,1) interaction model

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 21:04
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
LOG(M1)=(C(11)+C(12)*LOG(GDP(-1))+C(13)*(LOG(GDP(-1))-LOG(RS(-1)))^2)*DRS1+(C(21)+C(22)*LOG(GDP(-1))+C(23)*(LOG(GDP(-1))-LOG(RS(-1)))^2)*DRS2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	1.832853	0.028004	65.44875	0.0000
C(12)	0.627598	0.004988	125.8296	0.0000
C(13)	0.008582	0.001258	6.824200	0.0000
C(21)	0.732657	0.224125	3.268961	0.0013
C(22)	0.783349	0.036724	21.33073	0.0000
C(23)	0.013704	0.001513	9.055282	0.0000

R-squared	0.998206	Mean dependent var	5.816642
Adjusted R-squared	0.998154	S.D. dependent var	0.753241
S.E. of regression	0.032364	Akaike info criterion	-3.990593
Sum squared resid	0.181206	Schwarz criterion	-3.883753
Log likelihood	363.1580	Hannan-Quinn criter.	-3.947270
Durbin-Watson stat	0.238093		

Figure 4.89 Statistical results based on a two-piece modified CES model

Example 4.32. (A two-piece AR(3) CES model) Similar to Example 4.6, this example presents two-piece modified CES models in (4.104) with an endogenous variable log($m1$), exogenous variables log($gdp(-1)$) and log($rs(-1)$) and the dummy variables $Drs1$ and $Drs2$ of the variable RS. After using the trial-and-error methods, two alternative models are presented, with the statistical results given in Figures 4.89 and 4.90.

Both results show that the quadratic term (log($gdp(-1)$)-log($rs(-1)$))^2 has a significant effect on log($m1$) in both defined intervals. Since the basic model has a very small value of the DW-statistic, the AR(3) model should be considered as the better model, even though the indicator AR(3) is insignificant. Do this as an exercise to

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/20/07 Time: 21:08
Sample (adjusted): 1953Q1 1996Q4
Included observations: 176 after adjustments
Convergence achieved after 12 iterations
LOG(M1)=(C(11)+C(12)*LOG(GDP(-1))+C(13)*(LOG(GDP(-1))-LOG(RS(-1)))^2)*DRS1+(C(21)+C(22)*LOG(GDP(-1))+C(23)*(LOG(GDP(-1))-LOG(RS(-1)))^2)*DRS2+[AR(1)=C(1),AR(2)=C(2),AR(3)=C(3)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	1.743283	0.110678	15.75092	0.0000
C(12)	0.653557	0.019040	34.32570	0.0000
C(13)	0.004871	0.000878	5.544481	0.0000
C(21)	0.635469	0.361181	1.759421	0.0803
C(22)	0.803704	0.054781	14.67126	0.0000
C(23)	0.011671	0.001642	7.108972	0.0000
C(1)	0.622867	0.078037	7.981661	0.0000
C(2)	0.392920	0.085908	4.573749	0.0000
C(3)	-0.114597	0.077344	-1.481657	0.1403

R-squared	0.999678	Mean dependent var	5.833023
Adjusted R-squared	0.999663	S.D. dependent var	0.748997
S.E. of regression	0.013756	Akaike info criterion	-5.684941
Sum squared resid	0.031600	Schwarz criterion	-5.522814
Log likelihood	509.2748	Hannan-Quinn criter.	-5.619183
Durbin-Watson stat	1.985298		

Inverted AR Roots	.92	.24	-.53

Figure 4.90 Statistical results based on an AR(3) modified CES model

construct other alternative models by deleting the AR(3) indicator and do residual analyses to identify the limitation of each model. □

4.10 Final notes in developing models

4.10.1 Expert judgment

In the last three chapters, many alternative models have been presented with an endogenous variable $M1$, either univariate or multivariate linear models. These chapters have also demonstrated many more alternative models, so that an infinite number of models have been obtained. The author is very confident that many alternative models can also be presented based on any sets of three of five time series, namely two endogenous variables and three exogenous variables, in various fields.

In practice, however, a few of all possible alternative models are always considered, which, in fact, cannot represent the true population model. Refer to the special notes on this problem in Section 2.14. Since the true population model will never be known, then best judgment should always be used in developing several alternative models, which can be considered as acceptable models in a statistical sense as well as in a theoretical sense.

When talking about the judgment, Tukey (1962, quoted in Gifi, 1990, p. 23) stated the following three different kinds of judgment that are likely to be involved in almost every instance:

(a1) judgment based upon experience of the particular field of subject matter from which the data come;
(a2) judgment based upon a broad experience with how particular techniques of data analysis have worked out in a variety of fields of application;
(a3) judgment based upon abstract results about the properties of particular techniques, whether obtained by mathematical proofs or empirical sampling.

4.10.2 Other unexpected models

In the last three chapters examples of unexpected models have been presented, as well as ‘not recommended models’ or ‘not appropriate models,’ but they are acceptable models, in a statistical sense. In practice, at the first stage, an association model based on any set of variables should be defined based on a strong theoretical basis, since its statistical results will be highly dependent on the data set available, and the estimates of the model parameters could be unexpected statistical values. In other words, the data do not support a particular model(s). In some cases, EViews presents the ‘Near Singular Matrix’ or ‘Overflow’ error messages, as well as the note ‘Convergence not achieved after ... iterations,’ even though a ‘good’ model exists.

On the other hand, statistical results may be obtained with many of the independent variables having an insignificant adjusted effect. This type of statistical result does not directly mean that the model is a ‘*bad*’ model.

To overcome these problems, it is suggested that the following steps should be applied, besides using the trial-and-error methods in order to obtain alternative models, which are acceptable models in a statistical sense. Also refer to the special notes and comments presented in Section 2.14.

(1) To select a good linear predictor variable. A scatter plot or graph should be constructed with linear regression between the dependent variable and each of the numerical independent variables. In most cases, additive models would be applied. Refer to the basic scatter graphs presented in Chapter 1.
(2) On the other hand, the scatter graph with regression based on the transformed variables should also be observed. For example, consider the graph of $(Y \log(X))$ where Y is a ratio or percentage variable and X is a numerical variable with very large observed values.
(3) Each graph can be used to judge or identify whether an independent variable should be used or not as a linear predictor or explanatory variable. Refer to the previous Example 4.6, where it is stated that *RS* cannot be used as a linear predictor of $M1$ or $\log(M1)$, as well as the special notes and comments on scatter graphs presented in Section 1.4.
(4) The data may consist of several groups or time periods having different patterns of relationships between the independent and dependent variables. If this is the case, then a model may be presented with dummy variables and should be defined based on the numerical independent or dependent variables, or other external variables.

4.10.3 The principal component factor analysis

If there is a large dimensional multivariate time series, it is suggested that factor analysis should be applied in order to reduce the dimension of input or source variables, as well as the dimension of the output or downstream variables. In general, the principal component method will be used to construct a few orthogonal input factors as well as a few orthogonal output factors. EViews 4, 5 and 6 provide the principal component method.

Then, based on those input and output factors, it is easy to apply all the alternative models presented in this book, as well as models from other sources. For a detailed discussion on factor analysis, refer to Tsay (2002), Hair *et al.* (2006) and Timm (1975).

5

Special cases of regression models

5.1 Introduction

In the previous chapters, many alternative time series models have been presented that are based on the variables in the Demo_workfile. The author is very confident that those models will also be applicable for other data sets. This chapter will present selected models based on selected other data sets. By selecting a specific set of variables, it is possible to present special cases of regression models.

For the first group of special cases, four selected variables, *ivmaut*, *ivmdep*, *ivmmae* and *mmdep*, are used in the POOL1_workfile of the EViews Examples Files, as presented in Figure 5.1. These variables are interesting because of their specific patterns of growth curves, as shown in Figure 5.2.

In order to present specific cases of growth curve patterns, a new variable '*time*' should be generated as follows:

(1) After opening the POOL1 workfile, click *Quick/Generate Series* ... and enter time=@trend(1968:01) in the window available on the screen. Then click *OK*, which gives an additional name '*time*' in the data set with a value 0 in 1968 : 01.
(2) Another time variable $t = \text{time} + 1$ is also defined, with a value 1 in 1968 : 01. This variable is needed if $\log(t)$ is used as an independent variable.
(3) To check the new variables, block the variables and then click *View/Show... OK*.

In the following sections, several alternative growth curve models are presented.

5.2 Specific cases of growth curve models

Observing the growth curve of the dated variable *MMDEP* in Figure 5.2, there should be confidence that a third-degree polynomial growth curve model can be applied, using the time t-variable as an independent variable. For a comparative study, present the following alternative regressions are presented.

Time Series Data Analysis Using EViews IGN Agung

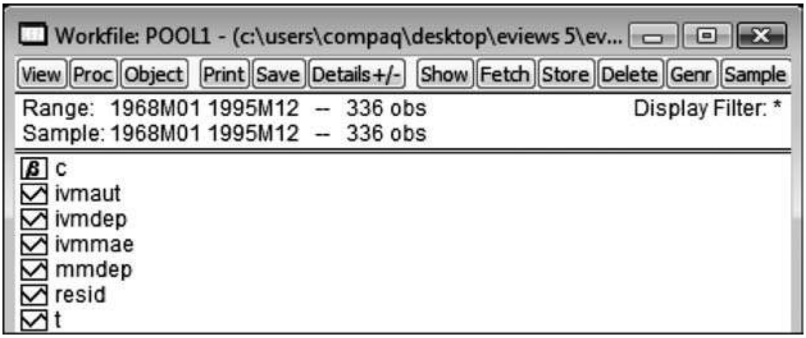


Figure 5.1 Selected variables in POOL1.wf1

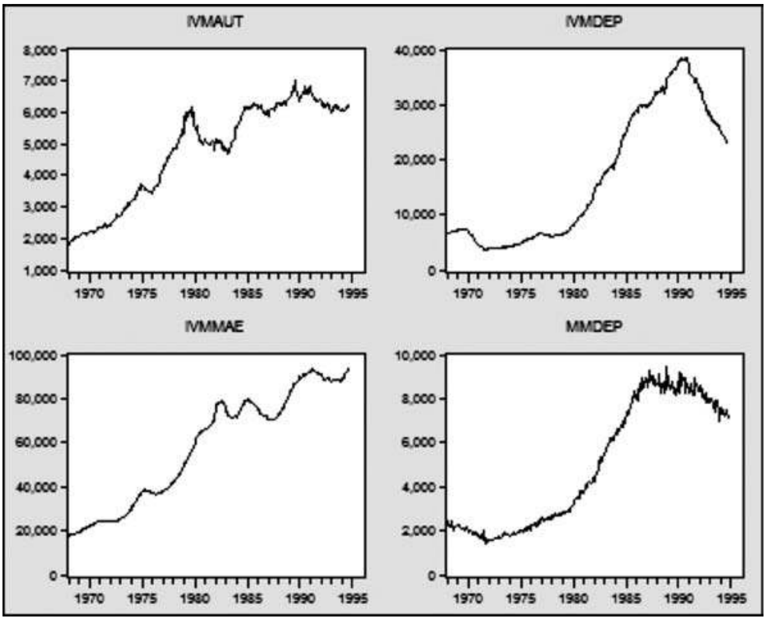


Figure 5.2 Growth curves of the variables *IVMAUT*, *IVMDEP*, *IVMMAE* and *MMDEP*, in POOL1.wf1

5.2.1 Basic polynomial model

The first model presented is a basic polynomial model with the time t as an exogenous variable. By entering the variable series

$$mmdep\ c\ t\ t^2\ t^3 \tag{5.1}$$

the statistical results in Figure 5.3 are obtained, with its residual graph in Figure 5.4. Note that this model is not the same as the classical growth model presented in (2.3).

Dependent Variable: MMDEP
Method: Least Squares
Date: 10/21/07 Time: 08:04
Sample (adjusted): 1968M01 1994M10
Included observations: 322 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	3350.879	102.6951	32.62939	0.0000
T	-75.65706	2.749189	-27.51977	0.0000
T^2	0.778097	0.019762	39.37380	0.0000
T^3	-0.001575	4.02E-05	-39.14967	0.0000

R-squared	0.974290	Mean dependent var	4925.146
Adjusted R-squared	0.974047	S.D. dependent var	2826.488
S.E. of regression	455.3448	Akaike info criterion	15.09233
Sum squared resid	65933765	Schwarz criterion	15.13922
Log likelihood	-2425.865	Hannan-Quinn criter.	15.11105
F-statistic	4016.846	Durbin-Watson stat	0.231263
Prob(F-statistic)	0.000000		

Figure 5.3 Statistical results based on the growth model in (5.1)

For this reason, this model will not be named 'growth model,' but the third-degree polynomial model of *MMDEP* on the time t. As a comparison, the growth model of the variable *MMDEP* should be presented using the semilog model with the equation specification as follows:

$$\log(mmdep)\ c\ t\ t^2\ t^3 \tag{5.2}$$

However, it should always be remembered that this basic model is not an appropriate model for statistical inference, which corresponds to a small value of the DW-statistic with the sign ($\pm$) of the error terms having systematic changes over time. Hence, an AR(1) model is presented in the following subsection. Note that, since there is only one observation at each time point, then there will always be systematic changes of the positive and negative values of the error terms, based on any basic regressions with the time t as an independent variable.

However, for the estimation in the sense of fitted values, this polynomial model can be considered as a good model because its R-squared value is very large (= 0.974 290).

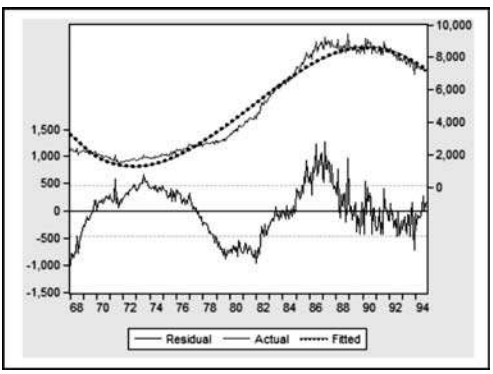


Figure 5.4 Residual graph of the regression in Figure 5.3

5.2.2 An AR(1) regression model

Corresponding to the previous basic regression model, Figure 5.5 presents the results based on the AR(1) third-degree polynomial model, as follows:

$$mmdep\ c\ t\ t^2\ t^3\ ar(1) \tag{5.3}$$

Note that this model has $DW = 2.74$ compared to a very low value of 0.231 263 for the basic regression model in (5.1).

However, the residual graph in Figure 5.6 shows heterogeneity of the error terms, which should be considered as a limitation of this model. Refer to the following subsection.

Dependent Variable: MMDEP
Method: Least Squares
Date: 10/21/07 Time: 08:07
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 10 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3835.557	486.6775	7.881107	0.0000
T	-85.63111	11.88780	-7.203279	0.0000
T^2	0.834473	0.079813	10.45534	0.0000
T^3	-0.001669	0.000155	-10.78432	0.0000
AR(1)	0.876701	0.026110	33.57776	0.0000

R-squared	0.994449	Mean dependent var	4933.449
Adjusted R-squared	0.994378	S.D. dependent var	2826.965
S.E. of regression	211.9570	Akaike info criterion	13.56610
Sum squared resid	14196547	Schwarz criterion	13.62484
Log likelihood	-2172.359	Hannan-Quinn criter.	13.58955
F-statistic	14151.99	Durbin-Watson stat	2.740423
Prob(F-statistic)	0.000000		

Inverted AR Roots .88

Figure 5.5 Statistical results based on the AR(1) polynomial model in (5.3), using the LS estimation method

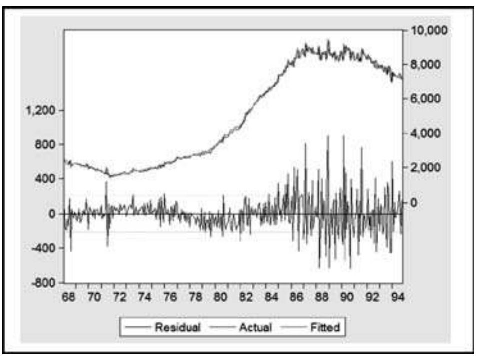


Figure 5.6 Residual graph of the regression in Figure 5.5

5.2.3 Heteroskedasticity-consistent covariance (White)

In the previous models the assumption has been made that the error terms have constant variance, called homoskedasticity. However, in many cases, the homoskedasticity assumption is not appropriate. Hence, a modified estimation method should be used.

In order to take into account the unknown heteroskedasticity of the error terms of a model, White (1980, p. 277), in the EViews 4 User's Guide, has derived a heteroskedasticity consistent covariant matrix estimator, given by

$$\hat{\sum}_W = \frac{T}{T-k}(X'X)^{-1}\left(\sum_{t=1}^{T} u_t^2 x_t x'_t\right)(X'X)^{-1} \tag{5.4}$$

where T is the number of observations, k is the number of exogenous or independent variables and u_t is the least squares residual. This matrix provides correct estimates of the coefficient covariance in the presence of heteroskedasticity of an unknown form, but it uses the assumption that the residuals of the estimated equation are serially uncorrelated. However, EViews 6 (User's Guide II, p.158) presents a general multiple regression with two independent variables as an illustrative example.

In order to take into account the two problems of unknown heteroskedaticity and the serial correlation of the residuals, Newey and West (1987a, 1987b, EViews 6 User's Guide II, p. 36) have proposed a more general covariance estimator that is consistent in the presence of both heteroskedasticity and autocorrelation of an unknown form.

The processes of the analysis are:

(1) After entering the variable series *mmdep c t* $t^2 t^3$ *ar*(1) in the '*Equation specification*' window, click *Option* . . . , which gives the options of the estimation methods, as presented in Figure 5.7.

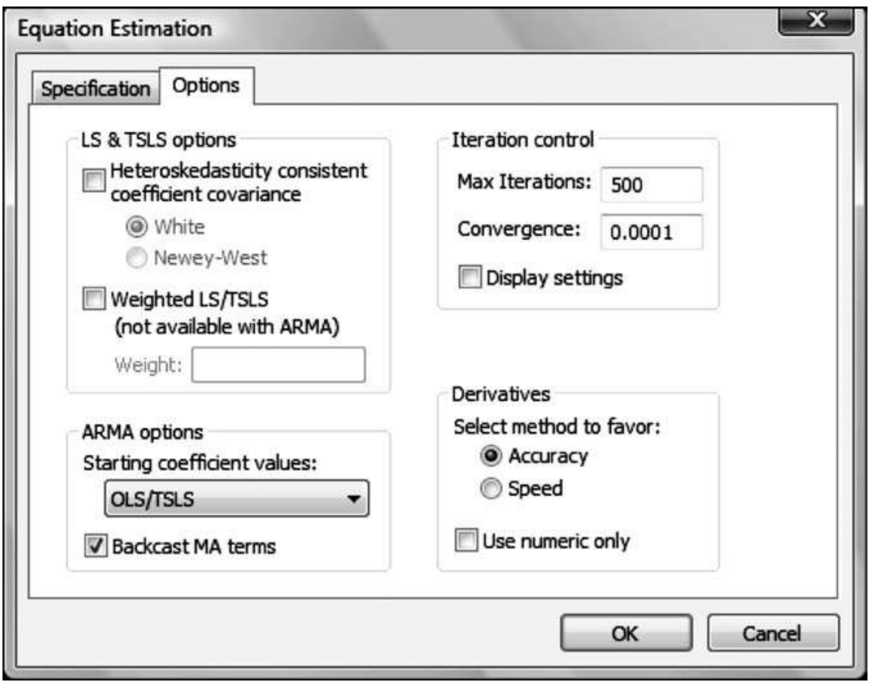


Figure 5.7 The LS and TLS options of the estimation methods

Dependent Variable: MMDEP
Method: Least Squares
Date: 10/21/07 Time: 08:34
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 10 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	3835.557	356.7307	10.75197	0.0000
T	-85.63111	9.067009	-9.444252	0.0000
T^2	0.834473	0.060919	13.69808	0.0000
T^3	-0.001669	0.000117	-14.23645	0.0000
AR(1)	0.876701	0.028561	30.69592	0.0000

R-squared	0.994449	Mean dependent var	4933.449
Adjusted R-squared	0.994378	S.D. dependent var	2826.965
S.E. of regression	211.9570	Akaike info criterion	13.56610
Sum squared resid	14196547	Schwarz criterion	13.62484
Log likelihood	-2172.359	Hannan-Quinn criter.	13.58955
F-statistic	14151.99	Durbin-Watson stat	2.740423
Prob(F-statistic)	0.000000		

Inverted AR Roots .88

Figure 5.8 Statistical results based on the AR(1) polynomial model in (5.3), using the Newey–West estimation method

(2) Click the *Het...* option and then select either the White or Newey–West option.
(3) By clicking *OK*, the statistical results in Figure 5.8 are obtained. Note that the estimation method uses iteration.
(4) Using the White or the Newey-West methods does not change the point estimates of the parameters, but the standard errors. Hence, the same regression functions will be obtained as in the previous results in Figure 5.5, by using the LS estimation method, as well as the residual graph in Figure 5.9, but with different values of the *t*-statistic.

5.3 Seemingly causal models

In general, based on the time series data, the relationship can also be studied between a group of independent (exogenous or source) variables with a dependent (an endogenous or downstream) variable, as well as the growth curve models. However, the relationships between dated independent variables with a dated dependent variable may not have any meaning, which has been mentioned in Chapter 1. For this reason,

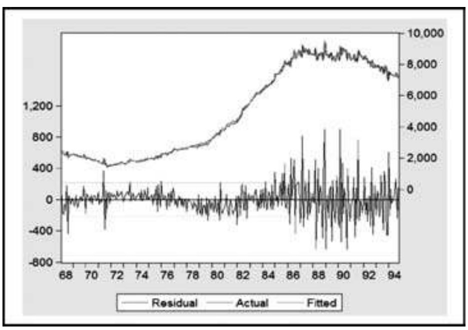


Figure 5.9 Residual graph of the regression in Figure 5.8

here alternative models, called seemingly causal models, will be presented as in Chapter 4, which look like showing a causal relationship between selected groups of a set of exogenous variables with an endogenous variable.

The following examples present statistical results of some possible types of relationships between an endogenous and a set of exogenous variables in the basic regression, based on the selected variable in the POOL1_workfile. Since this involves working with the time series data, the AR(1) models in the examples are presented using the Newey–West estimation method to take into account the unknown form of the autocorrelation and heteroskedasticity of their error terms.

5.3.1 Autoregressive models

Because of the time series data, this subsection will present directly examples of autoregressive models. The problem in constructing or defining an autoregressive model is to find an appropriate or a good or the best AR(p) model, besides selecting relevant exogenous variables corresponding to each endogenous variable.

Based on experience in doing the data analyses, a start should be made with an AR (1) model. If the AR(1) model is considered as not a good model, e.g. based on the value of its DW-statistic or the residual plot, then the procedure should be to move to an AR(2) model, then an AR(3) model and so on, until the highest AR(p) model is obtained that is not significant. Then the AR($p-1$) model should be used as the best model. However, in some cases, selected or unordered AR indicators may be used out of the set of AR(1) up to AR(p) indicators. Alternative models are presented in the following examples.

Example 5.1. (AR(p) interaction models) Here, alternative AR(p) two-way interaction models are presented having the dependent variable *mmdep* and three independent variables *ivmaut*, *ivmdep* and *ivmaut*ivmdep*. Figure 5.10 presents statistical results based on a basic regression with its residual graph in Figure 5.11.

Dependent Variable: MMDEP
Method: Least Squares
Date: 10/21/07 Time: 08:40
Sample (adjusted): 1968M01 1994M10
Included observations: 322 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1487.599	269.2769	-5.524422	0.0000
IVMDEP	0.499535	0.041398	12.06671	0.0000
IVMAUT	0.517195	0.052827	9.790328	0.0000
IVMDEP*IVMAUT	-4.65E-05	6.52E-06	-7.127950	0.0000

R-squared	0.986116	Mean dependent var	4925.146
Adjusted R-squared	0.985985	S.D. dependent var	2826.488
S.E. of regression	334.6116	Akaike info criterion	14.47616
Sum squared resid	35604852	Schwarz criterion	14.52305
Log likelihood	-2326.662	Hannan-Quinn criter.	14.49488
F-statistic	7528.768	Durbin-Watson stat	0.484799
Prob(F-statistic)	0.000000		

Figure 5.10 Statistical results based on a basic two-way interaction model

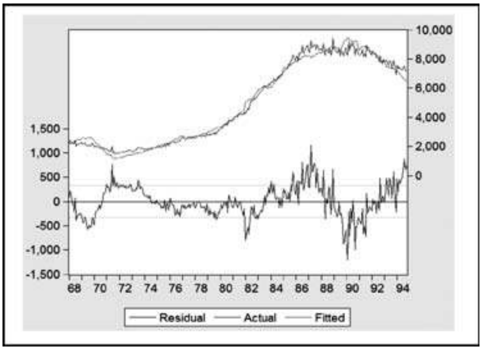


Figure 5.11 Residual graph of the regression in Figure 5.10

Dependent Variable: MMDEP
Method: Least Squares
Date: 10/21/07 Time: 08:42
Sample (adjusted): 1968M03 1994M10
Included observations: 320 after adjustments
Convergence achieved after 21 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-880.2019	437.9332	-2.009900	0.0453
IVMDEP	0.377168	0.052774	7.146877	0.0000
IVMAUT	0.483728	0.105234	4.596696	0.0000
IVMDEP*IVMAUT	-2.95E-05	8.33E-06	-3.545799	0.0005
AR(1)	0.481150	0.051145	9.407527	0.0000
AR(2)	0.409732	0.049442	8.287132	0.0000

R-squared	0.995016	Mean dependent var	4941.328
Adjusted R-squared	0.994937	S.D. dependent var	2827.860
S.E. of regression	201.2173	Akaike info criterion	13.46522
Sum squared resid	12713354	Schwarz criterion	13.53588
Log likelihood	-2148.435	Hannan-Quinn criter.	13.49343
F-statistic	12538.23	Durbin-Watson stat	2.276204
Prob(F-statistic)	0.000000		

Inverted AR Roots	.92	-.44

Figure 5.12 Statistical results based on an AR(2) two-way interaction model

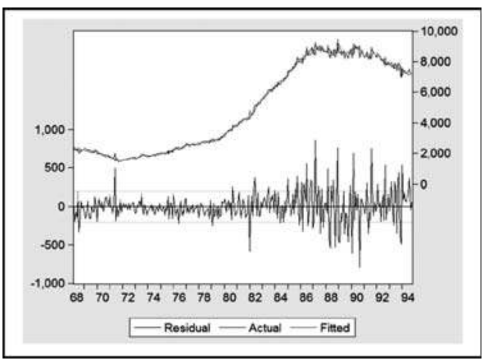


Figure 5.13 Residual graph of the regression in Figure 5.12

Dependent Variable: MMDEP
Method: Least Squares
Date: 12/06/07 Time: 08:59
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 14 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-6638.041	29208.60	-0.227263	0.8204
IVMDEP	0.127543	0.051624	2.470601	0.0140
IVMAUT	-0.051415	0.162054	-0.317268	0.7513
IVMDEP*IVMAUT	-3.65E-06	7.51E-06	-0.486048	0.6273
AR(1)	0.350099	0.052030	6.728734	0.0000
AR(2)	0.249119	0.054044	4.609600	0.0000
AR(3)	0.402949	0.052206	7.718395	0.0000

R-squared	0.995956	Mean dependent var	4949.552
Adjusted R-squared	0.995878	S.D. dependent var	2828.468
S.E. of regression	181.5897	Akaike info criterion	13.26308
Sum squared resid	10288144	Schwarz criterion	13.34570
Log likelihood	-2108.461	Hannan-Quinn criter.	13.29607
F-statistic	12806.66	Durbin-Watson stat	2.081074
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	-.33+.54i	-.33-.54i

Estimated AR process is nonstationary

Figure 5.14 Statistical results based on an AR(3) two-way interaction model

Note that the regression in Figure 5.10 shows that the adjusted effect of each variable is significant. However, its $DW = 0.484\,799$, as well as its residual graph in Figure 5.11, indicate an autoregressive problem. For this reason, alternative autoregression models need to be applied. Then an AR(2) regression was found, as presented in Figure 5.12 with its residual graph in Figure 5.13, with $DW = 2.276\,204$. This indicates that the AR(2) regression is better than the basic regression, in a statistical sense.

For further illustration purposes, Figures 5.14 to 5.16 present the statistical results based on AR(3) models, with a special note '*Estimated AR process is nonstationary*.' Hence, these models are unacceptable time series models, corresponding to the data

Dependent Variable: MMDEP
Method: Least Squares
Date: 12/06/07 Time: 08:46
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 24 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-5475.587	22647.14	-0.241778	0.8091
IVMDEP	0.139023	0.036820	3.775718	0.0002
IVMDEP*IVMAUT	-5.55E-06	4.51E-06	-1.230558	0.2194
AR(1)	0.349764	0.051935	6.734689	0.0000
AR(2)	0.249827	0.053933	4.632133	0.0000
AR(3)	0.402792	0.052127	7.727187	0.0000

R-squared	0.995954	Mean dependent var	4949.552
Adjusted R-squared	0.995889	S.D. dependent var	2828.468
S.E. of regression	181.3508	Akaike info criterion	13.25737
Sum squared resid	10293981	Schwarz criterion	13.32819
Log likelihood	-2108.551	Hannan-Quinn criter.	13.28566
F-statistic	15408.48	Durbin-Watson stat	2.082463
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	-.33+.54i	-.33-.54i

Estimated AR process is nonstationary

Figure 5.15 Statistical results based on a nonhierarchical model

Dependent Variable: MMDEP
Method: Least Squares
Date: 12/06/07 Time: 08:48
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 15 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-8420.589	31971.76	-0.263376	0.7924
IVMAUT	-0.321459	0.117961	-2.725135	0.0068
IVMDEP*IVMAUT	1.32E-05	2.98E-06	4.435979	0.0000
AR(1)	0.368470	0.052134	7.067752	0.0000
AR(2)	0.235909	0.054490	4.329426	0.0000
AR(3)	0.397677	0.052142	7.626788	0.0000

R-squared	0.995877	Mean dependent var	4949.552
Adjusted R-squared	0.995811	S.D. dependent var	2828.468
S.E. of regression	183.0721	Akaike info criterion	13.27627
Sum squared resid	10490323	Schwarz criterion	13.34709
Log likelihood	-2111.565	Hannan-Quinn criter.	13.30455
F-statistic	15118.91	Durbin-Watson stat	2.044227
Prob(F-statistic)	0.000000		

Inverted AR Roots 1.00 -.32-.55i -.32+.55i
Estimated AR process is nonstationary

Figure 5.16 Statistical results based on another nonhierarchical model

used in this analysis, even though the results in Figure 5.16 show that each of the independent variables and the AR indicators are significant.

Note that there might be nothing wrong with the models, but the data used gives unacceptable estimates. The author is very confident that acceptable estimates can be obtained using these models based on other data sets.

Finally, the output in Figure 5.17 based on an AR(3) model is obtained, but does not present the note '*Estimated AR process is nonstationary*,' even though one of the presented inverted AR roots is equal to 1.00. It is certainly true that this value is in fact

Dependent Variable: MMDEP
Method: Least Squares
Date: 10/21/07 Time: 08:57
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 15 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	14066.14	25980.53	0.541411	0.5886
IVMDEP	0.103460	0.019188	5.391770	0.0000
IVMAUT	-0.121339	0.089078	-1.362163	0.1741
AR(1)	0.349593	0.056407	6.197737	0.0000
AR(2)	0.245736	0.080724	3.044151	0.0025
AR(3)	0.402300	0.064187	6.267650	0.0000

R-squared	0.995961	Mean dependent var	4949.552
Adjusted R-squared	0.995897	S.D. dependent var	2828.468
S.E. of regression	181.1808	Akaike info criterion	13.25550
Sum squared resid	10274686	Schwarz criterion	13.32632
Log likelihood	-2108.252	Hannan-Quinn criter.	13.28378
F-statistic	15437.53	Durbin-Watson stat	2.076256
Prob(F-statistic)	0.000000		

Inverted AR Roots 1.00 -.32+.55i -.32-.55i

Figure 5.17 Statistical results based on an AR(3) additive model

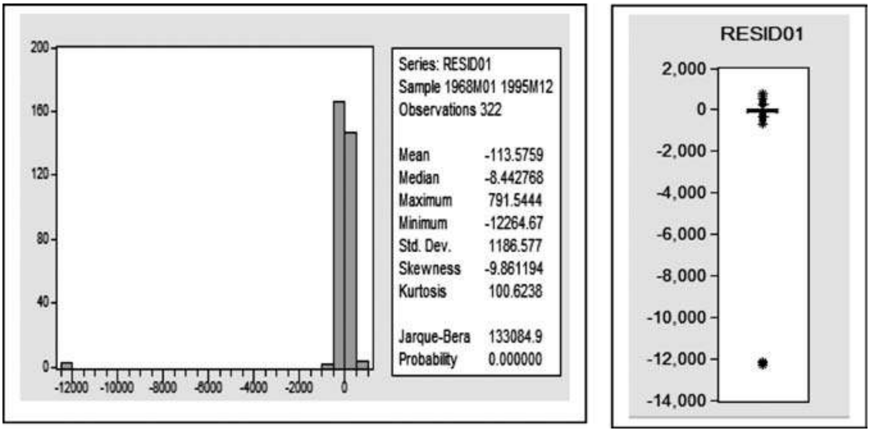


Figure 5.18 Residual histogram and box plot of the regression in Figure 5.17

strictly less than one or it has a value $= 1.00 - \varepsilon$. For this reason and the sufficiently large $DW = 2.076\,256$, this output is an acceptable estimate with respect to the data used in the data analysis.

In order to study the limitation or weakness of the estimates of the AR(3) model in Figure 5.17, Figure 5.18 presents the residuals graph and box plot. Both the histogram and box plot show that there are some *far outliers* and it was found that the first three error terms are very large. For this reason, other data analyses should be carried out. However, do this as an exercise, e.g. by deleting the outliers. □

Example 5.2. (Unexpected AR models) Based on the AR(3) interaction model presented in the previous example, further experimentation has been done for illustration purposes. Figure 5.19 presents statistical results based on the AR(4) interaction model using EViews 6 and Figure 5.20 presents statistical results based on the same model using EViews 5. However, EViews 6 presents the note 'Estimated AR process is nonstationary,' but the output of EViews 5 does not present the statement. Therefore, it can be said that these outputs are unexpected outputs. For a comparison, refer to the inconsistent results presented in Example 2.39, specifically the statistical results in Figure 2.96 and Example 2.40. These outputs demonstrate that different statistical results could be obtained using EViews 4 or 5 compared to the statistical results presented in this book, which in general use EViews 6.

On the other hand, the model considered is an unexpected or unusual model, since it has four unordered indicators AR(2), AR(3), AR(5) and AR(6). The statistical results are obtained by entering the following equation specification:

$$mmdep\ c\ ivmaut\ ivmdep\ ivmaut{*}ivmaut\ ar(2)\ ar(3)\ ar(5)\ ar(6) \qquad (5.5)$$

□

Dependent Variable: MMDEP
Method: Least Squares
Date: 10/21/07 Time: 09:13
Sample (adjusted): 1968M07 1994M10
Included observations: 316 after adjustments
Convergence achieved after 43 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-11753.03	57784.31	-0.203395	0.8390
IVMAUT	0.028726	0.114123	0.251713	0.8014
IVMDEP	0.182247	0.051531	3.536656	0.0005
IVMDEP*IVMAUT	-9.76E-06	8.23E-06	-1.186100	0.2365
AR(2)	0.328442	0.084572	3.883602	0.0001
AR(3)	0.450907	0.064105	7.033935	0.0000
AR(5)	0.129546	0.074674	1.734817	0.0838
AR(6)	0.093340	0.049655	1.879770	0.0611

R-squared	0.995525	Mean dependent var	4975.709
Adjusted R-squared	0.995423	S.D. dependent var	2829.034
S.E. of regression	191.3907	Akaike info criterion	13.37150
Sum squared resid	11282165	Schwarz criterion	13.46658
Log likelihood	-2104.697	Hannan-Quinn criter.	13.40949
F-statistic	9788.116	Durbin-Watson stat	1.511452
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	.25-.52i	.25+.52i	-.45
	-.53-.59i	-.53+.59i		

Estimated AR process is nonstationary

Figure 5.19 Statistical results based on the model in (5.5), using EViews 6

Sample(adjusted): 1968:07 1994:10
Included observations: 316 after adjusting endpoints
Convergence achieved after 19 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10564.11	17990.52	0.587204	0.5575
IVMAUT	0.014384	0.110983	0.129604	0.8970
IVMDEP	0.179291	0.051470	3.483379	0.0006
IVMAUT*IVMDEP	-9.37E-06	8.19E-06	-1.143840	0.2536
AR(2)	0.326649	0.084649	3.858868	0.0001
AR(3)	0.449206	0.063854	7.034870	0.0000
AR(5)	0.127879	0.074191	1.723646	0.0858
AR(6)	0.091960	0.049957	1.840779	0.0666

R-squared	0.995536	Mean dependent var	4975.709
Adjusted R-squared	0.995435	S.D. dependent var	2829.034
S.E. of regression	191.1439	Akaike info criterion	13.36892
Sum squared resid	11253087	Schwarz criterion	13.46400
Log likelihood	-2104.289	F-statistic	9813.522
Durbin-Watson stat	1.512248	Prob(F-statistic)	0.000000

Inverted AR Roots	1.00	.25 -.52i	.25+.52i	-.45
	-.53 -.59i	-.53+.59i		

Figure 5.20 Statistical results based on the model in (5.5), using EViews 5

Example 5.3. (An AR(1) Cobb–Douglas model) In this example, a Cobb– Douglas additive AR(1) model is presented, having a dependent variable log(*mmdep*) and three independent variables log(*ivmaut*), log(*ivmdep*) and log(*ivmmae*). Based on the statistical results in Figure 5.21, the following findings are obtained:

(1) Each of the independent variables has a positive adjusted effect on the dependent variable, at a significant level of 0.05 based on the *t*-test, with $DW = 2.586\,209$ and *R*-squared $= 0.995\,544$.

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 09:37
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 6 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.706918	0.227613	-3.105796	0.0021
LOG(IVMAUT)	0.143777	0.080333	1.789755	0.0745
LOG(IVMDEP)	0.621510	0.023872	26.03466	0.0000
LOG(IVMMAE)	0.179563	0.075254	2.386082	0.0176
AR(1)	0.731046	0.039432	18.53923	0.0000

R-squared	0.995544	Mean dependent var	8.312279
Adjusted R-squared	0.995488	S.D. dependent var	0.641394
S.E. of regression	0.043086	Akaike info criterion	-3.435801
Sum squared resid	0.586613	Schwarz criterion	-3.377056
Log likelihood	556.4461	Hannan-Quinn criter.	-3.412346
F-statistic	17649.65	Durbin-Watson stat	2.586206
Prob(F-statistic)	0.000000		

Inverted AR Roots .73

Figure 5.21 Statistical results based on a Cobb–Douglas model

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	3.778327	(1, 316)	0.0528
Chi-square	3.778327	1	0.0519

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2) + C(3) + C(4)	-0.055150	0.028373

Restrictions are linear in coefficients.

Figure 5.22 A constant return-to-scale Wald test for the model in Figure 5.21

(2) Corresponding to the CD production function, the Wald statistic in Figure 5.22 shows that the null hypothesis of $C(2) + C(3) + C(4) = 1$ (i.e. a constant return-to-scale production function) is rejected. The result shows that $C(2)$, $C(3)$ and $C(4)$ have positive estimated values and $C(2) + C(3) + C(4)$ is significantly less than one. Hence, in fact, the regression function is a decreasing return to the scale function.

(3) Compared to the previous model, this model has smaller values of AIC and SC and hence it may be concluded that this model is preferred to the previous model, in a statistical sense. □

Example 5.4. (An AR(p) CES model) As an illustration, a CES (constant elasticity of substitution) AR(1) model is presented, having the dependent variable log(*mmdep*) and two main independent variables log(*ivmaut*) and log(*ivmdep*). An AR(p) CES model having an output and two input variables, in general, can be presented as

$$\begin{aligned}\log(y) = {} & c(1) + c(2)*\log(x1) + c(3)*\log(x2) \\ & + c(4)*(\log(x1))^2 + c(5)*\log(x1)*\log(x2) + c(6)*(\log(x2))^2 \\ & + [ar(1) = c(7), \ldots, ar(p) = c(7+p)]\end{aligned} \quad (5.6)$$

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 09:44
Sample (adjusted): 1968M05 1994M10
Included observations: 318 after adjustments
Convergence achieved after 30 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-29.01601	10.23145	-2.835964	0.0049
LOG(IVMAUT)	9.464430	2.602436	3.636758	0.0003
LOG(IVMDEP)	-1.412003	0.775092	-1.821724	0.0695
LOG(IVMAUT)^2	-0.667371	0.186831	-3.572061	0.0004
LOG(IVMAUT)*LOG(IVMDEP)	0.218653	0.148896	1.468487	0.1430
LOG(IVMDEP)^2	0.010429	0.057356	0.181832	0.8558
AR(1)	0.299757	0.056746	5.282457	0.0000
AR(2)	0.283204	0.057229	4.948601	0.0000
AR(3)	0.252331	0.056691	4.450954	0.0000
AR(4)	0.059717	0.056030	1.065791	0.2874

R-squared	0.996653	Mean dependent var	8.317632
Adjusted R-squared	0.996555	S.D. dependent var	0.642022
S.E. of regression	0.037683	Akaike info criterion	-3.688289
Sum squared resid	0.437356	Schwarz criterion	-3.569986
Log likelihood	596.4380	Hannan-Quinn criter.	-3.641038
F-statistic	10190.04	Durbin-Watson stat	2.013532
Prob(F-statistic)	0.000000		

Inverted AR Roots	.95	-.19+.44i	-.19-.44i	-.27

Figure 5.23 Statistical results based on an AR(4) CES model

For the analysis, the following series of variables are entered:

$$\begin{aligned} &\log(mmdep)\ c\log(ivmaut)\ \log(ivmdep)\ \log(ivmaut)^{\wedge}2 \\ &\log(ivmaut)^{*}\log(ivmdep)\ \log(ivmdep)^{\wedge}2\ ar(1)\ldots ar(p) \end{aligned} \tag{5.7}$$

Figure 5.23 presents statistical results based on an AR(4) CES model and Figure 5.24 presents statistical results based on its reduced model, namely the AR(3) CES model. The AR(3) model should be a preferred model.

Now, since log(*ivmdep*)^2 has a very large p-value = 0.99, then a reduced model should be obtained. For this reason, in most cases, log(*ivmdep*)^2 would be deleted from the model. Do this as an exercise.

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 09:46
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 18 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-26.24975	9.584585	-2.738746	0.0065
LOG(IVMAUT)	8.722965	2.463461	3.540940	0.0005
LOG(IVMDEP)	-1.340396	0.758971	-1.766069	0.0784
LOG(IVMAUT)^2	-0.632018	0.178875	-3.533302	0.0005
LOG(IVMAUT)*LOG(IVMDEP)	0.233949	0.150053	1.559109	0.1200
LOG(IVMDEP)^2	0.000103	0.057516	0.001798	0.9986
AR(1)	0.312239	0.054464	5.732906	0.0000
AR(2)	0.294757	0.054462	5.412147	0.0000
AR(3)	0.272829	0.053455	5.103848	0.0000

R-squared	0.996626	Mean dependent var	8.315690
Adjusted R-squared	0.996539	S.D. dependent var	0.641949
S.E. of regression	0.037765	Akaike info criterion	-3.687065
Sum squared resid	0.442119	Schwarz criterion	-3.580837
Log likelihood	597.0868	Hannan-Quinn criter.	-3.644641
F-statistic	11447.06	Durbin-Watson stat	2.020290
Prob(F-statistic)	0.000000		

Inverted AR Roots	.94	-.31+.44i	-.31-.44i

Figure 5.24 Statistical results based on an AR(3) CES model

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 09:59
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 22 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-27.36947	9.530222	-2.871860	0.0044
LOG(IVMAUT)	8.387862	2.398650	3.496910	0.0005
LOG(IVMDEP)	-0.774316	0.647872	-1.195167	0.2329
LOG(IVMAUT)^2	-0.484796	0.143645	-3.374955	0.0008
LOG(IVMDEP)^2	0.075787	0.034425	2.201489	0.0284
AR(1)	0.316704	0.054484	5.812742	0.0000
AR(2)	0.299434	0.054472	5.497013	0.0000
AR(3)	0.268333	0.053602	5.006023	0.0000

R-squared	0.996600	Mean dependent var	8.315690
Adjusted R-squared	0.996523	S.D. dependent var	0.641949
S.E. of regression	0.037851	Akaike info criterion	-3.685563
Sum squared resid	0.445568	Schwarz criterion	-3.591138
Log likelihood	595.8473	Hannan-Quinn criter.	-3.647853
F-statistic	13022.62	Durbin-Watson stat	2.017075
Prob(F-statistic)	0.000000		

Inverted AR Roots	.94	-.31-.43i	-.31+.43i

Figure 5.25 Statistical results based on a reduced AR(3) CES model

However, for illustration purposes, experimentation has been done by deleting other variable(s). By deleting the interaction factor from the model in Figure 5.23, even though it has a much smaller p-value, the statistical results in Figure 5.25 are obtained . This result shows that $\log(ivmdep)^2$ is significant with a p-value $= 0.0284$, which has the largest p-value in the full models in Figure 5.24. This finding proves that an independent variable having a greater p-value should not be deleted from a model in order to obtain an acceptable model. It is suggested that the variable which is considered less important, in a theoretical sense or based on best judgment, should be deleted. Even though $\log(ivmdep)$ is insignificant with a p-value $= 0.2329$, it may be kept in the model. Otherwise another reduced model, such as that in Figure 5.26, could be found. □

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 10:11
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 13 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-26.74273	8.916015	-2.999404	0.0029
LOG(IVMAUT)	7.376678	2.143617	3.441229	0.0007
LOG(IVMAUT)^2	-0.425219	0.129001	-3.296246	0.0011
LOG(IVMDEP)^2	0.035116	0.001614	21.76198	0.0000
AR(1)	0.310533	0.054507	5.697075	0.0000
AR(2)	0.294002	0.054491	5.395391	0.0000
AR(3)	0.261902	0.053666	4.880221	0.0000

R-squared	0.996587	Mean dependent var	8.315690
Adjusted R-squared	0.996521	S.D. dependent var	0.641949
S.E. of regression	0.037863	Akaike info criterion	-3.687970
Sum squared resid	0.447292	Schwarz criterion	-3.605348
Log likelihood	595.2312	Hannan-Quinn criter.	-3.654974
F-statistic	15182.94	Durbin-Watson stat	2.013476
Prob(F-statistic)	0.000000		

Inverted AR Roots	.93	-.31-.43i	-.31+.43i

Figure 5.26 Statistical results based on another reduced AR(3) CES model

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 10:29
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.772498	0.230773	-3.347436	0.0009
LOG(IVMAUT)	0.186391	0.075987	2.452921	0.0147
LOG(IVMDEP)	0.809002	0.082486	9.807712	0.0000
(LOG(IVMAUT)-LOG(IVMDEP))^2	-0.082777	0.044562	-1.857581	0.0642
AR(1)	0.695415	0.040074	17.35310	0.0000

R-squared	0.995500	Mean dependent var	8.312279
Adjusted R-squared	0.995443	S.D. dependent var	0.641394
S.E. of regression	0.043297	Akaike info criterion	-3.426022
Sum squared resid	0.592378	Schwarz criterion	-3.367277
Log likelihood	554.8765	Hannan-Quinn criter.	-3.402566
F-statistic	17477.12	Durbin-Watson stat	2.536623
Prob(F-statistic)	0.000000		
Inverted AR Roots	.70		

Figure 5.27 Statistical results based on the modified CES model in (5.8)

Example 5.5. (An AR(1) modified CES model) Corresponding to the modified CES model in (4.104), Figure 5.27 presents an AR(1) modified CES model of the model in (5.6) with the following equation:

$$\begin{aligned} \log(mmdep) = \; & c(1) + c(2)^*\log(ivmaut) + c(3)^*\log(ivmdep) \\ & + c(4)^*(\log(ivmaut) - \log(ivmdep))^2 + [ar(1) = c(5)] \end{aligned} \tag{5.8}$$

If a condition is that $c(4) = 0$, the Cobb–Douglas AR(1) model with the statistical results in Figure 5.28 are obtained. Furthermore, by using additional AR indicators, namely AR(2) or AR(3), or both, in the modified CES model, unexpected statistical results would occur. Do this as an exercise. □

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 10:31
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.522827	0.208606	-2.506284	0.0127
LOG(IVMAUT)	0.313060	0.037589	8.328599	0.0000
LOG(IVMDEP)	0.657778	0.017861	36.82861	0.0000
AR(1)	0.722677	0.038650	18.69813	0.0000

R-squared	0.995458	Mean dependent var	8.312279
Adjusted R-squared	0.995415	S.D. dependent var	0.641394
S.E. of regression	0.043430	Akaike info criterion	-3.422933
Sum squared resid	0.597925	Schwarz criterion	-3.375936
Log likelihood	553.3807	Hannan-Quinn criter.	-3.404168
F-statistic	23158.74	Durbin-Watson stat	2.572271
Prob(F-statistic)	0.000000		
Inverted AR Roots	.72		

Figure 5.28 Statistical results based on a reduced model of (5.8)

5.4 Lagged variable models

This section presents alternative models using lagged variables. Two alternative models will be considered: (i) linear models for an endogenous variable, called the basic lagged-variable model or the pure autoregressive model and (ii) linear models for an endogenous and a set of exogenous variables, called the general lagged-variable model.

5.4.1 The basic lagged-variable model

The basic lagged-variable model, namely the LVAR(p) model, can be presented in the following general equation:

$$Y_t = c(1) + \sum_{i=1}^{p} c(1+i)Y_{t-i} + \varepsilon_t \tag{5.9}$$

Note that EViews has been using the letter 'c' for the model parameter, where $c(1)$ indicates the intercept and $c(1+i)$, $i = 1,2,\ldots,p$, are the coefficients of the lagged variables. In fact, corresponding to the basic lagged model in (5.9), a pure AR(p) model may be obtained, which can be presented as

$$\begin{aligned} Y_t &= c(1) + u_t \\ u_t &= \sum_{i=1}^{p} \rho_i u_{t-i} + \varepsilon_t \end{aligned} \tag{5.10}$$

Example 5.6. (Basic lagged-variable models) Figure 5.29 presents the statistical results using two basic lagged-variable models, namely LV(2) models. In order to take into account the unknown heteroskedasticity and autocorrelation of the error terms, the

Dependent Variable: LOG(IVMAUT)
Method: Least Squares
Date: 10/21/07 Time: 10:40
Sample (adjusted): 1968M03 1994M10
Included observations: 320 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.068174	0.019954	3.416540	0.0007
LOG(IVMAUT(-1))	0.994165	0.055955	17.76738	0.0000
LOG(IVMAUT(-2))	-0.001830	0.055546	-0.032948	0.9737

R-squared	0.998241	Mean dependent var	8.411218
Adjusted R-squared	0.998230	S.D. dependent var	0.388015
S.E. of regression	0.016324	Akaike info criterion	-5.383055
Sum squared resid	0.084470	Schwarz criterion	-5.347727
Log likelihood	864.2888	Hannan-Quinn criter.	-5.368948
F-statistic	89960.21	Durbin-Watson stat	1.992839
Prob(F-statistic)	0.000000		

(a)

Dependent Variable: LOG(IVMDEP)
Method: Least Squares
Date: 10/21/07 Time: 10:42
Sample (adjusted): 1968M03 1994M10
Included observations: 320 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007003	0.012107	0.578416	0.5634
LOG(IVMDEP(-1))	1.402347	0.051407	27.27955	0.0000
LOG(IVMDEP(-2))	-0.402846	0.051401	-7.837286	0.0000

R-squared	0.999481	Mean dependent var	9.431413
Adjusted R-squared	0.999478	S.D. dependent var	0.812019
S.E. of regression	0.018557	Akaike info criterion	-5.126587
Sum squared resid	0.109165	Schwarz criterion	-5.091259
Log likelihood	823.2539	Hannan-Quinn criter.	-5.112480
F-statistic	305240.7	Durbin-Watson stat	2.260890
Prob(F-statistic)	0.000000		

(b)

Figure 5.29 Statistical results based on LV(2) models with dependent variables: (a) log (*ivmaut*) and (b) log(*ivmdep*), using the Newey–West estimation method

Newey–West estimation method is used. Based on these results, the following notes and conclusions are presented:

(1) In fact, this figure presents the results based on two LV(2) models with the dependent variables log($ivmaut_t$) and log($ivmdep_t$) respectively. The statistical results are obtained by using or entering the following general equation specification:

$$\log(y)\ c\log(y(-1))\log(y(-2)) \tag{5.11}$$

(2) The result based on the first model shows that the second lagged variable log ($ivmaut(-2)$) does not have a significant partial (adjusted) effect with a large p-value of 0.98. In a statistical sense, this suggests that the model should be reduced. On the other hand, the result of the second model suggests the use of an additional lagged variable(s), since both lagged variables have significant adjusted (partial) effects. However, the analysis based on modified models will not be done here. Do this as an exercise. □

Example 5.7. (Comparison between the lagged-variable model and the auto-regressive model) Corresponding to the two models in (5.9) and (5.10), Figures 5.30 and 5.31 present the statistical results by using the following two alternative equation specifications:

$$\log(mmdep)\ c\log(mmdep(-1))\log(mmdep(-2))\log(mmdep(-3)) \tag{5.12a}$$

and

$$\log(mmdep)\ c\ ar(1)ar(2)ar(3) \tag{5.12b}$$

The model in (5.12a) is an LV(3) model and the model in (5.12b) is an AR(3) model with a dependent variable log($mmdep$). Note that except for the value of $C = C(1)$

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 10:53
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.012819	0.030222	0.424147	0.6717
LOG(MMDEP(-1))	0.470473	0.058428	8.052229	0.0000
LOG(MMDEP(-2))	0.306929	0.068932	4.452605	0.0000
LOG(MMDEP(-3))	0.221805	0.067791	3.271873	0.0012

R-squared	0.996566	Mean dependent var	8.315690
Adjusted R-squared	0.996533	S.D. dependent var	0.641949
S.E. of regression	0.037799	Akaike info criterion	-3.700617
Sum squared resid	0.450057	Schwarz criterion	-3.653404
Log likelihood	594.2484	Hannan-Quinn criter.	-3.681762
F-statistic	30468.88	Durbin-Watson stat	1.975753
Prob(F-statistic)	0.000000		

Figure 5.30 Statistical results based on the LV(3) model in (5.12a)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 10:47
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 5 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

	Coefficient	Std. Error	t-Statistic	Prob.
C	16.15816	33.97830	0.475544	0.6347
AR(1)	0.470473	0.058428	8.052229	0.0000
AR(2)	0.306929	0.068932	4.452605	0.0000
AR(3)	0.221805	0.067791	3.271873	0.0012

R-squared	0.996566	Mean dependent var	8.315690
Adjusted R-squared	0.996533	S.D. dependent var	0.641949
S.E. of regression	0.037799	Akaike info criterion	-3.700617
Sum squared resid	0.450057	Schwarz criterion	-3.653404
Log likelihood	594.2484	Hannan-Quinn criter.	-3.681762
F-statistic	30468.88	Durbin-Watson stat	1.975753
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	-.26+.39i	-.26-.39i

Figure 5.31 Statistical results based on the AR(3) model in (5.12b)

(i.e. *the intercept parameter*), both figures show equal values of the summary statistics, as follows:

(1) The coefficient of log(*mmdep*(−1) equals the coefficient of AR(1), which is the first (partial) autocorrelation, say $\rho_1 = C(2)$ in the models in (5.9) and (5.10), and likewise for the other model parameters $C(3) = \rho_2$ and $C(4) = \rho_3$.
(2) The values of the *t*-test for the adjusted effect of each independent variable, *R*-squared, *DW*-statistic and the *F*-test, are found.
(3) However, the estimation equations are differently written or printed, as presented above. For example, the term $C(2)^*\log(mmdep(-1))$ in the first model corresponds to the term $[AR(1) = C(2)]$ in the second model.
(4) Based on the estimation equations above, *C*(1) in the first model is an intercept parameter, but *C*(1) in the second model corresponds to an *adjusted mean* or *average* of the endogenous variable log(*mmdep*), which can be presented as follows:

$$\begin{aligned} \log(mmdep_t) &= c(1) + u_t \\ u_t &= \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \varepsilon_t \end{aligned} \tag{5.13}$$

On the other hand, *C*(1) may also be named as an intercept of the AR(3) model in (5.12b), because the average value of log(*mmdep*) is 8.310 450 with the Std Err = 0.035 735, while the estimated value of *C*(1) is 16.158 159 52. □

Example 5.8. (Illustration of the AR(*p*) models) Figure 5.32 presents statistical results based on an AR(4) model with an endogenous variable log(*ivmmae*), and its reduced or modified model in Figure 5.33 is obtained by deleting the indicators AR(2) and AR(3), since both have large *p*-values. Note that both models have DW-statistics of 2.09 and 2.20 respectively. Based on these two models only, the reduced model may

Dependent Variable: LOG(IVMMAE)
Method: Least Squares
Date: 10/21/07 Time: 10:58
Sample (adjusted): 1968M05 1994M10
Included observations: 318 after adjustments
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	11.76470	0.529343	22.22510	0.0000
AR(1)	1.162090	0.054442	21.34554	0.0000
AR(2)	0.088207	0.085363	1.033321	0.3023
AR(3)	0.014563	0.085280	0.170768	0.8645
AR(4)	-0.266620	0.054117	-4.926732	0.0000

R-squared	0.999750	Mean dependent var	10.86448
Adjusted R-squared	0.999747	S.D. dependent var	0.516775
S.E. of regression	0.008215	Akaike info criterion	-6.749997
Sum squared resid	0.021126	Schwarz criterion	-6.690845
Log likelihood	1078.250	Hannan-Quinn criter.	-6.726371
F-statistic	313493.7	Durbin-Watson stat	2.090444
Prob(F-statistic)	0.000000		

Inverted AR Roots	.99	.85	-.34+.45i	-.34-.45i

Figure 5.32 Statistical results based on an AR(4) model of log(*ivmmae*)

be considered to be a better model. Furthermore, note that this reduced model is an uncommon or unexpected model, since it has unordered autoregressive indicators.

On the other hand, by doing other processes, by deleting the AR(4) even though it has a significant effect, the results based on the AR(3) and AR(2) models, as presented in Figures 5.34 and 5.35 are obtained . These processes again demonstrate that a significant indicator or exogenous variables can be deleted from a model in order to obtain a statistically acceptable reduced model. Now there are four alternative models. Do you think the AR(2) is the best model?

For further illustration purposes, Figure 5.36 presents the growth curve of log (*ivmmae*) and the residual graph and box plot of the AR(2) model. Note that the residual graph, as well as the box plot, show that there are far outliers, which cannot be identified based on the growth curve of log(*ivmmae*).

Dependent Variable: LOG(IVMMAE)
Method: Least Squares
Date: 10/21/07 Time: 11:00
Sample (adjusted): 1968M05 1994M10
Included observations: 318 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	11.77192	0.534589	22.02050	0.0000
AR(1)	1.225675	0.020807	58.90666	0.0000
AR(4)	-0.227426	0.020530	-11.07791	0.0000

R-squared	0.999749	Mean dependent var	10.86448
Adjusted R-squared	0.999748	S.D. dependent var	0.516775
S.E. of regression	0.008210	Akaike info criterion	-6.757444
Sum squared resid	0.021234	Schwarz criterion	-6.721953
Log likelihood	1077.434	Hannan-Quinn criter.	-6.743269
F-statistic	627763.1	Durbin-Watson stat	2.204744
Prob(F-statistic)	0.000000		

Inverted AR Roots	.99	.83	-.30+.43i	-.30-.43i

Figure 5.33 Statistical results based on an unexpected model of log(*ivmmae*)

Dependent Variable: LOG(IVMMAE)
Method: Least Squares
Date: 10/21/07 Time: 11:02
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	11.85888	0.474422	24.99649	0.0000
AR(1)	1.249019	0.053316	23.42690	0.0000
AR(2)	0.068083	0.088033	0.773382	0.4399
AR(3)	-0.319283	0.052991	-6.025195	0.0000

R-squared	0.999734	Mean dependent var	10.86119
Adjusted R-squared	0.999732	S.D. dependent var	0.519313
S.E. of regression	0.008503	Akaike info criterion	-6.684371
Sum squared resid	0.022774	Schwarz criterion	-6.637159
Log likelihood	1070.157	Hannan-Quinn criter.	-6.665516
F-statistic	395293.6	Durbin-Watson stat	2.173542
Prob(F-statistic)	0.000000		

Inverted AR Roots	.99	.71	-.45

Figure 5.34 Statistical results based on an AR(3) model of log(*ivmmae*)

Dependent Variable: LOG(IVMMAE)
Method: Least Squares
Date: 10/21/07 Time: 11:03
Sample (adjusted): 1968M03 1994M10
Included observations: 320 after adjustments
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	11.95057	0.394197	30.31626	0.0000
AR(1)	1.365757	0.052047	26.24089	0.0000
AR(2)	-0.368710	0.051810	-7.116631	0.0000

R-squared	0.999707	Mean dependent var	10.85788
Adjusted R-squared	0.999705	S.D. dependent var	0.521865
S.E. of regression	0.008960	Akaike info criterion	-6.582776
Sum squared resid	0.025449	Schwarz criterion	-6.547448
Log likelihood	1056.244	Hannan-Quinn criter.	-6.568669
F-statistic	540928.2	Durbin-Watson stat	2.230351
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	.37

Figure 5.35 Statistical results based on an AR(2) model of log(*ivmmae*)

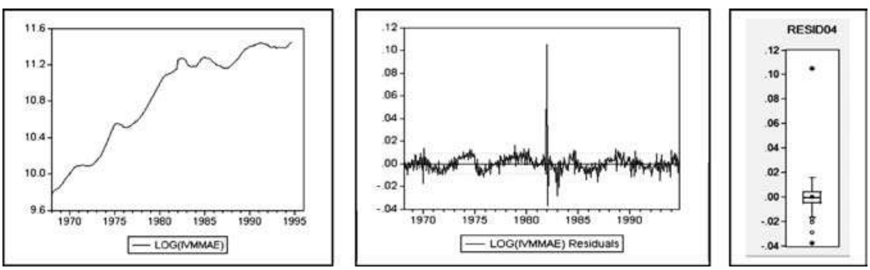


Figure 5.36 Growth curve of log(*ivmmae*), the residual graph and the box plot of its AR(2) model

Dependent Variable: D(D(LOG(IVMMAE)))
Method: Least Squares
Date: 10/21/07 Time: 11:11
Sample (adjusted): 1968M05 1994M10
Included observations: 318 after adjustments
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.74E-05	0.000228	-0.120464	0.9042
AR(1)	-0.731446	0.052341	-13.97453	0.0000
AR(2)	-0.368744	0.052290	-7.051874	0.0000

R-squared	0.382967	Mean dependent var	-1.84E-05
Adjusted R-squared	0.379049	S.D. dependent var	0.010828
S.E. of regression	0.008532	Akaike info criterion	-6.680543
Sum squared resid	0.022932	Schwarz criterion	-6.645052
Log likelihood	1065.206	Hannan-Quinn criter.	-6.666368
F-statistic	97.75373	Durbin-Watson stat	2.169888
Prob(F-statistic)	0.000000		

Inverted AR Roots	-.37+.48i	-.37-.48i

Figure 5.37 Statistical results based on an AR(2) model of $d(d(\log(ivmmae)))$

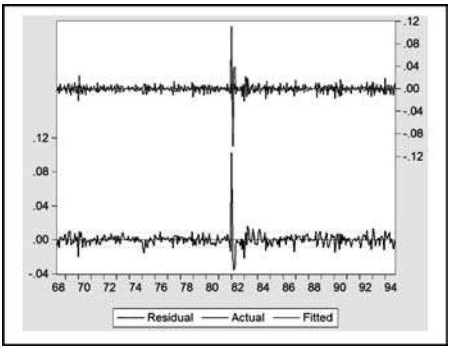


Figure 5.38 Residual graph of the regression in Figure 5.37

On the other hand, the residual graph indicates heterogeneity of the error terms. For this reason, modified AR(2) models are presented with endogenous variables: the first and second differences of log(*ivmmae*), namely *d*(log(*ivmmae*)), and *d*(*d*(log(*ivmmae*))). Then the AR(2) model of *d*(*d*(log(*ivmmae*))) is obtained, which is considered to be the best model, with the statistical results given in Figure 5.37 and its residual graph in Figure 5.38. Based on these results, the following notes and conclusions are presented:

(1) The residual graph indicates that there is a breakpoint or an outlier. However, the AR (2) model of the second difference can be considered as an accep model. Note that this model is quite different from the model based on the original variable log(*ivmmae*).
(2) By observing the raw data set, it is found that *d*(*d* log(*ivmmae*)) has a maximum value of 0.110 771 at 1982 : 1 and a minimum value of −0.101 873 at 1982 : 2, which should be considered as outliers. Hence, there can be three alternative data analyses, as follows:
   (i) The first data analysis is based on the subset of data without the outliers.

(ii) The second data analysis is based on a modified data set, which is constructed by replacing the outliers with the average of the observed values or the average of adjacent observed values of the outliers, or by interpolation.

(iii) The third data analysis is based on the original model, by adding dummy variables of the two outliers as independent variables, which are defined as $DO1 = 1$ if $d(d\,\log(ivmmae)) = 0.110\,771$ and $DO1 = 0$ if otherwise, and $DO2 = 1$ if $d(d\,\log(ivmmae)) = -0.101\,873$ and $DO2 = 0$ if otherwise. Do this as an exercise.

(3) Corresponding to the model in Figure 5.37, the model with dummy variables will be considered as follows:

$$\begin{aligned} d(d\log(ivmmae_t)) = c(1) + c(2)DO1 + c(3)DO2 \\ + [ar(1) = c(4), ar(2) = c(5)] + \varepsilon_t \end{aligned} \tag{5.14}$$

This model, in fact, represents a set of four models, corresponding to the four possible values of the two dummy variables, which are $(DO1, DO2) = (0,0), (1,0), (0,1)$ and $(1,1)$, with the following equations respectively:

$$\begin{aligned} d(d\log(ivmmae_t)) &= c(1) + [ar(1) = c(4), ar(2) = c(5)] + \varepsilon_t \\ d(d\log(ivmmae_t)) &= c(1) + c(2) + [ar(1) = c(4), ar(2) = c(5)] + \varepsilon_t \\ d(d\log(ivmmae_t)) &= c(1) + c(3) + [ar(1) = c(4), ar(2) = c(5)] + \varepsilon_t \\ d(d\log(ivmmae_t)) &= c(1) + c(2) + c(3) + [ar(1) = c(4), ar(2) = c(5)] + \varepsilon_t \end{aligned} \tag{5.15}$$

Note that, compared to the first model, which is the model based on the subdata set without the outliers, the parameters $c(2)$, $c(3)$ and $\{c(2) + c(3)\}$ represent the effects of each outlier and both outliers. □

Example 5.9. (Autocorrelation (AC) and partial autocorrelation (PAC)) Corresponding to the alternative AR(2) models presented in the previous examples, namely Examples 5.7 and 5.8, Figure 5.39 presents illustrative graphs for the variable log(*ivmmae*) and three residual correlograms, namely the correlograms of log(*ivmmae*), *d*(log(*ivmmae*) and *d*(*d*(log(*ivmmae*). The process of constructing a graph has been presented in Chapter 1, such as click *Show*. . . , insert the corresponding endogenous variable and then click *OK* to present the data on the screen. Then select click *View/Graph*. . . or *View/Correlogram*. . . options.

Please note that the growth curve of log(*ivmmae*) is nonlinear and is a waving curve. However, its correlogram shows that only the first partial autocorrelation (PAC) is significant. On the other, the PACs of its first difference are significantly positive at higher orders or levels, but the PACs of its second differences are significantly negative at higher levels. Furthermore, note that their significant differences of the aucorrelations are at a certain level k, namely ρ_k These findings indicate that the models based on the endogenous variables log(*ivmmae*), *d*(log(*ivmmae*) and *d*(*d*(log (*ivmmae*) are in fact representing three different models. □

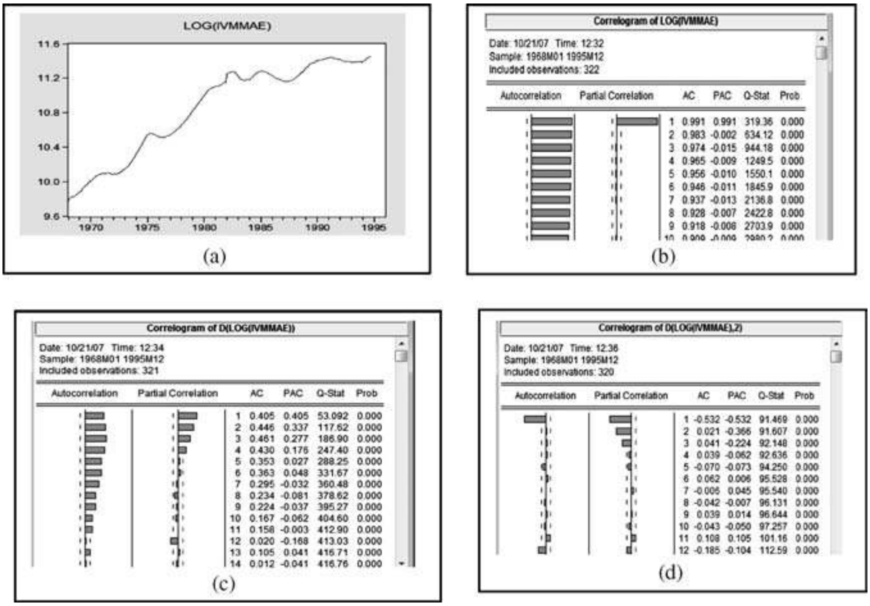


Figure 5.39 Illustrative graphs for the endogenous variable log(*ivmmae*): (a) the growth curve of log(*ivmmae*), (b) correlogram of log(*ivmae*), (c) correlogram of *d* log(*ivmae*) and (d) correlogram of *d*(*d* log(*ivmae*))

5.4.2 Some notes

Based on various examples, the following question and notes are presented:

(a) Should all lagged variables, Y_{t-1}, Y_{t-2}, ... and Y_{t-p}, be used for each selected value of p?
(b) On the other hand, only the lagged variables might be used, which happen to have a significant effect(s), based on a time series. In a statistical sense, this process could be done easily. However, if a researcher is using this procedure, then it could be said that he/she has been highly dependent on the sample statistics to develop a model. It is suggested that a researcher should be using personal best judgment (see Section 4.10.1), since sample data could lead to an unexpected conclusion of a testing hypothesis.
(c) This is similar for the autoregressive (AR) models. Refer to the growth model presented by Yaffee and McGee (2000) in Example 4.22. They present a model with three unordered autoregressive indicators, which are AR(1), AR(5) and AR(6).

5.4.3 Generalized lagged-variable autoregressive model

This subsection will present examples of seemingly causal models having a combination of the lagged (endogenous) variables and autoregressive indicators as independent variables, namely LVAR(p,q), starting with the simplest model for $p = q = 1$. The simplest model considered has the general equation

$$\begin{aligned} Y_t &= c(1) + c(2)^* Y_{t-1} + u_t \\ u_t &= \rho_1 u_{t-1} + \varepsilon_t \end{aligned} \tag{5.16}$$

or

$$Y_t = c(1) + c(2)^* Y_{t-1} + c(3)^* \mu_{t-1} + \varepsilon_t \tag{5.17}$$

In the data analysis, the following estimation equation is used or entered:

$$Y_t = c(1) + c(2)^* Y_{t-1} + [Ar(1) = c(3)] \tag{5.18}$$

Example 5.10. (Comparing simple models) For a first comparison, Figure 5.40 presents statistical results using two alternative models having the following equation specifications:

$$\log(mmdep) = C(11) + C(12)^* \log(mmdep(-1)) \tag{5.19}$$

$$\log(mmdep) = C(21) + [AR(1) = C(22)] \tag{5.20}$$

Both statistical results show that the estimated values of $C(12)$ and $C(22)$ are equal to 0.997 736. Based on the first model in (5.19), $C(12)$ indicates the effect of log ($mmdep(-1)$) on log($mmdep$) and $C(22)$ indicates the first-order autocorrelation or serial correlation of the error terms of the model in (5.20), which can be considered as an AR(1) mean model of log($mmdep$). Are they equal, in a mathematical statistics sense? Or are they equal up to certain decimal points? A theoretical explanation of these findings has not yet been found.

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:20
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
LOG(MMDEP)=C(11) + C(12)*LOG(MMDEP(-1))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.022448	0.031178	0.720001	0.4721
C(12)	0.997736	0.003741	266.6730	0.0000

R-squared	0.995534	Mean dependent var	8.312279
Adjusted R-squared	0.995520	S.D. dependent var	0.641394
S.E. of regression	0.042929	Akaike info criterion	-3.452335
Sum squared resid	0.587881	Schwarz criterion	-3.428836
Log likelihood	556.0997	Hannan-Quinn criter.	-3.442952
F-statistic	71114.50	Durbin-Watson stat	2.858407
Prob(F-statistic)	0.000000		

(a)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:21
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 3 iterations
LOG(MMDEP)=C(21)+[AR(1)=C(22)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(21)	9.916307	2.860085	3.467137	0.0006
C(22)	0.997736	0.003741	266.6730	0.0000

R-squared	0.995534	Mean dependent var	8.312279
Adjusted R-squared	0.995520	S.D. dependent var	0.641394
S.E. of regression	0.042929	Akaike info criterion	-3.452335
Sum squared resid	0.587881	Schwarz criterion	-3.428836
Log likelihood	556.0997	Hannan-Quinn criter.	-3.442952
F-statistic	71114.50	Durbin-Watson stat	2.858407
Prob(F-statistic)	0.000000		
Inverted AR Roots	1.00		

(b)

Figure 5.40 Statistical results based on (a) the LV(1)_MODEL in (5.19) and (b) the AR(1)_Model in (5.20)

On the other hand, it has been known that $C(12)$ is not equal to the correlation coefficient between log(*mmdep*) and log(*mmdep*(−1)), which is in fact equal to 0.997 765. For a further comparison, an LVAR(1,1) model is applied in the following equation with the dependent variable log(*mmdep*):

$$\log(mmdep) = C(1) + C(2)*\log(mmdep(-1)) + [AR(1) = C(3)] \qquad (5.21)$$

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/21/07 Time: 13:43
Sample (adjusted): 1968M03 1994M10
Included observations: 320 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.009600	0.019620	0.489318	0.6250
LOG(MMDEP(-1))	0.999265	0.002354	424.4545	0.0000
AR(1)	-0.433969	0.050421	-8.606836	0.0000

R-squared	0.996394	Mean dependent var	8.313917
Adjusted R-squared	0.996371	S.D. dependent var	0.641726
S.E. of regression	0.038656	Akaike info criterion	-3.658879
Sum squared resid	0.473698	Schwarz criterion	-3.623551
Log likelihood	588.4206	Hannan-Quinn criter.	-3.644771
F-statistic	43797.47	Durbin-Watson stat	2.188296
Prob(F-statistic)	0.000000		

Inverted AR Roots	-.43

Figure 5.41 Statistical results based on the LVAR(1,1) model in (5.21)

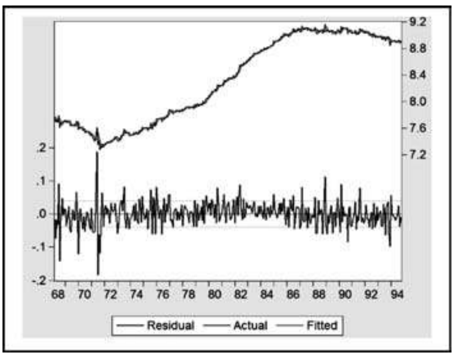


Figure 5.42 Residual graph of the regression in Figure 5.41

Based on the results in Figure 5.41, the following notes and conclusions can be made:

(1) The effect of log(*mmdep*(−1)) is 0.999 265, which is a little bit larger than the effect based on the model in (5.19), and the estimated value of the first-order autocorrelation of the error terms, $C(3)$, is a negative value of −0.433 969.

However, note that, based on this model, $C(3)$ is not the first-order serial correlation of log(*mmdep*), but of the error term u_t in the following model, which is in fact the same as the model in (5.21):

$$\begin{aligned} \log(mmdep) &= C(1) + C(2)^{*}\log(mmdep(-1)) + \mu_t \\ \mu_t &= C(3)^{*}\mu_{t-1} + \varepsilon_t \end{aligned} \tag{5.22}$$

(2) The DW-statistic of 2.188 296 is sufficient to accept the null hypothesis of no first-order serial correlation of the error terms ε_t. This hypothesis also can be tested by using the BG serial correlation LM test.
(3) $R^2 = 0.996\,394$, which is very close to one, indicates that the fitted values of the model are very close to the observed values.
(4) However, the residual graph in Figure 5.42 indicates that there is a breakpoint or an outlier. Corresponding to this problem, refer to Example 5.8. □

Example 5.11. (Models with exogenous variables) As an extension of the previous models, Figure 5.43 presents the statistical results based on the following LVAR(1,1) model with two exogenous variables log(*ivmaut*) and log(*ivmaut*(−1)):

$$\begin{aligned} \log(mmdep) &= c(1) + c(2)^{*}\log(mmdep(-1)) + c(3)^{*}\log(ivmaut) \\ &\quad + c(4)^{*}\log(ivmaut(-1)) + [ar(1) = c(5)] \end{aligned} \tag{5.23}$$

This figure shows that each of the exogenous variables log(*ivmaut*) and log (*ivmaut*(−1)) have an insignificant adjusted effect with a large *p*-value. One of the main reasons for this is that the two variables have a high correlation, which is 0.999 127. Hence, they should not be used as independent variables of the model, and a reduced model should be presented by deleting one of them.

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:33
Sample (adjusted): 1968M03 1994M10
Included observations: 320 after adjustments
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.099759	0.035836	-2.783726	0.0057
LOG(MMDEP(-1))	0.984455	0.004466	220.4546	0.0000
LOG(IVMAUT)	-0.072139	0.119199	-0.605196	0.5455
LOG(IVMAUT(-1))	0.099820	0.118976	0.838996	0.4021
AR(1)	-0.446221	0.050174	-8.893501	0.0000

R-squared	0.996558	Mean dependent var	8.313917
Adjusted R-squared	0.996515	S.D. dependent var	0.641726
S.E. of regression	0.037885	Akaike info criterion	-3.693001
Sum squared resid	0.452120	Schwarz criterion	-3.634121
Log likelihood	595.8802	Hannan-Quinn criter.	-3.669489
F-statistic	22802.89	Durbin-Watson stat	2.229381
Prob(F-statistic)	0.000000		

Inverted AR Roots -.45

Figure 5.43 Statistical results based on the LVAR(1,1) model in (5.23)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:34
Sample (adjusted): 1968M03 1994M10
Included observations: 320 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.103846	0.035096	-2.958905	0.0033
LOG(MMDEP(-1))	0.984652	0.004444	221.5835	0.0000
LOG(IVMAUT(-1))	0.027940	0.007278	3.839061	0.0001
AR(1)	-0.448314	0.050043	-8.958669	0.0000

R-squared	0.996554	Mean dependent var	8.313917
Adjusted R-squared	0.996522	S.D. dependent var	0.641726
S.E. of regression	0.037847	Akaike info criterion	-3.698094
Sum squared resid	0.452644	Schwarz criterion	-3.650990
Log likelihood	595.6951	Hannan-Quinn criter.	-3.679285
F-statistic	30464.98	Durbin-Watson stat	2.232328
Prob(F-statistic)	0.000000		
Inverted AR Roots	-.45		

Figure 5.44 Statistical results based on on the LVAR(1,1) in (5.21)

Comparing the two possible reduced models, it is found that the best reduced model is the model having the independent variable log(*ivmaut*(−1)), with lower values of AIC and SC. The results are presented in Figure 5.44.

By using the trial-and-error methods, two acceptable models have been found, an LVAR(1,2) model and an LV(3) model, with the statistical results presented in Figure 5.45, using the two variables *MMDEP* and *IVMAUT*. His example has demonstrated that various LVAR(p, q) models could be applied based on only two variables, but their estimates are highly dependent on the data set used. Refer to the special notes and comments presented in Section 2.14. □

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 08/03/08 Time: 10:53
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.099435	0.027527	-3.612309	0.0004
LOG(MMDEP(-1))	0.986210	0.003423	288.1204	0.0000
LOG(IVMAUT)	0.025867	0.005671	4.561021	0.0000
AR(1)	-0.567540	0.054577	-10.39894	0.0000
AR(2)	-0.257923	0.054434	-4.738293	0.0000

R-squared	0.996773	Mean dependent var	8.315690
Adjusted R-squared	0.996732	S.D. dependent var	0.641949
S.E. of regression	0.036698	Akaike info criterion	-3.756621
Sum squared resid	0.422885	Schwarz criterion	-3.697605
Log likelihood	604.1811	Hannan-Quinn criter.	-3.733052
F-statistic	24247.81	Durbin-Watson stat	1.995370
Prob(F-statistic)	0.000000		
Inverted AR Roots	-.28+.42i	-.28-.42i	

(a)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 08/03/08 Time: 10:51
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.181199	0.051104	-3.545712	0.0005
LOG(MMDEP(-1))	0.419265	0.054577	7.682130	0.0000
LOG(MMDEP(-2))	0.301421	0.056663	5.319541	0.0000
LOG(MMDEP(-3))	0.254452	0.053800	4.729567	0.0000
LOG(IVMAUT)	0.046861	0.010516	4.456030	0.0000

R-squared	0.996770	Mean dependent var	8.315690
Adjusted R-squared	0.996729	S.D. dependent var	0.641949
S.E. of regression	0.036716	Akaike info criterion	-3.755664
Sum squared resid	0.423290	Schwarz criterion	-3.696649
Log likelihood	604.0285	Hannan-Quinn criter.	-3.732096
F-statistic	24224.55	Durbin-Watson stat	1.998004
Prob(F-statistic)	0.000000		

(b)

Figure 5.45 Statistical results based on (a) the LVAR(1,2) model and (b) the LV(3) model with a dependent variable log(*mmdep*)

Example 5.12. (The AR(p) and LV(p) models with exogenous variables) To study the differences between an AR(p) model and an LV(p) model, Figures 5.46 and 5.47 present the statistical results based on AR(3) and LV(3) models having

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:44
Sample (adjusted): 1968M06 1994M10
Included observations: 317 after adjustments
Convergence achieved after 14 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	13.70381	11.88665	1.152875	0.2499
LOG(IVMAUT)	-0.127116	0.132303	-0.960800	0.3374
LOG(IVMAUT(-1))	0.226331	0.149413	1.514798	0.1308
LOG(IVMAUT(-2))	-0.148295	0.132069	-1.122864	0.2624
AR(1)	0.467827	0.055459	8.435469	0.0000
AR(2)	0.311549	0.058913	5.288280	0.0000
AR(3)	0.219247	0.055446	3.954241	0.0001

R-squared	0.996616	Mean dependent var	8.319558
Adjusted R-squared	0.996550	S.D. dependent var	0.642116
S.E. of regression	0.037715	Akaike info criterion	-3.695703
Sum squared resid	0.440942	Schwarz criterion	-3.612699
Log likelihood	592.7689	Hannan-Quinn criter.	-3.662547
F-statistic	15214.96	Durbin-Watson stat	1.985361
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	-.27-.39i	-.27+.39i

Figure 5.46 Statistical results based on the AR(3) model in (5.24)

exactly the same exogenous variables, namely log($ivmaut_t$), log($ivmaut_{t-1}$) and log ($ivmaut_{t-2}$). Here, the Newey–West estimation method is used in order to take into account the unknown autocorrelation and heteroskedasticity of the error terms, with the following equation specifications:

$$\begin{aligned} \log(mmdep_t) &= c(1) + c(2)\log(ivmaut_t) + c(3)\log(ivmaut_{t-1}) + c(4)\log(ivamut_{t-2}) \\ &\quad + [ar(1) = c(5), ar(2) = c(6), ar(3) = c(7)] + \varepsilon_t \end{aligned} \tag{5.24}$$

$$\begin{aligned} \log(mmdep_t) &= c(1) + c(2)\log(ivmaut_t) + c(3)\log(ivmaut_{t-1}) \\ &\quad + c(4)\log(ivmaut_{t-2}) + c(5)\log(mmdep_{t-1}) \\ &\quad + c(6)\log(mmdep_{t-2}) + c(7)\log(mmdep_{t-3}) + \varepsilon_t \end{aligned} \tag{5.25}$$

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:42
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.178460	0.051446	-3.468856	0.0006
LOG(IVMAUT)	-0.114231	0.126675	-0.901761	0.3679
LOG(IVMAUT(-1))	0.277622	0.178982	1.551113	0.1219
LOG(IVMAUT(-2))	-0.116752	0.126603	-0.922195	0.3571
LOG(MMDEP(-1))	0.426092	0.054756	7.781658	0.0000
LOG(MMDEP(-2))	0.297551	0.056728	5.245236	0.0000
LOG(MMDEP(-3))	0.251406	0.053808	4.672286	0.0000

R-squared	0.996795	Mean dependent var	8.315690
Adjusted R-squared	0.996734	S.D. dependent var	0.641949
S.E. of regression	0.036689	Akaike info criterion	-3.751006
Sum squared resid	0.419967	Schwarz criterion	-3.668385
Log likelihood	605.2855	Hannan-Quinn criter.	-3.718010
F-statistic	16174.21	Durbin-Watson stat	1.995747
Prob(F-statistic)	0.000000		

Figure 5.47 Statistical results based on the LV(3) model in (5.25)

Based on the statistical results in Figures 5.46 and 5.47, the following notes and conclusions are presented:

(1) In both models, log(*ivmaut*) and log(*ivmaut*(−2)) have insignificant adjusted effects. The main reason for these findings should be the multicolinearity between the three exogenous variables log(*ivmaut*), log(*ivmaut*(−1)) and log(*ivmaut* (−2)). As a result, an attempt should be made to obtain a modified or reduced model by deleting either one of these variables, at each stage of the analysis. Do this as an exercise and several unexpected results will be seen.
(2) The values of the DW-statistics are sufficient to accept the null hypothesis of no first-order serial correlation of the error terms in both models. □

Example 5.13. (Translog linear model of a set of exogenous variables) The models in the previous examples could easily be extended to models having a set of exogenous variables, specifically using the first lagged exogenous variables. After doing experimentation, the four best translog linear models or Cobb–Douglas SCM (i.e. seemingly causal model) were found, as presented in Figures 5.48 to 5.51. In a statistical sense, these models are acceptable models, but the model in Figure 5.51 should be considered as an unusual or unexpected model, since it has log(*mmdep*(−2)) and AR(2) as independent variables, without log(*mmdep*(−1)) and AR(1).

However, it is certain that there are many other acceptable or unexpected models that can be presented using the four time series variables *ivmaut*, *ivmdep*, *ivmmae* and *mmdep*.

On the other hand, in the process of doing experimentation a poor or worst model may be found. For example, the results based on the following model is unacceptable, in a statistical sense, since each independent has an insignificant adjusted effect, but the joint effects of all independent variables is significant based on the *F*-statistic with a *p*-value = 0.000, as presented in Figure 5.52. However, by replacing the indicator

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:51
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.266786	0.054199	-4.922308	0.0000
LOG(IVMAUT(-1))	0.037702	0.021335	1.767195	0.0782
LOG(IVMDEP(-1))	0.076948	0.023969	3.210343	0.0015
LOG(IVMMAE(-1))	0.043116	0.018228	2.365424	0.0186
LOG(MMDEP(-1))	0.367219	0.054823	6.698257	0.0000
LOG(MMDEP(-2))	0.260167	0.056574	4.598724	0.0000
LOG(MMDEP(-3))	0.223690	0.053891	4.150802	0.0000

R-squared	0.996938	Mean dependent var	8.315690
Adjusted R-squared	0.996879	S.D. dependent var	0.641949
S.E. of regression	0.035862	Akaike info criterion	-3.796582
Sum squared resid	0.401256	Schwarz criterion	-3.713960
Log likelihood	612.5548	Hannan-Quinn criter.	-3.763586
F-statistic	16930.84	Durbin-Watson stat	1.964370
Prob(F-statistic)	0.000000		

Figure 5.48 Statistical results based on an LV(3) model of log(*mmdep*)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/24/07 Time: 14:48
Sample (adjusted): 1968M05 1994M10
Included observations: 318 after adjustments
Convergence achieved after 18 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.863444	0.439095	-1.966418	0.0501
LOG(IVMAUT(-1))	0.222371	0.088165	2.522212	0.0122
LOG(IVMDEP(-1))	0.617279	0.038097	16.20289	0.0000
LOG(IVMMAE(-1))	0.136663	0.092029	1.485004	0.1386
AR(1)	0.370891	0.054672	6.783898	0.0000
AR(2)	0.252435	0.057064	4.423689	0.0000
AR(3)	0.256120	0.054316	4.715328	0.0000

R-squared	0.997169	Mean dependent var	8.317632
Adjusted R-squared	0.997115	S.D. dependent var	0.642022
S.E. of regression	0.034486	Akaike info criterion	-3.874786
Sum squared resid	0.369858	Schwarz criterion	-3.791973
Log likelihood	623.0909	Hannan-Quinn criter.	-3.841710
F-statistic	18260.04	Durbin-Watson stat	1.999084
Prob(F-statistic)	0.000000		

Inverted AR Roots .93 -.28+.44i -.28-.44i

Figure 5.49 Statistical results based on an AR model of log(*mmdep*)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/25/07 Time: 04:27
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 6 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.145689	0.030743	-4.738909	0.0000
LOG(IVMAUT(-1))	0.016914	0.012201	1.386299	0.1666
LOG(IVMDEP(-1))	0.034175	0.014072	2.428533	0.0157
LOG(IVMMAE(-1))	0.025344	0.010321	2.455649	0.0146
LOG(MMDEP(-1))	0.928976	0.021177	43.86740	0.0000
AR(1)	-0.548924	0.057156	-9.604030	0.0000
AR(2)	-0.238050	0.056743	-4.195234	0.0000

R-squared	0.996898	Mean dependent var	8.315690
Adjusted R-squared	0.996838	S.D. dependent var	0.641949
S.E. of regression	0.036095	Akaike info criterion	-3.783608
Sum squared resid	0.406496	Schwarz criterion	-3.700986
Log likelihood	610.4854	Hannan-Quinn criter.	-3.750612
F-statistic	16711.93	Durbin-Watson stat	1.983398
Prob(F-statistic)	0.000000		

Inverted AR Roots -.27-.40i -.27+.40i

Figure 5.50 Statistical results based on an LVAR(1,2) model of log(*mmdep*)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/25/07 Time: 04:29
Sample (adjusted): 1968M05 1994M10
Included observations: 318 after adjustments
Convergence achieved after 6 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.294236	0.049139	-5.987813	0.0000
LOG(IVMAUT(-1))	0.043361	0.019570	2.215759	0.0274
LOG(IVMDEP(-1))	0.096277	0.022973	4.190944	0.0000
LOG(IVMMAE(-1))	0.049168	0.016495	2.980805	0.0031
LOG(MMDEP(-2))	0.818878	0.034143	23.98346	0.0000
AR(2)	-0.244789	0.059653	-4.103558	0.0001

R-squared	0.996117	Mean dependent var	8.317632
Adjusted R-squared	0.996055	S.D. dependent var	0.642022
S.E. of regression	0.040324	Akaike info criterion	-3.565031
Sum squared resid	0.507331	Schwarz criterion	-3.494048
Log likelihood	572.8399	Hannan-Quinn criter.	-3.536680
F-statistic	16008.94	Durbin-Watson stat	1.574317
Prob(F-statistic)	0.000000		

Figure 5.51 Statistical results based on an unexpected model

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/25/07 Time: 04:38
Sample (adjusted): 1968M04 1994M10
Included observations: 319 after adjustments
Convergence achieved after 16 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.193236	0.266452	-0.725218	0.4689
LOG(IVMAUT(-1))	0.030843	0.043264	0.712919	0.4764
LOG(IVMDEP)	0.067582	0.093822	0.720321	0.4719
LOG(IVMMAE)	0.030833	0.046002	0.670265	0.5032
LOG(MMDEP(-1))	0.651432	1.757180	0.370726	0.7111
LOG(MMDEP(-2))	0.224144	1.588473	0.141106	0.8879
AR(1)	-0.255839	1.748784	-0.146296	0.8838

R-squared	0.996820	Mean dependent var	8.315690
Adjusted R-squared	0.996759	S.D. dependent var	0.641949
S.E. of regression	0.036549	Akaike info criterion	-3.758636
Sum squared resid	0.416775	Schwarz criterion	-3.676014
Log likelihood	606.5025	Hannan-Quinn criter.	-3.725640
F-statistic	16298.49	Durbin-Watson stat	2.071352
Prob(F-statistic)	0.000000		

Inverted AR Roots	-.26

Figure 5.52 Statistical results based on an LVAR(2,1) model of log(*mmdep*)

Dependent Variable: LOG(MMDEP)
Method: Least Squares
Date: 10/25/07 Time: 04:39
Sample (adjusted): 1968M05 1994M10
Included observations: 318 after adjustments
Convergence achieved after 6 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.208061	0.046311	-4.492696	0.0000
LOG(IVMAUT(-1))	0.028360	0.018608	1.524101	0.1285
LOG(IVMDEP)	0.058671	0.021506	2.728151	0.0067
LOG(IVMMAE)	0.034493	0.015797	2.183523	0.0297
LOG(MMDEP(-1))	0.413606	0.055788	7.413947	0.0000
LOG(MMDEP(-2))	0.471716	0.059189	7.969669	0.0000
AR(2)	-0.195320	0.067531	-2.892302	0.0041

R-squared	0.996832	Mean dependent var	8.317632
Adjusted R-squared	0.996771	S.D. dependent var	0.642022
S.E. of regression	0.036481	Akaike info criterion	-3.762299
Sum squared resid	0.413892	Schwarz criterion	-3.679486
Log likelihood	605.2055	Hannan-Quinn criter.	-3.729223
F-statistic	16311.83	Durbin-Watson stat	2.110963
Prob(F-statistic)	0.000000		

Figure 5.53 Statistical results based on an unexpected model of log(*mmdep*)

AR(1) with AR(2), an acceptable estimate can be obtained of the model in Figure 5.53, which is really unexpected. □

5.5 Cases based on the US domestic price of copper

Based on time series data, namely 'the US domestic price of copper, 1951–1980,' as an exercise, Gujarati (2003, Table 12.7, p. 499) proposed the application of a translog linear (Cobb–Douglas) model as follows:

$$\ln(P_t) = \beta_1 + \beta_2 \ln(G_t) + \beta_3 \ln(I_t) + \beta_4 \ln(L_t) + \beta_5 \ln(H_t) + \beta_6 \ln(A_t) + \mu_t \quad (5.26)$$

where $P =$ 12-month average US domestic price of copper (cents per pound), $G =$ annual gross national product (\$ billions), $I =$ 12-month average index of industrial production, $L = 12$-month average London Metal Exchange price of copper (pounds sterling), $H =$ number of housing starts per year (thousands of units) and $A = 12$-month average price of aluminum (cents per pound).

Based on this data set, a workfile has been developed, called Gujarati_12.7. In this section, several alternative models will be presented, besides the model (5.26) proposed by Gujarati, as illustrative examples.

5.5.1 Graphical representation

Graphical representation based on pairs of variables should be considered as the first stage of analysis in developing a regression, even though it is difficult to predict or judge what type of relationship will occur in the corresponding multidimensional space.

To study the relationship between the exogenous variable P and the other five variables, G, I, L, H and A, Figure 5.54 presents the growth curve of each variable. The first question to arise is 'What type of a growth curve equation or model could be a good fit for each of the variables'? Then, how can a true population growth model be predicted or defined?

On the other hand, in most cases, without using or considering a bivariate graph, a researcher directly defines a model to present the relationship between the variables,

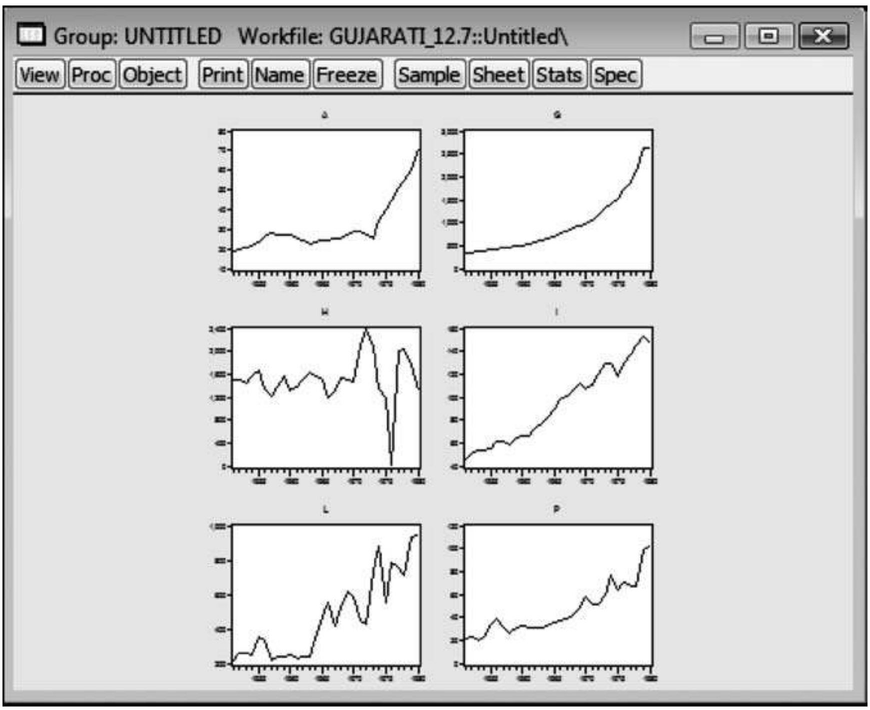


Figure 5.54 Growth curves of the variables A, G, H, I, L and P in the US domestic price of copper data, 1951–1980

and he/she assumes that the model is the true population model. This is also the case for the Cobb–Douglas functions presented above. Data analysis based on any defined models can easily be done using a package program, including EViews. However, some cases there will be error messages, which are highly dependent on the data set, as well as the starting coefficient values in the iterative procedure.

Example 5.14. (Scatter plots with regression lines) For further exercises and discussion, look at the graphical relationship between the pairs of variables, (G, P), (H, P), (A, P) and (I, P), presented in Figure 5.55, since P will be taken as an endogenous variable and the others are exogenous variables. Note that the scatter graph of (L, P) is not presented.

The process of obtaining the scatter graphs with regression lines using EViews 6 are as follows:

(1) Present the variables on the screen in a series: P, G, H, A and I.
(2) Click *View/Graph...*, which produces the options window in Figure 5.56
(3) Then by clicking *OK*, the graphs will appear on the screen.

Based on these graphs, the following notes and conclusions are obtained:

(a) Two of the observed values of the P-variable can be considered as out of the others as a group, or they might be outliers. Hence, two possible alternative data analyses

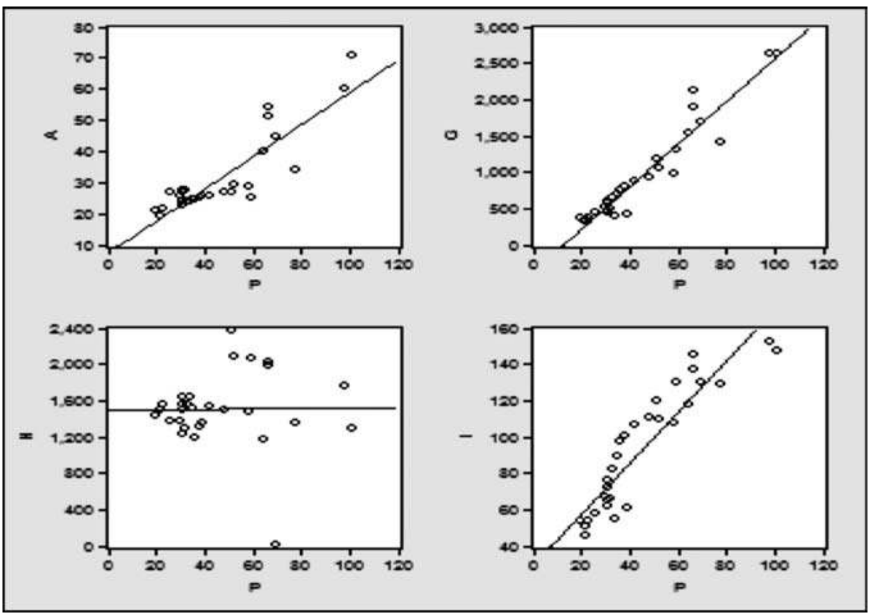


Figure 5.55 Scatter graphs with regression lines of G, H, A and I on P

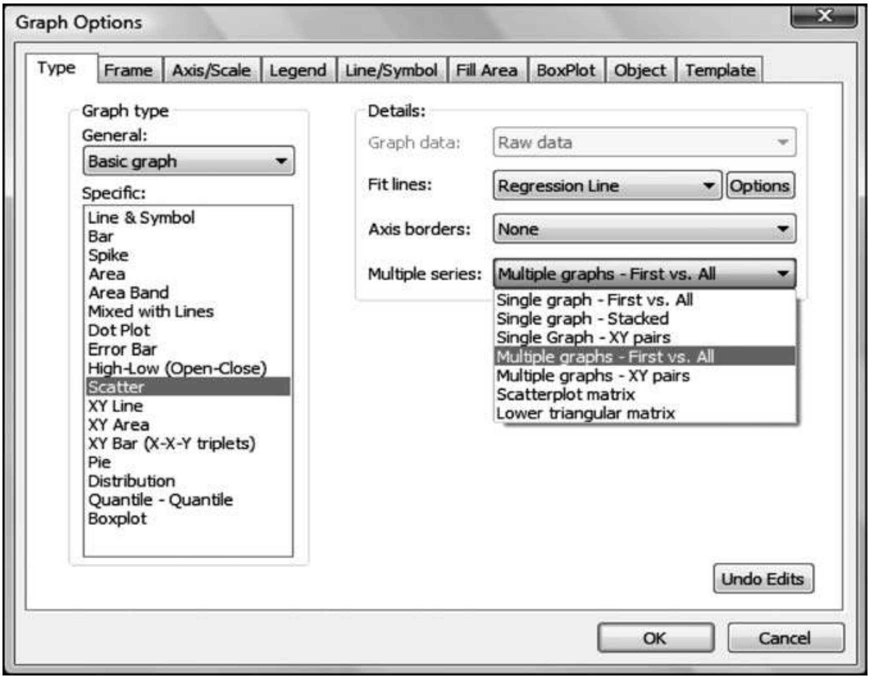


Figure 5.56 The graph options in EViews 6

can be presented. The first data analysis uses the whole unit of observations and the second does not use the two outliers.

(b) The scatter plot (*H*, *P*) shows that the *H*-variable may not need to be used as an independent variable, because the regression line is almost horizontal, even though in a multidimensional space this could be different.

(c) On the other hand, because of the time series data, an autoregressive model should be used, but the Newey–West estimation method as presented in the previous section as well as in the previous chapters could also be used. □

The following examples present cases of regression models based on specific selected groups of the six variables.

5.5.2 Seemingly causal model

Even though Gujarati proposed a translog linear model in (5.26) with five independent variables, in this section alternative models are presented, starting with the simplest model with one exogenous variable.

Since the US_DPOC data is a time series data set, the autoregressive models can be directly applied using the Newey–West estimation method in order to anticipate the unknown forms of the serial or autocorrelation and heteroskadisticity of the error terms.

5.5.2.1 Simplest seemingly causal models

A seemingly causal model (SCM) is defined as the simplest SCM if the model uses an exogenous variable, including its lags. An SCM will be called a simple model if and only if the SCM is defined based on only two time series, namely an endogenous variable Y_t and an exogenous variable X_t. Here, the findings from the experimentation are presented in order to obtain several simple SCMs using the endogenous variable P and each of the variables G, L, I and A as an independent variable.

A model is defined as an acceptable model if and only if its DW-statistic is around two and the p-value of each independent variable is strictly less than 0.20. The reason for selecting this upper bound is that the corresponding independent variable would have either a negative or a positive significant adjusted effect on the dependent variable, at a significant level of $\alpha = 0.10$. For a comparison, in fact Hosmer and Lemeshow (2000, p. 95) stated that 'Any variable whose univariable test has a p-value < 0.25 is a candidate for the multivariable model along with all variables of known clinical importance.'

Example 5.15. (Simple AR(p) SCM with one exogenous variable) By using the trial-and-error methods four acceptable simple SCMs with the endogenous variable log (P) have been obtained, as presented in Figures 5.57 to 5.60. The regressions in Figures 5.57 and 5.58 are considered to be the simplest linear regressions in a two-dimensional coordinate system or space, since each of the regressions has only one independent variable, namely log($G(-1)$) and log(I) respectively, since each of these regressions has only one exogenous variable X_t.

On the other hand, the regressions in Figures 5.59 and 5.60 are considered to be the simplest linear regressions in a three-dimensional space, since each of the regressions has only two exogenous variables, namely {log(L),log($L-1$))} and {log(A),log(A (-1))} respectively.

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:18
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 4 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=2)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.616326	0.265513	-2.321270	0.0295
LOG(G(-1))	0.659218	0.038686	17.04009	0.0000
AR(1)	0.483233	0.185927	2.599043	0.0160
AR(2)	-0.436270	0.205766	-2.120227	0.0450

R-squared	0.930796	Mean dependent var	3.795079
Adjusted R-squared	0.921769	S.D. dependent var	0.407289
S.E. of regression	0.113918	Akaike info criterion	-1.370727
Sum squared resid	0.298477	Schwarz criterion	-1.178752
Log likelihood	22.50482	Hannan-Quinn criter.	-1.313643
F-statistic	103.1169	Durbin-Watson stat	1.974403
Prob(F-statistic)	0.000000		

Inverted AR Roots	.24+.61i	.24-.61i

Figure 5.57 Statistical results based on an AR(2) simplest model of log(P) on log($G(-1)$)

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:21
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 7 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=2)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.268870	1.269894	-0.999194	0.3286
LOG(I)	1.122048	0.272135	4.123138	0.0004
AR(1)	1.010192	0.156543	6.453111	0.0000
AR(2)	-0.767743	0.236574	-3.245248	0.0037
AR(3)	0.449311	0.178030	2.523792	0.0193

R-squared	0.924533	Mean dependent var	3.795079
Adjusted R-squared	0.910812	S.D. dependent var	0.407289
S.E. of regression	0.121634	Akaike info criterion	-1.210022
Sum squared resid	0.325487	Schwarz criterion	-0.970052
Log likelihood	21.33529	Hannan-Quinn criter.	-1.138666
F-statistic	67.37992	Durbin-Watson stat	1.992062
Prob(F-statistic)	0.000000		

Inverted AR Roots	.77	.12-.75i	.12+.75i

Figure 5.58 Statistical results based on an AR(3) simplest model of log(P) on log(A)

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:23
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 8 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=2)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.814823	0.440130	-1.851322	0.0776
LOG(L)	0.507367	0.086729	5.850060	0.0000
LOG(L(-1))	0.255508	0.047881	5.336309	0.0000
AR(1)	0.883785	0.168514	5.244579	0.0000
AR(2)	-0.326939	0.193523	-1.689403	0.1053

R-squared	0.948653	Mean dependent var	3.795079
Adjusted R-squared	0.939317	S.D. dependent var	0.407289
S.E. of regression	0.100332	Akaike info criterion	-1.595096
Sum squared resid	0.221462	Schwarz criterion	-1.355126
Log likelihood	26.53379	Hannan-Quinn criter.	-1.523740
F-statistic	101.6133	Durbin-Watson stat	1.878303
Prob(F-statistic)	0.000000		

Inverted AR Roots	.44-.36i	.44+.36i

Figure 5.59 Statistical results based on an AR(2) simplest model of log(P) on {log(L), log($L(-1)$)}

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:25
Sample (adjusted): 1955 1980
Included observations: 26 after adjustments
Convergence achieved after 12 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=2)

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.429762	2.947485	1.502895	0.1485
LOG(A)	0.983898	0.267721	3.675083	0.0015
LOG(A(-1))	-0.845582	0.278225	-3.039208	0.0065
AR(1)	1.034495	0.171850	6.019765	0.0000
AR(2)	-0.671580	0.320605	-2.094727	0.0491
AR(3)	0.583477	0.180339	3.235445	0.0041

R-squared	0.932431	Mean dependent var	3.820699
Adjusted R-squared	0.915539	S.D. dependent var	0.392541
S.E. of regression	0.114081	Akaike info criterion	-1.304646
Sum squared resid	0.260288	Schwarz criterion	-1.014316
Log likelihood	22.96040	Hannan-Quinn criter.	-1.221042
F-statistic	55.19903	Durbin-Watson stat	1.956454
Prob(F-statistic)	0.000000		

Inverted AR Roots	.97	.03-.78i	.03+.78i

Figure 5.60 Statistical results based on an AR(3) simplest model of log(P) on {log(L), log($L(-1)$)}

For each of these simple models, exercises in detailed residual analysis can be done, which have been presented in the previous example, especially to study whether an external variable should be used as an additional variable to improve the '*quality*' of the model. Do this as an exercise.

However, corresponding to the findings of these four models, we would define a general equation can be defined for the AR(p) SCM with an endogenous variable Y_t and an exogenous variable X_t, as follows:

$$
\begin{aligned}
Y_t &= c(1) + \sum_{i=0}^{k} c(2+i)^{*} X_{t-i} + u_t \\
u_t &= \sum_{i=1}^{p} \rho_i u_{t-i} + \varepsilon_t
\end{aligned}
\tag{5.27}
$$

□

5.5.3 Generalized translog linear model

Associated with the time series data, the generalized translog linear models or Cobb–Douglas type models or a constant elasticity function should have at least two exogenous variables and their lagged variables, including the lags of endogenous variables. The following examples present a data analysis using autoregressive translog linear models.

Example 5.16. (The model proposed by Gujarati) Figure 5.61 presents statistical results based on an AR(3) and an AR(2) translog linear model, which are autoregressive modified models of the model in (5.26) proposed by Gujarati. Note that the AR(2) model is a reduced model of the AR(3) model. Based on this reduced model, the following conclusions can be derived:

(1) Log(H) has an insignificant adjusted effect with a very large p-value of 0.9506, which is confirmed by the scatter plot with regression presented above. On the

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:28
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 14 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=2)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.006173	0.490438	-2.051580	0.0551
LOG(A)	0.270937	0.241686	1.121027	0.2770
LOG(G)	0.248806	0.416329	0.597619	0.5575
LOG(H)	-0.000836	0.007081	-0.118084	0.9073
LOG(I)	0.049821	0.571107	0.087235	0.9314
LOG(L)	0.325027	0.103922	3.127597	0.0058
AR(1)	0.673959	0.136544	4.935821	0.0001
AR(2)	-0.394006	0.228260	-1.726131	0.1014
AR(3)	-0.012157	0.217541	-0.055882	0.9561

R-squared	0.962850	Mean dependent var	3.795079
Adjusted R-squared	0.946339	S.D. dependent var	0.407289
S.E. of regression	0.094348	Akaike info criterion	-1.622451
Sum squared resid	0.160228	Schwarz criterion	-1.190505
Log likelihood	30.90309	Hannan-Quinn criter.	-1.494011
F-statistic	58.31522	Durbin-Watson stat	2.000581
Prob(F-statistic)	0.000000		

Inverted AR Roots	.35-.54i	.35+.54i	-.03

(a)

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:30
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 14 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.529450	0.517084	-2.957837	0.0078
LOG(A)	0.412663	0.231186	1.784985	0.0894
LOG(G)	-0.003661	0.355383	-0.010303	0.9919
LOG(H)	0.000389	0.006209	0.062724	0.9506
LOG(I)	0.456581	0.483996	0.943357	0.3568
LOG(L)	0.306178	0.101855	3.006029	0.0070
AR(1)	0.777510	0.143929	5.402047	0.0000
AR(2)	-0.477150	0.197793	-2.412368	0.0256

R-squared	0.959118	Mean dependent var	3.765864
Adjusted R-squared	0.944809	S.D. dependent var	0.428531
S.E. of regression	0.100674	Akaike info criterion	-1.518909
Sum squared resid	0.202704	Schwarz criterion	-1.138279
Log likelihood	29.26472	Hannan-Quinn criter.	-1.402547
F-statistic	67.03028	Durbin-Watson stat	1.871026
Prob(F-statistic)	0.000000		

Inverted AR Roots	.39+.57i	.39-.57i

(b)

Figure 5.61 Statistical results based on (a) an AR(3) translog linear model and (b) an AR(2) translog linear model

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:39
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 13 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.522857	0.446401	-3.411411	0.0026
LOG(A)	0.411731	0.224135	1.836982	0.0804
LOG(G)	-0.001687	0.335117	-0.005035	0.9960
LOG(I)	0.455126	0.464617	0.979571	0.3384
LOG(L)	0.304958	0.087580	3.482055	0.0022
AR(1)	0.777397	0.140296	5.541112	0.0000
AR(2)	-0.477306	0.192209	-2.483263	0.0215

R-squared	0.959116	Mean dependent var	3.765864
Adjusted R-squared	0.947434	S.D. dependent var	0.428531
S.E. of regression	0.098250	Akaike info criterion	-1.590282
Sum squared resid	0.202715	Schwarz criterion	-1.257231
Log likelihood	29.26395	Hannan-Quinn criter.	-1.488465
F-statistic	82.10739	Durbin-Watson stat	1.871909
Prob(F-statistic)	0.000000		
Inverted AR Roots	.39-.57i	.39+.57i	

Figure 5.62 Statistical results based on an AR(2) reduced model

other hand, log(G) and log(I) also have an insignificant adjusted effect with a p-value sufficiently large. As a result, a reduced or modified model should be found.

(2) Corresponding to the statement about the variable H above, at the first stage log (H) is deleted and then the results in Figure 5.62 are obtained, which shows that log(G) and log(I) are insignificant. Therefore, a reduced model may be obtained by deleting log(G) or log(I) or both. Note that the p-value of log(G) is greater than that of log(I), so in general log(G) should be deleted from the model.

(3) However, here it needs to be demonstrated that a contradictory method can be applied, since the impact of multicollinearity of the independent variables is unpredictable. When log(I) is deleted, the statistical results in Figure 5.63 are

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 05:32
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 7 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.175608	0.263434	-4.462631	0.0002
LOG(A)	0.246027	0.114700	2.144958	0.0433
LOG(G)	0.306328	0.086928	3.523940	0.0019
LOG(L)	0.337403	0.092289	3.655928	0.0014
AR(1)	0.751968	0.159510	4.714234	0.0001
AR(2)	-0.406751	0.188537	-2.157408	0.0422

R-squared	0.957196	Mean dependent var	3.765864
Adjusted R-squared	0.947468	S.D. dependent var	0.428531
S.E. of regression	0.098219	Akaike info criterion	-1.615826
Sum squared resid	0.212233	Schwarz criterion	-1.330354
Log likelihood	28.62157	Hannan-Quinn criter.	-1.528554
F-statistic	98.39412	Durbin-Watson stat	1.938137
Prob(F-statistic)	0.000000		
Inverted AR Roots	.38-.52i	.38+.52i	

Figure 5.63 Statistical results based on an AR(2) Cobb–Douglas model

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	2.460694	(1, 22)	0.1310
Chi-square	2.460694	1	0.1167

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2) + C(3) + C(4)	-0.110241	0.070277

Restrictions are linear in coefficients.

(a)

t-statistic:

t_0 = Value/Std.Error
= - 0.110241/0.070277
= - 1.568664

Prob(2-tailed) = 0.1310

(b)

Figure 5.64 (a) The wald test for H_0: $c(2) + c(3) + c(4) = 1$ and (b) the t-statistic for testing the one-sided hypothesis

obtained. This reduced model should be considered as a good or best fit model or an acceptable model, in a statistical sense, since the DW-statistic = 1.938 137 and each of the independent variables, as well as the AR indicators, is significant.

(4) Since the estimated values of the parameters $c(2)$, $c(3)$ and $c(4)$ are positive, this model is confirmed with the basic Cobb–Douglas production function. As a result, the hypothesis of '*a constant return to scale*' of the function can be tested with the null hypothesis H_0: $C(2) + C(3) + C(4) = 1$. At a significant level of 0.10, the null hypothesis is accepted based on the F-statistic of 2.460 694 with $df = (1, 22)$ and p-value = 0.1310, as presented in Figure 5.64.

(5) For illustration purposes, find the t-statistic as presented in Figure 5.64. Based on the Wald test in Figure 5.64(a), it is easy to compute the t-statistic as presented in Figure 5.64(b).

(6) Note that the Prob(2-tailed) of the t-statistic equals the Prob(F-statistic) of 0.1310 with $df = (1, 22)$. Corresponding to the negative value of the t-statistic, for illustration purposes, if a left-side hypothesis is considered, then

$$H_0 : C(2) + C(3) + C(4) \geq 1 \text{ versus } H_1 : C(2) + C(3) + C(4) < 1 \qquad (5.28)$$

Then, at a significant level of 0.10, the null hypothesis is rejected based on the t-statistic of $-1.568\,664$ with $df = 22$ and p-value = 0.1310/2 = 0.0655 < 0.10. □

Example 5.17. (Modified translog linear models) Note that all previous models have presented values of the DW-statistic of less than two. For this reason, further exercises are done to obtain simple SCMs having larger values of DW. Figure 5.65 presents statistical results based on two alternative models.

Furthermore, note that the first model in Figure 5.65(a) is an AR(2) model with independent variables $\log(G(-1))$ and $\log(L(-1))$. However, $\log(L(-1))$ has an insignificant adjusted effect with a p-value of 0.2071.

The other model in Figure 5.65(b) is also an AR(2) model with independent variables, $\log(L)$, $\log(L(-1))$ and $\log(A(-1))$, where each of them has a positive

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 06:28
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 12 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.790565	0.289532	-2.730494	0.0122
LOG(G(-1))	0.548261	0.101324	5.410954	0.0000
LOG(L(-1))	0.151749	0.127695	1.188378	0.2474
AR(1)	0.374680	0.197903	1.893252	0.0716
AR(2)	-0.409826	0.201544	-2.033431	0.0542

R-squared	0.935262	Mean dependent var	3.795079
Adjusted R-squared	0.923492	S.D. dependent var	0.407289
S.E. of regression	0.112657	Akaike info criterion	-1.363369
Sum squared resid	0.279213	Schwarz criterion	-1.123399
Log likelihood	23.40548	Hannan-Quinn criter.	-1.292013
F-statistic	79.45828	Durbin-Watson stat	2.086557
Prob(F-statistic)	0.000000		

Inverted AR Roots	.19-.61i	.19+.61i

(a)

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 06:43
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.314005	0.417797	-3.145081	0.0049
LOG(L)	0.475734	0.072964	6.520130	0.0000
LOG(L(-1))	0.195805	0.080205	2.441313	0.0236
LOG(A(-1))	0.308925	0.143847	2.147599	0.0436
AR(1)	0.710048	0.185498	3.827801	0.0010
AR(2)	-0.315758	0.180454	-1.749792	0.0948

R-squared	0.955329	Mean dependent var	3.795079
Adjusted R-squared	0.944693	S.D. dependent var	0.407289
S.E. of regression	0.095784	Akaike info criterion	-1.660305
Sum squared resid	0.192667	Schwarz criterion	-1.372341
Log likelihood	28.41412	Hannan-Quinn criter.	-1.574678
F-statistic	89.81995	Durbin-Watson stat	2.041559
Prob(F-statistic)	0.000000		

Inverted AR Roots	.36+.44i	.36-.44i

(b)

Figure 5.65 Statistical results based on alternative AR(2) Cobb–Douglas models

significant adjusted effect on log(P). These conclusion can easily be obtained using the t-test available in the output. For example, to test the right-hand hypothesis

$$\begin{aligned} H_0 &: C(4) \leq 0 \\ H_1 &: C(4) > 0 \end{aligned} \tag{5.29}$$

then $t_0 = 1.969\,638$ with a p-value $= 0.0622/2 = 0.0311$. Hence, the null hypothesis is rejected at a significant level of $\alpha = 0.05$.

Furthermore, note these two models can be considered as the AR(2) Cobb–Douglas models, since their independent variables have positive coefficients.

For further illustrations, Figure 5.66 presents alternative lagged-variable autoregressive models, namely LVAR(1,2) translog linear models. Note that the indicator AR(1) is insignificant with large p-values in both models. For this reason, both models should be modified. Do this as an exercise. □

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 06:35
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 10 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.571307	0.272908	-2.093405	0.0486
LOG(P(-1))	0.397487	0.252316	1.575356	0.1301
LOG(G(-1))	0.288820	0.168609	1.712959	0.1014
LOG(L(-1))	0.156442	0.096489	1.621341	0.1199
AR(1)	0.044179	0.237071	0.186354	0.8540
AR(2)	-0.520381	0.225756	-2.305057	0.0315

R-squared	0.938442	Mean dependent var	3.795079
Adjusted R-squared	0.923786	S.D. dependent var	0.407289
S.E. of regression	0.112440	Akaike info criterion	-1.339661
Sum squared resid	0.265499	Schwarz criterion	-1.051698
Log likelihood	24.08543	Hannan-Quinn criter.	-1.254035
F-statistic	64.02857	Durbin-Watson stat	2.258987
Prob(F-statistic)	0.000000		

Inverted AR Roots	.02-.72i	.02+.72i

(a)

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 06:42
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 11 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.710567	0.380177	-1.869045	0.0763
LOG(P(-1))	0.539109	0.229831	2.345673	0.0294
LOG(L)	0.390303	0.093413	4.178276	0.0005
LOG(L(-1))	-0.059834	0.135716	-0.440881	0.6640
LOG(A(-1))	0.141289	0.136207	1.037313	0.3120
AR(1)	0.157236	0.267710	0.587335	0.5636
AR(2)	-0.354043	0.251120	-1.409855	0.1739

R-squared	0.958057	Mean dependent var	3.795079
Adjusted R-squared	0.945474	S.D. dependent var	0.407289
S.E. of regression	0.095105	Akaike info criterion	-1.649255
Sum squared resid	0.180900	Schwarz criterion	-1.313297
Log likelihood	29.26494	Hannan-Quinn criter.	-1.549357
F-statistic	76.13980	Durbin-Watson stat	2.170909
Prob(F-statistic)	0.000000		

Inverted AR Roots	.08+.59i	.08-.59i

(b)

Figure 5.66 Statistical results based on alternative LVAR(1,2) translog linear models

5.5.4 Constant elasticity of substitution models

A constant elasticity of substitution (CES) model is, in fact, a nonlinear model. However, it could be estimated using a reduced linear model, i.e. a translog quadratic model, which is an approximation obtained based on a Taylor series expansion (Agung, Pasay and Sugiharso, 1994, pp. 53–54).

Example 5.18. (A CES model) Figure 5.67 presents statistical results based on a full AR(2) CES (constant elasticity of substitution) model having the endogenous variable P and exogenous variables I and A. It also shows its reduced model, which will be considered as a special reduced model.

Based on this figure the following notes and conclusions are obtained:

(1) The full model shows that $\log(I)^{*}\log(A)$ is insignificant with a p-value that is smaller than $\log(A)^2$. However, for illustration purposes, in order to obtain a special reduced model it is preferable to delete $\log(I)^{*}\log(A)$, for the following reasons:
   - For a fixed value $\log(P)$, the full regression function represents a quadratic graph in a two-dimensional orthogonal coordinate system with axes $\log(I)$ and $\log(A)$.
   - By doing a transformation, namely a rotation of the coordinate system, a quadratic function can always be obtained without the interaction factor.

(2) Each of the independent variables in the reduced model is significant, so this model is an acceptable or a good fit model, in a statistical sense.

(3) However, for illustration purposes, if $\log(A)^2$ is deleted from the full model, there will also be a good fit model as presented in Figure 5.68, with each of the independent variables having a significant adjusted effect. Now, which one should be choosen as the best fit model?

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 07:35
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.411638	4.199824	0.812329	0.4262
LOG(I)	-6.071109	2.064600	-2.940574	0.0081
LOG(A)	6.456739	1.677022	3.850123	0.0010
LOG(I)^2	1.077145	0.375260	2.870397	0.0095
LOG(I)*LOG(A)	-0.814818	0.728232	-1.118900	0.2764
LOG(A)^2	-0.317160	0.445412	-0.712059	0.4847
AR(1)	0.431379	0.175886	2.452614	0.0235
AR(2)	-0.775902	0.190886	-4.064742	0.0006

R-squared	0.962633	Mean dependent var	3.765864
Adjusted R-squared	0.949554	S.D. dependent var	0.428531
S.E. of regression	0.096248	Akaike info criterion	-1.608810
Sum squared resid	0.185275	Schwarz criterion	-1.228181
Log likelihood	30.52335	Hannan-Quinn criter.	-1.492448
F-statistic	73.60435	Durbin-Watson stat	1.887722
Prob(F-statistic)	0.000000		

Inverted AR Roots	.22-.85i	.22+.85i

(a)

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 07:37
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.052838	4.389756	0.923249	0.3664
LOG(I)	-5.700347	2.164856	-2.633130	0.0155
LOG(A)	5.580644	1.683845	3.314226	0.0033
LOG(I)^2	0.733354	0.246509	2.974954	0.0072
LOG(A)^2	-0.733942	0.235868	-3.111663	0.0053
AR(1)	0.526522	0.172436	3.053428	0.0060
AR(2)	-0.788805	0.200962	-3.925153	0.0008

R-squared	0.960683	Mean dependent var	3.765864
Adjusted R-squared	0.949449	S.D. dependent var	0.428531
S.E. of regression	0.096349	Akaike info criterion	-1.629365
Sum squared resid	0.194945	Schwarz criterion	-1.296314
Log likelihood	29.81112	Hannan-Quinn criter.	-1.527548
F-statistic	85.51942	Durbin-Watson stat	1.754095
Prob(F-statistic)	0.000000		

Inverted AR Roots	.26+.85i	.26-.85i

(b)

Figure 5.67 Statistical results based on (a) a CES model of $\log(P)$ and (b) its special reduced model

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 07:58
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.194921	4.128448	0.773879	0.4476
LOG(I)	-5.927459	2.005080	-2.956221	0.0075
LOG(A)	6.419706	1.608717	3.990575	0.0007
LOG(I)^2	1.235979	0.306536	4.032080	0.0006
LOG(I)*LOG(A)	-1.285383	0.339489	-3.786227	0.0011
AR(1)	0.384662	0.163547	2.352000	0.0285
AR(2)	-0.741588	0.175738	-4.219856	0.0004

R-squared	0.961773	Mean dependent var	3.765864
Adjusted R-squared	0.950851	S.D. dependent var	0.428531
S.E. of regression	0.095003	Akaike info criterion	-1.657494
Sum squared resid	0.189538	Schwarz criterion	-1.324443
Log likelihood	30.20492	Hannan-Quinn criter.	-1.555677
F-statistic	88.05897	Durbin-Watson stat	1.971276
Prob(F-statistic)	0.000000		

Inverted AR Roots	.19+.84i	.19-.84i

Figure 5.68 Statistical results based on the reduced CES model in Figure 5.67

(4) The residual graph in Figure 5.69 shows that the ± signs of the error terms of the model in Figure 5.68 in fact have a systematic change over time, especially before 1970. Therefore, the model may need to be modified. By using the AR(3) model, a 'better' residual graph was found, but the AR(3) indicator is insignificant with a p-value $= 0.6461$. On the other hand, by using $\log(P(-1))$ a better residual graph is obtained, but $\log(P(-1))$ is insignificant with a p-value $= 0.8420$. For these reasons this AR(2) reduced CES model is presented.

(5) Similar to the case presented in Figure 5.67, Figure 5.70 presents statistical results based on another CES model and its special reduced model. Note that these models use the first lagged exogenous variables, compared to the models in Figure 5.67. Furthermore, if experimentation is performed by deleting other independent variable(s), then unexpected statistical results will be found.

(6) To generalize the above results, the quadratic function

$$F(x,y) = Ax^2 + 2Bxy + Cy^2 + Dx + Ey + E \tag{5.30}$$

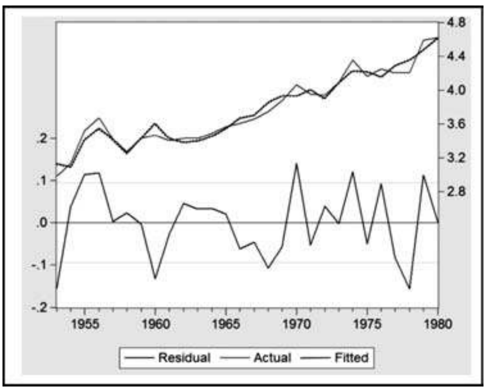


Figure 5.69 Residual graph of the regression in Figure 5.68

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 09:20
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 16 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-4.887468	4.582742	-1.066494	0.2996
LOG(G(-1))	-1.335773	1.231416	-1.084745	0.2916
LOG(L(-1))	3.613570	1.843996	1.959640	0.0649
LOG(G(-1))^2	0.318805	0.365242	0.872861	0.3936
LOG(G(-1))*LOG(L(-1))	-0.389723	0.844176	-0.461661	0.6496
LOG(L(-1))^2	-0.074592	0.542922	-0.137389	0.8922
AR(1)	0.423650	0.188493	2.247559	0.0367
AR(2)	-0.443603	0.205031	-2.163595	0.0434

R-squared	0.949991	Mean dependent var	3.795079
Adjusted R-squared	0.931567	S.D. dependent var	0.407289
S.E. of regression	0.106546	Akaike info criterion	-1.399290
Sum squared resid	0.215688	Schwarz criterion	-1.015338
Log likelihood	26.89041	Hannan-Quinn criter.	-1.285121
F-statistic	51.56182	Durbin-Watson stat	2.134601
Prob(F-statistic)	0.000000		

Inverted AR Roots	.21-.63i	.21+.63i

(a)

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 09:25
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 13 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.314910	4.384949	-1.212080	0.2396
LOG(G(-1))	-1.487155	1.158444	-1.283753	0.2139
LOG(L(-1))	3.928631	1.671776	2.349974	0.0292
LOG(G(-1))^2	0.154146	0.084997	1.813548	0.0848
LOG(L(-1))^2	-0.316561	0.139345	-2.271782	0.0343
AR(1)	0.439432	0.176229	2.493531	0.0215
AR(2)	-0.461027	0.196484	-2.346380	0.0294

R-squared	0.949427	Mean dependent var	3.795079
Adjusted R-squared	0.934255	S.D. dependent var	0.407289
S.E. of regression	0.104432	Akaike info criterion	-1.462151
Sum squared resid	0.218120	Schwarz criterion	-1.126193
Log likelihood	26.73904	Hannan-Quinn criter.	-1.362253
F-statistic	62.57832	Durbin-Watson stat	2.131695
Prob(F-statistic)	0.000000		

Inverted AR Roots	.22+.64i	.22-.64i

(b)

Figure 5.70 Statistical results based on (a) a CES model of log(P) and (b) its special reduced model

represents the full CES model. It has been known that this function has the following characteristics:

- By using a rotation of the coordinate system a new quadratic function can always be obtained, namely $G(x^*, y^*)$, without the interaction x^*y^*. For this reason, at the first stage, it is suggested that the interaction factor xy should be deleted in order to obtain a reduced model based on a full CES model, as demonstrated in Figures 5.67 and 5.70.
- The function could have a minimum or maximum value under the first-order necessary conditions: $\partial F/\partial x = \partial F/\partial y = 0$ or $F_x = F_y = 0$.
- The minimum value is obtained if F_x, $F_y > 0$ and $F_{xy}^2 - F_xF_y < 0$ and the maximum value is obtained if F_x, $F_y < 0$ and $F_{xy}^2 - F_xF_y < 0$.
- The function also could have a saddle point if $F_{xy}^2 - F_xF_y = 0$. □

Example 5.19. (A modified CES model) Figure 5.71 presents the results based on a modified CES model in (4.104) with an endogenous variable P and exogenous variables I and A. Compared to the CES model in the previous examples, it could be said that this modified CES model is a worst model, since it has a greater values of the AIC and SC as well as its reduced model.

However, if there were only these two models, it could be concluded that the reduced model is a good fit or the best fit model, in a statistical sense, since $DW = 1.740\,485$ could be considered sufficient and each of the independent variables is significant. □

Example 5.20. (An advanced CES model) Figure 5.72 presents statistical results based on an advanced CES model with an endogenous variable P and exogenous variables G, L and A, and one out of several possible reduced models. Compared to all previous CES models, this reduced model has the smallest AIC and SC statistics. In

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 09:32
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 18 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.276691	0.615923	-2.072808	0.0501
LOG(I)	0.508439	0.854318	0.595141	0.5578
LOG(A)	0.757765	0.839380	0.902768	0.3764
(LOG(I)-LOG(A))^2	0.133414	0.356658	0.374067	0.7119
AR(1)	0.606306	0.119721	5.064300	0.0000
AR(2)	-0.495916	0.192939	-2.570318	0.0175

R-squared	0.943758	Mean dependent var	3.765864
Adjusted R-squared	0.930976	S.D. dependent var	0.428531
S.E. of regression	0.112585	Akaike info criterion	-1.342801
Sum squared resid	0.278860	Schwarz criterion	-1.057329
Log likelihood	24.79922	Hannan-Quinn criter.	-1.255529
F-statistic	73.83392	Durbin-Watson stat	1.721953
Prob(F-statistic)	0.000000		

Inverted AR Roots	.30+.64i	.30-.64i

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/25/07 Time: 09:31
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 6 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.994065	0.265054	-3.750431	0.0010
LOG(A)	1.264487	0.069240	18.26244	0.0000
(LOG(I)-LOG(A))^2	0.355449	0.043076	8.251716	0.0000
AR(1)	0.546710	0.124586	4.388209	0.0002
AR(2)	-0.497858	0.184061	-2.704851	0.0126

R-squared	0.943432	Mean dependent var	3.765864
Adjusted R-squared	0.933594	S.D. dependent var	0.428531
S.E. of regression	0.110430	Akaike info criterion	-1.408443
Sum squared resid	0.280478	Schwarz criterion	-1.170550
Log likelihood	24.71821	Hannan-Quinn criter.	-1.335717
F-statistic	95.89763	Durbin-Watson stat	1.740485
Prob(F-statistic)	0.000000		

Inverted AR Roots	.27-.65i	.27+.65i

Figure 5.71 Statistical results based on a modified CES model and its reduced model

this case, it was found that each of the independent variables of the reduced model has a significant adjusted effect on log(*P*). However, in general, it may be possible to keep some independent variables having insignificant adjusted effects in an acceptable model.

Note that the reduced model is obtained by deleting two independent variables having the two largest *p*-values. This statistical process is widely used in most cases. However, should this process be used in all cases? The answer is certainly *'No,'* because the largest *p*-value may not directly mean that the corresponding independent variable is unimportant theoretically and substantively. Refer to the contradictory process in developing a reduced model presented in the previous examples. This

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/28/07 Time: 08:21
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 13 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.892034	3.340330	-0.865793	0.3994
LOG(A)	1.219705	1.697720	0.718437	0.4828
LOG(G)	3.497597	1.416231	2.469651	0.0252
LOG(L)	-3.289355	1.187625	-2.769691	0.0137
LOG(A)*LOG(G)	-3.944784	0.810015	-4.870014	0.0002
LOG(A)*LOG(L)	2.255681	0.579702	3.891103	0.0013
LOG(G)*LOG(L)	-0.625361	0.512050	-1.221289	0.2397
LOG(A)^2	1.795935	0.608832	2.949806	0.0094
LOG(G)^2	1.033599	0.270736	3.817739	0.0015
LOG(L)^2	0.017052	0.354146	0.048149	0.9622
AR(1)	0.279158	0.158385	1.762524	0.0971
AR(2)	-0.803449	0.160709	-4.999393	0.0001

R-squared	0.985233	Mean dependent var	3.765864
Adjusted R-squared	0.975081	S.D. dependent var	0.428531
S.E. of regression	0.067646	Akaike info criterion	-2.251520
Sum squared resid	0.073216	Schwarz criterion	-1.680575
Log likelihood	43.52128	Hannan-Quinn criter.	-2.076976
F-statistic	97.04800	Durbin-Watson stat	2.194812
Prob(F-statistic)	0.000000		

Inverted AR Roots	.14+.89i	.14-.89i

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/28/07 Time: 08:24
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 10 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.598964	2.980859	-0.871884	0.3948
LOG(G)	4.402299	0.682177	6.453308	0.0000
LOG(L)	-3.699809	0.896592	-4.126525	0.0006
LOG(A)*LOG(G)	-4.350922	0.535795	-8.120499	0.0000
LOG(A)*LOG(L)	2.534759	0.376886	6.725535	0.0000
LOG(G)*LOG(L)	-0.670641	0.200385	-3.346769	0.0036
LOG(A)^2	2.130334	0.381904	5.578187	0.0000
LOG(G)^2	1.086625	0.189658	5.729395	0.0000
AR(1)	0.246309	0.141880	1.736038	0.0996
AR(2)	-0.804777	0.146310	-5.500477	0.0000

R-squared	0.984730	Mean dependent var	3.765864
Adjusted R-squared	0.977095	S.D. dependent var	0.428531
S.E. of regression	0.064856	Akaike info criterion	-2.360837
Sum squared resid	0.075714	Schwarz criterion	-1.885050
Log likelihood	43.05172	Hannan-Quinn criter.	-2.215384
F-statistic	128.9736	Durbin-Watson stat	2.236350
Prob(F-statistic)	0.000000		

Inverted AR Roots	.12-.89i	.12+.89i

Figure 5.72 Statistical results based on an advanced CES model and one of several possible reduced models

statement corresponds to Enders' statement: 'the more parameters estimated, the greater the parameter uncertainty' (Enders, 2004, p. 106). □

5.5.5 *Models for the first difference of an endogenous variable*

The general model considered in this subsection is

$$\begin{aligned} dY_t &= c(10) + \sum_{i=1}^{p} c(1p)^* dY_{t-i} + \sum_{k=1}^{K} c(2k)^* Xk_t + u_t \\ u_t &= \sum_{j=1}^{q} \rho_j u_{t-j} + \varepsilon_t \end{aligned} \tag{5.31}$$

where $d(Y_t) = Y_t - Y_{t-1}$ is the first difference of an endogenous variable Y_t, X_k is the kth exogenous variable and ρ_j is the jth partial serial correlation of the error term u_t.

Furthermore, note that

$$d\log(Y_t) = \log(Y_t) - \log(Y_{t-1}) = r_t \tag{5.32}$$

represents the return rate of the series Y_t at time t or the *exponential growth rate* of Y_t within the time interval $[t-1, t]$, which can be derived as follows:

$$r_t = \log(Y_t/Y_{t-1}) \rightarrow Y_t/Y_{t-1} = \exp(r_t) \rightarrow Y_t = Y_{t-1}\exp(r_t), t = 1, 2, \ldots, T \tag{5.33}$$

From the author's point of view, a data analysis based on the first difference and the exponential growth rate of Y_t would be a completely different data analysis based on the original series Y_t or $\log(Y_t)$. Hence, there would be different and unexpected results, as presented in the following examples.

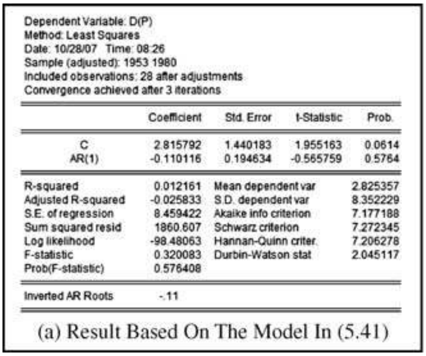
Example 5.21. (Simple models for $d(P_t)$) Figure 5.73 presents statistical results based on two simple models for the first difference of P_t with the following equations:

$$\begin{aligned} d(P_t) &= c(1) + u_t \\ u_t &= \rho_1 u_{t-1} + \varepsilon_t \end{aligned} \tag{5.34}$$

$$d(P_t) = c(1) + c(2)^* d(P_{t-1}) + u_t \tag{5.35}$$

The results in Figure 5.73 are obtained by entering the equation specifications '*d*(*p*) *c ar*(1)' and '*d*(*p*) *c d*(*p*(−1))' respectively. Based on these results the following notes and conclusions are made:

(1) The model in (5.34) is a first-order autoregressive mean model or AR(1) mean model (see the mean model in Chapter 4) and the model in (5.35) is a first lagged (endogenous)-variable model.
(2) Except for the intercepts, both models give the same statistical estimated values, such as the first-order autocorrelation $c(2) = \rho_1 = -0.110\,106$ and the values of $DW = 2.045\,117 > 2$.



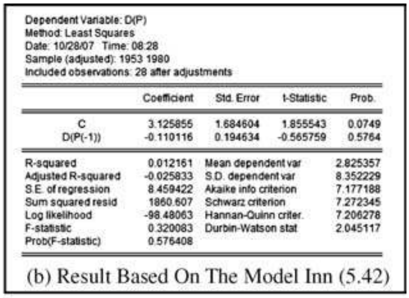
Dependent Variable: D(P)
Method: Least Squares
Date: 10/28/07 Time: 08:26
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.815792	1.440183	1.955163	0.0614
AR(1)	-0.110116	0.194634	-0.565759	0.5764

R-squared	0.012161	Mean dependent var	2.825357
Adjusted R-squared	-0.025833	S.D. dependent var	8.352229
S.E. of regression	8.459422	Akaike info criterion	7.177188
Sum squared resid	1860.607	Schwarz criterion	7.272345
Log likelihood	-98.48063	Hannan-Quinn criter.	7.206278
F-statistic	0.320083	Durbin-Watson stat	2.045117
Prob(F-statistic)	0.576408		

Inverted AR Roots -.11

(a) Result Based On The Model In (5.41)



Dependent Variable: D(P)
Method: Least Squares
Date: 10/28/07 Time: 08:28
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	3.125855	1.684604	1.855543	0.0749
D(P(-1))	-0.110116	0.194634	-0.565759	0.5764

R-squared	0.012161	Mean dependent var	2.825357
Adjusted R-squared	-0.025833	S.D. dependent var	8.352229
S.E. of regression	8.459422	Akaike info criterion	7.177188
Sum squared resid	1860.607	Schwarz criterion	7.272345
Log likelihood	-98.48063	Hannan-Quinn criter.	7.206278
F-statistic	0.320083	Durbin-Watson stat	2.045117
Prob(F-statistic)	0.576408		

(b) Result Based On The Model Inn (5.42)

Figure 5.73 Statistical results based on (a) an AR(1) model in (5.34) and (b) an LV(1) model in (5.35) of the first difference $d(P)$

(3) The null hypotheses H_0: $c(2)=0$ and H_0: $\rho_1=0$ are accepted based on a p-value $=0.3391$. Hence, in a statistical sense, consideration should be given to using modified models. Do this as an exercise.
(4) However, *the negative values of the 'Adjusted R-squared'* indicates that the models are improper or poor time series models.
(5) For a comparison, the following example presents simple models for the return rate of P_t, that is $r_t = d(\log(P_t))$. □

Example 5.22. (Simple models for $d(\log(P_t))$) Table 5.1 presents a summary of the statistical results based on three simple models for the first difference of $d(\log(P_t)) = r_t$ as follows:

$$\begin{aligned} r_t &= d(\log(P_t)) = c(1) + u_t \\ u_t &= \rho_1 u_{t-1} + \varepsilon_t \end{aligned} \tag{5.36}$$

Table 5.1 Summary of the statistical results based on the models in (5.36) to (5.38)

Dependent variable: $r = d(\log(P))$
Newey–West HAC standard errors and covariance (lag truncation = 3)

	Model (5.36)		Model (5.37)		Model (5.38)	
Variable	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
C	0.054 108	0.0778	0.053 745	0.0958	0.060 942	1.0000
$r(-1)$	—	—	0.006 711	0.9729	−0.002 588	1.0000
AR(1)	0.006 711	0.9729	—	—	−0.002 586	1.0000
R-squared	0.000 045		0.000 045		0.000 022	
Adjusted R^2	−0.038 415		−0.038 415		−0.083 310	
DW	1.961 179		1.961 179		1.929 867	
F-statistic	0.001 172	0.9730	0.001 172	0.9730	0.000 262	0.999 738

Dependent Variable: R
Method: Least Squares
Date: 12/11/07 Time: 10:37
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.052108	0.040146	-1.297984	0.2053
DLOG(G)	1.466216	0.451856	3.244874	0.0031

R-squared	0.157383	Mean dependent var	0.052864
Adjusted R-squared	0.126175	S.D. dependent var	0.149382
S.E. of regression	0.139640	Akaike info criterion	-1.033028
Sum squared resid	0.526481	Schwarz criterion	-0.938732
Log likelihood	16.97891	Hannan-Quinn criter.	-1.003496
F-statistic	5.043030	Durbin-Watson stat	1.844224
Prob(F-statistic)	0.033112		

Dependent Variable: R
Method: Least Squares
Date: 12/11/07 Time: 10:34
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.034293	0.044442	-0.771627	0.4476
DLOG(G)	0.691424	0.579722	1.192681	0.2442
DLOG(A)	0.426439	0.168200	2.535313	0.0179
DLOG(L)	0.365594	0.103688	3.525887	0.0017

R-squared	0.480032	Mean dependent var	0.052864
Adjusted R-squared	0.417636	S.D. dependent var	0.149382
S.E. of regression	0.113997	Akaike info criterion	-1.377843
Sum squared resid	0.324884	Schwarz criterion	-1.189251
Log likelihood	23.97872	Hannan-Quinn criter.	-1.318778
F-statistic	7.693302	Durbin-Watson stat	1.729562
Prob(F-statistic)	0.000833		

Figure 5.74 Statistical results based on two acceptable models of $R_t = d(\log(P_t))$

$$d(\log(P_t)) = c(1) + c(2)*(d(\log(P_t)))_{t-1} + u_t$$

or (5.37)

$$r_t = c(1) + c(2)*r_{t-1} + u_t$$

and an LVAR(1,1) model

$$\begin{aligned} r_t &= d(\log(P_t)) = c(1) + c(2)*r_{t-1} + u_t \\ u_t &= \rho_1 u_{t-1} + \varepsilon_t \end{aligned} \quad (5.38)$$

which is in fact the AR(1) model of the model in (5.37).

The results are obtained by entering equation specifications '*r c ar*(1),' '*r c r*(−1)' and '*r c r*(−1) *ar*(1)' respectively, after a new variable, namely $r = d(\log(p))$, has been generated. Based on these results, the same notes and conclusions are presented as in the previous example, especially corresponding to the negative values of the adjusted *R*-squared value. Hence, these models are poor models.

On the other hand, it is also surprising that the probabilities of each *t*-statistic of the model in (5.38) are equal to one. Therefore, this model is the worst model among the three poor models in Table 5.1.

For a comparison, Figure 5.74 presents statistical results based on two acceptable growth rate models, in a statistical sense, with endogenous $R_t = d(\log(P_t))$. Note that $d\log(G)$ is significant in the first model, but is insignificant in the second model. This result demonstrates or shows the unexpected or unpredictable impact of correlation between the independent variables $d\log(G)$, $d\log(A)$ and $d\log(L)$. □

5.5.6 Unexpected findings

The scatter plot with regression of *P* on *H*, in Figure 5.55, shows that *H* is not a good explanatory variable for *P*. However, after doing further experimentation on the relationship between the endogenous variable log(*P*) and the exogenous variable log (*H*), unexpected statistical results have been found in the application of the seemingly causal or explanatory models, without using the time *t* as an exogenous variable of the models. Note the following examples.

Example 5.23. (Unexpected results based on the series (P_t, H_t)) The scatter graph or plot with regression of (H, P) in Figure 5.55 shows that H cannot be a good linear predictor of P. It is also easy to show that the coefficient correlation of bivariate $\{H_t, P_t\} = \{H_i, P_i\}$ with $H_i \leq H_{i+1}$ is not significant.

Figure 5.75 presents statistical results based on a basic translog linear model by using the OLS (ordinary least squares) estimation method and the Newey–West estimation method, based on the following equation specification:

$$\log(P)\ c\ \log(H) \tag{5.39}$$

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/28/07 Time: 08:40
Sample: 1951 1980
Included observations: 30

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.181698	0.471266	8.873321	0.0000
LOG(H)	-0.064826	0.065331	-0.992277	0.3296

R-squared	0.033970	Mean dependent var	3.721145
Adjusted R-squared	-0.000531	S.D. dependent var	0.447149
S.E. of regression	0.447267	Akaike info criterion	1.293020
Sum squared resid	5.601345	Schwarz criterion	1.386433
Log likelihood	-17.39530	Hannan-Quinn criter.	1.322903
F-statistic	0.984613	Durbin-Watson stat	0.174411
Prob(F-statistic)	0.329561		

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/28/07 Time: 08:41
Sample: 1951 1980
Included observations: 30
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.181698	0.096774	43.21096	0.0000
LOG(H)	-0.064826	0.021716	-2.985119	0.0058

R-squared	0.033970	Mean dependent var	3.721145
Adjusted R-squared	-0.000531	S.D. dependent var	0.447149
S.E. of regression	0.447267	Akaike info criterion	1.293020
Sum squared resid	5.601345	Schwarz criterion	1.386433
Log likelihood	-17.39530	Hannan-Quinn criter.	1.322903
F-statistic	0.984613	Durbin-Watson stat	0.174411
Prob(F-statistic)	0.329561		

Figure 5.75 Statistical results based on the basic model in (5.39), by using the OLS and Newey–West estimation methods

These statistical results show that, except for the values of the Std Err, t-statistic and its probability, all other statistics have the same values. Since this involves a time series data, then the Newey–West estimation method should be used to test the hypothesis. However, since there is a negative adjusted R-squared value, then this model is a poor model.

Even though both results in Figure 5.75 are poor, here contradictory results should be noted when using the two estimation methods, i.e. the effect of log(H) on log(P). Based on the OLS estimation method, the result shows that log(H) has an insignificant effect on log(P), but it has a significant effect when based on the Newey–West estimation method.

Hence an alternative model(s) should be found. By using an AR(1) model with the equation specification

$$\log(P)\ c\ \log(H) ar(1) \tag{5.40}$$

the statistical results in Figure 5.76 are obtained using the OLS and Newey–West estimation methods. Based on these results the following notes and conclusions are presented:

(1) These results also show contradictory conclusions, where log(H) has an insignificant effect on log(P) based on the OLS estimation method, but based on the Newey–West estimation method it has a significant effect.
(2) Since there is a positive adjusted R-squared value and a DW-statistic of 1.903 414, this model is an acceptable model, in a statistical sense.

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/28/07 Time: 08:44
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	6.444532	9.531948	0.676098	0.5049
LOG(H)	-0.010508	0.015826	-0.663987	0.5125
AR(1)	0.980291	0.069300	14.14565	0.0000

R-squared	0.886219	Mean dependent var	3.743045
Adjusted R-squared	0.877466	S.D. dependent var	0.438383
S.E. of regression	0.153455	Akaike info criterion	-0.813119
Sum squared resid	0.612261	Schwarz criterion	-0.671675
Log likelihood	14.79023	Hannan-Quinn criter.	-0.768821
F-statistic	101.2544	Durbin-Watson stat	1.903414
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Dependent Variable: LOG(P)
Method: Least Squares
Date: 10/28/07 Time: 08:43
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 9 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
C	6.444532	8.102798	0.795346	0.4336
LOG(H)	-0.010508	0.002044	-5.140250	0.0000
AR(1)	0.980291	0.058517	16.75228	0.0000

R-squared	0.886219	Mean dependent var	3.743045
Adjusted R-squared	0.877466	S.D. dependent var	0.438383
S.E. of regression	0.153455	Akaike info criterion	-0.813119
Sum squared resid	0.612261	Schwarz criterion	-0.671675
Log likelihood	14.79023	Hannan-Quinn criter.	-0.768821
F-statistic	101.2544	Durbin-Watson stat	1.903414
Prob(F-statistic)	0.000000		

Inverted AR Roots	.98

Figure 5.76 Statistical results based on the AR(1) model in (5.40), by using the OLS and Newey–West estimation methods

(3) In order to compare other characteristics or the limitations of the results in Figures 5.75 and 5.76, Figures 5.77 and 5.78 present their residual graphs respectively. Based on these graphs, it could be said that the AR(1) model in (5.40) is a better model, corresponding to the residual graph in Figure 5.78. On

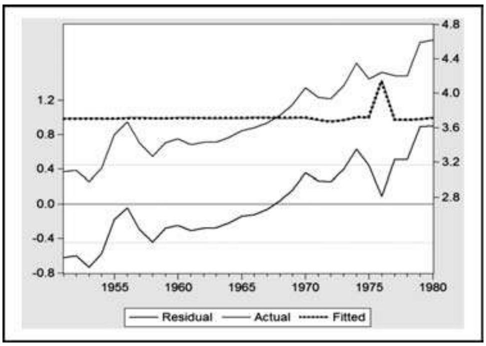


Figure 5.77 Residual graph of the regression in Figure 5.75

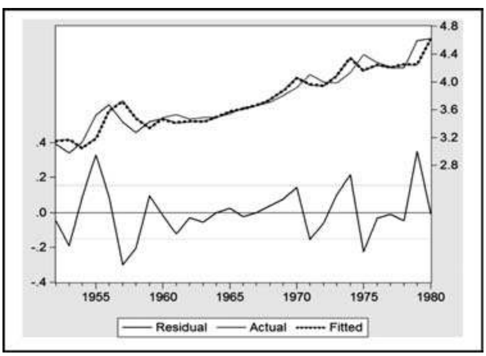


Figure 5.78 Residual graph of the regression in Figure 5.76

the other hand, it is also well known that the basic regression in (5.39) cannot be used for inferential statistical analysis, since the time series data are used.

(4) It is not easy to know which estimation method to use, since both estimation methods in Figure 5.76 will give exactly the same residual graphs. Since the Newey–West estimation method takes into account the unknown autocorrelation and heterogeneity of the error terms, then there would be a preference to apply this method instead of the OLS. Compare this with the results of the following example. □

Example 5.24. (Simple models based on the series (P_t, G_t)) For a comparison study, here a simple model is considered, as follows:

$$\log(P_t) = c(1) + c(2)^*\log(G_t) + \mu_t \tag{5.41}$$

Table 5.2 presents a summary of statistical results obtained using three estimation methods, the OLS, Newey–West and White estimation methods.

Compared to the model in the previous example, based on the series (P_t, H_t), this model gives different results. Using the OLS, Newey–West and White estimation methods:

(1) The adjusted R-squared value is positive, which indicates that the model is an acceptable model or a good fit model. Note that, except for the t-statistic, all other statistics have the same estimated values.

Table 5.2 Summary of statistical results based on the model in (5.41), by using the OLS, Newey–West and White estimation methods

Dependent variable: log(P)
Date: 06/10/07 Time: 06:51
Sample: 1951–1980
Included observations: 30

Variable	Coefficient	t-statistic OLS	t-statistic Newey-West	t-statistic White
C	−0.793 353	−3.065 716	−2.654 005	−2.817 953
log(G)	0.677 000	17.52 080	15.842 458	16.50 839
R-squared	0.916 413	Mean dependent variable		3.721 145
Adjusted R-squared	0.913 427	SD dependent variable		0.447 149
Std Err of regression	0.131 566	Akaike information criterion		−1.154 282
Sum squared residual	0.484 666	Schwarz criterion		−1.060 869
Log likelihood	19.31 423	F-statistic		306.9786
Durbin–Watson statistic	1.146 215	Prob.(F-statistic)		0.000 000

(2) Since the three estimation methods show that log(G) has a significant effect on log (P), then either one of the estimation methods will give the same conclusion in testing the hypothesis. This conclusion is confirmed by observing the scatter graph with regression presented in Figure 5.55.
(3) The aim is to find which one would be preferred in the time series data analysis. Since the Newey–West estimation method takes into account the unknown autocorrelation and heteroskedasticity of the error terms, then this method should be chosen.
(4) In order to improve the quality of the simple model, as well as to increase the DW-statistic, a first-order autoregressive model should be applied as follows:

$$\begin{aligned} \log(P_t) &= c(1) + c(2)^*\log(G_t) + \mu_t \\ \mu_t &= \rho\mu_{t-1} + \varepsilon_t \end{aligned} \tag{5.42}$$

or higher-order autoregressive models. Do this as an exercise.

Furthermore, based on all the variables in the US domestic price of copper, many lagged-variable autoregressive models, namely LVAR(p,q)_SCMs, can be applied, either additive, two-way interaction or three-way interaction models, which could give unexpected estimates because of the unpredictable correlations and multicollinearity of the independent variables. Refer to the special notes and comments presented in Section 2.14. □

5.5.7 Multivariate linear seemingly causal models

By using the multivariate series $(P_t, G_t, I_t, L_t, H_t, A_t)$, various multivariate SCMs could be applied, even those only based on a path diagram defined on the six time series, which have been demonstrated in the last three chapters, including various simultaneous causal models.

Example 5.25. (A simultaneous causal model) Figure 5.79 presents the statistical results under the assumption that log(P) and log(G) have simultaneous causal effects. This model is a first-order autoregressive or AR(1) simultaneous causal model, with two other exogenous variables, log(A) and log(L), and their interaction. Based on this figure, the following notes and conclusions are given:

(1) Since many of the variables of the full model are insignificant, an attempt should be made to try to obtain a reduced model, by deleting some of the independent variables from each regression.
(2) By using the trial-and-error methods, the statistical results based on a reduced model are obtained, with the first regression an AR(2) interaction translog model and the second regression an AR(1) additive translog model. Based on this reduced model, the following notes and conclusions are presented:
    - Based on the first regression, the following function is found:

$$\begin{aligned} \log(p) = & -10.006 + 3.625\log(a) + [1.451 - 0.447\log(a) + 0.048\log(l)]^*\log(g) \\ & + [ar(1) = 0.582,\ ar(2) = -0.560] \end{aligned} \tag{5.43}$$

Estimation Method: Iterative Least Squares
Date: 11/06/07 Time: 13:16
Sample: 1952 1980
Included observations: 30
Total system (balanced) observations 58
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-6.837295	6.809584	-1.004069	0.3208
C(12)	0.982972	0.937870	1.048090	0.3003
C(13)	3.125299	1.899631	1.645214	0.1071
C(14)	-0.262984	1.234613	-0.213009	0.8323
C(15)	-0.381631	0.258023	-1.479060	0.1462
C(16)	0.091423	0.182480	0.501002	0.6189
C(1)	0.413570	0.208006	1.988257	0.0530
C(21)	5.543698	1.325629	4.181936	0.0001
C(22)	-0.283904	0.385208	-0.737014	0.4650
C(23)	-0.509578	0.591252	-0.861862	0.3934
C(24)	0.013702	0.244176	0.056114	0.9555
C(25)	0.105546	0.150067	0.703324	0.4856
C(26)	0.001426	0.062381	0.022856	0.9819
C(2)	1.028978	0.015534	66.24127	0.0000

Determinant residual covariance 7.18E-06

Equation: LOG(P)=C(11)+C(12)*LOG(G)+C(13)*LOG(A)+C(14)*LOG(L)
+C(15)*LOG(G)*LOG(A)+C(16)*LOG(G)*LOG(L)+[AR(1)=C(1)]
Observations: 29

R-squared	0.955079	Mean dependent var	3.743045
Adjusted R-squared	0.942828	S.D. dependent var	0.438383
S.E. of regression	0.104820	Sum squared resid	0.241720
Durbin-Watson stat	1.561067		

Equation: LOG(G)=C(21)+C(22)*LOG(P)+C(23)*LOG(A)+C(24)*LOG(L)
+C(25)*LOG(P)*LOG(A)+C(26)*LOG(P)*LOG(L)+[AR(1)=C(2)]
Observations: 29

R-squared	0.997630	Mean dependent var	6.698337
Adjusted R-squared	0.996984	S.D. dependent var	0.621429
S.E. of regression	0.034128	Sum squared resid	0.025625
Durbin-Watson stat	2.099448		

Estimation Method: Iterative Least Squares
Date: 11/06/07 Time: 13:36
Sample: 1952 1980
Included observations: 30
Total system (unbalanced) observations 57
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-10.00596	3.888389	-2.573291	0.0134
C(12)	1.451022	0.504365	2.876931	0.0061
C(13)	3.624613	1.270016	2.853990	0.0065
C(14)	-0.446795	0.162671	-2.746609	0.0086
C(15)	0.048265	0.013767	3.505842	0.0010
C(16)	0.581998	0.189441	3.072182	0.0036
C(17)	-0.560467	0.195749	-2.863197	0.0063
C(21)	4.747759	0.649165	7.313636	0.0000
C(22)	0.102894	0.042040	2.447512	0.0183
C(23)	-0.110664	0.085636	-1.292267	0.2027
C(24)	1.038091	0.011173	92.91482	0.0000

Determinant residual covariance 5.71E-06

Equation: LOG(P)=C(11)+C(12)*LOG(G)+C(13)*LOG(A)+C(14)*LOG(G)
*LOG(A)+C(15)*LOG(G)*LOG(L)+[AR(1)=C(16), AR(2)=C(17)]
Observations: 28

R-squared	0.964798	Mean dependent var	3.765864
Adjusted R-squared	0.954740	S.D. dependent var	0.428531
S.E. of regression	0.091168	Sum squared resid	0.174543
Durbin-Watson stat	1.932544		

Equation: LOG(G)=C(21)+C(22)*LOG(P)+C(23)*LOG(A)+[AR(1)=C(24)]
Observations: 29

R-squared	0.997498	Mean dependent var	6.698337
Adjusted R-squared	0.997198	S.D. dependent var	0.621429
S.E. of regression	0.032896	Sum squared resid	0.027054
Durbin-Watson stat	2.249599		

Figure 5.79 Statistical results based on an AR(1) simultaneous causal interaction model and its reduced model

where log(g) has a significant effect on log(p), which is dependent on the function [1.451 − 0.447 log(a) + 0.048 log(l)], since each of the independent variables log(g), log(g)*log(a) and log(g)*log(l) is significant based on the t-statistics.

- The second regression shows that log(p) has a significant positive adjusted effect on log(g) with a p-value = 0.0183.
- Therefore, it can be concluded that the data supports the assumption that log(p) and log(g) have simultaneous causal effects. However, it should always be remembered that simultaneous causality between a pair of variables should be defined based on a theoretical and substantial basis.

(3) As an additional exercise, define your own path diagram, either with or without a simultaneous causal effect(s). Then, based on the path diagram write an additive, two-way interaction or three-way interaction model, as presented in the previous chapters. Those models can use either the original variable, the transformed variable, such as the bounded semilog or translog models, or the lagged endogenous and exogenous variables. □

5.6 Return rate models

By considering the classical exponential growth model as

$$Y_t = Y_0 \exp(r^* t) \tag{5.44}$$

or

$$\log(Y_t) = \log(Y_0) + r^* t \tag{5.45}$$

which has been presented in Section 2.2, it is easy to derive a time series R_t, as follows:

$$\log(Y_t) = \log(Y_{t-1}) + R_t, \quad \text{for} \quad t = 1, 2, \ldots, T \tag{5.46}$$

or

$$R_t = \log(Y_t) - \log(Y_{t-1}) = d(\log(Y_t)), \quad \text{for} \quad t = 1, 2, \ldots, T \tag{5.47}$$

Note that R_t is in fact the return rate or the growth rate of the endogenous variable Y_t at the time point t. This can be compared to other types of return rates in econometrics, such as the return of asset (ROA), return of investment (ROI) and return of equity (ROE), which have been widely used or considered in time series models. However, they are defined as the ratios of two indicators or variables.

Hence, by using the R_t series in modeling, in fact a different aspect of the Y_t series is being modeled. For this reason, a specific model is used or proposed, namely the *return rate model* (*RRM*), if the model has an endogenous variable R_t.

Furthermore, by using R_t as an endogenous variable, it is easy to apply all types of models presented in this chapter and previous chapters, such as the continuous and discontinuous growth models, models with trend and time-related effects, seemingly causal models (SCMs), models with dummy variables and the system equations, as well as the models presented in the following chapters. The models can easily be derived from the previous models by using R_t for the Y_t in the univariate linear models and by using R_{gt} for the Y_{gt}, $g = 1, 2, \ldots, G$ in the multivariate linear models.. For this reason, examples based on a model having R_t as an endogenous variable will not be presented here.

However, since in general the return rates can have negative values, then for the translog linear models, the bounded growth model should be used. For example, based on the multivariate autoregressive model (MAR) presented in Chapter 2, the following general autoregressive return rate model (AR_RRM) would be obtained:

$$\begin{aligned} \log\left(\frac{R_{gt} - L_g}{U_g - R_{gt}}\right) &= \left\{\sum_{k=1}^{K} C(gk)^* X_{gk}\right\} + \Theta_g{}^* t + \mu_{gt} \\ \mu_{gt} &= \rho_g \mu_{g(t-1)} + \varepsilon_{gt}, \quad \text{for} \quad g = 1, 2, \ldots, G \end{aligned} \tag{5.48}$$

where $X_{g1}, X_{g2}, \ldots, X_{gK}$ are multivariate independent or cause variables with $X_{g1} = 1$ for all g, L_g and U_g are lower and upper bounds of all possible values of the random variable R_g respectively and Θ_g is the adjusted growth rate of the return rate variable R_g. The values of L_g and U_g should be subjectively selected by the researchers, and the lower bound L_g in general will be negative. One of the author's students, Kernen (2003), has been using a bounded multiple regression having a negative lower bound.

For basic illustration purposes, Figure 5.80 presents the growth curves of the variables *M*1, *GDP*, *PR* and *RS* in Demo.wf1, based on a subsample 1990Q1 to 1996Q4, and Figure 5.81 presents the growth curves of their return rates, namely *d*(log(*M*1)), *d*(log(*GDP*)), *d*(log(*PR*)) and *d*(log(*RS*)).

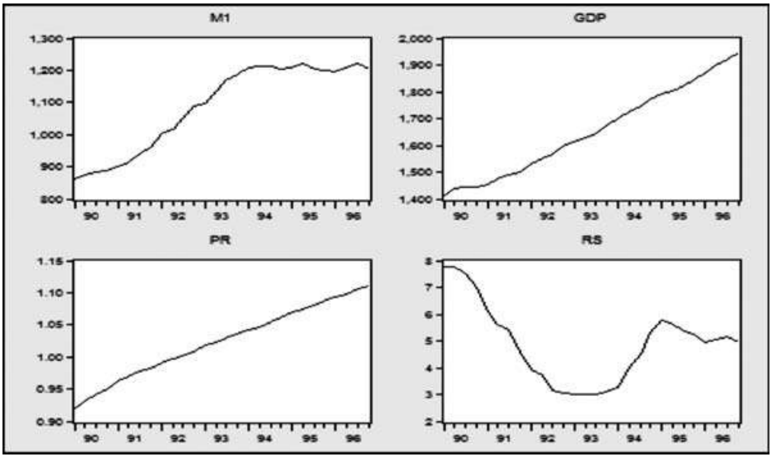


Figure 5.80 Growth curves of the time series of $M1$, GDP, PR and RS, based on a subsample 1990Q1 to 1996Q4

Note that these figures clearly show the differences between the return rate growth curves and the growth curves of their original variables. For this reason, a model of the return rates should be quite different from the model of the original variables. However, all time series models presented in the previous chapters should be applicable, by using R_t as an endogenous variable instead of Y_t.

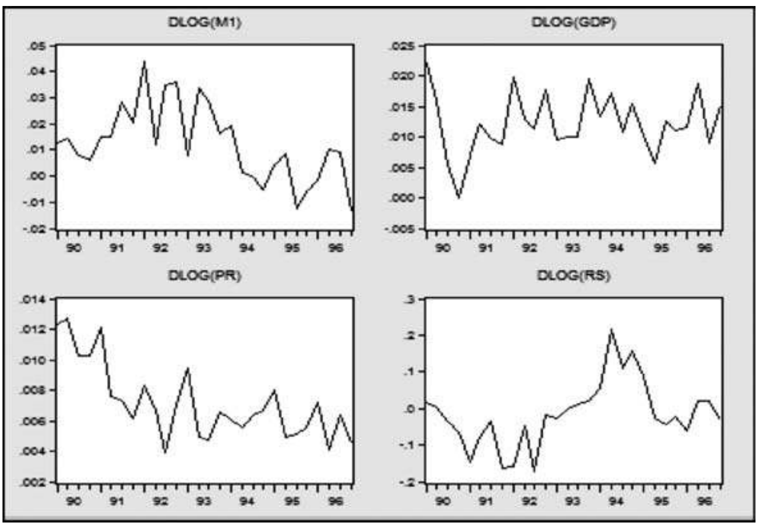


Figure 5.81 Growth curves of the return rates of $M1$, GDP, PR and RS, based on a subsample 1990Q1 to 1996Q4

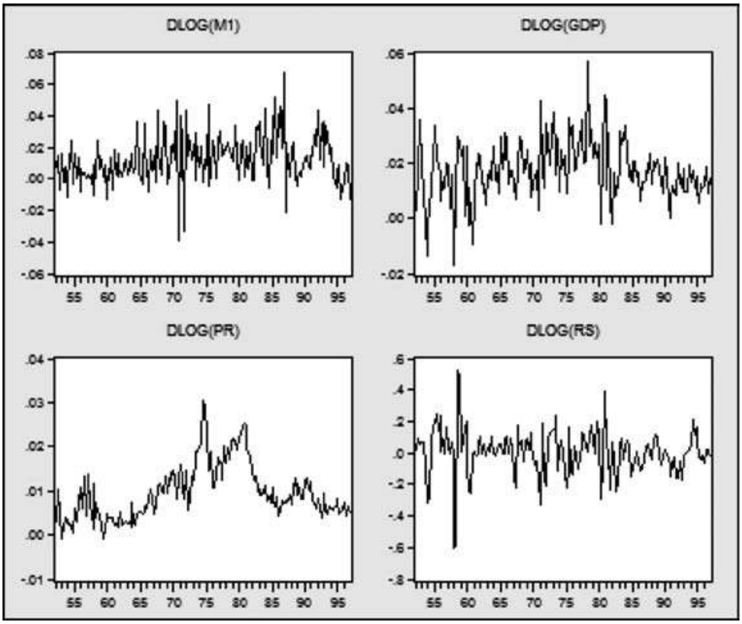


Figure 5.82 Growth curves of the return rates of $M1$, GDP, PR and RS, based on the whole sample 1952Q1 to 1996Q4

For a more detailed comparison, Figure 5.82 presents the growth curves of the return rates based on the whole sample: 1952Q1 to 1996Q4, which can be compared to the growth curves of the original variables, as presented in Chapter 2.

5.7 Cases based on the BASICS workfile

All models that have been presented can easily be applied using any subsets of variables in BASICS.wf1. In this section, however, special cases are considered. So far, it was found that there is an acceptable functional relationship between any endogenous variable and exogenous variable(s). However, in some or many cases, it was easily identified or visually observed that the function cannot be accepted as a good explanatory model, or even as a causal model. Refer to the scatter graphs presented in Figures 4.28 and 4.30, which present the possible causal relationships between the components of bivariate variables.

By observing the scatter graph of a bivariate (X_t, Y_t), in some or many cases it is very difficult to define a regression model or a statistical function, as

$$Y_t = f(\theta, X_t) + \mu_t, \quad t = 1, 2, \ldots, T \tag{5.49}$$

where $f(\theta, X_t)$ is a known function having a finite number of parameters θ and μ_t is an unknown error term or disturbance. For illustration purposes, note the following cases, based on BASICS.wf1.

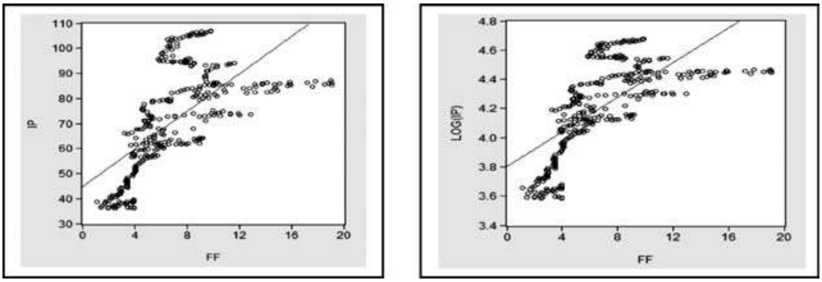


Figure 5.83 Scatter graphs with regressions of *IP* and log(*IP*) on *FF*

Example 5.26. (Industry production index and federal fund) Figure 5.83 presents two scatter graphs with regressions of an endogenous variable *IP* (industry production total index) and log(*IP*) on an exogenous variable *FF* (interest rate: federal funds, % per annum). Based on this figure, the following notes and conclusions are presented:

(1) Neither the simple linear regression nor the semilogarithmic regression, presented in the graphs, can be considered as an acceptable regression or a good fit model.
(2) Even though IP_t and FF_t are time series variables, the graphs represent the graphs of (FF_i, IP_i) with $FF_i \leq FF_{i+1}$ for all i. As a result, for a value FF_i, there could be several observations or values of the endogenous variable *IP*. Therefore, in a mathematical sense, there cannot be a functional relationship between *IP* and *FF*.
(3) Even though *FF* has a significant effect on *IP*, as well as on log(*IP*), the simple models should not be considered as an acceptable model or a good fit model. □

Example 5.27. (Scatter graphs based on the bivariate (X_t, Y_t)) The scatter graph of these variables, which is, in fact, the graph of (X_i, Y_i) for $X_i \leq X_{i+1}$, has been presented in Figure 4.30. For further illustration and discussion, Figure 5.84 presents two additional scatter graphs with regression lines.

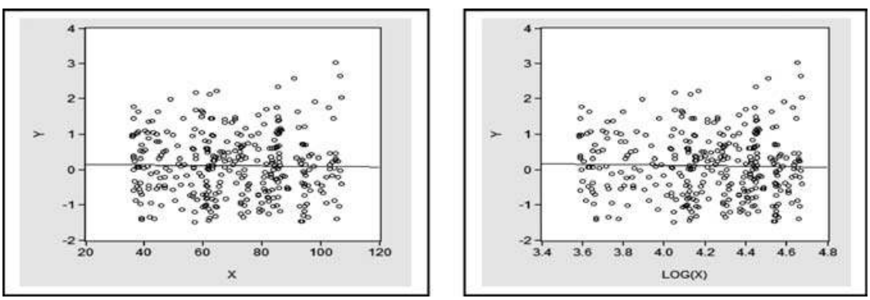


Figure 5.84 Scatter graphs with regression lines of *Y* on *X* and log(*X*)

Dependent Variable: Y
Method: Least Squares
Date: 10/28/07 Time: 10:08
Sample: 1959M01 1989M12
Included observations: 340

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.380279	2.043476	0.186094	0.8525
X	-0.002827	0.095326	-0.029658	0.9764
X^2	-9.22E-05	0.001405	-0.065602	0.9477
X^3	9.73E-07	6.60E-06	0.147472	0.8828

R-squared	0.002900	Mean dependent var	0.108655
Adjusted R-squared	-0.006003	S.D. dependent var	0.883724
S.E. of regression	0.886373	Akaike info criterion	2.608337
Sum squared resid	263.9807	Schwarz criterion	2.653383
Log likelihood	-439.4172	Hannan-Quinn criter.	2.626286
F-statistic	0.325692	Durbin-Watson stat	1.969849
Prob(F-statistic)	0.806791		

Figure 5.85 A third-degree polynomial model of *Y* on *X*

(a) *Case 1: Scatter Graph with Simple Regressions*
If $f(\theta, X_t) = a + bX$, for all t, is defined, then a simple linear regression function will be obtained, based on any data set. In the case where X_t is a positive variable, then a simple linear regression may also be obtained if $f(\theta, X_t) = c + d^*\log(X)$, for all t, is defined. Both regression functions are presented in Figure 5.84, along with their scatter plots.

Furthermore, it has been found that each regression has a very small value of R-squared of 0.000 213 and 0.000 847 respectively, and the variable X, as well as $\log(X)$, has an insignificant adjusted effect on Y. On the other hand, the regressions have negative adjusted R-squared values of $-0.002\,745$ and $-0.002\,470$, so these models are not acceptable time series models. For a comparison see the following case model.

(b) *Case 2: A Third-Degree Polynomial Regression*
Figure 5.85 presents the statistical results based on a third-degree polynomial of Y on X. The function has a very small R-squared value, and each of the independent

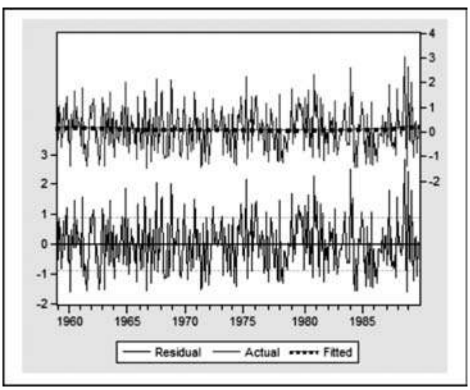


Figure 5.86 Residual graph of the regression in Figure 5.85

variables has an insignificant adjusted effect. The statistical result also shows a negative value of the adjusted R-squared value. Therefore, this model is an unacceptable model even though its residual graph indicates that the regression is a good regression, as presented in Figure 5.86. In other words, the residual graph does not clearly show that the $\pm$ signs of the error terms have a systematic change over time. This case has demonstrated that a residual graph could indicate that the corresponding model is a good model, based on the residual analysis, but the statistical results present a poor estimate. □

5.7.1 Special notes

By observing the scatter graph(s), in some or many cases, it was found that it is very difficult or (almost) impossible to define an explicit function $f(\theta, X_t)$ such that the corresponding regression has a good fit or a sufficiently large value of R-squared. For this reason, the nonparametric estimation method or nonparametric regression should be used (Huitema,1980; Hardle, 1999), including the simplest or very basic moving average estimation method, which will be discussed in Chapter 11.

6

VAR and system estimation methods

6.1 Introduction

Corresponding to time series data, a first-order autoregressive multivariate linear model or an AR(1) multivariate regression can be presented using a general equation:

$$\begin{aligned} y_{g,t} &= X_{g,t}{*}C_g + \mu_{g,t} \\ \mu_{g,t} &= \rho_g \mu_{g,t-1} + \varepsilon_{g,t} \end{aligned} \tag{6.1}$$

where y_g is the gth endogenous or dependent variable, X_g is an exogenous or independent multivariate or multivariable, say $X_g = (X_{g1}, X_{g2}, \ldots, X_{gK})$, of the gth regression, $C_g = (C(g,1), \ldots, C(g,K))'$ is a $K \times 1$ vector of model parameters and ε_g is the error term and ρ_g is the first autocorrelation or serial correlation of the gth regression, for $g = 1, 2, \ldots, G$.

If the multivariate $X_g = (X_1, X_2, \ldots, X_K)$ for all g, then all multiple regressions will have the same exogenous variables. In this case the system can be written as

$$\begin{aligned} y_{g,t} &= X_t{*}C_g + \mu_{g,t} \\ \mu_{g,t} &= \rho_g \mu_{g,t-1} + \varepsilon_{g,t} \end{aligned} \tag{6.2}$$

Note that the components of the multivariate $X_g = (X_{g1}, X_{g2}, \ldots, X_{gK})$ could be any type of measured variables, including various main exogenous variables and the lags of each endogenous or dependent variables, as well as their selected two-way or three-way interactions.

EViews provides several alternative estimation methods for a multivariate time series model, such as the least squares estimates (LS) using the system of equations,

Time Series Data Analysis Using EViews IGN Agung

the VAR (vector autoregression), the VEC (vector error correction) and the system equation estimation methods. Note that the model in (6.1) can be easily extended to a higher-order autoregressive multivariate model, say the AR(p) multivariate regression, either using the original observed variables or transformed variables, such as the natural logarithm of the endogenous or exogenous variable(s), or both. Furthermore, it has been well known that there are several or many possible types of linear association or structural equation models that could be defined, based on a specific multivariate data set.

This chapter, in general, will present examples of alternative bivariate linear models, based on two endogenous variables, say Y_1 and Y_2, and a set of exogenous variables, namely X_k, $k = 1, 2, \ldots, K$. It is expected that all models could be applicable or used in various fields. Furthermore, those models can easily be extended for multivariate endogenous variables.

As a generalization, the symbol Y will be used for selected endogenous variables and the symbol X for selected exogenous variables. Hence, all variables in the data sets used for the illustrations will be defined or selected as the Y-variables for the endogenous and the X-variables for the exogenous variables. By using the symbols $\boldsymbol{Y}$ and $\boldsymbol{X}$ for the multivariate endogenous and exogenous variables respectively, it is proposed that alternative multivariate linear models or system of equations presented in the following sections and examples could be applicable for any sets of variables in various fields of study.

To illustrate this, in the following sections cases will be presented based on the data set in the Demo_Modified workfile, which have been used in the previous chapters, and other selected time series data with a limited number of detailed examples.

6.2 The VAR models

EViews versions 4, 5 and 6 provide a specific VAR estimation method or VAR function, which can be used to apply specific lagged endogenous multivariate models, called the vector autoregressive (VAR) models. Figure 6.1 presents the options of the VAR specification or estimation method of a basic VAR model with endogenous variables $Y1$ and $Y2$. This presentation can be obtained by selecting *Quick/Esimate VAR ...* or *Object/New Object/VAR ... OK,* after opening any EViews workfiles.

For illustration purposes, Figure 6.2 presents a representation of a basic or default bivariate VAR model having endogenous $Y1$ and $Y2$, with '*Lag intervals of Endogenous*': 1 2. Both equations in Figure 6.2 show that the model parameters are presented, as well as recorded, in EViews, by using the symbol $C(i, j)$.

Corresponding to the pth-order lagged-variable models, namely the LV(p) models presented in Chapter 2, specifically the model in (2.26) for $q = 0$, this VAR model can be considered as a special case of the LV(p) models. Furthermore, the VAR model with exogenous variables can also be considered as a special case of the general MAR(p)_T growth model in (2.74), for $p = 0$.

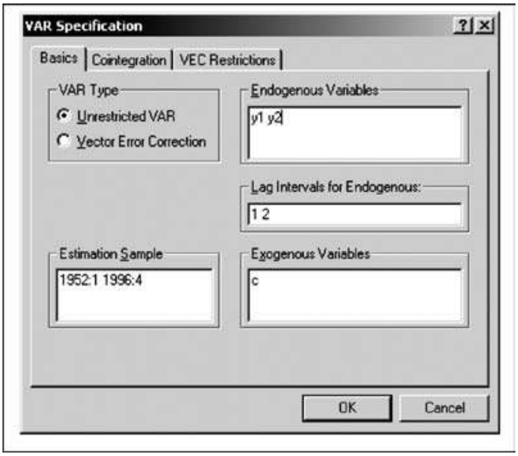


Figure 6.1 The VAR specification of a basic bivariate VAR model

6.2.1 *The basic VAR model*

Based on two endogenous variables, namely $Y1$ and $Y2$, the basic VAR model has the following general equation:

$$Y1_t = \alpha_1 + \sum_{j=1}^{k} \beta_{1j} Y1_{t-j} + \sum_{j=1}^{k} \delta_{1j} Y2_{t-j} + u_{1t} \tag{6.3a}$$

$$Y2_t = \alpha_2 + \sum_{j=1}^{k} \beta_{2j} Y1_{t-j} + \sum_{j=1}^{k} \delta_{2j} Y2_{t-j} + u_{2t} \tag{6.3b}$$

```
Estimation Proc:
================================
LS 1 2 Y1 Y2  @ C

VAR Model:
================================
Y1 =   C(1,1)*Y1(-1) + C(1,2)*Y1(-2)
      + C(1,3)*Y2(-1) + C(1,4)*Y2(-2)
       + C(1,5)

Y2 =  C(2,1)*Y1(-1) + C(2,2)*Y1(-2)
       + C(2,3)*Y2(-1) + C(2,4)*Y2(-2)
       + C(2,5)
```

Figure 6.2 The representation of the VAR model in Figure 6.1

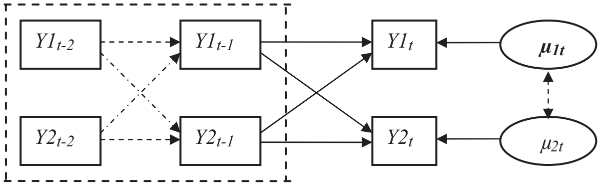


Figure 6.3 The path diagram of a VAR model in (6.3) for $k = 2$

where $\boldsymbol{Y}_{t-j} = (Y1, Y2)_{t-j}$ is the jth lagged variable of $\boldsymbol{Y}_t$, and it is assumed that each of the error terms does not have serial correlations or autocorrelations. In general, these assumptions could be accepted because the model has been using the lagged dependent variables. Thus, the statistical results can be obtained by entering only the endogenous variables '$Y1$ $Y2$' with selected options on the lags of the endogenous variables, as presented in Figure 6.1.

For $j = 2$, the causal association or path diagram between the endogenous variables in model (6.3) can be presented as in Figure 6.3. The correlation between the error terms μ_{1t} and μ_{2t} indicates that the endogenous variables have a type of relationship.

Since both regressions represent the first lagged variables $Y1_{t-1}$ and $Y2_{t-1}$ as the cause factors of $Y1$ and $Y2$, then it may also be considered that $Y1_{t-2}$ and $Y2_{t-2}$ are the cause factors of $Y1_{t-1}$ and $Y2_{t-1}$. However, the model could not show these causal relationships explicitly. For this reason, dotted lines are used between the four variables $Y1_{t-1}$, $Y1_{t-2}$, $Y2_{t-1}$ and $Y2_{t-2}$.

On the other hand, note that their multicollinearity should have (unpredictable) effects on the parameter estimates, as well as the testing hypotheses (refer to the special notes in Section 2.14).

The model in (6.3) is considered as a *bilateral causality model*, because of the two exogenous variables $Y1$ and $Y2$. Four types of causality can be distinguished, as follows (Gujarati, 2003, p. 697):

(1) *Unidirectional causality from Y2 to Y1* is indicated if the estimated coefficients on the lagged $Y2$ in (6.3a) are statistically different from zero as a group (i.e. $\Sigma \beta_{1j} \neq 0$) and the set of estimated coefficients on lagged $Y1$ in (6.3b) is not statistically different from zero (i.e. $\Sigma\ \beta_{2j} \neq 0$).
(2) Conversely, *unidirectional causality from Y1 to Y2* exists if the set of the lagged $Y2$ coefficient in (6.3a) is not statistically different from zero as a group (i.e. $\Sigma \beta_{1j} = 0$) and the set of estimated coefficients on lagged $Y1$ in (6.3b) is statistically different from zero (i.e. $\Sigma\ \beta_{2j} \neq 0$).
(3) *Feedback or bilateral causality* is suggested when the sets of lagged $Y1$ and $Y2$ coefficients are statistically significantly different from zero in both regressions.
(4) Finally, *independence* is suggested when the sets of lagged $Y1$ and $Y2$ coefficients are not statistically significant in both regressions.

6.2.2 The VAR models with exogenous variables

A VAR model based on only two endogenous variables and multivariate exogenous variables can be presented as

$$
\begin{aligned}
Y_{1,t} &= \alpha_1 + \sum_{j=1}^{J}\beta_{1j}Y_{1,t-j} + \sum_{j=1}^{J}\delta_{1j}Y_{2,t-j} + \sum_{k=1}^{K}\lambda_{1k}X_k + u_{1t} \\
Y_{2,t} &= \alpha_2 + \sum_{j=1}^{J}\beta_{2j}Y_{1,t-j} + \sum_{j=1}^{J}\delta_{2j}Y_{2,t-j} + \sum_{k=1}^{K}\lambda_{2k}X_k + u_{2t}
\end{aligned}
\tag{6.4}
$$

where $\boldsymbol{Y}_{t-j} = (Y_1, Y_2)_{t-j}$ is the *j*th lagged variable of $\boldsymbol{Y_t}$ and X_k is the *k*th exogenous variable, and it is assumed that each of the error terms does not have serial correlations or autocorrelations. These assumptions could be accepted because the model has been using the lagged dependent variables.

The exogenous variables, X_k, can be any variable that has been presented in the previous chapters, such as the time *t*, pure exogenous variables, the lags of exogenous variables, the environmental variable and dummy variables. Compared to the multivariate models presented in Chapter 2, the VAR model can be considered as a special case of the MAR(*p*)_T model in (2.74). For this reason, the analysis using any VAR models can be done using the '*System Equation*', which has been demonstrated in the previous chapters.

6.2.3 Cases based on the demo_modified workfile

In order to generalize the models presented in the following examples, the endogenous variables $Y1 = M1$ and $Y2 = GDP$ and the exogenous variables $X1 = PR$ and $X2 = RS$ are defined. For the data analysis EViews 6 has been used.

Example 6.1. (A basic VAR model) Figure 6.4 presents the result by applying a bivariate VAR model. The process of the data analysis based on a basic VAR model is as follows:

(1) Click *Objects/New Objects*, which gives the VAR specification block presented above, with the VAR type '*Unrestricted VAR*.'
(2) By entering the variables *Y*1 and *Y*2 and clicking *OK*, the statistical results will appear on the screen, as presented in Figure 6.4.
(3) Note that the default of the lag interval of the endogenous variable entered is '1 2.' The lag intervals for the endogenous variables can be modified, as well as the estimation sample; '*k k*,' for $k = 0, 1, \ldots,$ may also be used.
(4) The default of the exogenous variables used is *C* (a constant variable), as presented in the window.
(5) Stability of a VAR model.

Vector Autoregression Estimates
Date: 12/16/07 Time: 07:40
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y1	Y2
Y1(-1)	1.187960	0.029337
	(0.07717)	(0.05060)
	[15.3941]	[0.57978]
Y1(-2)	-0.241175	-0.038042
	(0.07609)	(0.04989)
	[-3.16968]	[-0.76251]
Y2(-1)	-0.026848	1.356673
	(0.10910)	(0.07153)
	[-0.24609]	[18.9653]
Y2(-2)	0.065305	-0.343179
	(0.11233)	(0.07365)
	[0.58139]	[-4.65949]
C	4.624977	2.063100
	(1.60007)	(1.04916)
	[2.89048]	[1.96643]

R-squared	0.999375	0.999900
Adj. R-squared	0.999361	0.999897
Sum sq. resids	13165.68	5660.386
S.E. equation	8.723656	5.720052
F-statistic	69206.32	430815.3
Log likelihood	-635.5901	-560.4633
Akaike AIC	7.197642	6.353521
Schwarz SC	7.287018	6.442897
Mean dependent	448.5793	638.5360
S.D. dependent	345.1043	564.4308

Determinant resid covariance (dof adj.)	2481.321
Determinant resid covariance	2343.878
Log likelihood	-1195.743
Akaike information criterion	13.54768
Schwarz criterion	13.72643

Figure 6.4 Statistical results based on a basic VAR model in (6.4) for $k = 2$

In order to obtain additional results or conduct further analysis, select *View*; alternative options can then be seen on the screen, as presented in Figure 6.5. Selecting the *AR Roots Table* gives the result in Figure 6.6, and selecting the *AR Roots Graph* gives the graphical representation of the roots in Figure 6.7 using a complex coordinate system.

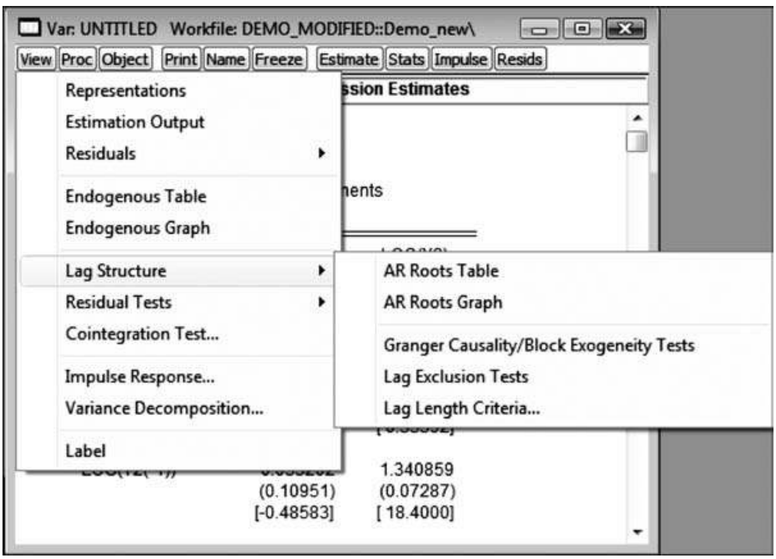


Figure 6.5 Options of the lag structure

VAR Stability Condition Check

Roots of Characteristic Polynomial
Endogenous variables: Y1 Y2
Exogenous variables: C
Lag specification: 1 2
Date: 06/11/07 Time: 15:14

Root	Modulus
1.013002	1.013002
0.940880	0.940880
0.295375 - 0.046881i	0.299073
0.295375 + 0.046881i	0.299073

Warning: At least one root outside the unit circle.
VAR does not satisfy the stability condition.

Figure 6.6 The VAR stability check of the VAR model in Figure 6.4

Note that Figure 6.6 presents two complex roots of 0.296 375 – 0.046 881*i* and 0.296 375 + 0.046 881*i*, with an equal modulus of 0.299 073, and two real roots. Furthermore, one of the roots is outside the unit circle, which indicates that the VAR model is not stable, as shown in Figure 6.7. Therefore, this model is a poor VAR model. As a result, further analysis does not need to be done and a modified VAR model needs to be found, which will be presented in the following example. □

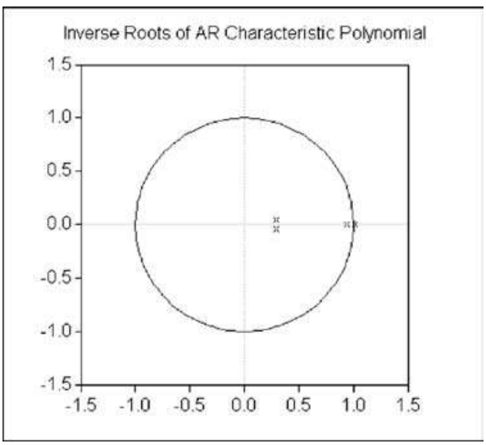


Figure 6.7 The graph of the AR roots in Figure 6.5

Example 6.2. (Other basic bivariate VAR models) After doing some experimentation, alternative VAR models were found that satisfy the stability condition. The first model is a basic VAR model using the endogenous variables log($Y1$) and log($Y2$), as

Vector Autoregression Estimates
Date: 12/16/07 Time: 08:10
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG(Y1)	LOG(Y2)
LOG(Y1(-1))	0.840360	-0.039682
	(0.07682)	(0.05112)
	[10.9399]	[-0.77628]
LOG(Y1(-2))	0.107297	0.017784
	(0.07509)	(0.04997)
	[1.42900]	[0.35592]
LOG(Y2(-1))	-0.053202	1.340859
	(0.10951)	(0.07287)
	[-0.48583]	[18.4000]
LOG(Y2(-2))	0.095777	-0.324236
	(0.11247)	(0.07484)
	[0.85162]	[-4.33231]
C	0.064637	0.039629
	(0.01850)	(0.01231)
	[3.49374]	[3.21882]

R-squared	0.999634	0.999908
Adj. R-squared	0.999626	0.999905
Sum sq. resids	0.036584	0.016201
S.E. equation	0.014542	0.009677
F-statistic	118236.6	467867.1
Log likelihood	503.0331	575.5281
Akaike AIC	-5.595877	-6.410429
Schwarz SC	-5.506501	-6.321053
Mean dependent	5.822083	6.008518
S.D. dependent	0.751831	0.995104

Determinant resid covariance (dof adj.)	1.91E-08
Determinant resid covariance	1.81E-08
Log likelihood	1081.664
Akaike information criterion	-12.04117
Schwarz criterion	-11.86242

Figure 6.8 Statistical results based on a basic bivariate VAR model of {log($Y1$), log($Y2$)}

presented in Figure 6.8, with the stability condition check presented in Figure 6.9. The other model uses the bivariates $\{dY1, dY2\}$. However, their statistical results are not presented.

This output presents the observed values of the t- and F-statistics for testing specific hypotheses. Note that the t-statistic can also be used to test one-sided hypotheses. Based on this result, the following should be noted:

```
Estimation Proc:
===============================
LS 1 2 LOG(Y1) LOG(Y2)  @ C

VAR Model:
===============================
LOG(Y1) = C(1,1)*LOG(Y1(-1)) + C(1,2)*LOG(Y1(-2))
          + C(1,3)*LOG(Y2(-1)) + C(1,4)*LOG(Y2(-2)) + C(1,5)

LOG(Y2) = C(2,1)*LOG(Y1(-1)) + C(2,2)*LOG(Y1(-2))
          + C(2,3)*LOG(Y2(-1)) + C(2,4)*LOG(Y2(-2)) + C(2,5)

VAR Model - Substituted Coefficients:
===============================
LOG(Y1) = 0.8403600893*LOG(Y1(-1)) + 0.1072972616*LOG(Y1(-2)) -
0.05320159724*LOG(Y2(-1)) + 0.09577722874*LOG(Y2(-2)) + 0.06463733726

LOG(Y2) =  - 0.03968193379*LOG(Y1(-1)) + 0.01778400987*LOG(Y1(-2)) +
1.340858557*LOG(Y2(-1)) - 0.3242358467*LOG(Y2(-2)) + 0.03962883563
```

Figure 6.9 Representation of the basic bivariate VAR model in Figure 6.8

(1) The VAR estimates do not present the p-values for testing the corresponding parameters. However, based on each value of the t-statistics, it is easy to conclude whether or not a lagged variable has a significant adjusted effect on the corresponding dependent variable, by using a critical point of $t_0 = 2$ or 1.96. For example, if $|t_0| > 2$, or 1.96, then it can be concluded that the corresponding independent variable has a significant adjusted (partial) effect.
(2) For example, corresponding to the exogenous variable log(y1(−1)), H_0: C(2, 1) = 0 is accepted based on the t-statistic of −0.776 28. Hence, it has an insignificant adjusted effect on log(Y2). The others can easily be identified.
(3) Since some of the endogenous variables have insignificant effects, then a reduced model could be produced by deleting at least one of them. However, this process cannot be done by using the VAR function, since all regressions in a VAR model should have exactly the same set of exogenous variables. In order to obtain a reduced model, the '*System*' function or option should be used, which will be presented in a following relevant section.
(4) The information criteria, AIC and SC, can be used for model selection in order to determine the lag length of the VAR model, with smaller values of the information criterion being preferred.
(5) In order to test hypotheses using the Wald test, the parameters of a VAR model should be identified, as presented in Figure 6.9, which can be obtained by selecting *View/Representation*. Note that the model parameters are presented and saved by using the symbol $C(i, j)$, for $i = 1$ and $i = 2$, $j = 1, 2, \ldots, 5$, where $C(i, 5)$, $i = 1$ and $i = 2$, are the intercept parameters. These parameters should be used to write the hypotheses. However, in practice, the model parameters may be presented by using other symbols, such as β_{ij} or others.
(6) Further statistical analysis based on this model will be presented sequentially in the following examples. □

Example 6.3. (The lag structure analysis) Corresponding to the basic models of {log(Y1), log(Y2)} in Figure 6.8, this example presents a detailed lag structure analysis or options as presented in Figure 6.5, such as follows:

(1) *The AR Roots of a Characteristic Polynomial*
Figure 6.10 presents four real-valued AR roots of the VAR model in Figure 6.8, with a statement that the VAR model satisfies the stability condition, and Figure 6.11 presents the graph of the roots using a complex coordinate system. Note that this graph in fact presents four points in the real (or horizontal) axis.
(2) *The VAR Granger Causality Tests*
By selecting *View/Lag Structure/Granger causality* . . ., the result in Figure 6.12. Based on this result, the following notes and conclusions are obtained:
   - The null hypothesis that log(Y2) is not a Granger-cause of log(Y1), that is H_0: $C(13) = C(14) = 0$, is rejected based on the chi-squared test of 20.813 99, with $df = 2$ and a p-value = 0.0000. Note that, in fact, this is carried out to test the

Roots of Characteristic Polynomial
Endogenous variables: LOG(Y1) LOG(Y2)
Exogenous variables: C
Lag specification: 1 2
Date: 12/16/07 Time: 08:14

Root	Modulus
0.994575	0.994575
0.984923	0.984923
0.318636	0.318636
-0.116916	0.116916

No root lies outside the unit circle.
VAR satisfies the stability condition.

Figure 6.10 The AR roots of the VAR model in Figure 6.8

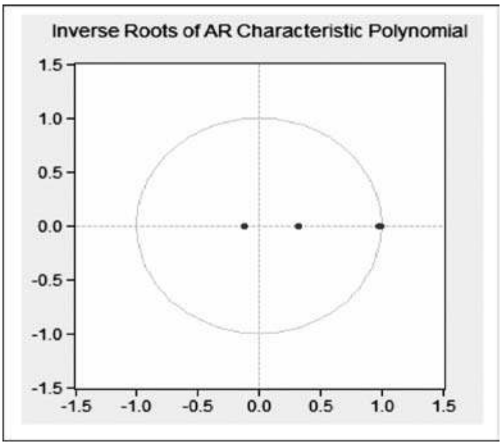


Figure 6.11 The graph of the AR roots in Figure 6.10

joint effects of log($Y2(-1)$) and log($Y2(-2)$) on log($Y1$), and similarly for the following test.

- The null hypothesis that log($Y1$) is not a Granger-cause of log($Y2$), that is H_0: $C(21) = C(22) = 0$, is rejected based on the chi-squared test of 6.944 871, with $df = 2$ and a p-value $= 0.0000$.
- Therefore, it can be concluded that Granger causality can run in two ways. In other words, log($Y2$) is significantly a Granger-cause of log($Y1$) and log($Y1$) is significantly a Granger-cause of log($Y2$). Corresponding to the causality forms proposed by Gujarati, log($Y1$) and log($Y2$) have a *feedback or bilateral causality*.

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 12/16/07 Time: 11:05
Sample: 1952Q1 1996Q4
Included observations: 178

Dependent variable: LOG(Y1)

Excluded	Chi-sq	df	Prob.
LOG(Y2)	20.81399	2	0.0000
All	20.81399	2	0.0000

Dependent variable: LOG(Y2)

Excluded	Chi-sq	df	Prob.
LOG(Y1)	6.944871	2	0.0310
All	6.944871	2	0.0310

Figure 6.12 The VAR Granger causality tests for the model in Figure 6.8

VAR Lag Exclusion Wald Tests
Date: 12/16/07 Time: 11:06
Sample: 1952Q1 1996Q4
Included observations: 178

Chi-squared test statistics for lag exclusion:
Numbers in [ ] are p-values

	LOG(Y1)	LOG(Y2)	Joint
Lag 1	121.0524	342.5987	475.0591
	[0.000000]	[0.000000]	[0.000000]
Lag 2	3.341007	18.98471	24.82025
	[0.188152]	[7.54e-05]	[5.47e-05]
df	2	2	4

Figure 6.13 The VAR lag exclusion tests for the model in Figure 6.8

(3) *The VAR Lag Exclusion Wald Tests*

Based on the result in Figure 6.13, the following findings are given:

- Lag 1, namely each of the first lags log($y1(-1)$) and log($y2(-1)$), as well as their joint effects, are significant with a p-value = 0.000 000.
- Log($y2(-2)$ has a significant effect based on the chi-squared test of 18.984 71 with a p-value = 7.54e-05.
- The joint effects of log($y1(-2)$) and log($y2(-2)$) is significant, based on the chi-squared test of 24.820 25 with a p-value = 5.47e-05.

VAR Lag Order Selection Criteria
Endogenous variables: LOG(Y1) LOG(Y2)
Exogenous variables: C
Date: 12/16/07 Time: 11:07
Sample: 1952Q1 1996Q4
Included observations: 172

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-69.15966	NA	0.007842	0.827438	0.864037	0.842287
1	1036.759	2173.258	2.14e-08	-11.98556	-11.87577	-11.94102
2	1047.788	21.41811	1.97e-08	-12.06730	-11.88431*	-11.99306
3	1055.183	14.18703	1.89e-08	-12.10678	-11.85058	-12.00283
4	1061.156	11.32246	1.85e-08	-12.12973	-11.80034	-11.99608
5	1069.324	15.29004	1.76e-08	-12.17818	-11.77560	-12.01484
6	1075.880	12.12185*	1.71e-08*	-12.20791*	-11.73213	-12.01487*
7	1076.441	1.023860	1.78e-08	-12.16792	-11.61894	-11.94518
8	1077.552	2.002111	1.84e-08	-12.13433	-11.51215	-11.88189

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figure 6.14 Statistical values of the VAR lag order selection criteria for the model in Figure 6.4

(4) *The Lag Order Selection Criteria*
By selecting *Lag Structure/Lag Length Criteria* ..., the statistical results in Figure 6.14 are obtained. This figure shows that the lags of order two are sufficient, which conforms with the model above, based on the SC statistic. However the LR, FPE, AIC and HQ statistics select the lags of order six.

Hence, it is possible to observe the VAR model with the lags interval '1 6'. Do this as an exercise. □

Example 6.4. (Cointegration test) By selecting *View/Cointegration Test* ..., the six options presented in Figure 6.15 appear as well as two windows to insert selected exogenous variables and lag intervals. However, it is very difficult to identify or define the best possible selection. For this reason, it is suggested that the default options given in Figure 6.15 should be used. Corresponding to the VAR model in Figure 6.8, by clicking *OK*, the statistical results in Figure 6.16 are obtained.

This figure shows that there is one cointegration equation at the 0.05 level based on the trace test, as well as the maximum eigenvalue test. Based on this finding, the VEC model of $\{\log(Y1), \log(Y2)\}$ should be applied, which will be presented in Section 6.3. □

Example 6.5. (Residual tests) By selecting *View/Residual Tests* ..., the alternative options presented in Figure 6.17 are obtained. Since alternative residual analyses have been presented in the previous chapters, here only some selected analyses will be presented. However, to consider the residual analysis, refer to the special notes and comments in Section 2.14.3.

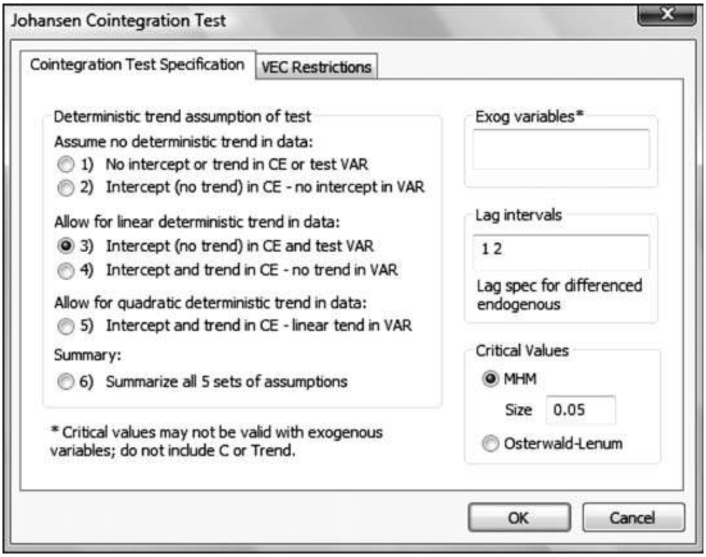


Figure 6.15 Alternative options of the Johansen cointegration test

Date: 12/16/07 Time: 22:21
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Trend assumption: Linear deterministic trend
Series: LOG(Y1) LOG(Y2)
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.120329	25.39339	15.49471	0.0012
At most 1	0.015142	2.700684	3.841466	0.1003

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.120329	22.69271	14.26460	0.0019
At most 1	0.015142	2.700684	3.841466	0.1003

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

LOG(Y1)	LOG(Y2)
-11.03183	8.780112
5.080987	-2.868983

Unrestricted Adjustment Coefficients (alpha):

D(LOG(Y1))	0.005085	0.000414
D(LOG(Y2))	0.001290	-0.001059

Figure 6.16 Statistical results of the default option for the cointegration test

(1) *Correlograms*
Figure 6.18 presents the correlograms of the basic VAR model with endogenous variables log($Y1$) and log($Y2$) with lag intervals of endogenous '1 2', with the statistical results presented in Figure 6.8. Note that Figure 6.18 presents four correlograms, which show that one or two of the corresponding *population autocorrelations* (or *autocorrelation parameters*) are significant. For example, the first graph shows that one of the autocorrelations is outside the interval with two standard error bounds and the second graph shows that two of the autocorrelations are outside the interval.

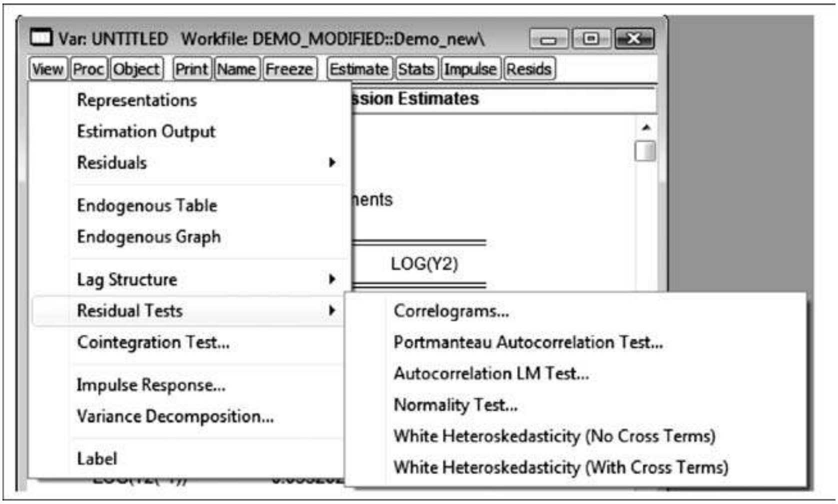


Figure 6.17 Alternative options of the residual tests

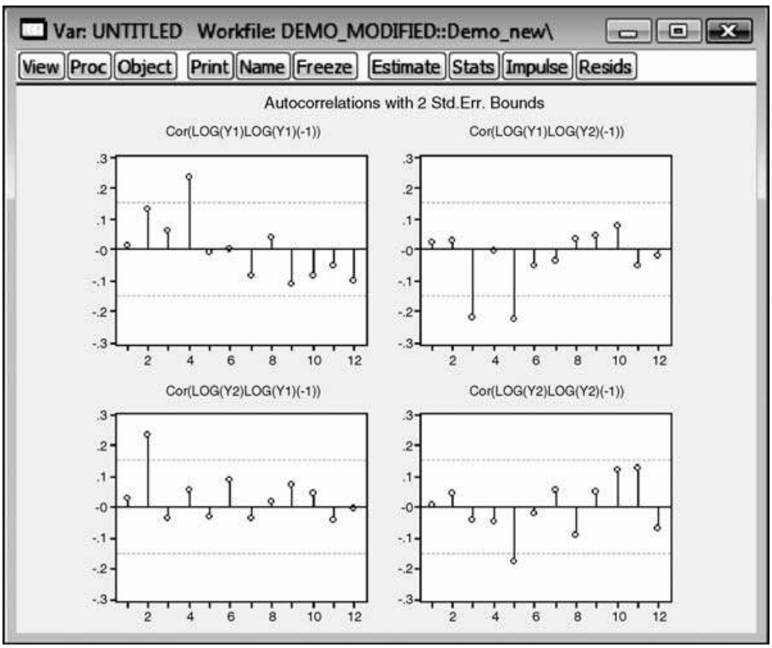


Figure 6.18 Residual correlograms of the VAR model in Figure 6.8

(2) *White Heteroskedasticity Tests*

Figure 6.19 presents two alternative statistical results for testing the residual heteroskedasticity of the basic VAR model with endogenous variables log($Y1$) and log($Y2$). Based on this figure, the following conclusions are obtained:

VAR Residual Heteroskedasticity Tests: No Cross Terms (only levels and squares)
Date: 12/16/07 Time: 23:23
Sample: 1952Q1 1996Q4
Included observations: 178

Joint test:

Chi-sq	df	Prob.
41.98151	24	0.0130

Individual components:

Dependent	R-squared	F(8,169)	Prob.	Chi-sq(8)	Prob.
res1*res1	0.118377	2.836487	0.0056	21.07109	0.0070
res2*res2	0.086936	2.011374	0.0478	15.47453	0.0505
res2*res1	0.049049	1.089603	0.3728	8.730712	0.3655

VAR Residual Heteroskedasticity Tests: Includes Cross Terms
Date: 12/16/07 Time: 23:25
Sample: 1952Q1 1996Q4
Included observations: 178

Joint test:

Chi-sq	df	Prob.
45.76708	33	0.0688

Individual components:

Dependent	R-squared	F(11,166)	Prob.	Chi-sq(11)	Prob.
res1*res1	0.127799	2.211192	0.0160	22.74823	0.0192
res2*res2	0.098702	1.652615	0.0885	17.56891	0.0921
res2*res1	0.049996	0.794198	0.6457	8.899354	0.6312

Figure 6.19 Two alternative White heteroskedasticity tests for the residuals of the basic VAR model in Figure 6.8

- At a 5% significant level, the joint test of 'No Cross Terms' shows that the residuals are heterogeneous, but at the 10% significant level, both joint tests show that the residuals are heterogeneous.
- The null hypothesis H_0: $\rho(res2, res1) = 0$ or Cov$(res2, res1) = 0$ is accepted, based on either the F-statistic or the chi-squared-statistic, based on both tests.
- If there are contradictory conclusions based on the two tests, then one would have to be selected as the final conclusion. Since both tests are acceptable, in a statistical sense, then either one of the tests could be used. □

Example 6.6. (Stability status of the VAR models with exogenous variables) The two previous examples presented a basic VAR model with endogenous variables $\{Y1, Y2\}$, which is not stable, and a basic VAR model with endogenous $\{\log(Y1), \log(Y2)\}$, which is a stable VAR model.

After doing experimentation, it was found that the stability status of a VAR model is unpredictable, which is not consistent with the basic VAR models in the previous examples. Table 6.1 presents alternative VAR models having endogenous variables $\{Y1, Y2\}$ or $\{\log(Y1), \log(Y2)\}$ and various sets of endogenous variables, with their stability status.

This table presents only a limited number of all possible VAR models corresponding to all types of AR(p) models with linear trend or time-related effects, as well as SCMs (i.e. seemingly causal models). These models could easily be extended to more advanced VAR models, such as by inserting an *environmental* or *instrumental* variable, namely Z_t, as presented in Section 4.4, and using the natural logarithm of the X-variables as independent variables, especially for the endogenous variables $\log(Y1)$ and $\log(Y2)$.

Note that some of the models in this table are considered as '*not recommended*' or '*improper*' models, since the endogenous variables are not consistent with their lagged variables used in the models. For example, in model c.3 with endogenous variables $\{\log(y1), \log(y2)\}$, the exogenous variables use $y1(-1)$ and $y2(-1)$ in the form of $t^*y1(-1)$ and $t^*y2(-1)$.

Table 6.1 VAR models with various sets of exogenous variables and their stability status

Model	Alternative VAR models/exogenous variables	Endogenous variables {Y1, Y2}	{log(Y1), log(Y2)}
(a)	*Basic VAR model*		
(a.1)	*C*	Not stable	Stable
(b)	*VAR models with trend*		
(b.1)	*C t*	Not stable	Stable
(b.2)	$C\ t\ t^2 t^3$	Not stable	Stable
(b.3)	*C t X*1 *X*2	Stable	Not stable
(b.4)	*C t X*1 *X*2 *X*1***X*2	Stable	Not stable
(c)	*Models with time-related effects*		
(c.1)	*C t X*1 *X*2 *t***X*1 *t***X*2	Stable	Stable
(c.2)	*C t X*1 *X*2 *X*1**X*2 *T***X*1 *t***X*2 *t***X*1**X*2	Stable	Stable
(c.3)	*C t t**(*y*1(−1)) t*(*y*2(−1))	Not stable	Not stable[a]
(c.4)	*C t t**log(*y*1(−1)) *t**log(*y*2(−1))	Stable[a]	Not stable
(c.5)	*C t t**log(*Y*1(−1)) *t**log(*Y*1(−2)) *t**log(*Y*2(−1)) *t**log(*Y*2(−2))	Not stable[a]	Stable
(c.6)	*C t t***Y*1(−1) *t***Y*1(−2) *t***Y*2(−1) *t***Y*2(−2)	Not stable	Stable[a]
(c.7)	*C t X*1 t**X*1 *t**(*Y*1(−1)) *t**(*Y*1(−2)) *t**(*Y*2(−1)) *t**(*Y*2(−2))	Not stable	Not stable[a]
(c.8)	*C t X*1 *X*2 *t***X*1 *t***X*2 *t**log(*Y*1(−1)) *t**log(*Y*2(−))	Stable[a]	Stable
(d)	*Seemingly causal models*		
(d.1)	*C X*1 *X*2	Stable	Not stable
(d.2)	*C X*1 *X*2 *X*1(−1) *X*2(−1)	Stable	Not stable
(d.3)	*C X*1 *X*2 *X*1**X*2	Stable	Not stable
(d.4)	*C X*1(−1) *X*2(−1) *X*1(−1)**X*2(−1)	Stable	Not stable
(d.5)	*C X*1 *X*2 *X*1**X*2 *X*1(−1) *X*2(−1) *X*1(−1)* *X*2(−1)	Stable	Not stable

Source: Outputs Using EViews 5.
[a]Not recommended models.

The data analysis can easily be done by inserting the relevant set of exogenous variables in the '*Exogenous Variables*' window, as presented in Figure 6.1. The characteristics of each VAR model can be studied by using the same methods presented in Example 6.2.

However, note that all regressions in a VAR model should have the same set of exogenous variables. Hence, if an exogenous variable needs to be deleted or inserted, then the variable will be deleted from or inserted into all regressions. Corresponding to some problems in using a large number of variables in a VAR model, refer to the special notes and comments in Section 2.14. □

Example 6.7. (A VAR model corresponding to the model in (2.78)) Corresponding to the multivariate model with trend in (2.78), here alternative VAR models are presented that are based on the three variables, $M1$, GDP and PR. Using these variables many alternative VAR models can be defined, similar to the VAR models that have been presented in the previous examples, since their lags, as well as the time t, can be used. One of the VAR models is presented in Figure 6.20:

(1) This model is a stable (i.e. no root outside the unit circle) VAR model with endogenous variables log($M1$) and log(GDP) and exogenous variables t, log(PR) and t^*log(PR), and can be considered as a translog linear model with trend and a time-related effect.
(2) Even though log($M1(-2)$) has an insignificant effect on both log($M1$) and log (GDP), this variable cannot be deleted by using the VAR estimation method, since log($GDP(-2)$) should be in the model, corresponding to the option '1 2' lagged interval. However, it is possible to modify the exogenous variables.
(3) Since the time t has insignificant effects on both endogenous variables with t-statistics of $-0.479\,78$ and $0.576\,24$ respectively, this variable may be deleted from the model. The reduced model shows that log(pr) and t^*log(pr) have significant effects on only the second endogenous variable, log(gdp). However, the results are not presented. Note that each regression in the VAR model should have the same set of exogenous variables. Hence, there could be two alternative models based on these set of variables, since each regression of a VAR model should have the same set of exogenous variables. The first is the

Vector Autoregression Estimates
Date: 12/18/07 Time: 13:16
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG(M1)	LOG(GDP)
LOG(M1(-1))	0.844473	-0.027650
	(0.07731)	(0.04850)
	[10.9232]	[-0.57016]
LOG(M1(-2))	0.133905	0.078550
	(0.07895)	(0.04953)
	[1.69601]	[1.58603]
LOG(GDP(-1))	-0.095101	1.220551
	(0.11747)	(0.07369)
	[-0.80957]	[16.5640]
LOG(GDP(-2))	0.109092	-0.321783
	(0.11887)	(0.07456)
	[0.91777]	[-4.31556]
C	0.092662	0.378800
	(0.25501)	(0.15996)
	[0.36337]	[2.36805]
T	-0.000189	0.000142
	(0.00039)	(0.00025)
	[-0.47978]	[0.57624]
LOG(PR)	0.030028	0.105042
	(0.04878)	(0.03060)
	[0.61561]	[3.43302]
T*LOG(PR)	-0.000164	-0.000417
	(0.00015)	(9.2E-05)
	[-1.12291]	[-4.55121]
R-squared	0.999637	0.999919
Adj. R-squared	0.999622	0.999915
Sum sq. resids	0.036288	0.014279
S.E. equation	0.014610	0.009165
F-statistic	66933.34	298081.1
Log likelihood	503.7553	586.7670
Akaike AIC	-5.570285	-6.503000
Schwarz SC	-5.427283	-6.359998
Mean dependent	5.822083	6.008518
S.D. dependent	0.751831	0.995104

Determinant resid covariance (dof adj.)	1.74E-08
Determinant resid covariance	1.59E-08
Log likelihood	1093.158
Akaike information criterion	-12.10290
Schwarz criterion	-11.81690

Figure 6.20 A stable VAR model with trend and time-related effect

VAR Stability Condition Check

Roots of Characteristic Polynomial
Endogenous variables: M1 GDP
Exogenous variables: C T PR T*PR
Lag specification: 1 2
Date: 12/19/07 Time: 05:29

Root	Modulus
0.948774 - 0.065844i	0.951056
0.948774 + 0.065844i	0.951056
0.302589	0.302589
0.090248	0.090248

No root lies outside the unit circle.
VAR satisfies the stability condition.

VAR Stability Condition Check

Roots of Characteristic Polynomial
Endogenous variables: M1 GDP
Exogenous variables: C T T*PR
Lag specification: 1 2
Date: 12/19/07 Time: 05:33

Root	Modulus
0.929162 - 0.029326i	0.929625
0.929162 + 0.029326i	0.929625
0.334146	0.334146
0.188703	0.188703

No root lies outside the unit circle.
VAR satisfies the stability condition.

Figure 6.21 Alternative stable VAR models with endogenous variables $M1$ and GDP and exogenous variables t, PR and t^*PR

full VAR model and its reduced model with exogenous variables log(*pr*) and t^*log(*pr*).

(4) For a comparison, a stable VAR model has been found with endogenous variables $M1$ and GDP and exogenous variables t, pr and t^*pr. Its three possible reduced models with exogenous variables, (i) t and t^*pr, (ii) pr and t^*pr and (iii) t^*pr, are also stable VAR models. Two of the stability tests are presented in Figure 6.21. □

Example 6.8. (Possible reduced VAR models) The models presented in this example should be considered as modifications of the VAR models above. The models still have selected lagged endogenous variables in both regressions, but with different sets of exogenous variables. In order to meet this objective, the system estimation method or function should be used, as already presented in the previous chapters. Find the following alternative models:

(1) *A Reduced Model of the Model in* Figure 6.20
The results in Figure 6.20 show that some of the independent variables are insignificant, with very small values of the t-statistic. It should be possible to obtain a reduced model. Corresponding to the results in Figure 6.20, an attempt has been made to obtain regressions having different sets of exogenous variables. Figure 6.22 presents the statistical results using the SUR estimation method based on a reduced model, by using the '*System*' function or option, which has been demonstrated in the previous chapters. Note that the two regressions in this multivariate model have different sets of independent variables.

(2) *Another Modified Model*
Figure 6.23 presents statistical results based on a modified model. This is not a reduced model of the model in Figure 6.20, since the first regression in the MAR model is an AR(1) model. The second regression is not an AR(1) model, because the indicator AR(1) is insignificant with a large p-value if it is in the model. Compare this with the model in Figure 6.22. □

Estimation Method: Seemingly Unrelated Regression
Date: 12/19/07 Time: 06:12
Sample: 1952Q3 1996Q4
Included observations: 178
Total system (balanced) observations 356
Linear estimation after one-step weighting matrix

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.189882	0.086351	2.198957	0.0285
C(12)	0.842593	0.073604	11.44771	0.0000
C(13)	0.131973	0.072991	1.808054	0.0715
C(17)	0.041355	0.015663	2.640322	0.0087
C(18)	-0.000143	4.09E-05	-3.486108	0.0006
C(21)	0.303376	0.090410	3.355560	0.0009
C(23)	0.055993	0.017168	3.261396	0.0012
C(24)	1.222644	0.070210	17.41409	0.0000
C(25)	-0.315315	0.068954	-4.572844	0.0000
C(27)	0.096642	0.025668	3.765100	0.0002
C(28)	-0.000425	8.76E-05	-4.850450	0.0000
Determinant residual covariance		1.60E-08		

Equation: LOG(M1)=C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(17)
*LOG(PR)+C(18)*T*LOG(PR)
Observations: 178

R-squared	0.999635	Mean dependent var	5.822083
Adjusted R-squared	0.999627	S.D. dependent var	0.751831
S.E. of regression	0.014522	Sum squared resid	0.036485
Durbin-Watson stat	1.968786		

Equation: LOG(GDP)=C(21)+C(23)*LOG(M1(-2))+C(24)*LOG(GDP(-1))
+C(25)*LOG(GDP(-2))+C(27)*LOG(PR)+C(28)*T*LOG(PR)
Observations: 178

R-squared	0.999918	Mean dependent var	6.008518
Adjusted R-squared	0.999916	S.D. dependent var	0.995104
S.E. of regression	0.009129	Sum squared resid	0.014335
Durbin-Watson stat	1.983828		

Figure 6.22 Statistical results based on a reduced model of the model in Figure 6.20

Estimation Method: Seemingly Unrelated Regression
Date: 12/19/07 Time: 06:29
Sample: 1952Q3 1996Q4
Included observations: 178
Total system (unbalanced) observations 355
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 8 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.140410	0.063447	2.213025	0.0276
C(12)	1.241225	0.124054	10.00549	0.0000
C(13)	-0.260245	0.122517	-2.124152	0.0344
C(17)	0.030148	0.011660	2.585551	0.0101
C(18)	-9.68E-05	3.32E-05	-2.920876	0.0037
C(19)	-0.378373	0.117840	-3.210894	0.0014
C(21)	0.280959	0.090057	3.119810	0.0020
C(22)	0.048495	0.017115	2.833432	0.0049
C(24)	1.216271	0.071683	16.96746	0.0000
C(25)	-0.298909	0.069364	-4.309287	0.0000
C(27)	0.087907	0.025381	3.463560	0.0006
C(28)	-0.000385	8.55E-05	-4.507492	0.0000
Determinant residual covariance		1.61E-08		

Equation: LOG(M1)=C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(17)
*LOG(PR)+C(18)*T*LOG(PR)+[AR(1)=C(19)]
Observations: 177

R-squared	0.999640	Mean dependent var	5.827503
Adjusted R-squared	0.999629	S.D. dependent var	0.750468
S.E. of regression	0.014452	Sum squared resid	0.035717
Durbin-Watson stat	2.017898		

Equation: LOG(GDP)=C(21)+C(22)*LOG(M1(-1))+C(24)*LOG(GDP(-1))
+C(25)*LOG(GDP(-2))+C(27)*LOG(PR)+C(28)*T*LOG(PR)
Observations: 178

R-squared	0.999917	Mean dependent var	6.008518
Adjusted R-squared	0.999915	S.D. dependent var	0.995104
S.E. of regression	0.009194	Sum squared resid	0.014540
Durbin-Watson stat	2.032484		

Figure 6.23 A modified model of the VAR model in Figure 6.20

Example 6.9. (A simultaneous causal model) Under the assumption that the time series $M1$ and GDP have a simultaneous causal relationship, to present the data analysis the 'System' function or option should be used. Refer to the simultaneous seemingly causal models presented in Sections 4.5 and 4.6.

Corresponding to each of the VAR model in Figure 6.20, many alternative simultaneous causal models could be obtained by inserting an independent variable $\log(gdp)$ in the first regression and $\log(m1)$ in the second regression. For an illustration, Figure 6.24 presents a simultaneous causal model that is directly derived from to the VAR model in Figure 6.20. Note that five of the independent variables are insignificant with large p-values. For illustration purposes, the following tests are performed by using the Wald tests:

(1) The joint effects of $\log(gdp)$, $\log(gdp(-1))$ and $\log(gdp(-2))$ on $\log(m1)$ are investigated, with the null hypothesis H_0: $C(13) = C(14) = C(15) = 0$. This null hypothesis is rejected based on the chi-squared-statistic of 20.926 15 with $df = 3$ and a p-value $= 0.0001$.

Estimation Method: Seemingly Unrelated Regression
Date: 12/19/07 Time: 06:52
Sample: 1952Q3 1996Q4
Included observations: 178
Total system (balanced) observations 356
Linear estimation after one-step weighting matrix

	Coefficient	Std. Error	t-Statistic	Prob.
C(10)	-0.107805	0.249454	-0.432166	0.6659
C(11)	0.529218	0.116051	4.560216	0.0000
C(12)	0.859106	0.074512	11.52983	0.0000
C(13)	0.092335	0.076569	1.205909	0.2287
C(14)	-0.741038	0.181268	-4.088080	0.0001
C(15)	0.279385	0.120396	2.320561	0.0209
C(16)	-0.000264	0.000379	-0.696267	0.4867
C(17)	-0.025562	0.048525	-0.526787	0.5987
C(18)	5.66E-05	0.000149	0.380979	0.7035
C(20)	0.359504	0.154087	2.333119	0.0202
C(21)	0.208239	0.045664	4.560216	0.0000
C(22)	-0.203502	0.060561	-3.360297	0.0009
C(23)	0.050666	0.048079	1.053792	0.2927
C(24)	1.240355	0.071086	17.44858	0.0000
C(25)	-0.344500	0.071970	-4.786713	0.0000
C(26)	0.000181	0.000238	0.763347	0.4458
C(27)	0.098789	0.029495	3.349414	0.0009
C(28)	-0.000383	8.85E-05	-4.323858	0.0000
Determinant residual covariance		1.26E-08		

Equation: LOG(M1)=C(10)+C(11)*LOG(GDP)+C(12)*LOG(M1(-1))+C(13)
*LOG(M1(-2))+C(14)*LOG(GDP(-1))+C(15)*LOG(GDP(-2))+C(16)*T
+C(17)*LOG(PR)+C(18)*T*LOG(PR)
Observations: 178

R-squared	0.999638	Mean dependent var	5.822083
Adjusted R-squared	0.999621	S.D. dependent var	0.751831
S.E. of regression	0.014630	Sum squared resid	0.036171
Durbin-Watson stat	1.991830		

Equation: LOG(GDP)=C(20)+C(21)*LOG(M1)+C(22)*LOG(M1(-1))+C(23)
*LOG(M1(-2))+C(24)*LOG(GDP(-1))+C(25)*LOG(GDP(-2))+C(26)*T
+C(27)*LOG(PR)+C(28)*T*LOG(PR)
Observations: 178

R-squared	0.999919	Mean dependent var	6.008518
Adjusted R-squared	0.999915	S.D. dependent var	0.995104
S.E. of regression	0.009177	Sum squared resid	0.014233
Durbin-Watson stat	1.985041		

Figure 6.24 Statistical results based on a simultaneous causal model derived directly from the VAR model in Figure 6.20

Similarly, the joint effects of log($m1$), log($m1(-1)$) and log($m1(-2)$) on log (gdp) are investigated, with the null hypothesis H_0: $C(21) = C(22) = C(23) = 0$. This null hypothesis is rejected based on the chi-squared-statistic of 30.542 02 with $df = 3$ and a p-value = 0.0000.

These testing hypotheses could be considered as an extension of the Granger causality tests based on the VAR model. These tests could be called *generalized Granger causality* (GGC) tests.

(2) On the other hand, in order to generalise the causality forms proposed by Gujarati (2003, p. 697), the total coefficient parameters of log(gdp), log($gdp(-1)$) and log ($gdp(-2)$) are tested, with the null hypothesis H_0: $C(13) + C(14) + C(15) = 0$. This null hypothesis is accepted based on the chi-squared-statistic of 1.141 332 with $df = 1$ and a p-value = 0.2854. However, the null hypothesis of the total coefficients of log($m1$), log($m1(-1)$) and log($m1(-2)$), or H_0: $C(21) + C(22) + C(23) = 0$, is rejected based on the chi-squared-statistic of 10.764 61 with $df = 1$ and a p-value = 0.0010. These findings show that there is a unidirectional causality from log(gdp) to log($m1$).

Since the results in Figure 6.24 present several large p-values, an attempt has been made to find better models in a statistical sense. By using the trail-and-error method, a reduced model was obtained, as shown in Figure 6.25, which is a statistically acceptable model with respect to the DW-statistics and each of the independent variables is significant. Figure 6.26 presents an acceptable AR(1) model. □

Example 6.10. (A basic VAR model of four time series) By using the four variables log($M1$), log(GDP), log(PR) and RS as endogenous variables of a basic VAR model, the statistical results were easily obtained, and several testing hypotheses

Estimation Method: Weighted Least Squares
Date: 12/19/07 Time: 06:48
Sample: 1952Q3 1996Q4
Included observations: 178
Total system (balanced) observations 356
Linear estimation after one-step weighting matrix

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.057800	0.016179	3.572606	0.0004
C(12)	0.829465	0.074937	11.06880	0.0000
C(13)	0.121764	0.072604	1.677087	0.0944
C(14)	0.039891	0.008478	4.705301	0.0000
C(21)	0.286725	0.087439	3.279153	0.0011
C(22)	0.057823	0.016522	3.499755	0.0005
C(23)	1.208002	0.070918	17.03372	0.0000
C(24)	-0.300300	0.069336	-4.331061	0.0000
C(25)	0.092793	0.024330	3.813949	0.0002
C(26)	-0.000412	7.97E-05	-5.166391	0.0000

Determinant residual covariance 1.63E-08

Equation: LOG(M1)=C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(14)
*LOG(GDP)
Observations: 178

R-squared	0.999636	Mean dependent var	5.822083
Adjusted R-squared	0.999629	S.D. dependent var	0.751831
S.E. of regression	0.014474	Sum squared resid	0.036451
Durbin-Watson stat	1.941525		

Equation: LOG(GDP)=C(21)+C(22)*LOG(M1)+C(23)*LOG(GDP(-1))+C(24)
*LOG(GDP(-2))+C(25)*LOG(PR)+C(26)*T*LOG(PR)
Observations: 178

R-squared	0.999919	Mean dependent var	6.008518
Adjusted R-squared	0.999917	S.D. dependent var	0.995104
S.E. of regression	0.009083	Sum squared resid	0.014191
Durbin-Watson stat	1.984844		

Figure 6.25 A reduced model of the model in Figure 6.24

were conducted. This example only presents selected testing hypotheses, without presenting the complete statistical results of the model.

(1) *Stability of the VAR model*
The VAR model does not have a root outside the unit circle, with four real roots and four complex roots. Hence the model is a stable VAR model. However, if the original endogenous variables *M*1, *GDP PR* and *RS* are used, there will be eight complex roots, where two roots have a modulus of 1.011 75. Therefore, the model is an unstable VAR model.

(2) *Granger Cause Causality*
Figure 6.27 presents the statistical results for the VAR Granger causality tests. Based on these results, conclusions can be made on the Granger cause causality (GCC) for each pair of the variables. For an example, note the following:
   (a) The joint effect of log($GDP(-1)$) and log($GDP(-2)$) on log($M1$) is significant, based on the chi-squared-statistic of 28.558 02 with $df = 2$ and a p-value = 0.0000. The joint effect of log($M1(-1)$) and log($M1(-2)$) on log(GDP) is significant, based on the chi-squared-statistic of 15.003 65 with

Estimation Method: Weighted Least Squares
Date: 12/19/07 Time: 07:04
Sample: 1952Q4 1996Q4
Included observations: 178
Total system (balanced) observations 354
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 5 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.080661	0.025071	3.217343	0.0014
C(12)	0.522998	0.104225	5.017972	0.0000
C(13)	0.408593	0.097432	4.193617	0.0000
C(14)	0.055644	0.013544	4.108436	0.0000
C(15)	0.323590	0.110088	2.939373	0.0035
C(21)	0.282267	0.087496	3.226052	0.0014
C(22)	0.057185	0.016524	3.460766	0.0006
C(23)	1.211494	0.070949	17.07546	0.0000
C(24)	-0.302594	0.069349	-4.363356	0.0000
C(25)	0.091714	0.024337	3.768473	0.0002
C(26)	-0.000412	7.97E-05	-5.170890	0.0000

Determinant residual covariance 1.60E-08

Equation: LOG(M1)=C(11)+C(12)*LOG(M1(-1))+C(13)*LOG(M1(-2))+C(14)
*LOG(GDP)+[AR(1)=C(15)]
Observations: 177

R-squared	0.999642	Mean dependent var	5.827503
Adjusted R-squared	0.999633	S.D. dependent var	0.750468
S.E. of regression	0.014369	Sum squared resid	0.035512
Durbin-Watson stat	1.985677		

Equation: LOG(GDP)=C(21)+C(22)*LOG(M1)+C(23)*LOG(GDP(-1))+C(24)
*LOG(GDP(-2))+C(25)*LOG(PR)+C(26)*T*LOG(PR)+[AR(1)=C(26)]
Observations: 177

R-squared	0.999918	Mean dependent var	6.017065
Adjusted R-squared	0.999916	S.D. dependent var	0.991352
S.E. of regression	0.009111	Sum squared resid	0.014110
Durbin-Watson stat	1.986388		

Figure 6.26 An AR(1) model of the model in Figure 6.25

$df = 2$ and a p-value $= 0.0000$. Hence, the GCC of the variables $\log(m1)$ and $\log(GDP)$ runs in two ways. Since there are other exogenous variables in each regression, then the GCC may be considered as the *adjusted GCC*, and similarly for the variables $\log(m1)$ and RS.

(b) The joint effect of $\log(PR(-1))$ and $\log(PR(-2))$ on $\log(M1)$ is insignificant, based on the chi-squared-statistic of 2.569 528 with $df = 2$ and a p-value $=$ 0.2767. The joint effect of $\log(M1(-1))$ and $\log(M1(-2))$ on $\log(PR)$ is insignificant, based on the chi-squared-statistic of 0.036 559 with $df = 2$ and a p-value $= 0.9819$. Hence, the GCC between the variables $\log(m1)$ and $\log(PR)$ are insignificant. In other words, there is no GCC between $\log(m1)$ and $\log(PR)$.

(c) The joint effect of the first and second lagged variables of $\log(GDP)$, $\log(PR)$ and RS on $\log(M1)$ is significant, based on the chi-squared-statistic of 54.038 44 with $df = 6$ and a p-value $= 0.0000$. Similarly, the first and second lagged variables of another set of three variables has a significant effect on the fourth variable.

(d) For a later comparison with the VEC model, this VAR model has $AIC = -18.899\,13$ and $SC = -18.255\,63$. □

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 12/19/07 Time: 17:50
Sample: 1952Q1 1996Q4
Included observations: 178

Dependent variable: LOG(M1)

Excluded	Chi-sq	df	Prob.
LOG(GDP)	28.55802	2	0.0000
LOG(PR)	2.569528	2	0.2767
RS	28.60838	2	0.0000
All	54.03844	6	0.0000

Dependent variable: LOG(GDP)

Excluded	Chi-sq	df	Prob.
LOG(M1)	15.00365	2	0.0006
LOG(PR)	13.30577	2	0.0013
RS	18.86469	2	0.0001
All	31.18951	6	0.0000

Dependent variable: LOG(PR)

Excluded	Chi-sq	df	Prob.
LOG(M1)	0.036559	2	0.9819
LOG(GDP)	5.279166	2	0.0714
RS	9.001282	2	0.0111
All	23.16239	6	0.0007

Dependent variable: RS

Excluded	Chi-sq	df	Prob.
LOG(M1)	9.809286	2	0.0074
LOG(GDP)	3.144904	2	0.2075
LOG(PR)	3.863921	2	0.1449
All	28.02164	6	0.0001

Figure 6.27 The Granger causality tests for the basic VAR model with endogenous variables $\log(M1)$, $\log(GDP)$, $\log(PR)$ and RS

6.2.4 The VAR models with dummy variables

All models with dummy variables, including the growth models by states presented in Chapter 2, can easily be extended to the VAR models with dummy variables. For illustration purposes, selected VAR models are presented in the following examples. The dummy variables considered are the dummy variables $Drs1$ and $Drs2$, which have previously been defined based on the variable RS, namely $Drs1 = 1$ for $t \leq 119$ and $Drs1 = 0$ otherwise, and $Drs2 = 1$ for $Drs1 = 0$ and $Drs2 = 0$ for $Drs1 = 1$.

Example 6.11. (The simplest VAR model of {*Y*1, *Y*2} with dummy variables) Figure 6.28 presents statistical results, with the t-statistic in $[\cdot]$, based on a VAR model of the bivariate endogenous variables $\{Y1, Y2\}$ with a dummy variable $Drs1$, which can be considred as the simplest VAR model with a dummy variable(s). This model is a stable VAR model; in fact, it is a first lagged endogenous variable bivariate model.

The main objective of this model is to study the differential effects of the first lagged variables $Y1(-1)$ and $Y2(-1)$ on their corresponding endogenous variables, between the two defined time periods. As an exercise, write the regression functions within each of the time periods. Based on the analysis the following results have been found:

(1) The model is a stable VAR model having two complex roots with a modulus of 0.992 364.
(2) Figure 6.29 presents a part of the output '*Cointegration Test* ....' Based on this result, the following findings are presented:
- The trace test indicates that there is no integration at the 0.05 level.

Vector Autoregression Estimates
Date: 12/20/07 Time: 05:42
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y1	Y2
Y1(-1)	1.006526	0.040613
	(0.02761)	(0.01685)
	[36.4528]	[2.41058]
Y2(-1)	-0.009684	0.978012
	(0.01988)	(0.01213)
	[-0.48710]	[80.6247]
C	19.13921	13.16192
	(5.01546)	(3.06030)
	[3.81605]	[4.30086]
DRS1	-19.68925	-18.91291
	(9.14553)	(5.58036)
	[-2.15288]	[-3.38919]
DRS1*Y1(-1)	-0.001127	0.004375
	(0.09585)	(0.05849)
	[-0.01175]	[0.07480]
DRS1*Y2(-1)	0.018153	0.027792
	(0.05361)	(0.03271)
	[0.33863]	[0.84965]

R-squared	0.999377	0.999913
Adj. R-squared	0.999359	0.999911
Sum sq. resids	13190.98	4911.157
S.E. equation	8.732036	5.328057
F-statistic	55527.66	399358.9
Log likelihood	-638.8313	-550.4032
Akaike AIC	7.204819	6.216795
Schwarz SC	7.311659	6.323635
Mean dependent	446.7856	635.4611
S.D. dependent	344.9693	564.3446

Determinant resid covariance (dof adj.)	2164.532
Determinant resid covariance	2021.855
Log likelihood	-1189.233
Akaike information criterion	13.42160
Schwarz criterion	13.63528

Figure 6.28 A VAR model of {*Y*1, *Y*2} with a dummy variable

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05	Prob.**
None	0.074633	13.72893	14.26460	0.0606
At most 1	0.004853	0.861151	3.841466	0.3534

Max-eigenvalue test indicates no cointegration at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

1 Cointegrating Equation(s): Log likelihood -1160.367

Normalized cointegrating coefficients (standard error in parentheses)

Y1	Y2
1.000000	-0.588799
	(0.04507)

Figure 6.29 Cointegration test of the VAR model in Figure 6.28

- However, the result shows that there is one integrating equation, with a normalized cointegrating coefficient, namely $Y1 - 0.588\,799Y2$. Hence, a VEC model should be applied, which will be presented in the following section.

(3) For an extension of the VAR model with dummy variables, it is easy to modify all of the models with dummy variables presented in Chapter 3. Therefore, many

VAR models with dummy variables could be obtained, in addition to the selection of various lag intervals. □

Example 6.12. (Modification of the model in (4.68)) The model in (4.68) is an additive bivariate SCM (i.e. seemingly causal model) having the independent variables log(m1) and log(gdp) and exogenous variables log(pr) and log(pr (-1)), as well as the lags of the endogenous variables. The analysis used the system equation. This example presents a modified model, which is a VAR model as presented in Figure 6.30. Based on this result, the following notes and conclusions are made:

Vector Autoregression Estimates
Date: 12/20/07 Time: 06:40
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG(M1)	LOG(GDP)
LOG(M1(-1))	0.826538	-0.018671
	(0.07789)	(0.05096)
	[10.6118]	[-0.36636]
LOG(M1(-2))	0.111515	0.012945
	(0.07541)	(0.04934)
	[1.47879]	[0.26236]
LOG(GDP(-1))	-0.030212	1.302768
	(0.11197)	(0.07327)
	[-0.26981]	[17.7813]
LOG(GDP(-2))	0.080471	-0.294228
	(0.11506)	(0.07528)
	[0.69940]	[-3.90826]
C	0.076668	-0.015671
	(0.10097)	(0.06606)
	[0.75935]	[-0.23721]

C	0.076668	-0.015671
	(0.10097)	(0.06606)
	[0.75935]	[-0.23721]
LOG(PR)	-0.266950	0.379992
	(0.23423)	(0.15326)
	[-1.13969]	[2.47941]
LOG(PR(-1))	0.266826	-0.387773
	(0.23114)	(0.15123)
	[1.15441]	[-2.56405]
R-squared	0.999637	0.999911
Adj. R-squared	0.999624	0.999908
Sum sq. resids	0.036298	0.015540
S.E. equation	0.014569	0.009533
F-statistic	78526.42	321414.8
Log likelihood	503.7306	579.2339
Akaike AIC	-5.581243	-6.429594
Schwarz SC	-5.456117	-6.304468
Mean dependent	5.822083	6.008518
S.D. dependent	0.751831	0.995104

Determinant resid covariance (dof adj.)	1.85E-08
Determinant resid covariance	1.70E-08
Log likelihood	1086.886
Akaike information criterion	-12.05490
Schwarz criterion	-11.80464

Figure 6.30 A VAR model of {log(M1), log(GDP)} with exogenous variables log(PR) and log($PR(-1)$)

(1) Each of the exogenous variables log(pr) and log($pr(-1)$) has a significant adjusted effect on log(gdp). Hence, these variables should be acceptable or good explanatory variables of the VAR model.
(2) Even though log($m1(-2)$) has an insignificant adjusted effect on log(m1) and log(gdp), it cannot be deleted from the VAR model, since log($gdp(-2)$) has a significant adjusted effect on log(gdp).
(3) Furthermore, it has been found that the trace test indicates one integrating equation at the 0.05 level. The cointegrating equation of the endogenous variables is 'log(m1) – 0.825 759 log(gdp).' Hence, consideration should be given to using or applying a VEC model, which will be presented in the following section.
(4) For a comparison, conduct additional or further data analysis based on the model in (4.68). □

6.2.5 Selected VAR models based on the US domestic price of copper data

In this subsection time series models are presented based on two endogenous variables, namely Y_1 and Y_2, and three exogenous variables, say X_1, X_2 and X_3. By using the symbols $\boldsymbol{Y}$ and $\boldsymbol{X}$ for endogenous and exogenous multidimensional variables respectively, it is proposed that alternative multivariate linear models or systems of equations presented in the following subsections and examples could be applicable for any sets of variables in various fields of study. Hence, each multivariate linear model using the variables Y_1, Y_2, X_1, X_2 and X_3 presented below should be considered as an acceptable model in various fields.

However, for illustrative examples, the US domestic price of copper data presented in Chapter 4 will be used, with the Y-variables and X-variables defined or selected as follows:

$Y_1 =$ 12-month average US domestic price of copper (cents per pound);
$Y_2 =$ 12-month average price of aluminum (cents per pound);
$X_1 =$ annual gross national product (\$ billions);
$X_2 =$ 12-month average index of industrial production;
$X_3 =$ 12-month average London Metal Exchange price of copper (pounds sterling).

6.2.5.1 Application of continuous VAR models with trend

Corresponding to the hypothetical path diagram in Figure 2.89, the continuous VAR linear models will be presented as follows:

(a) *The VAR Additive Models*
Corresponding to the multivariate continuous models presented based on the path diagram in Figure 2.89, here alternative VAR additive models will be presented as modifications of the multivariate models presented in Chapter 2. By entering the two endogenous variables $Y1$ and $Y2$ and the endogenous variables t, $X1$, $X2$ and $X3$, with the 'Lag Interval for Endogenous' as '1 1,' the statistical results based on the following VAR model will be obtained:

$$\begin{aligned} Y1_t &= c(1,1)Y1_{t-1}+c(1,2)+c(1,3)t+c(1,4)X1+c(1,5)X2+c(1,6)X3+\mu 1_t \\ Y2_t &= c(2,1)Y2_{t-1}+c(2,2)+c(2,3)t+c(2,4)X1+c(2,5)X2+c(2,6)X3+\mu 2_t \end{aligned} \tag{6.5}$$

The association or structural relationship of all variables in this VAR model can be presented as the path diagram in Figure 6.31.

Compare this path diagram with the path diagram in Figure 2.89. Figure 6.31 represents the following characteristics:

(1) All the arrows from each of the independent variables to both dependent variables indicate that each of the independent variables, in general, are considered as source variables, which could not be pure cause factors. For example, it cannot be said that the time t is a cause factor of any endogenous variables.

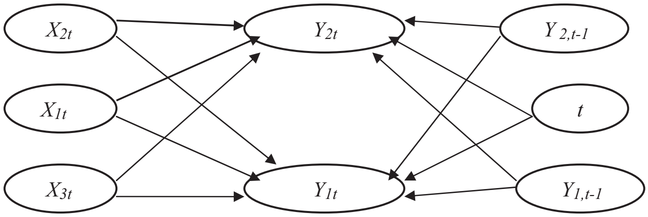


Figure 6.31 The path diagram of the VAR model in (6.5)

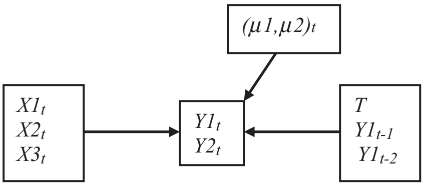


Figure 6.32 A simplified path diagram of the path diagram in Figure 6.31

(2) The diagram does not present an arrow or a line between any pairs of independent variables. However, their correlations and multicollinearity have unpredictable impact(s) on the estimates of the model parameters. Refer to the special notes in Section 2.14.2.
(3) The model in (6.4) can easily be extended to higher-level lagged intervals for endogenous variables. Here, the empirical results are not presented, since it will not be a problem when doing the data analysis.
(4) For simplicity the path diagram in Figure 6.31 will be presented as the path diagram in Figure 6.32. This path diagram shows that the six exogenous or independent variables are defined as having direct '*effects*' on both endogenous variables $Y1_t$ and $Y2_t$. In fact, in general, they are source variables (they are not pure cause factors).

As a further extension for a modification of the MAR additive model in (2.83), a VAR model is presented of the four variables $Y1$, $Y2$, $X1$ and $X3$, with '1 1' as the lag interval for endogenous and exogenous variables t and $X2$, as follows:

$$\begin{aligned}
Y1 &= C(1,1)*Y1(-1)+C(1,2)*Y2(-1)+C(1,3)*X1(-1)+C(1,4)*X3(-1)\\
&\quad +C(1,5)+C(1,6)*T+C(1,7)*X2+\mu 1\\
Y2 &= C(2,1)*Y1(-1)+C(2,2)*Y2(-1)+C(2,3)*X1(-1)+C(2,4)*X3(-1)\\
&\quad +C(2,5)+C(2,6)*T+C(2,7)*X2+\mu 2\\
X1 &= C(3,1)*Y1(-1)+C(3,2)*Y2(-1)+C(3,3)*X1(-1)+C(3,4)*X3(-1)\\
&\quad +C(3,5)+C(3,6)*T+C(3,7)*X2+\mu 3\\
X3 &= C(4,1)*Y1(-1)+C(4,2)*Y2(-1)+C(4,3)*X1(-1)+C(4,4)*X3(-1)\\
&\quad +C(4,5)+C(4,6)*T+C(4,7)*X2+\mu 3
\end{aligned} \tag{6.6}$$

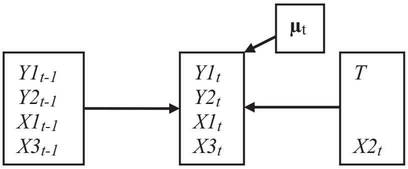


Figure 6.33 Path diagram of the VAR model in (6.6)

The association or structural relationship between the variables of this model can be presented as the path diagram in Figure 6.33. Note that all regressions in a VAR model should have the same set of independent variables. For this reason, the six independent variables are assumed or defined to have direct effects on each of the four dependent variables $Y1_t$, $Y2_t$, $X1_t$ and $X3_t$. In order to differentiate the type of variables, the lagged endogenous variables are presented on the left-hand side and the exogenous variables on the right-hand side in Figure 6.33.

(b) *The VAR Two-Way Interaction Models*

Corresponding to the two-way interaction models presented in the previous chapters, as well as the VAR models with interactions exogenous variables presented in Table 6.1, various VAR two-way interaction models can be defined based on the five defined variables, or any set of variables. For illustration purposes, the following sections present general VAR two-way interaction models with two endogenous variables only, namely $Y1$ and $Y2$.

b.1 The Two-Way Interaction VAR Models with Trend

The assumption that the exogenous variables $X1$, $X2$ and $X3$ have pairwise correlations or associations provides the set of independent variables t, $X1$, $X2$, $X1^*X2$, $X1^*X3$ and $X2^*X3$. For this reason, there is a general two-way interaction VAR model with trend, using '1 1' as the lag interval for endogenous variables, as follows:

$$
\begin{aligned}
Y1 = {} & C(1,1)^*Y1(-1) + C(1,2)^*Y2(-1) + C(1,3) + C(1,4)^*T \\
& + C(1,5)^*X1 + C(1,6)^*X2 + C(1,7)^*X3 \\
& + C(1,8)^*X1^*X2 + C(1,9)^*X1^*X3 + C(1,10)^*X2^*X3 \\
Y2 = {} & C(2,1)^*Y1(-1) + C(2,2)^*Y2(-1) + C(2,3) + C(2,4)^*T \\
& + C(2,5)^*X1 + C(2,6)^*X2 + C(2,7)^*X3 \\
& + C(2,8)^*X1^*X2 + C(2,9)^*X1^*X3 + C(2,10)^*X2^*X3
\end{aligned}
\tag{6.7}
$$

The statistical results based on this full model, as well as its possible reduced models, could easily be done as an exercise. Furthermore, this two-way interaction VAR model can be modified by using the transformed variables, such as the logarithm of the endogenous variables, as well as the exogenous variables and their lags.

b.2 The VAR Models with Time-Related Effects

The VAR models with time-related effects can have many or countable infinite alternative models having the endogenous variables $Y1$ and $Y2$. Some simple models could have the following set of exogenous variables:

(i) t, $X1$, $X2$, $X3$, t^*X1, t^*X2 and t^*X3, with various lag intervals for endogenous variables. The main objective to apply this model is to study or test the effects of each exogenous variable $X1$, $X2$ and $X3$, as well as their joint effects, which are dependent on the time t.

(ii) $t, t^*Y1(-1)$ and $t^*Y2(-1)$, specifically for the models with the lag interval for endogenous variables = '1 1.' The main objective to apply this model is to study or test the effects of each $Y1(-1)$ and $Y2(-1)$, as well as their joint effects, which are dependent on the time t. For other lag intervals, such as the lag interval of '1 2', additional two-way interaction factors $t^*Y1(-2)$ and $t^*Y2(-2)$ should be entered as exogenous variables.

(iii) Corresponding to the points in (i) and (ii) above, there could be a VAR model with the exogenous variables $t, X1, X2, X3, t^*X1, t^*X2, t^*X3, t^*Y1(-1)$ and $t^*Y2(-1)$, for the lag interval '1 1'.

(c) *The VAR Three-Way Interaction Models*

Under the assumption that the exogenous variables $X1, X2$ and $X3$ are completely correlated, the following VAR three-way interaction models may be presented:

(i) Corresponding to the path diagram in Figure 6.31 and the VAR additive model in (6.5), there may be a VAR three-way interaction model with trend using the exogenous variables t, $X1$, $X2$, $X3$, $X1^*X2$, $X1^*X3$, $X2^*X3$ and $X1^*X2^*X3$, which is a hierarchical VAR model with trend. In practice, a nonhierarchical three-way interaction VAR model could be obtained by deleting some of the main factors or two-way interactions.

(ii) Corresponding to the VAR two-way interaction model in (6.7), the most general VAR model with time-related effects can be derived from the model in (6.7) by entering additional exogenous variables t^*X1, t^*X2, t^*X3, t^*X1^*X2, t^*X1^*X3, t^*X2^*X3, $t^*Y1(-1)$ and $t^*Y2(-1)$ as exogenous variables. The interactions t^*X1^*X2, t^*X1^*X3 and t^*X2^*X3 indicate that the model is a VAR three-way interaction model.

(iii) Furthermore, under the assumption that $X1$, $X2$ and $X3$ have a complete association, a more advanced VAR interaction model can be proposed or defined, by inserting a four-way interaction $t^*X1^*X2^*X3$. In this case, a VAR four-way interaction model would be produced.

(d) *Special Notes on the VAR Models*

In fact, based on the five time series, namely $X1, X2, X3, Y1$ and $Y2$, a countable or an infinite number of VAR models could be defined, because a set of three, four or all of these variables could also be used as endogenous variables, as well as their transformations, such as their natural logarithms, their first differences and their return rates, and the lagged exogenous variables. In practice, however, there would only be a very limited number of VAR models, which are highly dependent on personal knowledge and judgment.

On the other hand, by using many or a large number of independent variables in any model, including the VAR models, there should be awareness of the unpredictable statistical results (refer to the special notes in Section 2.14).

Vector Autoregression Estimates
Date: 12/20/07 Time: 10:29
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG(Y1)	LOG(Y2)
LOG(Y1(-1))	0.828788	0.111851
	(0.11219)	(0.23650)
	[7.38753]	[0.47295]
LOG(Y2(-1))	0.164293	0.448637
	(0.07588)	(0.15995)
	[2.16522]	[2.80477]
C	0.436676	0.639498
	(1.25752)	(2.65090)
	[0.34725]	[0.24124]
T	0.004252	0.018995
	(0.01541)	(0.03248)
	[0.27597]	[0.58482]
LOG(X1)	0.190242	0.004676
	(0.19592)	(0.41301)
	[0.97101]	[0.01132]
LOG(X2)	-0.696156	-0.387443
	(0.26825)	(0.56548)
	[-2.59520]	[-0.68516]
LOG(X3)	0.224837	0.410353
	(0.06656)	(0.14031)
	[3.37794]	[2.92458]
R-squared	0.980088	0.949823
Adj. R-squared	0.974658	0.936138
Sum sq. resids	0.060759	0.270004
S.E. equation	0.052553	0.110783
F-statistic	180.4781	69.40805
Log likelihood	48.28871	26.66169
Akaike AIC	-2.847497	-1.355979
Schwarz SC	-2.517460	-1.025942
Mean dependent	3.382868	3.743045
S.D. dependent	0.330119	0.438383

Determinant resid covariance (dof adj.)	3.37E-05
Determinant resid covariance	1.94E-05
Log likelihood	75.03075
Akaike information criterion	-4.209017
Schwarz criterion	-3.548944

Figure 6.34 Statistical results based on the model in (6.8)

Example 6.13. (An application of the VAR additive model) Figure 6.34 presents statistical results based on a VAR translog linear model with trend, as follows:

$$\begin{aligned}\log(Y1) = {} & C(1,1)^*\log(Y1(-1)) + C(1,2)^*\log(Y2(-1)) + C(1,3) + C(1,4)^*T \\ & + C(1,5)^*\log(X1) + C(1,6)^*\log(X2) + C(1,7)^*\log(X3) \\ \log(Y2) = {} & C(2,1)^*\log(Y1(-1)) + C(2,2)^*\log(Y2(-1)) + C(2,3) + C(2,4)^*T \\ & + C(2,5)^*\log(X1) + C(2,6)^*\log(X2) + C(2,7)^*\log(X3)\end{aligned} \tag{6.8}$$

Based on this result, the following notes and conclusions are obtained:

(1) The (adjusted) growth rate of $Y1$ is 0.004 252 and 0.018 995 for $Y2$. Even though the time t has an insignificant adjusted effect, it should be kept in the model, since a study needs to be made of the growth rates of $Y1$ and $Y2$ or a VAR model with trend should be presented.
(2) Since log($X1$) has an insignificant effect on both endogenous variables, a reduced model may be obtained by deleting log($X1$). However, the result will not be presented. □

Example 6.14. (A VAR model with time-related effects) Figure 6.35 presents statistical results based on a VAR model with trend and time-related effects using endogenous variables $Y1$ and $Y2$, with the lag interval for endogenous variables

Vector Autoregression Estimates
Date: 12/20/07 Time: 11:17
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y1	Y2
Y1(-1)	0.910843	0.276742
	(0.36736)	(0.98979)
	[2.47944]	[0.27960]
Y2(-1)	0.104245	0.929357
	(0.19670)	(0.52999)
	[0.52996]	[1.75354]
C	16.78223	65.95625
	(16.8738)	(45.4641)
	[0.99457]	[1.45073]
T	0.056074	2.761764
	(0.49653)	(1.33782)
	[0.11293]	[2.06437]
X1	-0.047206	-0.211520
	(0.05256)	(0.14161)
	[-0.89819]	[-1.49370]
T*X1	0.001684	0.008414
	(0.00163)	(0.00438)
	[1.03495]	[1.91900]
T*Y1(-1)	-0.012636	-0.092915
	(0.01711)	(0.04611)
	[-0.73831]	[-2.01492]
T*Y2(-1)	0.000615	-0.034201
	(0.00882)	(0.02377)
	[0.06969]	[-1.43862]

R-squared	0.984023	0.959850
Adj. R-squared	0.978698	0.946467
Sum sq. resids	72.60162	527.0559
S.E. equation	1.859360	5.009780
F-statistic	184.7747	71.71966
Log likelihood	-54.45574	-83.19937
Akaike AIC	4.307293	6.289612
Schwarz SC	4.684478	6.666797
Mean dependent	31.28655	46.45862
S.D. dependent	12.73949	21.65240

Determinant resid covariance (dof adj.)	76.13230
Determinant resid covariance	39.92193
Log likelihood	-135.7589
Akaike information criterion	10.46613
Schwarz criterion	11.22050

Figure 6.35 A VAR model with time-related effects

of '1 1,' and exogenous variables t, $x1$, t^*x1, $t^*y1(-1)$ and $t^*y2(-1)$. This case shows that $t^*y1(-1)$ has a significant (adjusted) effect on $y2$. Since $t^*y2(-1)$ is insignificant in both regressions, as well as $x1$, then this may be a reduced model. Do this as an exercise. □

Vector Autoregression Estimates
Date: 12/20/07 Time: 14:04
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y1	Y2
Y1(-1)	1.013886	-0.673103
	(0.08878)	(0.27697)
	[11.4205]	[-2.43026]
Y2(-1)	0.097075	0.414527
	(0.04427)	(0.13811)
	[2.19291]	[3.00152]
C	-2.001417	28.45285
	(1.91075)	(5.96112)
	[-1.04745]	[4.77307]
T	-0.487795	0.328049
	(0.08725)	(0.27221)
	[-5.59066]	[1.20515]
X1*X3	3.81E-05	1.27E-05
	(1.1E-05)	(3.3E-05)
	[3.62143]	[0.38843]
X1*X2*X3	-2.11E-07	8.95E-08
	(6.8E-08)	(2.1E-07)
	[-3.10912]	[0.42325]

R-squared	0.990545	0.968144
Adj. R-squared	0.988490	0.961219
Sum sq. resids	42.96467	418.1782
S.E. equation	1.366759	4.263996
F-statistic	481.9287	139.7999
Log likelihood	-46.84891	-79.84409
Akaike AIC	3.644752	5.920282
Schwarz SC	3.927641	6.203171
Mean dependent	31.28655	46.45862
S.D. dependent	12.73949	21.65240

Determinant resid covariance (dof adj.)	33.70351
Determinant resid covariance	21.19995
Log likelihood	-126.5814
Akaike information criterion	9.557339
Schwarz criterion	10.12312

Roots of Characteristic Polynomial
Endogenous variables: Y1 Y2
Exogenous variables: C T X1*X3 X1*X2*X3
Lag specification: 1 1
Date: 12/20/07 Time: 14:06

Root	Modulus
0.870625	0.870625
0.557789	0.557789

No root lies outside the unit circle.
VAR satisfies the stability condition.

Figure 6.36 A three-way interaction VAR model and its stability check

Example 6.15. (A three-way interaction VAR model) After doing experimentation, the statistical results are obtained based on a nonhierarchical VAR three-way interaction model, as presented in Figure 6.36, p. 349 with the lag interval for endogenous variables of '1 1.' This figure shows that the three-way interaction $X1^{*}X2^{*}X3$ has a significant negative adjusted effect on $Y1$, but it has an insignificant effect on $Y2$ and similarly for the two-way interaction $X1^{*}X3$.

By using the system estimation method, a multivariate model can be obtained where the two regressions have different sets of independent variables, e.g. by deleting $X1^{*}X3$ or $X1^{*}X2^{*}X3$ from the second regression. □

Example 6.16. (The classical growth models of the five defined variables) By entering the endogenous variables log($Y1$), log($Y2$), log($X1$), log($X2$) and log($X3$) and the lag interval for endogenous variables of '0 0,' the statistical results are obtained based on a set of five classical growth models, as presented in Figure 6.37. This figure

Vector Autoregression Estimates
Date: 12/20/07 Time: 14:15
Sample: 1951 1980
Included observations: 30
Standard errors in ( ) & t-statistics in [ ]

	LOG(Y1)	LOG(Y2)	LOG(X1)	LOG(X2)	LOG(X3)
C	2.886780	2.978364	5.568688	3.817477	5.232134
	(0.07316)	(0.05648)	(0.03748)	(0.02155)	(0.08641)
	[39.4606]	[52.7299]	[148.588]	[177.180]	[60.5473]
T	0.031063	0.047921	0.070948	0.042059	0.050944
	(0.00412)	(0.00318)	(0.00211)	(0.00121)	(0.00487)
	[7.53811]	[15.0619]	[33.6079]	[34.6550]	[10.4660]
R-squared	0.669901	0.890136	0.975810	0.977217	0.796418
Adj. R-squared	0.658112	0.886212	0.974946	0.976403	0.789147
Sum sq. resides	1.068603	0.637028	0.280451	0.092692	1.491029
S.E. equation	0.195357	0.150834	0.100080	0.057536	0.230762
F-statistic	56.82303	226.8600	1129.488	1200.971	109.5365
Log likelihood	7.454521	15.21394	27.52016	44.12698	2.457807
Akaike AIC	-0.363635	-0.880929	-1.701344	-2.808465	-0.030520
Schwarz SC	-0.270222	-0.787516	-1.607931	-2.715052	0.062893
Mean dependent	3.368254	3.721145	6.668382	4.469389	6.021767
S.D. dependent	0.334108	0.447149	0.632279	0.374553	0.502544

Determinant resid covariance (dof adj.)	1.10E-10
Determinant resid covariance	7.76E-11
Log likelihood	136.3416
Akaike information criterion	-8.422773
Schwarz criterion	-7.955707

Figure 6.37 A set of classical growth models presented as a VAR model

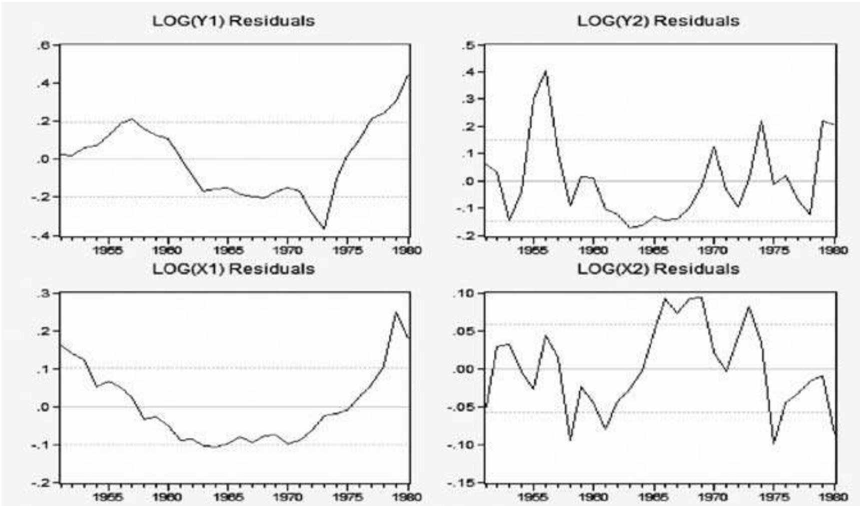


Figure 6.38 Four out of five residual graphs of the growth models in Figure 6.37

shows that each of the variables $Y1$, $Y2$, $X1$, $X2$ and $X3$ has significant growth rates during the observation time period, based on the standard t-statistic. For possible modified models, refer to the piecewise growth models presented in Chapter 3.

On the other hand, note that these growth models could be considered as unacceptable time series models, since their error terms are autocorrelated, as presented in Figure 6.38. To overcome these problems, a lag interval should be used for endogenous variables of '1 k.' For a comparison, Figure 6.39, p. 352 presents the residual graphs by using the lag interval '1 4' for the endogenous variables. This figure shows that the corresponding model is a better time series model compared to the growth model in Figure 6.37. For a comparison study, it is suggested that readers should use smaller lag intervals. □

6.2.5.2 Application of the VAR seemingly causal models

Each of the *seemingly causal models* (SCMs) presented in Chapter 4 can easily be modified to obtain a VAR model, namely VAR_SCM, which is a special type of the multivariate seemingly causal models. Furthermore, by deleting the time t from the additive, two-way and three-way interaction models presented in Chapter 2, as well as the VAR model in the previous sections, can easily provide various VAR_SCMs. Therefore, in this subsection, only a few illustrative examples will be presented.

Example 6.17. (Additive VAR_SCMs) Corresponding to the VAR translog linear model with trend in Figure 6.34, by deleting the time t from the model, an additive VAR_SCM can be obtained with the statistical results presented in Figure 6.40.

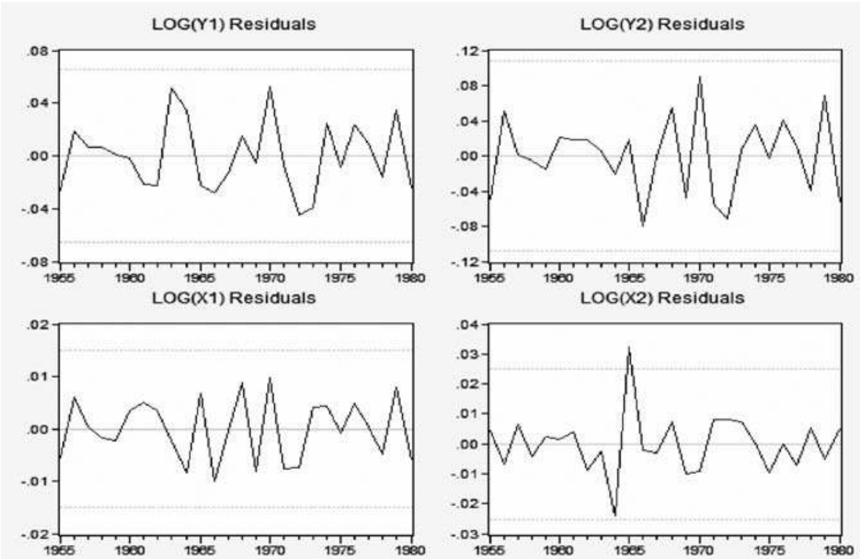


Figure 6.39 Residual graphs of a VAR model of the variables log(y1), log(Y2), log(X1), log(X2) and log(X3), using a lag interval of '1 4'

Vector Autoregression Estimates
Date: 12/20/07 Time: 15:04
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG(Y1)	LOG(Y2)
LOG(Y1(-1))	0.825916	0.099023
	(0.10944)	(0.23208)
	[7.54692]	[0.42667]
LOG(Y2(-1))	0.169485	0.471828
	(0.07202)	(0.15273)
	[2.35336]	[3.08932]
C	0.096064	-0.882117
	(0.23589)	(0.50025)
	[0.40724]	[-1.76335]
LOG(X1)	0.222207	0.147474
	(0.15481)	(0.32830)
	[1.43537]	[0.44920]
LOG(X2)	-0.641063	-0.141325
	(0.17553)	(0.37225)
	[-3.65207]	[-0.37965]
LOG(X3)	0.214490	0.364130
	(0.05388)	(0.11426)
	[3.98091]	[3.18679]

R-squared	0.980019	0.949043
Adj. R-squared	0.975676	0.937965
Sum sq. resids	0.060970	0.274201
S.E. equation	0.051486	0.109187
F-statistic	225.6211	85.67218
Log likelihood	48.23860	26.43801
Akaike AIC	-2.913007	-1.409518
Schwarz SC	-2.630118	-1.126629
Mean dependent	3.382868	3.743045
S.D. dependent	0.330119	0.438383

Determinant resid covariance (dof adj.)	3.14E-05
Determinant resid covariance	1.97E-05
Log likelihood	74.77184
Akaike information criterion	-4.329092
Schwarz criterion	-3.763315

Roots of Characteristic Polynomial
Endogenous variables: LOG(Y1) LOG(Y2)
Exogenous variables: C LOG(X1) LOG(X2) LOG(X3)
Lag specification: 1 1
Date: 12/20/07 Time: 15:08

Root	Modulus
0.868252	0.868252
0.429492	0.429492

No root lies outside the unit circle.
VAR satisfies the stability condition.

Figure 6.40 A VAR_SCM as a reduced model with trend in Figure 6.34 and its stability check

Vector Autoregression Estimates
Date: 09/09/06 Time: 03:32
Sample(adjusted): 1953 1980
Included observations: 28 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	Y1	Y2
Y1(-1)	1.060188	-1.123131
	(0.20908)	(0.74911)
	[5.07064]	[-1.49930]
Y1(-2)	-0.433036	0.816102
	(0.21025)	(0.75329)
	[-2.05962]	[1.08339]
Y2(-1)	0.048908	0.256682
	(0.06859)	(0.24576)
	[0.71301]	[1.04446]
Y2(-2)	-0.105689	-0.088756
	(0.05555)	(0.19904)
	[-1.90243]	[-0.44592]
C	10.43721	14.19638
	(4.22237)	(15.1279)
	[2.47189]	[0.93842]

Continued

X1(-1)	0.015313	0.033689
	(0.00454)	(0.01628)
	[3.36922]	[2.06889]
X2(-1)	-0.107986	0.049029
	(0.04914)	(0.17605)
	[-2.19766]	[0.27850]
R-squared	0.985440	0.934356
Adj. R-squared	0.981280	0.915601
Sum sq. resids	64.03626	821.9996
S.E. equation	1.746238	6.256424
F-statistic	236.8881	49.81823
Log likelihood	-51.31171	-87.04377
Akaike AIC	4.165122	6.717412
Schwarz SC	4.498173	7.050464
Mean dependent	31.71071	47.32179
S.D. dependent	12.76302	21.53564

Determinant Residual Covariance	91.08069
Log Likelihood (d.f. adjusted)	-142.6250
Akaike Information Criteria	11.18750
Schwarz Criteria	11.85360

Figure 6.41 Statistical results based on an additive SC_VAR model of $\{Y1, Y2\}$

For a modified model, Figure 6.41 presents the statistical results based on the additive VAR_SCM with endogenous variables $Y1$ and $Y2$. Note that this model has greater values of AIC and SC statistics compared to the previous model. Therefore, the previous model could be the preferred model. □

Example 6.18. (A VAR two-way interaction semilog model) Figure 6.42 presents a VAR two-way interaction semilog model, which shows that log($Y1(-1)$) and log ($Y2(-1)$) should be kept in both regressions, even though they are insignificant, since the lag interval of '1 1' is used. In order to modify this model, other lag intervals of '$n\,m$' may be used, either $n=m$ or $n<m$.

Vector Autoregression Estimates
Date: 12/21/07 Time: 08:52
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG(Y1)	LOG(Y2)
LOG(Y1(-1))	0.811118	0.083625
	(0.12737)	(0.26717)
	[6.36798]	[0.31300]
LOG(Y2(-1))	0.225696	0.506387
	(0.07192)	(0.15086)
	[3.13799]	[3.35659]
C	0.236114	1.146830
	(0.35449)	(0.74356)
	[0.66607]	[1.54235]
X2	-0.007738	0.003788
	(0.00258)	(0.00540)
	[-3.00325]	[0.70092]
X1*X2	1.30E-06	-2.57E-06
	(1.3E-06)	(2.7E-06)
	[1.00492]	[-0.94788]
X1*X3	-2.27E-07	3.46E-07
	(2.4E-07)	(5.1E-07)
	[-0.94320]	[0.68545]
X2*X3	6.38E-06	3.29E-06
	(2.8E-06)	(5.8E-06)
	[2.30341]	[0.56665]
R-squared	0.979575	0.949040
Adj. R-squared	0.974004	0.935142
Sum sq. resids	0.062326	0.274217
S.E. equation	0.053226	0.111644
F-statistic	175.8487	68.28522
Log likelihood	47.91951	26.43717
Akaike AIC	-2.822035	-1.340494
Schwarz SC	-2.491999	-1.010457
Mean dependent	3.382868	3.743045
S.D. dependent	0.330119	0.438383

Determinant resid covariance (dof adj.)	3.47E-05
Determinant resid covariance	2.00E-05
Log likelihood	74.59396
Akaike information criterion	-4.178894
Schwarz criterion	-3.518820

Figure 6.42 Statistical results based on a two-way interaction semilog VAR model

On the other hand, since only $X3$ and $X2^*X3$ have significant adjusted effects on log($Y1$), one or two other exogenous variables should be deleted in order to obtain an acceptable VAR model, in a statistical sense. Do this as an exercise. □

Example 6.19. (Another interaction VAR model) Corresponding to the CES or quadratic translog model in (4.103), Figure 6.43 presents the statistical results based on a VAR CES model with endogenous variables $Y1$ and $Y2$.

Vector Autoregression Estimates
Date: 12/21/07 Time: 09:07
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG(Y1)	LOG(Y2)
LOG(Y1(-1))	0.581453	-0.217655
	(0.14457)	(0.30168)
	[4.02203]	[-0.72147]
LOG(Y2(-1))	0.188224	0.474645
	(0.09710)	(0.20262)
	[1.93853]	[2.34254]
C	3.771497	5.832323
	(4.28572)	(8.94342)
	[0.88002]	[0.65214]
LOG(X1)	-4.170085	-2.063639
	(3.19842)	(6.67445)
	[-1.30380]	[-0.30919]
LOG(X2)	4.815407	1.002783
	(5.10323)	(10.6494)
	[0.94360]	[0.09416]
LOG(X1)^2	-0.063250	0.248355
	(0.64460)	(1.34514)
	[-0.09812]	[0.18463]
LOG(X1)*LOG(X2)	1.151063	-0.226846
	(2.42873)	(5.06826)
	[0.47394]	[-0.04476]
LOG(X2)^2	-1.415356	0.090694
	(2.24985)	(4.69499)
	[-0.62909]	[0.01932]
R-squared	0.971496	0.929612
Adj. R-squared	0.961995	0.906149
Sum sq. resids	0.086977	0.378761
S.E. equation	0.064357	0.134299
F-statistic	102.2487	39.62083
Log likelihood	43.08719	21.75390
Akaike AIC	-2.419806	-0.948545
Schwarz SC	-2.042621	-0.571360
Mean dependent	3.382868	3.743045
S.D. dependent	0.330119	0.438383

Determinant resid covariance (dof adj.)	6.44E-05
Determinant resid covariance	3.38E-05
Log likelihood	66.98481
Akaike information criterion	-3.516194
Schwarz criterion	-2.761824

Figure 6.43 Statistical results based on a CES VAR model of {Y1, Y2}, using the lag interval '1 1'

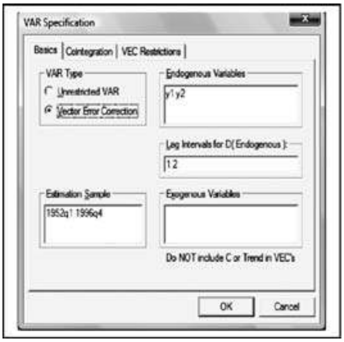
The result shows that log($X1$) and log($X2$) have significant adjusted effects on log ($Y1$), but only log($X1$) has a significant adjusted effect on log($Y2$). Therefore, in a statistical sense a reduced model should be made. Do this as an exercise. □

6.3 The vector error correction models

6.3.1 The basic VEC model

Figure 6.44 presents the window for doing analysis based on a VEC model and the equation of a basic VEC model with dependent variables $Y1$ and $Y2$, by using the default option of the '*Lag Interval for Endogenous*,' i.e. '1 2.' Furthermore, the window presents a note '*Do NOT include C or Trend in VECs*.' This basic VEC model has special characteristics as follows:

(1) Both regressions have the first differences of the endogenous variables $Y1$ and $Y2$, namely $D(Y1)$ and $D(Y2)$, as dependent variables. Hence, a VEC model can be considered as a special case of the multivariate models of the first differences.




```
Estimation Proc:
===============================
EC(C,1) 1 2 Y1 Y2

VAR Model:
===============================
D(Y1) = A(1,1)*(B(1,1)*Y1(-1) + B(1,2)*Y2(-1) + B(1,3))
        + C(1,1)*D(Y1(-1)) + C(1,2)*D(Y1(-2))
        + C(1,3)*D(Y2(-1)) + C(1,4)*D(Y2(-2)) + C(1,5)

D(Y2) = A(2,1)*(B(1,1)*Y1(-1) + B(1,2)*Y2(-1) + B(1,3))
        + C(2,1)*D(Y1(-1)) + C(2,2)*D(Y1(-2))
        + C(2,3)*D(Y2(-1)) + C(2,4)*D(Y2(-2)) + C(2,5)
```

Figure 6.44 The default options and a basic VEC model of $\{Y1, Y2\}$

(2) Both regressions have the same specific term or a linear combination of $Y1(-1)$ and $Y2(-1)$, namely '$B(1, 1)^{*}Y1(-1) + B(1, 2)^{*}Y2(-1) + B(1, 3)$,' which is called the '*Cointegrating Equation*,' as an independent variable of both regressions in the model.
(3) The other independent variables are the first and second lags of $D(Y1)$ and $D(Y2)$, namely $D(Y1(-1))$, $D(Y1(-2)$, $D(Y2(-1))$ and $D(Y2(-2))$, which are associated with the lag interval '1 2.'
(4) The lag intervals for D(Endogenous) can also be modified to '0 0' or '1 1,' in order to have the first two simplest VEC models. In these cases, the VEC models are as presented in Figure 6.45.
(5) Note that the three VEC models of the variables $\{Y1, Y2\}$ with the lag intervals for D(Endogenous) of '0 0,' '1 1' and '1 2' have the same form of '*Cointegrating Equation*,' namely '$B(1, 1)^{*}Y1(-1) + B(1, 2)^{*}Y2(-1) + B(1, 3)$,' but they will have different estimates. Do this as an exercise.

```
Estimation Proc:
===============================
EC(C,1) 0 0 Y1 Y2

VAR Model:
===============================
D(Y1) = A(1,1)*(B(1,1)*Y1(-1)
        + B(1,2)*Y2(-1) + B(1,3)) + C(1,1)

D(Y2) = A(2,1)*(B(1,1)*Y1(-1)
        + B(1,2)*Y2(-1) + B(1,3)) + C(2,1)
```

```
Estimation Proc:
===============================
EC(C,1) 1 1 Y1 Y2

VAR Model:
===============================
D(Y1) = A(1,1)*(B(1,1)*Y1(-1) + B(1,2)*Y2(-1) + B(1,3))
        + C(1,1)*D(Y1(-1)) + C(1,2)*D(Y2(-1)) + C(1,3)

D(Y2) = A(2,1)*(B(1,1)*Y1(-1) + B(1,2)*Y2(-1) + B(1,3))
        + C(2,1)*D(Y1(-1)) + C(2,2)*D(Y2(-1)) + C(2,3)
```

Figure 6.45 The first two simplest VEC models of $(Y1, Y2)$

Example 6.20. (The VEC specification is imposing one unit root) Figure 6.46(a) and (b) presents the statistical results of two VEC models that are imposing one

Roots of Characteristic Polynomial
Endogenous variables: Y1 Y2
Exogenous variables:
Lag specification: 1 2
Date: 12/21/07 Time: 19:40

Root	Modulus
1.000000	1.000000
0.829339	0.829339
-0.253440 - 0.769533i	0.810193
-0.253440 + 0.769533i	0.810193
0.790197	0.790197
0.283572	0.283572

VEC specification imposes 1 unit root(s).

(a)

Roots of Characteristic Polynomial
Endogenous variables: LOG(Y1) LOG(Y2)
Exogenous variables:
Lag specification: 1 2
Date: 12/21/07 Time: 19:45

Root	Modulus
1.000000	1.000000
0.877115	0.877115
-0.047593 - 0.763789i	0.765270
-0.047593 + 0.763789i	0.765270
0.327603 - 0.138659i	0.355739
0.327603 + 0.138659i	0.355739

VEC specification imposes 1 unit root(s).

(b)

Figure 6.46 Illustrations of the VEC specification which imposes one unit root: (a) the VEC model of ($Y1$, $Y2$) and (b) the VEC model of (log($Y1$), log($Y2$))

unit root. It was also found that the printout of any VEC models, either with or without exogenous variable(s), presents the message '*VEC specification imposes* 1 *unit root*(*s*).' □

Example 6.21. (A VEC model with lag specification '0 0') Figure 6.47 presents statistical results based on a VEC model having endogenous variables {log($Y1$), log ($Y2$)} with lag specification '0 0'. The regression function of this VEC model is

$$D(\log(Y1)) = -0.0654[\log(Y1(-1)) - 1.9058\log(Y2(-1)) + 3.6953] + 0.0454$$
$$D(\log(Y2)) = 0.0241[\log(Y1(-1)) - 1.9058\log(Y2(-1)) + 3.6953] + 0.0529 \tag{6.9}$$

Vector Error Correction Estimates
Date: 12/21/07 Time: 20:17
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(Y1(-1))	1.000000	
LOG(Y2(-1))	-1.905806	
	(0.44532)	
	[-4.27961]	
C	3.695295	
Error Correction:	**D(LOG(Y1))**	**D(LOG(Y2))**
CointEq1	-0.065360	0.024094
	(0.02389)	(0.04939)
	[-2.73542]	[0.48778]
C	0.045393	0.052864
	(0.01360)	(0.02812)
	[3.33653]	[1.87960]

R-squared	0.216995	0.008735
Adj. R-squared	0.187994	-0.027978
Sum sq. resids	0.144929	0.619358
S.E. equation	0.073265	0.151457
F-statistic	7.482522	0.237927
Log likelihood	35.68347	14.62311
Akaike AIC	-2.322998	-0.870559
Schwarz SC	-2.228701	-0.776263
Mean dependent	0.045393	0.052864
S.D. dependent	0.081305	0.149382

Determinant resid covariance (dof adj.)	0.000111
Determinant resid covariance	9.58E-05
Log likelihood	51.87064
Akaike information criterion	-3.163492
Schwarz criterion	-2.880603

```
Estimation Proc:
===============================
EC(C,1) 0 0 LOG(Y1) LOG(Y2)

VAR Model:
===============================
D(LOG(Y1)) = A(1,1)*(B(1,1)*LOG(Y1(-1)) + B(1,2)*LOG(Y2(-1)) + B(1,3)) + C(1,1)

D(LOG(Y2)) = A(2,1)*(B(1,1)*LOG(Y1(-1)) + B(1,2)*LOG(Y2(-1)) + B(1,3)) + C(2,1)
```

Figure 6.47 Statistical results based on the VEC model of {log($y1$), log($y2$)} with lag specification'0 0' with an estimated cointegrating equation $\log(Y1(-1)) - 1.9058\ \log(Y2(-1)) + 3.6953$, which has a significant negative effect on the first difference $D(\log(Y1))$, but has an insignificant positive effect on $D(\log(Y2))$. □

Example 6.22. (The system estimation of the VEC model in figure 6.47) For a comparison, Figure 6.48 presents statistical results of a similar bivariate model of the endogenous variables $D\log(Y1)$ and $D\log(Y2)$ on the cointegrating equation of the VEC model in Figure 6.47. This figure also shows that the cointegrating equation has a significant negative effect on $D\log(Y1)$ but an insignificant positive effect on $D\log(Y2)$. However, they have different values of t-statistics.

The advantages of using the system equation are that the Wald coefficient tests can be used, which cannot be done using the VEC model in Figure 6.47. In this case, a multivariate hypothesis of the cointegration equation effect can be tested on both endogenous variables, with the null hypothesis H_0: $C(11) = C(21) = 0$. This null hypothesis is rejected based on the chi-squared-statistic of 10.257 74 with $df = 2$ and a p-value $= 0.0059$. Therefore, it can be concluded that the cointegrating equation has a significant effect on the bivariate $\{D\log(y1), D\log(y2)\}$.

For further comparison, Figure 6.49 presents the statistical results based on a system equation, which can be considered similar to the VEC model in Figure 6.47, since it has the same endogenous and exogenous variables, but without the cointegrating equation. Based on this model, various hypotheses on the model parameters

System: SYS16
Estimation Method: Seemingly Unrelated Regression
Date: 12/21/07 Time: 20:50
Sample: 1952 1980
Included observations: 29
Total system (balanced) observations 58
Linear estimation after one-step weighting matrix

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.065360	0.023055	-2.834922	0.0064
C(12)	0.045393	0.013127	3.457902	0.0011
C(21)	0.024094	0.047661	0.505520	0.6153
C(22)	0.052864	0.027138	1.947975	0.0566

Determinant residual covariance 9.58E-05

Equation: D(LOG(Y1)) = C(11)*(LOG(Y1(-1)) - 1.90580576615*LOG(Y2(-1)) + 3.69529488378) + C(12)
Observations: 29

R-squared	0.216995	Mean dependent var	0.045393
Adjusted R-squared	0.187994	S.D. dependent var	0.081305
S.E. of regression	0.073265	Sum squared resid	0.144929
Durbin-Watson stat	1.242311		

Equation: D(LOG(Y2)) = C(21)*(LOG(Y1(-1)) - 1.90580576615*LOG(Y2(-1)) + 3.69529488378) + C(22)
Observations: 29

R-squared	0.008735	Mean dependent var	0.052864
Adjusted R-squared	-0.027978	S.D. dependent var	0.149382
S.E. of regression	0.151457	Sum squared resid	0.619358
Durbin-Watson stat	1.925670		

Figure 6.48 Statistical results based on the VEC model in Figure 6.47 using the system equation

System: SYS15
Estimation Method: Seemingly Unrelated Regression
Date: 12/21/07 Time: 20:44
Sample: 1952 1980
Included observations: 29
Total system (balanced) observations 58
Linear estimation after one-step weighting matrix

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.042777	0.083049	-0.515081	0.6087
C(12)	0.113871	0.057901	1.966645	0.0546
C(13)	-0.232045	0.153340	-1.513266	0.1363
C(21)	0.123166	0.170850	0.720903	0.4742
C(22)	-0.092824	0.119116	-0.779279	0.4393
C(23)	-0.015661	0.315455	-0.049647	0.9606

Determinant residual covariance 9.46E-05

Equation: DLOG(Y1)=C(11)*LOG(Y1(-1))+C(12)*LOG(Y2(-1)) +C(13)
Observations: 29

R-squared	0.219151	Mean dependent var	0.045393
Adjusted R-squared	0.159086	S.D. dependent var	0.081305
S.E. of regression	0.074558	Sum squared resid	0.144530
Durbin-Watson stat	1.275897		

Equation: DLOG(Y2)=C(21)*LOG(Y1(-1))+C(22)*LOG(Y2(-1)) +C(23)
Observations: 29

R-squared	0.021032	Mean dependent var	0.052864
Adjusted R-squared	-0.054274	S.D. dependent var	0.149382
S.E. of regression	0.153382	Sum squared resid	0.611675
Durbin-Watson stat	1.921521		

Figure 6.49 Statistical results based on a system equation, which is similar to the VEC model in Figure 6.47

could be tested, using the Wald tests, which cannot be tested using the VAR and VEC models. For example, note the following hypotheses:

(1) The joint effect of $\log(Y1(-1))$ and $\log(Y2(-1))$ on $D\log(Y1)$, with the univariate null hypothesis H_0: $C(11) = C(12) = 0$, is rejected based on the chi-squared-statistics of 8.139 081 with $df = 2$ and a p-value = 0.0171.
(2) The joint effect of $\log(Y1(-1))$ and $\log(Y2(-1))$ on both endogenous variables, with the multivariate null hypothesis H_0: $C(11) = C(12) = C(21) = C(22) = 0$, is rejected based on the chi-squared-statistic of 10.631 43 with $df = 4$ and a p-value = 0.0310.
(3) Figure 6.50 presents the options available for the VAR model, which does not present the '*Coefficient Tests*' option, compared to the options for the system equation. □

Example 6.23. (An extension of the basic VAR model) The VEC model in Figure 6.51 is an extension of the basic VAR model with endogenous variables $\{\log(Y1), \log(Y2)\}$. Based on the results in this figure, the following notes and conclusions are produced:

(1) The cointegrating equation has an insignificant effect on each of the endogenous variables.
(2) Even though only $D\log(Y2(-2))$ is significant, the other independent variables cannot be deleted by using the lag specification '1 2.' However, the

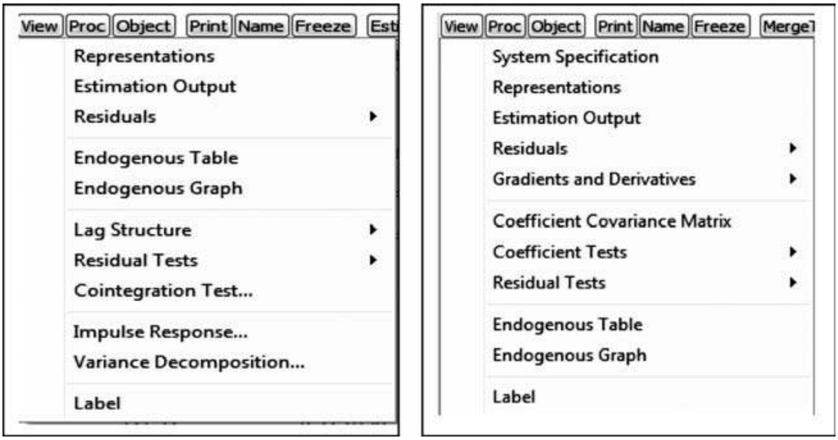


Figure 6.50 Some differential options available for the VAR model and the system equation

lag specification '2 2' may be used to obtain a modified model with the statistical result presented in Figure 6.52. Based on this result the following notes are made:

- The cointegration has a significant negative effect on $D\log(Y1)$.
- $D\log(Y2(-2))$ has a significant negative effect on $D\log(Y2)$.

Vector Error Correction Estimates
Date: 12/22/07 Time: 09:58
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
LOG(Y1(-1))	1.000000
LOG(Y2(-1))	-1.654289 (0.55096) [-3.00257]
C	2.811752

Error Correction:	D(LOG(Y1))	D(LOG(Y2))
CointEq1	-0.058672 (0.03393) [-1.72929]	0.025098 (0.06642) [0.37790]
D(LOG(Y1(-1)))	0.338550 (0.21957) [1.54191]	-0.543322 (0.42981) [-1.26411]
D(LOG(Y1(-2)))	0.064799 (0.21054) [0.30777]	0.581411 (0.41214) [1.41070]
D(LOG(Y2(-1)))	0.155045 (0.10954) [1.41544]	0.198778 (0.21442) [0.92703]
D(LOG(Y2(-2)))	-0.166845 (0.10736) [-1.55413]	-0.493838 (0.21015) [-2.34990]
C	0.026840 (0.01738) [1.54389]	0.070268 (0.03403) [2.06484]

R-squared	0.456908	0.351817
Adj. R-squared	0.327600	0.197488
Sum sq. resids	0.099719	0.382118
S.E. equation	0.068910	0.134893
F-statistic	3.533490	2.279651
Log likelihood	37.30530	19.16981
Akaike AIC	-2.318911	-0.975541
Schwarz SC	-2.030947	-0.687578
Mean dependent	0.045173	0.060815
S.D. dependent	0.084036	0.150579

Determinant resid covariance (dof adj.)	6.69E-05
Determinant resid covariance	4.05E-05
Log likelihood	59.92163
Akaike information criterion	-3.401602
Schwarz criterion	-2.729687

Figure 6.51 Statistical results based on a VEC model of {log($Y1$), log($Y2$} with lag specification '1 2'

Vector Error Correction Estimates
Date: 12/22/07 Time: 11:07
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(Y1(-1))	1.000000	
LOG(Y2(-1))	-10.02282	
	(2.72654)	
	[-3.67603]	
C	34.06205	
Error Correction:	**D(LOG(Y1))**	**D(LOG(Y2))**
CointEq1	-0.012072	0.004359
	(0.00412)	(0.00728)
	[-2.93067]	[0.59901]
D(LOG(Y1(-2)))	-0.047376	0.317129
	(0.19734)	(0.34860)
	[-0.24007]	[0.90971]
D(LOG(Y2(-2)))	-0.081205	-0.580546
	(0.11003)	(0.19437)
	[-0.73804]	[-2.98686]
C	0.050361	0.072348
	(0.01673)	(0.02956)
	[3.00950]	[2.44741]

R-squared	0.280422	0.300612
Adj. R-squared	0.186564	0.209387
Sum sq. resids	0.132125	0.412305
S.E. equation	0.075793	0.133889
F-statistic	2.987730	3.295292
Log likelihood	33.50660	18.14336
Akaike AIC	-2.185674	-1.047656
Schwarz SC	-1.993698	-0.855681
Mean dependent	0.045173	0.060815
S.D. dependent	0.084036	0.150579
Determinant resid covariance (dof adj.)		8.73E-05
Determinant resid covariance		6.33E-05
Log likelihood		53.88094
Akaike information criterion		-3.250440
Schwarz criterion		-2.770501

Figure 6.52 Statistical results based on a VEC model with lag specification '2 2'

- Even though $D\log(Y1(-2))$ has insignificant effects on both endogenous variables, it cannot be deleted in order to obtain a reduced model, using the VEC model. □

6.3.2 General equation of the basic VEC models

Based on the basic VEC models presented in the previous examples, the general equations of alternative basic VEC models can be derived, as follows:

(1) The Lag Intervals for *D*(Endogenous): '0 0'

$$D(Y_{g,t}) = A(g,1)^{*}Coint + C(g,1) + \mu_{g,t} \tag{6.10a}$$

or

$$D(Y_{g,t}) = A(g,1)^{*}\left\{\sum_{k=1}^{G} B(g,k)Y_{g,t-1} + B(g,G+1)\right\} + C(g,1) + \mu_{g,t} \tag{6.10b}$$

where $Y_{g,t}$ is the gth endogenous variable at the time t, for $g = 1, 2, \ldots, G$.

(2) The Lag Intervals for *D*(Endogenous): '1 1'

$$D(Y_{g,t}) = A(g,1)^{*}Coint + \sum_{k=1}^{G} C(g,k)^{*}D(Y_{g,t-1}) + C(g,G+1) + \mu_{g,t} \tag{6.11}$$

(3) The lag intervals for D(Endogenous): '1 2'

$$D(Y_{g,t}) = A(g,1)^{*}Coint + \sum_{k=1}^{G} C(g,2k-1)^{*}D(Y_{g,t-1}) + \sum_{k=1}^{G} C(g,2k)D(Y_{g,t-2}) + C(g,2G+1) + \mu_{g,t} \tag{6.12}$$

(4) Special Lag Intervals for D(Endogenous): '$p\,p$'
If there is a quarterly time series, the lag interval '4 4' for D(Endogenous) might be considered in order to match the quarters in recent and previous years, and if there is a monthly time series, the lag interval '12 12' might be used in order to match the months in recent and previous years for D(Endogenous). Hence, the following general equation, for $p = 4$ or $p = 12$, is obtained:

$$D(Y_{g,t}) = A(g,1)^{*}Coint + \sum_{k=1}^{G} C(g,k)^{*}D(Y_{g,t-p}) + C(g,G+1) + \mu_{g,t} \tag{6.13}$$

(5) VEC Models with Endogenous Variables
The general basic VEC models above can easily be extended to a VEC model with various exogenous variables.

Example 6.24. (A special basic VEC model) As an extension of the VAR models of {log(M1), log(GDP)} presented in the previous section, there is a need to present a special basic VEC model with endogenous variables log(m1) and log(gdp), and the lag specification '4 4.' The background of using this lag specification is to match the quarters in recent and previous years. Based on the statistical results presented in Figure 6.53, the following notes and conclusions can be made:

(1) The cointegrating equation has a significant negative effect on each of the endogenous variables: D(log(M1)) and D(log(GDP)) based on the t-statistics of −4.029 and −4.862 respectively.
(2) Dlog(M1(−4)) has a significant positive effect on Dlog(M1), but it has an insignificant positive effect on Dlog(GDP).
(3) Even though Dlog(GDP(−4)) has an insignificant effect on both endogenous variables, Dlog(GDP(−4)) cannot be deleted since the VEC model should have Dlog(M1(−4)) and Dlog(GDP(−4)) as a couple of independent variables. □

6.3.3 The VEC models with exogenous variables

Since there is a note 'Do NOT include C or Trend in VECs,' then the time t should not exist as a single exogenous variable. However, an experiment is conducted to use it

Vector Error Correction Estimates
Date: 12/22/07 Time: 15:18
Sample (adjusted): 1953Q2 1996Q4
Included observations: 175 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(M1(-1))	1.000000	
LOG(GDP(-1))	-0.773903	
	(0.01489)	
	[-51.9670]	
C	-1.169520	
Error Correction:	D(LOG(M1))	D(LOG(GDP))
CointEq1	-0.050776	-0.043066
	(0.01260)	(0.00886)
	[-4.02889]	[-4.86198]
D(LOG(M1(-4)))	0.291082	0.072222
	(0.07163)	(0.05034)
	[4.06389]	[1.43466]
D(LOG(GDP(-4)))	-0.125553	-0.128074
	(0.11262)	(0.07915)
	[-1.11482]	[-1.61805]
C	0.011131	0.018585
	(0.00228)	(0.00161)
	[4.87188]	[11.5742]

R-squared	0.185025	0.140687
Adj. R-squared	0.170727	0.125611
Sum sq. resids	0.034093	0.016840
S.E. equation	0.014120	0.009924
F-statistic	12.94077	9.332043
Log likelihood	499.2380	560.9522
Akaike AIC	-5.659863	-6.365168
Schwarz SC	-5.587525	-6.292830
Mean dependent	0.012685	0.017284
S.D. dependent	0.015505	0.010613

Determinant resid covariance (dof adj.)	1.94E-08
Determinant resid covariance	1.85E-08
Log likelihood	1061.440
Akaike information criterion	-12.01646
Schwarz criterion	-11.83562

VEC Stability Condition Check

Roots of Characteristic Polynomial
Endogenous variables: LOG(M1) LOG(GDP)
Exogenous variables:
Lag specification: 0 0
Date: 12/24/07 Time: 10:31

Root	Modulus
1.000000	1.000000
0.975537	0.975537

VEC specification imposes 1 unit root(s).

Figure 6.53 A special VEC model of $\{\log(M1), \log(GDP)\}$ with lag specification '4 4' and its stability condition check

and its interaction with other exogenous variable(s) as independent variable(s) of the model, in order to explore or study its statistical results. Note the following example.

The general equation of a VEC model with exogenous variables can easily be derived from the models in (6.10) to (6.12). For example, by using the lag intervals for D(Endogenous) of '0 0,' the general equation is obtained as follows:

$$D(Y_{g,t}) = A(g,1)^{*}Coint + C(g,1) + \sum_{k=1}^{K} C(g,k+1)X_{k,t} + \mu_{g,t} \qquad (6.14)$$

Since the equation of each VEC model with exogenous variables can easily be obtained or written based on the output, the general equation of the VEC model with other lag specifications will not be presented.

Example 6.25. (The simplest VEC model with interaction exogenous variables) By applying the simplest VEC model (i.e. the lag intervals for D(Endogenous) of '0 0' with the endogenous variables $\{Y1, Y2\}$ and the exogenous variables $X1$, $X2$ and $X1^{*}X2$, the statistical results in Figure 6.54 are obtained. Based on this output, the following notes and conclusions are presented:

(1) The VEC model, as well as its regression functions, can easily be obtained by selecting *View/Representations*.

Vector Error Correction Estimates
Date: 12/24/07 Time: 10:12
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
Y1(-1)	1.000000
Y2(-1)	0.020510
	(0.09074)
	[0.22601]
C	-453.5964

R-squared	0.511072	0.651347
Adj. R-squared	0.499832	0.643332
Sum sq. resids	8388.995	5484.106
S.E. equation	6.943531	5.614076
F-statistic	45.47015	81.26579
Log likelihood	-598.3225	-560.2790
Akaike AIC	6.741033	6.315967
Schwarz SC	6.830066	6.405000
Mean dependent	6.009006	10.39302
S.D. dependent	9.817988	9.400394

Determinant resid covariance (dof adj.)	1519.353
Determinant resid covariance	1435.659
Log likelihood	-1158.589
Akaike information criterion	13.07921
Schwarz criterion	13.29289

Error Correction:	D(Y1)	D(Y2)
CointEq1	-0.114387	-0.002244
	(0.01164)	(0.00941)
	[-9.82463]	[-0.23840]
C	-77.95909	-7.321388
	(7.46412)	(6.03499)
	[-10.4445]	[-1.21316]
X1	191.0468	29.41925
	(16.8224)	(13.6015)
	[11.3567]	[2.16295]
X2	2.060564	1.011915
	(0.57457)	(0.46456)
	[3.58625]	[2.17822]
X1*X2	-8.158089	-0.936817
	(1.06236)	(0.85895)
	[-7.67923]	[-1.09065]

Figure 6.54 Statistical results based on an interaction VEC model with lag specification '0 0'

(2) The '*cointegrating equation*' has a significant adjusted effect on $D(Y1)$, but it has an insignificant adjusted effect on $D(Y2)$.

(3) Each of the exogenous variables $X1$ and $X2$ has a significant adjusted effect on both $D(Y1)$ and $D(Y2)$. However, the interaction $X1^*X2$ has a significant adjusted effect only on $D(Y1)$.

(4) For illustration purposes, Figure 6.55 presents the VEC stability condition checks based on two VEC models of $\{Y1, Y2\}$. The first model is the interaction VEC model with exogenous variables and the second is the VEC model without exogenous variables, which has a root outside of the unit circle (= 1.013 952). Hence, the VEC model with exogenous variables should be considered as a better model. □

VEC Stability Condition Check

Roots of Characteristic Polynomial
Endogenous variables: Y1 Y2
Exogenous variables: X1 X2 X1*X2
Lag specification: 0 0
Date: 12/24/07 Time: 10:37

Root	Modulus
1.000000	1.000000
0.885567	0.885567

VEC specification imposes 1 unit root(s).

VEC Stability Condition Check

Roots of Characteristic Polynomial
Endogenous variables: Y1 Y2
Exogenous variables:
Lag specification: 0 0
Date: 12/24/07 Time: 10:35

Root	Modulus
1.013952	1.013952
1.000000	1.000000

VEC specification imposes 1 unit root(s).

Figure 6.55 The VEC stability condition checks based on two alternative VEC models of $\{Y1, Y2\}$, with and without exogenous variables, and lag specification '0 0'

Example 6.26. (An extension of the VAR model in figure 6.30) Refer to the VAR model of {log(*M*1), log(*GDP*)} with exogenous variables in Figure 6.30, where the cointegration test shows that it has one cointegrating equation between the endogenous variables. For this reason, here the corresponding VEC model is applied as an extension of the previous VAR model with additive exogenous variables.

Figure 6.56 presents the statistical results based on the corresponding VEC model, with its VAR stability condition check. Based on this output the following notes and conclusions are made:

(1) The cointegrating equation '$\log(M1(-1)) - 0.825\,759\ \log(GDP(-1)) - 0.860\,631$' has a significant negative effect on *D*log(*M*1), but it has an insignificant negative effect on *D*log(*GDP*).
(2) *D*(log(*M*1(−2))) has a significant positive effect on *D*log(*M!*).
(3) Each of *D*(log(*M*1(−2))) and *D*(log(*GDP*(−1))) has a significant positive effect on *D*log(*GDP*).
(4) The VAR stability condition check does not present a root outside the unit circle, besides the first unit root of 1 (one). Therefore, this VEC model should be considered as an acceptable model, in a statistical sense. □

Vector Error Correction Estimates
Date: 12/24/07 Time: 11:14
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(M1(-1))	1.000000	
LOG(GDP(-1))	-0.825759	
	(0.19198)	
	[-4.30126]	
C	-0.860631	
Error Correction:	**D(LOG(M1))**	**D(LOG(GDP))**
CointEq1	-0.064352	-0.005840
	(0.01518)	(0.00976)
	[-4.23990]	[-0.59867]
D(LOG(M1(-1)))	-0.103984	0.003792
	(0.07514)	(0.04830)
	[-1.38379]	[0.07850]
D(LOG(M1(-2)))	0.152841	0.168220
	(0.07484)	(0.04810)
	[2.04237]	[3.49727]
D(LOG(GDP(-1)))	-0.050504	0.305932
	(0.11780)	(0.07571)
	[-0.42874]	[4.04065]
D(LOG(GDP(-2)))	-0.103835	-0.046775
	(0.11897)	(0.07647)
	[-0.87275]	[-0.61167]
C	0.015642	0.005107
	(0.00352)	(0.00226)
	[4.44047]	[2.25578]
LOG(PR)	-0.309391	0.374554
	(0.23193)	(0.14907)
	[-1.33399]	[2.51255]
LOG(PR(-1))	0.306996	-0.376890
	(0.23147)	(0.14878)
	[1.32629]	[-2.53324]
R-squared	0.169379	0.273162
Adj. R-squared	0.134974	0.243056
Sum sq. resids	0.035070	0.014488
S.E. equation	0.014405	0.009259
F-statistic	4.923163	9.073431
Log likelihood	503.4490	581.6822
Akaike AIC	-5.598294	-6.482285
Schwarz SC	-5.454739	-6.338730
Mean dependent	0.012594	0.017396
S.D. dependent	0.015488	0.010642
Determinant resid covariance (dof adj.)		1.73E-08
Determinant resid covariance		1.57E-08
Log likelihood		1087.810
Akaike information criterion		-12.08824
Schwarz criterion		-11.76524

Roots of Characteristic Polynomial
Endogenous variables: LOG(M1) LOG(GDP)
Exogenous variables: LOG(PR) LOG(PR(-1))
Lag specification: 1 2
Date: 12/24/07 Time: 11:18

Root	Modulus
1.000000	1.000000
0.956161	0.956161
0.360043 - 0.254435i	0.440872
0.360043 + 0.254435i	0.440872
-0.392313	0.392313
-0.141515	0.141515

VEC specification imposes 1 unit root(s).

Figure 6.56 Statistical results based on a VEC model with exogenous variables, as an extension of the VAR model in Figure 6.30

Example 6.27. (A VEC model with additive exogenous variables) By applying a VEC model with endogenous variables $\{Y1, Y2\}$ and additive exogenous variables $X1$, $X2$, $X1(-1)$ and $X2(-1)$, the following regressions are obtained, with the t-statistics in [...]:

$$\begin{aligned}
D(Y1) &= \underset{[-6.288]}{-0.006}\{Y1(-1) + \underset{[6.532]}{12.703}\, Y2(-1) - 8380.971\} - \underset{[-6.209]}{65.963} \\
&\quad + \underset{[0.524]}{167.813}\, X1 - \underset{[-0.033]}{10.642}\, X1(-1) - \underset{[-2.825]}{2.269} X2 + \underset{[0.732]}{0.585} X2(-1) \\
&\quad R\text{-squared} = 0.511\,072, \text{ and Adj. } R\text{-squared} = 0.499\,832 \\
D(Y2) &= -\underset{[-3.017]}{0.002}\{Y1(-1) + \underset{[6.532]}{12.703}\, Y2(-1) - 8380.971\} - \underset{[-3.394]}{22.331} \\
&\quad + \underset{[3.140]}{622.421}\, X1 - \underset{[-2.790]}{558.983}\, X1(-1) + \underset{[5.684]}{2.827} X2 - \underset{[-6.799]}{3.364} X2(-1) \\
&\quad R\text{-squared} = 0.651\,347, \text{ and Adj. } R\text{-squared} = 0.643\,332
\end{aligned} \tag{6.15}$$

Based on these regressions the following notes and conclusions are made:

(1) The '*cointegrating equation*' has a significant adjusted effect on $D(Y1)$ and $D(Y2)$.
(2) Each of the exogenous variables has a significant adjusted effect on $D(Y2)$, but only $X2$ has a significant adjusted effect on $D(Y1)$. As a result, in a statistical sense, a reduced VEC model cannot be obtained by deleting any one of the exogenous variables, by using the VAR estimation method.
(3) Since this VEC model is a stable model, it should be an acceptable VEC model. □

Example 6.28. (A VEC model with interaction exogenous variable(s)) Even though there is a note or message '*Do NOT include C and Trend in VECs*,' an experimentation is carried out based on a VEC model with endogenous variables $\{\log(M1), \log(GDP)\}$ and exogenous variables '$t \log(pr)$ $t^* \log(pr)$' in order to explore the output and its possible problem. Figure 6.57(a) presents statistical results based on a special full VEC model with the lag specification '0 0,' and its reduced model is given in Figure 6.57(b).

Based on this result, the following notes and conclusions are presented:

(1) The interaction $t^* lg(pr)$ has a significant (adjusted) effect on $d(\log(gdp))$, but it has an insignificant effect on $d(\log(m1))$. In a statistical sense, this interaction may be deleted from the first regression. However, it cannot be done by using the VEC model. The system estimation should be used, which has been presented in the previous chapters, as well as in the previous examples.
(2) This also applies for the main factor $\log(pr)$.
(3) Since the time t has an insignificant effect on both $d(\log(m1))$ and $d(\log(gdp))$, then it may be deleted from both regressions, and a reduced VEC model can be found having exogenous variables $\log(pr)$ and $t^*\log(pr)$, presented also in Figure 6.57(b).

Vector Error Correction Estimates
Date: 12/24/07 Time: 13:35
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(M1(-1))	1.000000	
LOG(GDP(-1))	-1.312346 (0.49495) [-2.65145]	
C	2.047253	
Error Correction:	D(LOG(M1))	D(LOG(GDP))
CointEq1	-0.027462 (0.02776) [-0.98918]	0.046827 (0.01815) [2.57994]
C	0.035640 (0.03223) [1.10595]	0.070829 (0.02107) [3.36193]
T	-0.000260 (0.00016) [-1.63357]	-0.000159 (0.00010) [-1.52352]
LOG(PR)	0.003303 (0.03371) [0.09796]	0.069051 (0.02204) [3.13286]
T*LOG(PR)	-7.30E-05 (0.00014) [-0.52260]	-0.000407 (9.1E-05) [-4.45230]

R-squared	0.116012	0.207554
Adj. R-squared	0.095690	0.189337
Sum sq. resids	0.037348	0.015963
S.E. equation	0.014651	0.009578
F-statistic	5.708795	11.39331
Log likelihood	504.5093	580.5841
Akaike AIC	-5.581109	-6.431107
Schwarz SC	-5.492076	-6.342074
Mean dependent	0.012577	0.017312
S.D. dependent	0.015406	0.010638

Determinant resid covariance (dof adj.)	1.93E-08
Determinant resid covariance	1.82E-08
Log likelihood	1087.079
Akaike information criterion	-12.01205
Schwarz criterion	-11.79837

(a) A Special Full VEC Model

Vector Error Correction Estimates
Date: 12/24/07 Time: 13:39
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(M1(-1))	1.000000	
LOG(GDP(-1))	-1.564022 (0.24074) [-6.49670]	
C	3.552945	
Error Correction:	D(LOG(M1))	D(LOG(GDP))
CointEq1	-0.009870 (0.02526) [-0.39070]	0.047151 (0.01636) [2.88253]
C	0.003401 (0.02569) [0.13238]	0.061871 (0.01663) [3.71962]
LOG(PR)	-0.006275 (0.03804) [-0.16498]	0.075590 (0.02463) [3.06911]
T*LOG(PR)	-8.52E-05 (0.00014) [-0.60802]	-0.000407 (9.1E-05) [-4.48133]

R-squared	0.097312	0.206205
Adj. R-squared	0.081837	0.192597
Sum sq. resids	0.038138	0.015990
S.E. equation	0.014763	0.009559
F-statistic	6.288446	15.15327
Log likelihood	502.6357	580.4319
Akaike AIC	-5.571349	-6.440579
Schwarz SC	-5.500122	-6.369353
Mean dependent	0.012577	0.017312
S.D. dependent	0.015406	0.010638

Determinant resid covariance (dof adj.)	1.95E-08
Determinant resid covariance	1.86E-08
Log likelihood	1085.151
Akaike information criterion	-12.01286
Schwarz criterion	-11.83479

(b) A Reduced VEC Model

Figure 6.57 A special full VEC model with trend and time-related effect, and its reduced model

(4) The cointegrating equation has a significant effect on log(*GDP*) only, either based on the full or reduced models. □

6.3.4 Some notes and comments

Based on the previous examples, the following notes and comments on the VEC models are presented:

(1) By using the endogenous variables *Y*1 and *Y*2, the VEC estimation method will present a model of the first differences of the endogenous variables, namely *d*(*Y*1) and *d*(*Y*2).

(2) As a result, by using the endogenous variable log($Y1$) and log($Y2$), the model for the return rates of $Y1$ and $Y2$ are obtained, since $d(\log(Y1_t) = \log(Y1_t) - \log(Y1_{t-1}) = R1_t$ and $d(\log(Y2_t) = \log(Y2_t) - \log(Y2_{t-1}) = R2_t$ are the return rates or the exponential growth rates of $Y1_t$ and $Y2_t$. Similarly, by using or entering the first difference of the natural logarithm of a multivariate endogenous variables, multivariate models can be found of the return rates for the (original) endogenous variables.

(3) Considering a multivariate return rate model, it is highly likely that alternative time series models presented in the previous chapters could be applied, besides the VAR and the VEC models. For illustration purposes note the following examples.

Example 6.29. (Illustrative models for the return rates)

(a) *Autoregressive Return Rate Classical Growth Models*
Corresponding to the classical growth model in (2.3), illustrative examples of autoregressive return rate models or functions may be presented, as follows:

(a.1) *The return rate model of M1*
The following return rate function or summary shows that the growth rate of the return rate of $M1$ is 5.84E-05 and the exogenous variable t has a significant effect, since the observed t-statistic is greater than two:

$$\begin{aligned}
&d(\log(m1)) = \underset{[2.676]}{0.007\,187} + \underset{[2.293]}{5.84E{-}05}{*}t + [\mathrm{ar}(2) = \underset{[1.914]}{0.140\,870}] \\
&R^2 = 0.071, \quad \text{Adjusted } R^2 = 0.060, \quad DW = 2.120
\end{aligned} \tag{6.16}$$

(a.2) *The return rate model of GDP*
The following return rate function or summary shows that the growth rate of the return rate of *GDP* is 8.13E-06^ and the exogenous variable t has an insignificant effect, since the absolute observed value of the t-statistic is less than two:

$$\begin{aligned}
&d(\log(gdp)) = \underset{[6.850]}{0.016\,688} + \underset{[0.352]}{8.13E\text{-}06}{*}t + [ar(1) = \underset{[0.000]}{0.385\,614}] \\
&R^2 = 0.155, \quad \text{Adjusted } R^2 = 0.145, \quad DW = 2.042
\end{aligned} \tag{6.17}$$

(b) *Autoregressive Return Rate Models With Time-Related Effect*
Corresponding to the interaction models presented in Section 2.10.4, the following examples of AR(1) return rate models with time-related effect(s) are presented:

b.1 *The return rate model of M1 with time-related effect*
The following return rate function or summary shows that the interaction factor $t^*\log(pr)$ has a significant negative effect on the return rate of $M1$, since its t-statistic is less that -2.00 (or -1.96):

$$\begin{aligned} d(\log(m1)) = \underset{[2.52]}{0.0541} - \underset{[-1.899]}{0.0003} *t + \underset{[2.570]}{0.0325} *\log(pr) \\ - \underset{[-4.315]}{0.0002} *t*\log(pr) + [ar\,(1) = \underset{[-2.003]}{-0.1474}] \qquad (6.18) \\ R^2 = 0.133, \quad \text{Adjusted}\, R^2 = 0.113, \quad DW = 1.979 \end{aligned}$$

Hence, this model shows that the effect of $\log(pr)$ on the return rate of $M1$, namely $d(\log(M1))$, is significantly dependent on the time t, specifically on $(0.0325 - 0.0002^* t)$, since this function can be presented as

$$\begin{aligned} d(\log(m1)) = (0.0541 - 0.0003*t) + (0.0325 - 0.0002*t)*\log(pr) \qquad (6.19) \\ + [ar(1) = -0.1474] \end{aligned}$$

b.2 *The return rate model of GDP with time-related effect*
The following return rate function or summary shows that the interaction factor $t^*\log(pr)$ has a significant negative effect on the return rate of GDP, namely $d(\log(GDP))$, since its t-statistic is less that -2.00 (or -1.96):

$$\begin{aligned} d(\log(gdp)) = \underset{[1.72]}{0.0376} - \underset{[-1.074]}{0.0001} *t + \underset{[1.416]}{0.0183} *\log(pr) \\ - \underset{[-3.866]}{0.0002} *t*\log(pr) + [ar(1) = \underset{[3.866]}{0.2762}] \qquad (6.20) \\ R^2 = 0.288, \quad \text{Adjusted}\, R^2 = 0.210, \quad DW = 1.999 \end{aligned}$$

By presenting this function as

$$\begin{aligned} d(\log(gdp)) = (0.0376 - 0.0001*t) + (0.0183 - 0.0002*t)*\log(pr) \\ + [ar(1) = 0.2762] \qquad (6.21) \end{aligned}$$

it can easily be seen that the effect of $\log(pr)$ on $d(\log(GDP))$ is dependent on the time t. However, it cannot directly be concluded that its effect is significant, since $\log(pr)$ has an insignificant adjusted effect. A test should be made on the hypothesis on the joint effects $\log(pr)$ and $t^*\log(pr)$, using the Wald test. It was found that their joint effect is significant based on the F-statistic of 8.440 72 with $df = (2, 175)$ and a p-value = 0.0000. □

Example 6.30. (Additional experimentation with the VEC models) Corresponding to the VEC model with endogenous variables $\log(m1)$ and $\log(gdp)$ presented in the previous examples, experimentation has been conducted by entering the endogenous variables $D(\log(m1))$ and $D(\log(gdp))$ and exogenous variables t, $\log(pr(-1))$ and $t^*\log(pr(-1))$. Finally, a good acceptable model was found having the exogenous variables log($pr(-1)$) and $t^*\log(pr(-1))$, where each has a significant adjusted effect on $d(\log(m1), 2)$ and $d(\log(gdp), 2)$, as presented by the following regression functions with the t-statistics in [...]:

$$
\begin{aligned}
&d(\log(m1),2)\\
&\quad = -0.319 \underset{[-5.389]}{*} (d(\log(m1(-1))) + 2.573 \underset{[7.237]}{*} d(\log(gdp(-1))) - 0.0575)\\
&\qquad - \underset{[-8.301]}{0.634} * d(\log(m1(-1)),2) - 0.301 \underset{[-4.395]}{*} d(\log(m1(-2)),2)\\
&\qquad + \underset{[3.495]}{0.514} * d(\log(gdp(-1)),2) + \underset{[3.763]}{0.463} * d(\log(gdp(-2)),2) - \underset{[-1.411]}{0.0032}\\
&\qquad + \underset{[2.522]}{0.006\,875} * \log(pr(-1)) - \underset{[3.451]}{0.000\,184} * t * \log(pr(-1))\\
&\qquad R\text{-squared} = 0.541\,682, \quad \text{Adj. } R\text{-squared} = 0.522\,585\\
&d(\log(gdp),2)\\
&\quad = -\underset{[-6.437]}{0.235} * (d(\log(m1(-1))) + 2.573 \underset{[7.237]}{*} d(\log(gdp(-1))) - 0.0575)\\
&\qquad + \underset{[2.187]}{0.103} * d(\log(m1(-1)),2) + 0.119 \underset{[2.817]}{*} d(\log(m1(-2)),2)\\
&\qquad + \underset{[0.027]}{0.002} * d(\log(gdp(-1)),2) + 0.025 \underset{[0.323]}{*} d(\log(gdp(-2)),2) - \underset{[-1.465]}{0.0020}\\
&\qquad + \underset{[-3.452]}{0.005\,735} * \log(pr(-1)) - \underset{[-4.274]}{0.000\,141} * t * \log(pr(-1))\\
&\qquad R\text{-squared} = 0.368\,261, \quad \text{Adj. } R\text{-squared} = 0.341\,938
\end{aligned}
\tag{6.22}
$$

□

Example 6.31. (A comparison between multivariate basic VEC and VAR models) Figure 6.58 presents the roots of a characteristic polynomial based on a basic VEC model compared to a basic VAR model with the endogenous variables log ($m1$) log(gdp), log(pr) and rs, and the lag specification '1 2.'

Roots of Characteristic Polynomial
Endogenous variables: LOG(M1) LOG(GDP) LOG(P...
Exogenous variables:
Lag specification: 1 2
Date: 12/25/07 Time: 05:46

Root	Modulus
1.000000	1.000000
1.000000	1.000000
1.000000	1.000000
0.959136	0.959136
0.880716	0.880716
-0.007962 - 0.524032i	0.524093
-0.007962 + 0.524032i	0.524093
-0.506015	0.506015
0.311454 - 0.288611i	0.424617
0.311454 + 0.288611i	0.424617
-0.304685	0.304685
0.134452	0.134452

VEC specification imposes 3 unit root(s).

(a) Result Based On A VEC Model

Roots of Characteristic Polynomial
Endogenous variables: LOG(M1) LOG(GDP) LOG(P...
Exogenous variables: C
Lag specification: 1 2
Date: 12/25/07 Time: 05:51

Root	Modulus
0.996105	0.996105
0.968306	0.968306
0.947365	0.947365
0.886474	0.886474
0.507386 - 0.137772i	0.525758
0.507386 + 0.137772i	0.525758
-0.053089 - 0.125135i	0.135931
-0.053089 + 0.125135i	0.135931

No root lies outside the unit circle.
VAR satisfies the stability condition.

(b) Result Based On A VAR Model

Figure 6.58 The roots of a characteristic polynomial based on the VEC and VAR models with endogenous variables log($M1$), log(GDP), log(PR) and RS

Based on these outputs, the following notes are produced:

(1) The VEC model imposes three unit roots, compared to no unit root for the VAR model. Imposing a unit root is a special characteristic of the VEC models, as presented in the previous examples.
(2) It has been found that the number of unit roots imposed by the VEC model equals $(k-1)$, where k indicates the number of endogenous variables or the dimension of the multivariate independent variables. Therefore, it could be said that both models are acceptable time series models.
(3) On the other hand, was found that the cointegration test for both VEC and VAR models will give the same set of three cointegrating equations, as presented in Figure 6.59. The reason for this is that the cointegrating equations are defined or constructed based on the same set of endogenous variables. Note that by inserting the variables in a different order different forms of cointegrating equations can be obtained.
(4) Furthermore, based on all of the cointegrating equations presented in Figure 6.59, it is easy to generate new variables, such as those following:
   - *One Cointegrating Equation*

$$Coint1 = \log(m1) - 0.744\,557*\log(gdp) - 0.105\,846*\log(pr) + 0.021\,429*rs \tag{6.23}$$

   - *Two Cointegrating Equations*

$$\begin{aligned} Coint2a &= \log(m1) - 1.405\,418*\log(pr) + 0.090\,379*rs \\ Coint2b &= \log(gdp) - 1.745\,428*\log(pr) + 0.092\,606*rs \end{aligned} \tag{6.24}$$

Date: 12/25/07 Time: 05:53
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Trend assumption: Linear deterministic trend
Series: LOG(M1) LOG(GDP) LOG(PR) RS
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.165370	62.79209	47.85613	0.0011
At most 1 *	0.103151	30.79640	29.79707	0.0382
At most 2	0.049669	11.52682	15.49471	0.1811
At most 3	0.014079	2.509635	3.841466	0.1132

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level

1 Cointegrating Equation(s): Log likelihood 1720.420

Normalized cointegrating coefficients (standard error in parentheses)

LOG(M1)	LOG(GDP)	LOG(PR)	RS
1.000000	-0.744557	-0.105846	0.021429
	(0.09216)	(0.15311)	(0.00501)

2 Cointegrating Equation(s): Log likelihood 1730.055

Normalized cointegrating coefficients (standard error in parentheses)

LOG(M1)	LOG(GDP)	LOG(PR)	RS
1.000000	0.000000	-1.405418	0.090379
		(0.05781)	(0.01601)
0.000000	1.000000	-1.745428	0.092606
		(0.08422)	(0.02332)

3 Cointegrating Equation(s): Log likelihood 1734.563

Normalized cointegrating coefficients (standard error in parentheses)

LOG(M1)	LOG(GDP)	LOG(PR)	RS
1.000000	0.000000	0.000000	-2.289708
			(0.55173)
0.000000	1.000000	0.000000	-2.863291
			(0.68298)
0.000000	0.000000	1.000000	-1.693509
			(0.39955)

Figure 6.59 A part of the cointegration tests based on both the VEC and VAR models of {log ($M1$), log(GDP), log(PR), RS}

- *Three Cointegrating Equations:*

$$
\begin{aligned}
Coint3a &= \log(m1) - 2.289\,708{*}rs \\
Coint3b &= \log(gdp) - 2.863\,291{*}rs \\
Coint3c &= \log(m1) - 1.693\,509{*}rs
\end{aligned}
\tag{6.25}
$$

(5) For a comparison, an attempt has been made to obtain the cointegration equations using another procedure, as follows:
- Present the variables log(*M*1), log(*GDP*), log(*PR*) and *RS*.
- Then by selecting *View/Cointegration Tests* . . . and using the defaults options with the lag interval or specification '1 2,' exactly the same sets of cointegrating equations would be obtained. Furthermore, by using other lag specifications, different sets of cointegrating equations could be found. □

Example 6.32. (Characteristics of the cointegration series) Based on the six cointegration series, namely *Coint*1 up to *Coint*3*c*, computed in the previous Example 6.31, some of their characteristics are presented as follows:

(a) *Correlation Matrix*

Figure 6.60 presents the correlation matrix of the six *Coint*1 up to *Coint*3*c*. Based on this correlation matrix, the following notes and conclusions are presented:

Covariance Analysis: Ordinary
Date: 12/25/07 Time: 06:34
Sample: 1952Q1 1996Q4
Included observations: 180

Correlation t-Statistic Probability	COINT1	COINT2A	COINT2B	COINT3A	COINT3B	COINT3C
COINT1	1.000000					

COINT2A	-0.331850	1.000000				
	-4.693392	-----				
	0.0000	-----				
COINT2B	-0.580632	0.960712	1.000000			
	-9.514756	46.18121	-----			
	0.0000	0.0000	-----			
COINT3A	0.467558	-0.918942	-0.930682	1.000000		
	7.056855	-31.08631	-33.94166	-----		
	0.0000	0.0000	0.0000	-----		
COINT3B	0.452963	-0.922576	-0.929524	0.999475	1.000000	
	6.778559	-31.90302	-33.62990	411.4495	-----	
	0.0000	0.0000	0.0000	0.0000	-----	
COINT3C	0.464668	-0.923426	-0.933702	0.999934	0.999529	1.000000
	7.001193	-32.10217	-34.79157	1157.575	434.3579	-----
	0.0000	0.0000	0.0000	0.0000	0.0000	-----

Figure 6.60 Correlation matrix of a set of cointegration equations based on the variables {log (*M*1), log(*GDP*), log(*PR*), *RS*}

(a.1) *Coint*1 has significant negative correlations with each of *Coint*2*a* and *Coint*2*b*, but it has significant positive correlations with each of *Coint*3*a*, *Coint*3*b* and *Coint*3*c*.
(a.2) *Coint*2*a* and *Coint*2*b* has a significant positive correlation. However, each of them has significant negative correlations with each of *Coint*3*a*, *Coint*3*b* and *Coint*3*c*.
(a.3) Each pair of *Coint*3*a*, *Coint*3*b* and *Coint*3*c* has a significant positive correlation.
(a.4) These findings indicate that various time series models are presented based on a selected set of the cointegration series. Do this as an exercise.

(b) *The Growth Curves of Each Cointegration Series*
Figure 6.61 presents the growth curves of each of the defined cointegration series. Corresponding to the models presented in the previous chapters, similar models based on these six cointegration series may be applied. However, here additional examples based on this set of cointegration series will not be presented. Based on this figure the following notes are derived:
(1) The growth curves of *Coint*2*a* and *Coint*2*b* are very similar.
(2) The growth curves of *Coint*3*a* and *Coint*3*b* could not be differentiated.

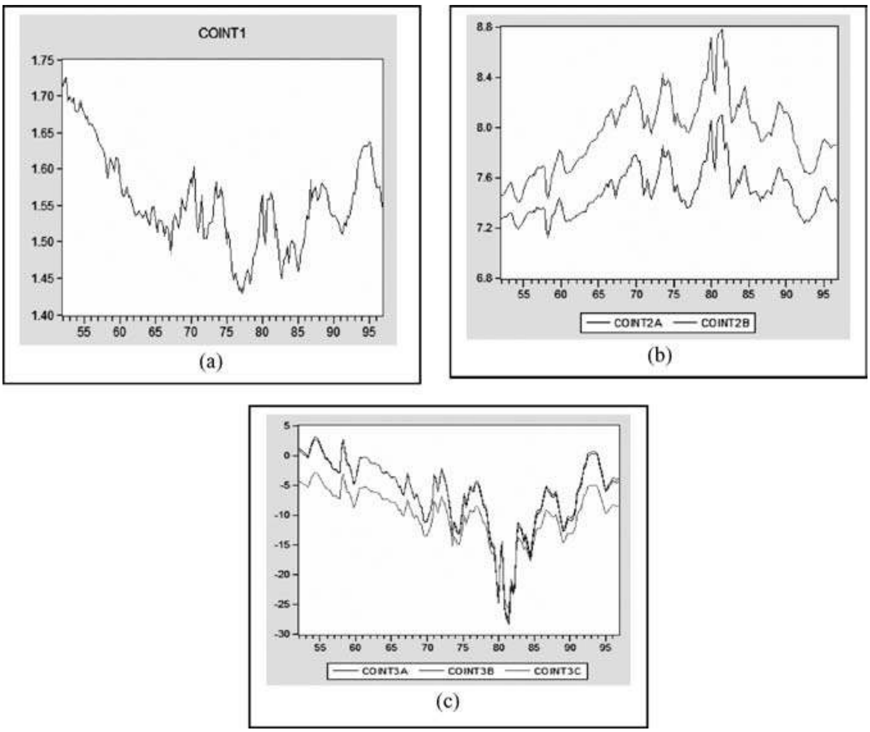


Figure 6.61 (a) Growth curve of *Coint*1, (b) growth curves of {*Coint*2*a*, *Coint*2*b*} and (c) growth curves of {*Coint*3*a*, *Coint*3*b*, *Coint*3*c*}

Table 6.2 Pairwise correlations between the set of original variables and each of their cointegrating equations

Correlation *t*-statistic Probability	*Coint*1	*Coint*2A	*Coint*2B	*Coint*3A	*Coint*3B	*Coint*3C
log(*M*1)	−0.293 296	0.245 274	0.297 988	−0.380 046	−0.352 059	−0.376 983
	−4.093 059	3.375 475	4.164 869	−5.481 756	−5.018 344	−5.430 217
	0.0001	0.0009	0.0000	0.0000	0.0000	0.0000
log(*GDP*)	−0.394 715	0.328 743	0.399 868	−0.470 238	−0.442 667	−0.467 186
	−5.731 531	4.644 107	5.820 486	−7.108 760	−6.586 381	−7.049 684
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
log(*PR*)	−0.338 677	0.276 742	0.338 500	−0.445 466	−0.418 292	−0.441 561
	−4.802 323	3.842 264	4.799 478	−6.638 290	−6.144 042	−6.565 931
	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000
RS	−0.478 081	0.902 102	0.919 245	−0.994 493	−0.990 823	−0.994 083
	−7.262 070	27.890 92	31.152 34	−126.6061	−97.798 70	−122.1002
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(c) *Selected Bivariate Correlations*

Table 6.2 shows that each of the variables log(*M*1), log(*GDP*), log(*PR*) and *RS* has either positive or negative significant correlations with each of the cointegration series *Coint*1 up to *Coint*3*c*.

These findings indicate that various time series models can be applied by using each of the variables log(*M*1), log(*GDP*), log(*PR*) and *RS* as dependent variable(s) and the cointegration series as independent variables, which will be presented in the following examples. Do this as an additional exercise. □

Example 6.33. (Experimentation based on a set of cointegrating equations using EViews 5) By using the trial-and-error methods, experimentation has been carried out with multivariate models having the dependent variables log(*m*1), log (*gdp*), log(*pr*) and *rs*. Some of the findings are as follows:

(1) By using the six cointegration series *Coint*1, *Coint*2*a*, *Coint*2*b*, *Coint*3*a*, *Coint*3*b* and *Coint*3*c* as independent variables, the '*Near Singular Error*' error message is obtained.
(2) By using a set of three cointegration series, such as *Coint*1, *Coint*2*a* and *Coint*3*a*, the error message is also obtained.
(3) By using only two of the three cointegration series, e.g. *Coint*1 and *Coint*2*a*, as independent variables, the fitted values are obtained, but with very low values of

the DW-statistic. As a result, experimentation needs to be done with multivariate autoregressive models (MARs), giving the following reasonable MARs with each of the regressions having a DW-statistic of 1.592 776, 1.424 904, 0.935 076 and 1.618 244 respectively:

$$\begin{aligned}
\log(m1) &= c(11) + c(12)^*Coint1 + c(13)^*Coint2a + [ar(1) = c(14)] \\
\log(gdp) &= c(21) + c(22)^*Coint1 + c(23)^*Coint2a + [ar(1) = c(24)] \\
\log(pr) &= c(31) + c(32)^*Coint1 + c(33)^*Coint2a + [ar(1) = c(34), \\
&\quad ar(3) = c(36), ar(5) = c(36)] \\
rs &= c(41) + c(42)^*Coint1 + c(43)^*Coint2a + [ar(1) = c(34), \\
&\quad ar(2) = c(35), ar(3) = c(36)]
\end{aligned} \tag{6.26}$$

Note that the third regression has unordered AR indicators, namely AR(1), AR(3) and AR(5), in order to have a greater value of the DW-statistic.

(4) Fitted regressions have also been found based on two MARs having independent variables: (i) *Coint*1 and *Coint*2*a* and (ii) *Coint*2*a* and *Coint*3*a*.

(5) Furthermore, by observing three MARs each having independent variable(s), (i) *Coint*1, (ii) *Coint*2*a* and *Coint*2*b* and (iii) *Coint*3*a*, *Coint*3*b* and *Coint*3*c*, the following findings can be observed:

- By using *Coint*1 as an independent variable, the following acceptable MAR has been obtained with each of the regressions having a DW-statistic of 2.333 838, 1.734 037, 2.018 027 and 2.069 476 respectively:

$$\begin{aligned}
\log(m1) &= c(11)+c(12)^*Coint1+[ar(1)=c(13),ar(2)=c(14)] \\
\log(gdp) &= c(21)+c(22)^*Coint1+[ar(1)=c(23),ar(2)=c(24),ar(3)=c(25)] \\
\log(pr) &= c(31)+c(32)^*Coint1+[ar(1)=c(33),ar(2)=c(34),ar(3)=c(35)] \\
rs &= c(41)+c(42)^*Coint1+[ar(1)=c(43),ar(2)=c(44)]
\end{aligned} \tag{6.27}$$

- By using *Coint*2*a* and *Coint*2*b* as independent variables, the following acceptable MAR has been obtained with each of the regressions having a DW-statistic of 2.728 314, 2.701 203, 1.962 607 and 2.515 312 respectively:

$$\begin{aligned}
\log(m1) &= c(11) + c(12)^*Coint2a + c(13)^*Coint2b \\
&\quad + [ar(1) = c(14), ar(2) = c(15)] \\
\log(gdp) &= c(21) + c(22)^*Coint2a + c(23)^*Coint2b \\
&\quad + [ar(1) = c(24), ar(2) = c(25)] \\
\log(pr) &= c(31) + c(32)^*Coint2a + c(33)^*Coint2b \\
&\quad + [ar(1) = c(34), ar(2) = c(35), ar(3) = c(36), ar(4) = c(37)] \\
rs &= c(41) + c(42)^*Coint2a + c(43)^*Coint2b \\
&\quad + [ar(1) = c(44), ar(2) = c(45), ar(3) = c(46)]
\end{aligned} \tag{6.28}$$

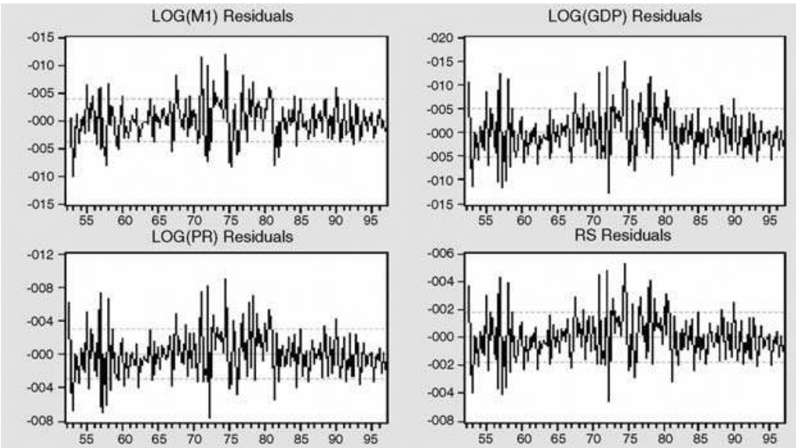


Figure 6.62 Residual graphs of the multivariate models in (6.28)

Note that by reducing the order of the autoregressives used in either one of the regressions, the values of the DW-statistics will decrease. □

Example 6.34. (Further experimentation) After doing further experimentation an acceptable MAR was found as follows:

$$\begin{aligned}\log(m1) &= c(11) + c(12)*Coint1 + c(13)*Coint2a + c(14)*Coint3a \\ &\quad + [ar(1) = c(15), ar(3) = c(17)] \\ \log(gdp) &= c(21) + c(22)*Coint1 + c(23)*Coint2a + c(24)*Coint3a \\ &\quad + [ar(1) = c(25), ar(2) = c(26)] \\ \log(pr) &= c(31) + c(32)*Coint1 + c(33)*Coint2a + c(34)*Coint3a \\ &\quad + [ar(1) = c(35), ar(2) = c(36)] \\ rs &= c(41) + c(42)*Coint1 + c(43)*Coint2a + c(44)*Coint3a \\ &\quad + [ar(1) = c(45), ar(2) = c(46)]\end{aligned} \tag{6.29}$$

During the experimentation, many unexpected results or cases were found. Some of those are as follows:

(1) Each of the independent variables, as well as the AR indicators, has a significant effect with a p-value of 0.000, by using the iterative least squares estimation method.
(2) The intercepts are $c(11) = 0.8836$ and $c(21) = c(22) = c(41) = 0.8036$. If the same AR(1) and AR(2) indicators are used in the first regression, then all intercepts would be equal to 0.8036.
(3) The values of the DW-statistic are 1.970 145 for the first regression and 2.523 325 for the others. However, by using the same AR(1) and AR(2) indicators in the first

regression, then the value of the DW-statistic would be 2.523 325 for the four regressions.

(4) On the other hand, by using an AR(1) multivariate model, where each regression has the AR(1) indicator only, the DW-statistic would be 0.803 109 for the four regressions.

(5) Figure 6.62 presents the residual graphs of the multivariate model in (6.29), with very small values of R-squared of 0.002 457, 0.004 458, 0.001 669 and 0.000 544 respectively. For this reason, alternative models may be found by using the lagged-variable autoregressive models, namely LVAR(p, q), in order to obtain a model with sufficiently large values of the DW-statistic. Do this as an exercise. □

Example 6.35. (Three-way interaction VEC model using EViews 6) Corresponding to the path diagram in Figure 6.3, by omitting the t-variable and using the trial-and-error methods an acceptable three-way interaction model was obtained, with the results presented in Figure 6.63. The following notes are presented:

(1) The characteristic roots indicate that the VEC model is an acceptable model, in a statistical sense.

(2) Since $D(Y1(-2)$ and $D(Y2(-2)$ have insignificant effects in both regressions, a reduced VEC model can be obtained by using the lag interval D(Endogenous) = '1 1.'

Vector Error Correction Estimates
Date: 12/25/07 Time: 08:45
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
Y1(-1)	1.000000	
Y2(-1)	0.652216	
	(0.20242)	
	[3.22212]	
C	-59.81807	
Error Correction:	D(Y1)	D(Y2)
CointEq1	-0.016251	-0.839405
	(0.05886)	(0.17282)
	[-0.27608]	[-4.85702]
D(Y1(-1))	0.253411	-0.397514
	(0.19327)	(0.56743)
	[1.31118]	[-0.70055]
D(Y1(-2))	-0.133406	0.463533
	(0.20386)	(0.59853)
	[-0.65439]	[0.77445]
D(Y2(-1))	0.055555	0.107309
	(0.05999)	(0.17614)
	[0.92601]	[0.60922]
D(Y2(-2))	-0.035569	0.098493
	(0.05095)	(0.14958)
	[-0.69814]	[0.65845]
C	3.183301	-15.18129
	(2.48177)	(7.28644)
	[1.28267]	[-2.08350]
X2	-0.100116	1.56E-05
	(0.03071)	(0.09015)
	[-3.26053]	[0.00017]
X1*X3	3.82E-05	1.66E-05
	(9.0E-06)	(2.6E-05)
	[4.23284]	[0.62535]
X1*X2*X3	-2.01E-07	8.75E-08
	(5.6E-08)	(1.6E-07)
	[-3.59606]	[0.53426]
R-squared	0.886705	0.853428
Adj. R-squared	0.836351	0.788285
Sum sq. resids	31.49620	271.4966
S.E. equation	1.322796	3.883702
F-statistic	17.60963	13.10082
Log likelihood	-40.39075	-69.47086
Akaike AIC	3.658574	5.812656
Schwarz SC	4.090519	6.244602
Mean dependent	1.849630	3.028519
S.D. dependent	3.269917	8.440537

Determinant resid covariance (dof adj.)	26.34688
Determinant resid covariance	11.70973
Log likelihood	-109.8383
Akaike information criterion	9.617655
Schwarz criterion	10.57753

Roots of Characteristic Polynomial
Endogenous variables: Y1 Y2
Exogenous variables: X2 X1*X3 X1*X2*X3
Lag specification: 1 2
Date: 12/25/07 Time: 08:49

Root	Modulus
1.000000	1.000000
0.414687 - 0.350964i	0.543269
0.414687 + 0.350964i	0.543269
-0.075108 - 0.301036i	0.310264
-0.075108 + 0.301036i	0.310264
0.117836	0.117836

VEC specification imposes 1 unit root(s).

Figure 6.63 A Three-way interaction VEC model and its characteristics roots

In the reduced model, it was found that $D(Y2(-1)$ has a significant effect on $D(Y2)$ only, based on the t-statistic = 2.382.

(3) Each of the exogenous variables $X2$, $X1^*X3$ and $X1^*X2^*X3$ has a significant adjusted effect on $D(Y1)$. By assuming that $X1$ is an important cause or source variable, then it can be concluded that the effect of $X1$ on $D(Y1)$ is significantly dependent on $X2$ and $X3$, in the form of (3.82E-05$X3$ − 2.01E-07$X2^*X3$).

(4) By using the VAR estimation method, it was found that the joint effects of $X2$, $X1^*X3$ and $X1^*X2^*X3$ cannot be tested on both endogenous variables $D(Y1)$ and $D(Y2)$. To test this multivariate hypothesis, the system estimation method should be applied, as presented in the following example. □

Example 6.36. (The system estimation method for a VEC model) Figure 6.64 presents the statistical results based on the VEC model in the previous Example 6.35, but here the SUR estimation method is used. Note that this figure shows that the cointegration equation in Figure 6.63, namely $CointEq1 = (Y1(-1) + 0.562\,216^*Y2(-1) - 59.818\,07)$, is used independently of both regressions. Based on this figure the following notes and conclusions are presented:

(1) Corresponding to the parameter $C(11)$, the $CointEq1$ has a significant negative effect on D(Y1), based on $t_0 = -2.667\,071$ with a p-value = 0.0112, but it is insignificant based on the VEC model in Figure 6.63. Therefore, the question arises: 'Why do these two estimation methods give contradictory conclusions?' To date there has not been an explanation for this.

(2) Corresponding to the parameter $C(21)$, the $CointEq1$ has a significant negative effect on $D(Y2)$, based on $t_0 = -5.985\,590$, and the VEC model presents $t_0 = -4.857\,02$ in Figure 6.63. This also produces the question 'Why do these two estimation methods give contradictory conclusions?'

Estimation Method: Seemingly Unrelated Regression
Date: 12/25/07 Time: 09:09
Sample: 1954 1980
Included observations: 27
Total system (balanced) observations 54
Linear estimation after one-step weighting matrix

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-0.077733	0.029145	-2.667071	0.0112
C(12)	0.367383	0.146402	2.509416	0.0165
C(13)	0.103184	0.040194	2.567123	0.0143
C(14)	-0.026087	0.158572	-0.164511	0.8702
C(15)	-0.030068	0.043304	-0.694335	0.4917
C(16)	-0.065299	0.012244	-5.333290	0.0000
C(17)	3.25E-05	6.71E-06	4.847040	0.0000
C(18)	-1.65E-07	4.12E-08	-4.000856	0.0003
C(21)	-0.546199	0.091252	-5.985590	0.0000
C(22)	-0.941049	0.458375	-2.053010	0.0470
C(23)	-0.119836	0.125846	-0.952238	0.3470
C(24)	-0.048274	0.496482	-0.097233	0.9231
C(25)	0.072257	0.135583	0.532933	0.5972
C(26)	-0.166026	0.038334	-4.331011	0.0001
C(27)	4.36E-05	2.10E-05	2.075397	0.0448
C(28)	-8.35E-08	1.29E-07	-0.647662	0.5211
Determinant residual covariance		15.75523		

Equation: D(Y1)=C(11)*(Y1(-1)+0.652216*Y2(-1)-59.81807)+C(12)*D(Y1(-1))+C(13)*D(Y2(-1))+C(14)*D(Y1(-2))+C(15)*D(Y2(-2))+C(16)*X2+C(17)*X1*X3+C(18)*X1*X2*X3
Observations: 27

R-squared	0.876349	Mean dependent var	1.849630
Adjusted R-squared	0.830794	S.D. dependent var	3.269917
S.E. of regression	1.345070	Sum squared resid	34.37504
Durbin-Watson stat	2.535760		

Equation: D(Y2)=C(21)*(Y1(-1)+0.652216*Y2(-1)-59.81807)+C(22)*D(Y1(-1))+C(23)*D(Y2(-1))+C(24)*D(Y1(-2))+C(25)*D(Y2(-2))+C(26)*X2+C(27)*X1*X3+C(28)*X1*X2*X3
Observations: 27

R-squared	0.818080	Mean dependent var	3.028519
Adjusted R-squared	0.751057	S.D. dependent var	8.440537
S.E. of regression	4.211339	Sum squared resid	336.9721
Durbin-Watson stat	1.738406		

Figure 6.64 Statistical results based on the VEC model in Figure 6.63 using the SUR estimation method

Wald Test:
System: Untitled

Test Statistic	Value	df	Probability
Chi-square	52.36761	3	0.0000

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(16)	-0.065299	0.012244
C(17)	3.25E-05	6.71E-06
C(18)	-1.65E-07	4.12E-08

Restrictions are linear in coefficients.

Wald Test:
System: Untitled

Test Statistic	Value	df	Probability
Chi-square	89.95820	3	0.0000

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(26)	-0.166026	0.038334
C(27)	4.36E-05	2.10E-05
C(28)	-8.35E-08	1.29E-07

Restrictions are linear in coefficients.

Figure 6.65 The Wald tests for finding the joint effect of $X1$, $X2$ and $X1^*X2$ on each of the endogenous variables $D(Y1)$ and $D(Y2)$

(3) For illustration purposes, Figure 6.65 presents the Wald tests to show the joint effect of the exogenous variables $X1$, $X2$ and $X1^*X2$ on each of the endogenous variable*s* $D(Y1)$ and $D(Y2)$.
(4) By using the system equation, alternative reduced models can be obtained, where each regression can have a different set of exogenous variables. Do this as an exercise. □

Example 6.37. (The return rate models using VEC) By entering log($y1$) and log ($y2$) as the endogenous variables of a VEC model, regressions are produced having dependent variables $D(\log(y1))$ and $D(\log(Y2))$, which are the returns rate series of $Y1$ and $Y2$. For this reason, a title or statement 'a VEC model as the return rate model' has been proposed.

Figure 6.66 presents a summary of the statistics of the model having exogenous variables log($x1$), log($x2$) and log($x3$), with the lag interval D(Endogenous) = '1 1,' with the t-statistic in [·]. These statistics show:

```
D(LOG(Y1)) =  - 0.101*( LOG(Y1(-1)) - 1.556*LOG(Y2(-1)) + 2.424) - 0.001*D(LOG(Y1(-1)))
                 [-1.73]                   [-2.59]                          [-0.01]

           + 0.051*D(LOG(Y2(-1))) + 0.267 + 0.149*LOG(X1) - 0.577*LOG(X2) + 0.228*LOG(X3)
             [0.59]                 [0.87]    [1.87]         [-3.28]          [4.15]
                          Adj. R-squared = 0.597313, F-stat =  7.674935


D(LOG(Y2)) = 0.265*( LOG(Y1(-1)) - 1.556*LOG(Y2(-1)) + 2.424) - 0.769*D(LOG(Y1(-1)))
              [2.36]                  [-2.59]                          [-2.40]

           + 0.130*D(LOG(Y2(-1))) - 1.661 + 0.053*LOG(X1) - 0.281*LOG(X2) + 0.437*LOG(X3)
             [0.78]                 [-2.80]   [0.35]         [-0.83]          [4.14]
                          Adj. R-squared =  0.559099, F-stat =  6.706371
```

Figure 6.66 Regression functions of the return rate model of ($Y1$, $Y2$) using the VEC model

(1) The cointegration equation has a significant effect on the return rate of $Y2$, namely $D(\log(Y2))$.
(2) By taking the t-statistic critical value of $|t_c| = 2$, it can be concluded that the *CointEq*1 has an insignificant effect on $D(\log(Y1))$, but a significant effect on $D(\log(Y2))$.
(3) Similar conclusions can easily be derived for each of the other exogenous variables.
(4) Furthermore, the testing hypothesis can be used to find the joint effect of a selected subset of the exogenous variables using the Wald test. Note that the system estimation method should be used, as demonstrated in the previous example. □

Example 6.38. (A special return rate model of five variables) By using log(y1), log(y2), log(x1), log(x2) and log(x3) as the endogenous variables of a VEC model, it is possible to obtain various alternative VEC models corresponding to the use of various lag intervals D(Endogenous) and sets of exogenous variables. Since there are only five variables in the US_DPOC (i.e. the US domestic price of copper) then the exogenous variables should be the lagged endogenous variables.

This example presents a special case of the return rate model, by using a VEC model with the lag interval D(Endogenous) = '0 0,' with the statistical results presented in Figure 6.67. Note that the regressions with dependent variables $D(\log(Y2))$ and $D(\log(X1))$ have negative adjusted R-squared, which indicate that the VEC models are poor VEC models.

For this reason the model needs to be modified using the trial-and-error methods. By using the lag specification '1 1,' it has been found that the five regressions have positive adjusted R-squared, but with small values. By using the lag specifications '1 2' or '2 2,' greater values of adjusted R-squared can be obtained.

Vector Error Correction Estimates
Date: 12/27/07 Time: 08:24
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
LOG(Y1(-1))	1.000000
LOG(Y2(-1))	-8.435208 (4.48935) [-1.87894]
LOG(X1(-1))	0.576966 (4.63317) [0.12453]
LOG(X2(-1))	25.70344 (6.69631) [3.83845]
LOG(X3(-1))	-18.71218 (2.91334) [-6.42293]
C	21.69831

Error Correction:	D(LOG(Y1))	D(LOG(Y2))	D(LOG(X1))	D(LOG(X2))	D(LOG(X3))
CointEq1	-0.011397 (0.00249) [-4.57963]	0.004488 (0.00603) [0.74387]	-9.32E-05 (0.00165) [-0.05650]	0.003315 (0.00208) [1.59639]	0.026152 (0.00795) [3.28764]
C	0.045393 (0.01153) [3.93545]	0.052864 (0.02796) [1.89046]	0.071594 (0.00764) [9.36752]	0.040767 (0.00962) [4.23591]	0.050048 (0.03687) [1.35752]
R-squared	0.437184	0.020082	0.000118	0.086247	0.285877
Adj. R-squared	0.416339	-0.016211	-0.036914	0.052404	0.259428
Sum sq. resids	0.104174	0.612268	0.045737	0.072523	1.064242
S.E. equation	0.062115	0.150588	0.041158	0.051827	0.198536
F-statistic	20.97302	0.553339	0.003192	2.548456	10.80860
Log likelihood	40.47116	14.79005	52.40702	45.72237	6.773759
Akaike AIC	-2.653184	-0.882072	-3.476346	-3.015336	-0.329225
Schwarz SC	-2.558887	-0.787776	-3.382050	-2.921040	-0.234928
Mean dependent	0.045393	0.052864	0.071594	0.040767	0.050048
S.D. dependent	0.081305	0.149382	0.040418	0.053241	0.230704

Determinant resid covariance (dof adj.)	3.21E-12
Determinant resid covariance	2.25E-12
Log likelihood	183.1663
Akaike information criterion	-11.59768
Schwarz criterion	-10.89045

Figure 6.67 A return rate VEC model of log(Y1), log(Y2), log(X1), log(X2) and log(X3), with lag specification '0 0'

On the other hand, by using the lag specification '1 3,' an error message '*insufficient number of observations*' is obtained.

Furthermore, the VEC model of log($Y1$), log($Y2(-1)$), log($X1(-1)$), log($X2$) and log($X3$), with the lag specification '0 0,' also gives a positive adjusted R-squared for the five regressions.

Perform the residual analysis and other tests for each of the VEC models as an exercise in order to explore the limitation of each model. Further analysis can also be done using the system equation to develop alternative multivariate models where regressions have different sets of exogenous or independent variables. □

6.4 Special notes and comments

Based on the previous illustrative examples in this chapter, as well as the previous chapters, some special notes and comments are presented, as follows:

(1) Based on the previous illustrative examples, it was found that the VAR and VEC models are special cases of the multivariate autoregressive models. Hence, the acronym MAR (i.e. *multivariate autoregressive*) model should be used instead of VAR, to represent a general time series model which is endogenous multivariate.
(2) All models presented in the previous chapters can be modified to the VAR or VEC models. Therefore, there could be various additive, two-way and three-way interaction VAR models, as well as the VAR model with dummy variables.
(3) The VAR model, as well as the VEC model, can be estimated by using the '*system*' function or estimation method. This is more flexible to use in developing a multivariate model, where the multiple regressions could have different sets of exogenous variables.
(4) Since it is believed that a set of regressions in any multivariate model should have different types or sets of cause or source variables, then the s*ystem* function (estimation method) is the preferred method used to develop alternative multivariate time series models. It is accepted that there should be a good or special reason why VAR or VEC models are applied where all multiple regressions in the VAR model have the same set of independent variables.

7

Instrumental variables models

7.1 Introduction

The application of (univariate) general linear models (GLMs) or multiple regressions presented in the previous chapters uses a basic assumption that the right-hand side variables in the models are uncorrelated with the disturbance terms. If this assumption is violated, then in order to estimate the model parameters an *instrumental variables model* should be used. The *instrument variables* (or *instruments* in short) are a set of variables, that need to be selected or defined such that they are both (i) correlated with the explanatory or independent variables of the GLM and (ii) uncorrelated with the disturbance or error terms.

When discussing a correlation between an exogenous variable and the error terms of any models, as well as the correlation between any pairs of numerical variables, it should always be noted that a pair of variables could have significant correlation, even though they are not substantially correlated. On the other hand, the correlation of a pair of variables could be insignificant based on a testing hypothesis, where they are in fact substantially correlated or associated. Since in hypothesis testing a sample data should be used, which is considered as a set of scores/measurements that happen to be selected or available for a researcher (Agung, 2004), the conclusion could be reached that the hypothesis testing could contradict the theoretical base. In other words, the data do not support a defined hypothesis.

Corresponding to the use of instrumental variables, it could said that there should be complete dependence on the conclusion of the testing hypothesis. If at least one of the exogenous variables of a model (or regression) has a significant correlation with the residual, then using an instrumental model should be considered. However, it has been found that there could be two possible types of modified models. The first is a model that is modified without using the instrumental variables and the second is one that is modified using instrumental variables.

Time Series Data Analysis Using EViews IGN Agung

However, by carrying out experimentation, it has been found that it is not easy to find a good set of instrumental variables and, moreover, the best set of instrumental variables for a basic model or regression. Refer to the special notes and comments on the true population model presented in Section 2.14 and observe the cases presented in the following examples.

In fact, there are at least two problems, namely *two stages of problems* (*TSOP*), in demonstrating or developing an instrumental model. First, a model should be developed having at least one exogenous variable that is significantly correlated with the residual of the model. Second, the best possible set of instrumental variables needs to be found. The second problem is exactly the same as the problem defining the true population model presented in Section 2.14.1. For these reasons, the TSLS estimation method could be considered as the process to use in order to solve two stages of problems in developing an instrumental model. Corresponding to the selection of a set of instrumental variables, Gujarati (2003, p. 527) stated: 'This task is much easier said than done.'

Note the following subsections, notes and examples, which present our experimentation in applying GLMs with instruments. For illustrative purposes, Demo.wf1 will again be used, as well as the set of five *X*- and *Y*-variables, which have been defined in the previous chapter based on the variables in the US domestic price of copper (US_DPOC) data set. Since the time series data will be used, the autoregressive models will be applied directly.

The steps of data analysis are as follows:

(1) By selecting *Quicks*/*Estimation Equation* ..., the window on the left-hand side in Figure 7.1 will appear. Then by selecting the TSLS estimation method, the window on the right-hand side will appear.
(2) The equation specification can be entered, as well as the list of instruments. Then by clicking *OK*, the statistical results will be obtained.
(3) For other alternative estimation methods, such as the White, the Newey–West and weighted LS/TLS estimation methods, click the *Options*.

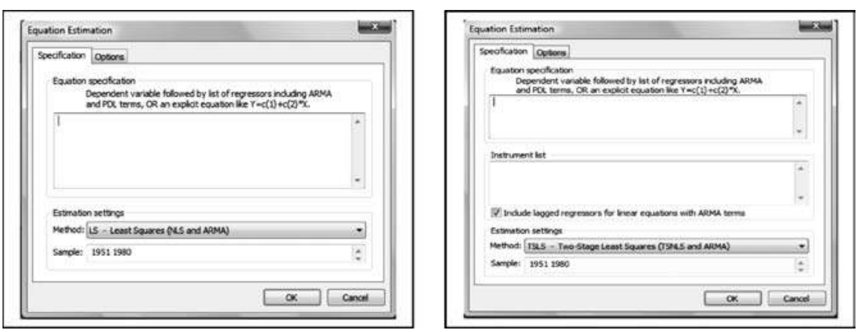


Figure 7.1 Windows for conducting the TSLS estimation method

7.2 Should we apply instrumental models?

The time series models presented in the previous chapters should be considered as the basic time series models, since the instrumental model will be considered as an advanced time series models. For this reason, it should be considered whether all models presented earlier should be improved or modified to the instrumental models.

Since an instrumental model should be applied if and only if at least one of the exogenous variables in a model is correlated with the error terms, then a further residual analysis should be done for each model before it is modified to be an instrumental model.

For this reason, further residual analysis has been conducted for selected models that have been presented in the previous chapters. The statistical results are presented in Table 7.1.

Table 7.1 Correlations between each exogenous variable of selected models with their corresponding error terms, with its *p*-value in [·]

Example	Dependent variable	Independent variables/Sig. *p*-value in [.] 1	2	3	AR
2.8	log(*m*1)	*t* [1.0000]	log(*m*1(−1)) [1.0000]	log(*m*1(−2)) [1.0000]	—
2.16	log(*m*1)	*t* [1.0000]	log(*gdp*) [1.0000]	log(*pr*) [1.0000]	1
4.5	POLI_1	POLI_1(−12) [0.3723]			1 and 2
4.12	log(*m*1)	log(*m*1(−1)) [0.8526]	Log(*m*1(−2)) [0.8536]	Log(*gdp*) [0.8259]	
		log(*gdp*(−1)) [0.8272]	*RS* [0.8733]		1
5.11	log(*mmdep*)	log(*mmdep*(−1)) [0.9998]	log(*ivmaut*) [0.9861]		1
5.15	log(*P*)	log(*A*) [0.8071]	log(*G*) [0.6635]	log(*L*) [0.6565]	1 and 2
5.17	log(*P*)	log(*I*) [0.6238]	log(*A*) [0.7421]	$(\log(I))^2$ [0.6390]	
		log(*I*)*log(*A*) [0.6801]	$(\log(A))^2$ [0.7544]		1 and 2
6.17	log(*Y*1), log(*Y*2)	log(*Y*1(−1)) [1.0000], log(*X*2) [1.0000]	log(*Y*2(−1)) [1.0000], log(*X*3) [1.0000]	log(*X*1) [1.0000]	
6.18	log(*Y*1), log(*Y*2)	log(*Y*1(−1)) [1.0000], *X*2 [1.0000]	log(*Y*2(−1)) [1.0000], *X*1**X*2 [1.0000]	log(*X*1) [1.0000], *X*1**X*3 [1.0000]	
		*X*2**X*3 [1.0000]			

Dependent Variable: M1
Method: Least Squares
Date: 12/29/07 Time: 16:21
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
GDP	0.481668	0.017773	27.10140	0.0000
PR	260.7521	25.21856	10.33969	0.0000

R-squared	0.985506	Mean dependent var	445.0064
Adjusted R-squared	0.985425	S.D. dependent var	344.8315
S.E. of regression	41.63098	Akaike info criterion	10.30662
Sum squared resid	308498.7	Schwarz criterion	10.34209
Log likelihood	-925.5954	Hannan-Quinn criter.	10.32100
Durbin-Watson stat	0.050382		

Covariance Analysis: Ordinary
Date: 12/29/07 Time: 16:28
Sample: 1952Q1 1996Q4
Included observations: 180

Correlation t-Statistic Probability	RESID01	GDP	PR
RESID01	1.000000 ----- -----		
GDP	-0.174072 -2.358417 0.0194	1.000000 ----- -----	
PR	-0.263093 -3.638274 0.0004	0.992475 108.1367 0.0000	1.000000 ----- -----

Figure 7.2 Statistical results based on a time series model through the origin

This Figure shows that each of the exogenous variables of the selected models is insignificantly correlated with their corresponding error terms with a large *p*-value, where some of them have a *p*-value of 1.0000. Therefore, these models do not need instrumental variables, and they should be considered as acceptable or good models without using instrumental variables.

Finally, for illustration purposes, an unusual and unexpected time series model was found with exogenous variables that are significantly correlated with the residual. The model is one without an intercept or model through the origin, as presented in Figure 7.2. This figure also presents the correlation matrix of the residual, namely *Resid*01, and the exogenous variables, *GDP* and *PR*. Since the exogenous variables have significant correlations with the residuals, this model should be modified using instrumental variables.

However, it has been recognized that this type of model can also be modified or improved by using additional exogenous variables instead of the instrumental variables, as shown by the following example.

Example 7.1. (Modified models without instrumental variables) Table 7.2 presents alternative time series models, where the independent variables *GDP* and *PR* have insignificant correlations with their corresponding residuals, compared to the unusual model in Table 7.2 as the first model in Table 7.2. Based on this Figure, the following notes are presented:

(1) Refer to the first two models inTable 7.2. The first model is a model without intercept, where the exogenous variables are significantly correlated with the residuals. By adding only the intercept parameter, the second model is obtained where the exogenous variables are insignificantly correlated with the residual, with the largest *p*-value of 1.0000.
(2) In fact, many other time series models presented in the previous chapters have been tried, but no one has found an exogenous variable that is significantly correlated with its corresponding residuals.

Table 7.2 Modified models with endogenous variable $M1$ and selected sets of exogenous variables

Number	Exogenous variables	Probability(t-statistic) of	
		$\rho(Resid, gdp)$	$\rho(Resid, pr)$
1	*gdp pr*	0.0194	0.0004
2	*C gdp pr*	1.0000	1.0000
3	*C gdp pr ar*(1)	0.3305	0.2925
4	*C gdp pr ar*(1) *ar*(2)	0.4615	0.4401
5	*C m*1(−1) *gdp pr*	1.0000	1.0000
6	*C m*1(−1) *gdp pr ar*(1)	0.3322	0.2950
7	*C m*1(−1) *m*1(−2) *gdp pr*	1.0000	1.0000
8	*C gdp pr rs ar*(1)	0.5306	0.4548
9	*C m*1(−1) *gdp pr rs*	1.0000	1.0000
10	*C gdp gdp*(−1) *pr pr*(−1)	1.0000	1.0000

(3) For a comparison, Figure 7.3 presents a translog linear model without the intercept of log($M1$) on log(GDP) and log(PR), and the correlation matrix of its residual and the exogenous variables. Note that the exogenous variables are insignificantly correlated with the residual, namely *Resid*13.

(4) Based on the models presented in Table 7.2 and the notes in points (1) and (2) above, as well as the model in Figure 7.3, it can be said that it is very difficult to find a common time series model, which should have an independent variable that is significantly correlated with its residuals. For this reason, in general, an instrumental variable model does not need to be applied, since (i) there is no good guide as to how to select the best set of instrumental variables and (ii) the model can be improved by using the additional independent variables and lagged variables presented in Table 7.2, as well

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 12/30/07 Time: 07:34
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
LOG(GDP)	0.928129	0.001859	499.3552	0.0000
LOG(PR)	-0.296269	0.010956	-27.04103	0.0000

R-squared	0.980193	Mean dependent var	5.811220
Adjusted R-squared	0.980082	S.D. dependent var	0.754650
S.E. of regression	0.106505	Akaike info criterion	-1.630205
Sum squared resid	2.019101	Schwarz criterion	-1.594728
Log likelihood	148.7185	Hannan-Quinn criter.	-1.615821
Durbin-Watson stat	0.023281		

Covariance Analysis: Ordinary
Date: 12/30/07 Time: 07:32
Sample: 1952Q1 1996Q4
Included observations: 180

Correlation t-Statistic Probability	RESID13	LOG(GDP)	LOG(PR)
RESID13	1.000000 — —		
LOG(GDP)	-0.057559 -0.769210 0.4428	1.000000 — —	
LOG(PR)	0.013590 0.181325 0.8563	0.994191 123.2414 0.0000	1.000000 — —

Figure 7.3 Statistical results based on a translog linear model of log($M1$) on log(GDP) and log(PR)

as the transformed variables presented in Figure 7.3 and models with dummy variables as presented in the following example. □

Example 7.2. (A modified model with a dummy variable) Corresponding to the unusual model in Figure 7.2, Figure 7.4 presents the statistical results based on a modified model with a dummy variable *Drs*1, where $Drs1 = 1$ for $t \leq 119$ and $Drs1 = 0$ otherwise. The model can be considered as a two-way interaction model, where each of the independent variables has an insignificant correlation with the residual, namely *Resid*14. □

Dependent Variable: M1
Method: Least Squares
Date: 12/30/07 Time: 08:01
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
GDP	0.759693	0.023013	33.01080	0.0000
PR	-163.7776	34.95309	-4.685639	0.0000
DRS1	115.2280	43.15657	2.669999	0.0083
DRS1*GDP	-0.070852	0.204711	-0.346108	0.7297
DRS1*PR	-69.79614	311.3015	-0.224207	0.8229

R-squared	0.994303	Mean dependent var	445.0064
Adjusted R-squared	0.994173	S.D. dependent var	344.8315
S.E. of regression	26.32340	Akaike info criterion	9.406178
Sum squared resid	121261.2	Schwarz criterion	9.494871
Log likelihood	-841.5560	Hannan-Quinn criter.	9.442140
Durbin-Watson stat	0.141758		

Covariance Analysis: Ordinary
Date: 12/30/07 Time: 08:02
Sample: 1952Q1 1996Q4
Included observations: 180

Correlation t-Statistic Probability	RESID14
RESID14	1.000000 ----- -----
GDP	0.003784 0.050482 0.9598
PR	0.005719 0.076300 0.9393
DRS1	0.004702 0.062734 0.9500
DRS1*GDP	0.003034 0.040474 0.9678
DRS1*PR	0.003912 0.052196 0.9584

Figure 7.4 Statistical results based on a modified two-way interaction model with a dummy variable *Drs*1 of the model in Figure 7.2

Example 7.3. (Modified models with instrumental variables) Table 7.2 presents models with the endogenous variable *M*1 and alternative sets of exogenous and instrumental variables, together with the Probability(*t*-statistic) in testing the correlations between *GDP* and *PR* with the corresponding residuals:

(1) The instrumental variables for the models 2, 3 and 4 are not sufficient to improve the first model in Table 7.2, since *GDP* is insignificantly correlated with the residuals, but *PR* is significantly correlated with the residuals.
(2) For this reason an attempt is made to apply the default options by entering only the exogenous variables *C*, *GDP* and *PR*, as well as the indicator AR(1), which gives the window in Figure 7.5. After entering the equation specification, by clicking *OK*, the statistical results based on the instrumental model 5a is obtained, with a statement 'Lagged dependent variable and regressors added to instrument list.'

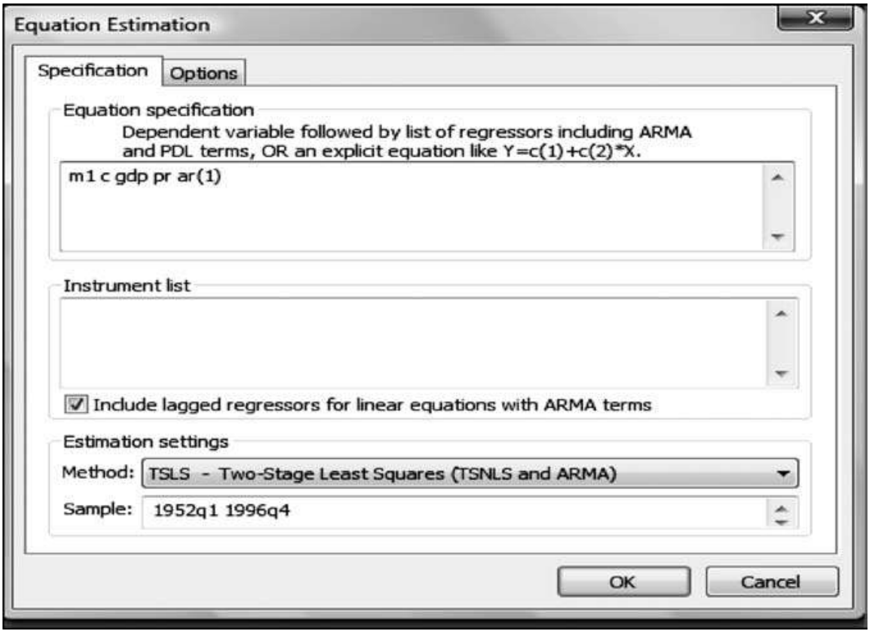


Figure 7.5 The default options for model 5a in Table 7.3

(3) From this point of view, if there is not a good reason to select a set of instrumental variables, it is suggested that a list of instruments should not be inserted, but to use the default option, since it is stated that the default option should be a good option.
(4) Note that the instrumental variable model 5b is in fact the same model as the model 5a.
(5) On the other hand, an error message may be obtained as presented in Figure 7.6, which indicates that additional instrumental variable(s) should be entered. For example, for the exogenous *C*, *GDP* and *PR*, by entering $M1(-1)$ as an instrumental variable, the error message will be obtained, and similarly if *C* and $M1(-1)$ are entered.
(6) Furthermore, note that the main objective of the models presented in Table 7.3 is to demonstrate that only various sets of instrumental variables can be used to modify or improve the unusual model in Figure 7.2. Note the following comparison:
   - The exogenous variables of the instrumental models 5a, 5b and 6 in Table 7.3 are exactly the same exogenous variables of model 3 without an instrumental variable in Table 7.2, which is an acceptable model in a statistical sense.
   - Similarly, the exogenous variables of the instrumental models 7, 8 and 9 in Table 7.3 are exactly the same exogenous variables of model 5 without an instrumental variable in Table 7.2.

Table 7.3 Illustrations of instrumental variable models corresponding to the model in Figure 7.2

Dependent variable: *M*1			Probability (*t*-statistic) of	
Number	Ind. Variables	Inst. Variables	ρ(*Resid*, *gdp*)	ρ(*Resid*, *pr*)
1	*gdp pr*	*Without instrument*	0.0194	0.0004
2	*gdp pr*	*C m*1(−1)	0.1944	0.0106
3	*gdp pr*	*C m*1(−1) *m*1(−2)	0.1872	0.0097
4	*C gdp pr*	*C m*1(−1) *m*1(−2)	0.1928	0.0139
5a	*C gdp pr ar*(1)	[a]	0.3300	0.2932
5b	*C gdp pr ar*(1)	*C m*1(−1) *gdp*(−1) *pr*(−1)	0.3300	0.2932
6	*C gdp pr ar*(1)	*C m*1(−1) *m*1(−2) *gdp*(−1) *pr*(−1)	0.5355	0.5493
7	*C m*1(−1) *gdp pr*	*C gdp*(−1) *pr*(−1) *rs*	0.9973	0.9931
8	*C m*1(−1) *gdp pr*	*C gdp*(−1) *pr*(−1) *rs rs*(−1)	0.9944	0.9942
9	*C m*1(−1) *gdp pr*	*C gdp*(−1) *pr*(−1) *rs Drs*1 *Drs*1[a]*rs*	0.9999	0.9975

[a]Instrument list: Lagged dependent variable and regressors added to the instrument list.

Figure 7.6 The error message of insufficient instruments

- These findings indicate that instrumental variables may be used even though the base model does not have an exogenous variable that is significantly correlated with its residual. However, it is suggested that an instrumental model should not be applied if an acceptable model could be developed without the instrumental variable. □

7.3 Residual analysis in developing instrumental models

It has been recognized that in developing acceptable instrumental models, a series of residual analyses should be conducted. This series of residual analyses has four specific main objectives, as follows:

(i) To test the correlation between the residuals and each independent variable of the model without the instruments.
(ii) To test each of the instrumental variables, whether or not it is qualified to become an instrumental variable. Note that this process is not done for the instruments in Table 7.3.
(iii) To test the correlation between the residuals and an external variable, which can be defined as an additional independent variable in order to improve the model.
(iv) After obtaining an instrumental model, the residual analysis should be conducted again to test whether or not each of the independent variables is insignificantly correlated with the residuals, as presented in Table 7.3.

7.3.1 Testing an hypothesis corresponding to the instrumental models

Based on the experimentation, it was found that a series of residual analyses should be conducted in order to test whether or not each exogenous variable of the basic model, as well as the instrumental model, has a significant (or insignificant) correlation with the residual series. The stages of data analysis should be done as follows:

(1) After defining a basic time series model, select *Quick/Estimate Equation* ... and then enter the corresponding equation specification. The OLS, the White or the Newey–West estimation methods could be used.
(2) By clicking *OK*, the statistical results appear on the screen. Then click the option '*Name*' in order to save the results.
(3) In order to make the residual series, click *Proc/Make Residual Series* ..., which then directly gives additional variables in the workfile, namely *Resid***.
(4) Then on the screen appear the variables *Resid*** and the exogenous variables, either the exogenous variables in or out of the model.
(5) By selecting *View/Covariance Analysis* ... the correlation matrix with the t-statistic and its probability (two-tailed) can be obtained. If there is no exogenous variable that has a significant correlation with the *Resid***, then the process is stopped. In other words, a set of instrument variables does not need to be found.
(6) Otherwise, two types of possible modifications could be done, as mentioned above. However, the first method of modification that could be suggested is to find additional exogenous variables, as presented in Examples 7.1 and 7.2.
(7) In fact, in developing an instrumental model *two stages of problems* (*TSOP*) are faced. First, an appropriate basic model needs to be found that can be modified or improved by using instrumental variables. The second problem

is to select the best possible set of instrumental variables. The true population model could never be known and nor could the true set of instrumental variables (refer to Section 2.14).

(8) On the other hand, it has been found to be very difficult to obtain a basic time series model that has at least one exogenous variable that is significantly correlated with the residual series of the model. Note that Table 7.1 presents no model that can be used as the basic model for developing instrumental models. For this reason, an unusual model is presented in Figure 7.2 as a basic model for illustration purposes.

(9) There is a basic important question, namely 'Can we directly apply an instrumental model, without doing a series of residual analysis on the basic model?' We are very confident that the answer to this question is 'We can't!,' since it is stated that the instrumental model should be applied if and only if at least one of the independent variables of the basic model has a significant correlation with its residual series.

(10) Finally, it is obvious that it is better to use or apply a time series model without instrumental variables unless there is a very good reason for using the instrumental model. This is because a more complex model, as well as a model having a large number of parameters, is most likely to have more uncertainty or unexpected estimates (refer to the multicollinearity problem in Section 2.14.3). For illustration purposes, the following examples present special instrumental models.

Example 7.4. (Special instrumental models) Figure 7.7(a) presents the statistical results of an AR(1) instrumental model with the statement '*Estimated AR Process is nonstationary*,' so this model is not an acceptable time series model. For this

Dependent Variable: M1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 05:16
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 31 iterations
Instrument list: C RS
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
GDP	0.837984	0.275819	3.038165	0.0027
PR	-767.4446	391.6811	-1.959361	0.0517
AR(1)	1.003109	0.006830	146.8735	0.0000

R-squared	0.999178	Mean dependent var	446.7856
Adjusted R-squared	0.999169	S.D. dependent var	344.9693
S.E. of regression	9.944883	Sum squared resid	17406.52
Durbin-Watson stat	1.487217	Second-Stage SSR	13697.05

Inverted AR Roots 1.00
Estimated AR process is nonstationary

(a)

Dependent Variable: M1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 05:31
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 6 iterations
Instrument list:
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
GDP	0.453248	0.105314	4.303771	0.0000
PR	297.2694	172.0282	1.728027	0.0857
AR(1)	0.974295	0.017406	55.97409	0.0000

R-squared	0.999281	Mean dependent var	446.7856
Adjusted R-squared	0.999272	S.D. dependent var	344.9693
S.E. of regression	9.305711	Sum squared resid	15240.94
Durbin-Watson stat	1.484398	Second-Stage SSR	12869.51

Inverted AR Roots .97

(b)

Figure 7.7 Statistical results based on (a) a special AR(1) instrumental model and (b) its reduced model reason, its modified or reduced model is presented, which is an acceptable instrumental model, presented in Figure 7.7(b). However, it may not be the best instrumental model. Do a further analysis to study the limitations of these two models and their possible modifications.

Furthermore, note the lagged dependent variable and regressors added to the instruments, so that the model in Figure 7.17(a) has the instruments '$C\ RS\ m1(-1)$ $gdp(-1)$ $PR(-1)$' and its reduced model has the instruments '$m1(-1)$ $gdp(-1)$ $PR(-1)$', which corresponds to the AR(1) model. If the option 'Lagged dependent variable and regressors added to the instruments' is not used, then the error message presented in Figure 7.6 would be obtained.

Without doing preliminary tests on the status or condition of the exogenous variables, various additional instrumental models could be developed by using the variables $M1$, GDP, PR and RS, as well as their transformed variables. □

Example 7.5. (An extension of the CD model in Example 5.3) Without doing preliminary tests corresponding to all SCMs (i.e. seemingly causal models) presented in the previous chapters, instrumental methods can easily be constructed. Therefore, many instrumental models could be obtained, since various sets of instrumental variables could be selected for each SCM. As an illustration, Figure 7.8 presents two instrumental models that should be considered as an extension of an SCM, namely the Cobb–Douglas (translog linear) model in Example 5.3, based on POOL1.wf1. Furthermore, various instrumental models could be obtained by using other variables in the workfile as instrumental variables. □

Dependent Variable: LOG(MMDEP)
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 06:00
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 5 iterations
Instrument list:
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.662694	0.220392	-3.006882	0.0029
LOG(IVMAUT)	0.194945	0.088619	2.199809	0.0285
LOG(IVMDEP)	0.643421	0.023243	27.68238	0.0000
LOG(IVMMAE)	0.116831	0.080475	1.451771	0.1476
AR(1)	0.720591	0.039317	18.32750	0.0000

R-squared	0.995529	Mean dependent var	8.312279
Adjusted R-squared	0.995473	S.D. dependent var	0.641394
S.E. of regression	0.043157	Sum squared resid	0.588563
F-statistic	17602.28	Durbin-Watson stat	2.572178
Prob(F-statistic)	0.000000	Second-Stage SSR	0.503861

Inverted AR Roots .72

Dependent Variable: LOG(MMDEP)
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 06:10
Sample (adjusted): 1968M02 1994M10
Included observations: 321 after adjustments
Convergence achieved after 5 iterations
Instrument list: LOG(IVMCON) LOG(IVMCST)
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.662263	0.219908	-3.011540	0.0028
LOG(IVMAUT)	0.201677	0.087597	2.302327	0.0220
LOG(IVMDEP)	0.644237	0.023183	27.78971	0.0000
LOG(IVMMAE)	0.110868	0.079717	1.390770	0.1653
AR(1)	0.720091	0.039267	18.33837	0.0000

R-squared	0.995527	Mean dependent var	8.312279
Adjusted R-squared	0.995471	S.D. dependent var	0.641394
S.E. of regression	0.043166	Sum squared resid	0.588814
F-statistic	17595.09	Durbin-Watson stat	2.571614
Prob(F-statistic)	0.000000	Second-Stage SSR	0.501476

Inverted AR Roots .72

Figure 7.8 Statistical results based on instrumental models as an extension of the Cobb–Douglas model in Example 5.3

7.3.2 Graphical representation of the residual series

Graphical representations of residual series can be used to perform an informal or visual analysis to study the good fit model (or the aptness of a model) and to decide whether or not an external variable should be used in the model.

Neter and Wasserman (1974, pp. 99–110) demonstrated the scatter graphs of residual series of a model against its fitted values, the exogenous variable(s) of the model and also the external variables (i.e. the variables that were not in the model). By observing this type of scatter graph, the following problems can be identified:

(1) The outlier(s) of the observed values.
(2) Whether or not a linear regression function is appropriate for the data set, since most of the time a researcher uses the first power of the variables as independent variables.
(3) Whether or not the variance of the residual series is heterogenic or is dependent on an exogenous variable; in general, whether or not the residual series is a function of a variable in the data set being analyzed.
(4) Whether or not an external variable or a variable outside the model should be use as an additional independent variable.

7.4 System equation with instrumental variables

For illustration purposes, the testing would be conducted whether or not the exogenous variables of the basic model used have significant correlations with their corresponding residual series. Here, only the method used to apply the system equation with the instrument variables will be presented, as shown in the following examples. However, experimentation has been conducted in order to present an instrumental model having acceptable or sound statistical results.

Example 7.6. (An AR simultaneous causal effects model) In this example a simple autoregressive simultaneous causal effects model having instrument variables is considered, with the following equations:

$$\begin{aligned}&\log(m1) = c(11) + c(12)^*\log(gdp) + [ar(1) = c(13)]\,@\,c\ \log(pr)\\&\log(gdp) = c(21) + c(22)^*\log(m1) + [ar(1) = c(23),\ ar(2) = c(24)]@\,c\,rs\end{aligned} \tag{7.1}$$

where the symbol @ is used to indicate that $c\log(pr)$ and $c\ rs$ are the instrumental variables of the first and secong models respectively.

In fact, this model is the result of experimentation in order to obtain sufficiently large values of the DW-statistic for each regression. Note that this model shows that $\log(m1)$ and $\log(gdp)$ have simultaneous causal effects.

The steps of data analysis are as follows:

(1) Click *Object/New Object .../System/OK*. Then the equation specification (7.1) can be entered, as presented in Figure 7.9.
(2) By clicking the option '*Estimate*', the window in Figure 7.10 appears on the screen.

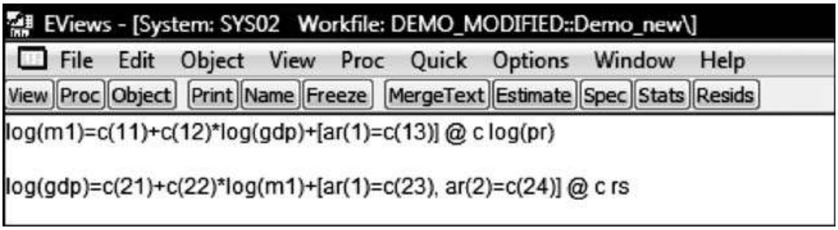


Figure 7.9 Equation specification of the instrumental model in (7.1)

(3) The TSLS estimation method should be used together with the default option 'Add lagged regressors . . . .' By clicking *OK*, the statistical results in Figure 7.11 are obtained. However, if this option is not used, the error message '*Near singular matrix*' will be obtained.

(4) By using the option 'Add lagged regressors . . . ,' the output shows the following specific characteristics:
  - For the first regression there are two additional instrument variables, namely log($m1(-1)$) and log($gdp(-1)$), which correspond to the AR(1) model, as well as the original instruments C and log(pr).
  - For the second regression there are four additional instruments, namely log($m1(-1)$), log($m1(-2)$), log($gdp(-1)$) and log($gdp(-2)$), which correspond to the AR(2) model, as well as the original instruments C and RS.

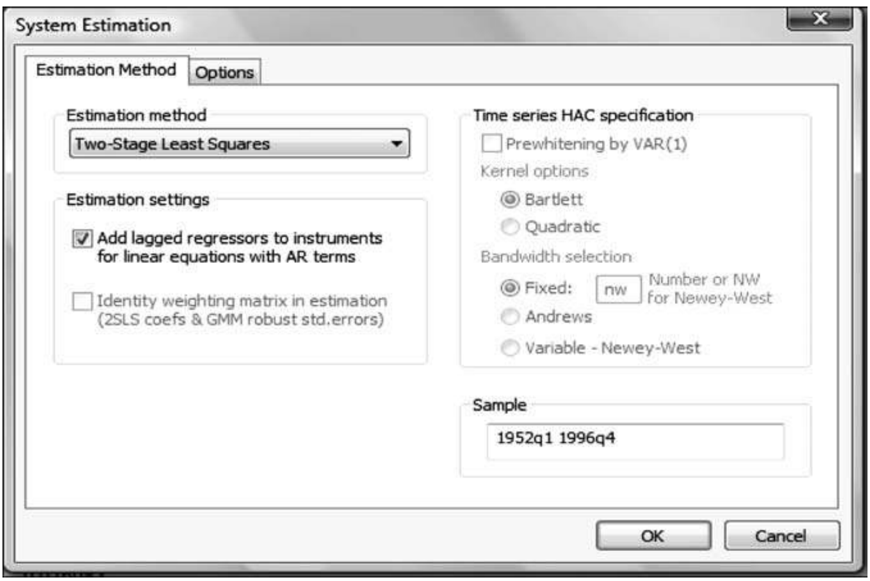


Figure 7.10 The estimation method, settings and options for the system estimation

System: SYS02
Estimation Method: Iterative Two-Stage Least Squares
Date: 12/30/07 Time: 14:11
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (unbalanced) observations 357
Convergence achieved after 51 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.269214	1.578607	0.170539	0.8647
C(12)	0.894798	0.195016	4.588320	0.0000
C(13)	0.983958	0.018754	52.46649	0.0000
C(21)	793.0155	50544.58	0.015689	0.9875
C(22)	0.104396	0.073601	1.418403	0.1570
C(23)	1.390346	0.069409	20.03125	0.0000
C(24)	-0.390358	0.069429	-5.622366	0.0000

Determinant residual covariance 1.73E-08

Equation: LOG(M1)=C(11)+C(12)*LOG(GDP)+[AR(1)=C(13)]
Instruments: C LOG(PR) LOG(M1(-1)) LOG(GDP(-1))
Observations: 179

R-squared	0.999555	Mean dependent var	5.816642
Adjusted R-squared	0.999550	S.D. dependent var	0.753241
S.E. of regression	0.015983	Sum squared resid	0.044958
Durbin-Watson stat	1.934898		

Equation: LOG(GDP)=C(21)+C(22)*LOG(M1)+[AR(1)=C(23), AR(2)=C(24)]
Instruments: C RS LOG(GDP(-1)) LOG(M1(-1)) LOG(GDP(-2)) LOG(M1(-2))
Observations: 178

R-squared	0.999908	Mean dependent var	6.008518
Adjusted R-squared	0.999907	S.D. dependent var	0.995104
S.E. of regression	0.009614	Sum squared resid	0.016083
Durbin-Watson stat	2.014735		

(a)

System: SYS02
Estimation Method: Weighted Two-Stage Least Squares
Date: 12/30/07 Time: 14:14
Sample: 1952Q2 1996Q4
Included observations: 180
Total system (unbalanced) observations 357
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 55 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.269232	1.565400	0.171990	0.8635
C(12)	0.894795	0.193384	4.627043	0.0000
C(13)	0.983957	0.018596	52.91092	0.0000
C(21)	793.0156	49973.43	0.015869	0.9873
C(22)	0.104396	0.072769	1.434614	0.1523
C(23)	1.390346	0.068625	20.26014	0.0000
C(24)	-0.390358	0.068645	-5.686608	0.0000

Determinant residual covariance 1.73E-08

Equation: LOG(M1)=C(11)+C(12)*LOG(GDP)+[AR(1)=C(13)]
Instruments: C LOG(PR) LOG(M1(-1)) LOG(GDP(-1))
Observations: 179

R-squared	0.999555	Mean dependent var	5.816642
Adjusted R-squared	0.999550	S.D. dependent var	0.753241
S.E. of regression	0.015983	Sum squared resid	0.044958
Durbin-Watson stat	1.934898		

Equation: LOG(GDP)=C(21)+C(22)*LOG(M1)+[AR(1)=C(23), AR(2)=C(24)]
Instruments: C RS LOG(GDP(-1)) LOG(M1(-1)) LOG(GDP(-2)) LOG(M1(-2))
Observations: 178

R-squared	0.999908	Mean dependent var	6.008518
Adjusted R-squared	0.999907	S.D. dependent var	0.995104
S.E. of regression	0.009614	Sum squared resid	0.016083
Durbin-Watson stat	2.014735		

(b)

Figure 7.11 Statistical results based on the model in (7.1) using (a) the TSLS and (b) the WTSLS estimation methods

(5) Without using this option, the statistical results can be obtained as if the lagged dependent variables were used as independent variables of the basic models. One of the statistical results is presented in Figure 7.12. However, if the option is used the statistical results are also obtained. Refer to the following example, which shows contradictory results. □

System: SYS04
Estimation Method: Two-Stage Least Squares
Date: 12/31/07 Time: 07:44
Sample: 1952Q2 1996Q4
Included observations: 179
Total system (balanced) observations 358

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.028923	0.024872	1.162868	0.2457
C(12)	0.974252	0.018190	53.56097	0.0000
C(13)	0.022183	0.013649	1.625219	0.1050
C(21)	0.053676	0.014220	3.774604	0.0002
C(22)	-0.029518	0.010170	-2.902504	0.0039
C(23)	1.022621	0.007682	133.1241	0.0000

Determinant residual covariance 2.10E-08

Equation: LOG(M1)=C(11)+C(12)*LOG(M1(-1)) +C(13)*LOG(GDP)
Instruments: C LOG(PR) LOG(PR(-1))
Observations: 179

R-squared	0.999626	Mean dependent var	5.816642
Adjusted R-squared	0.999622	S.D. dependent var	0.753241
S.E. of regression	0.014644	Sum squared resid	0.037742
Durbin-Watson stat	2.227520		

Equation: LOG(GDP)=C(21)+C(22)*LOG(M1)+C(23)*LOG(GDP(-1))
Instruments: C LOG(PR) LOG(PR(-1)) RS RS(-1)
Observations: 179

R-squared	0.999896	Mean dependent var	5.999972
Adjusted R-squared	0.999895	S.D. dependent var	0.998870
S.E. of regression	0.010224	Sum squared resid	0.018396
Durbin-Watson stat	1.372397		

Figure 7.12 Statistical results of a system instrumental model without using the option 'add lagged regressors ....'

Example 7.7. (A special case of the LVAR(1,1) instrumental models) By entering the following system equation specification in (7.2) and using the option 'Add lagged regressors ...,' the statistical results in Figure 7.13 are obtained, which

System: UNTITLED
Estimation Method: Iterative Two-Stage Least Squares
Date: 12/31/07 Time: 08:07
Sample: 1952Q3 1996Q4
Included observations: 179
Total system (balanced) observations 356
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.052711	0.015141	3.481248	0.0006
C(12)	0.956140	0.010443	91.55619	0.0000
C(13)	0.035839	0.007861	4.558863	0.0000
C(14)	-0.125165	0.076516	-1.635795	0.1028
C(21)	0.057703	0.016569	3.482605	0.0006
C(22)	-0.031769	0.011526	-2.756302	0.0062
C(23)	1.024081	0.008696	117.7685	0.0000
C(24)	0.315030	0.072372	4.352906	0.0000
Determinant residual covariance		1.81E-08		

Equation: LOG(M1)=C(11)+C(12)*LOG(M1(-1)) +C(13)*LOG(GDP(-1)) +[AR(1)=C(14)]
Instruments: C LOG(PR) LOG(PR(-1)) LOG(M1(-1)) LOG(M1(-2)) LOG(GDP(-2))
Observations: 178

R-squared	0.999633	Mean dependent var	5.822083
Adjusted R-squared	0.999627	S.D. dependent var	0.751831
S.E. of regression	0.014528	Sum squared resid	0.036724
Durbin-Watson stat	1.947382		

Equation: LOG(GDP)=C(21)+C(22)*LOG(M1(-1))+C(23)*LOG(GDP(-1)) +[AR(1)=C(24)]
Instruments: C LOG(PR) RS LOG(GDP(-1)) LOG(M1(-2)) LOG(GDP(-2))
Observations: 178

R-squared	0.999908	Mean dependent var	6.008518
Adjusted R-squared	0.999906	S.D. dependent var	0.995104
S.E. of regression	0.009650	Sum squared resid	0.016203
Durbin-Watson stat	1.991618		

Figure 7.13 A special case of the LVAR(1,1) instrumental models

shows additional instrumental lagged variables. On the other hand, if the option is not used, the error message '*Near singular matrix*' is obtained. This indicates that by using less instrumental variables, it is possible to obtain the error message.

Compared to the model in (7.1), the following model also has the lagged endogenous variables as the independent variables:

$$\begin{aligned} \log(m1) &= c(11) + c(12)^*\log(m1(-1)) + c(13)^*\log(gdp(-1)) + [ar(1) = c(14)] \\ &\quad @\ c\log(pr)\log(pr(-1)) \\ \log(gdp) &= c(21) + c(22)^*\log(m1(-1)) + c(23)^*\log(gdp(-1)) + [ar(1) = c(24)] \\ &\quad @\ c\log(pr)\ rs \end{aligned} \tag{7.2}$$

Without the instrumental variables, this model is a bivariate LVAR(1,1) model, i.e. the lagged-variable autoregressive (1,1) translog linear model. Therefore, this instrumental model is an extension of the LVAR(1,1) model. □

7.5 Selected cases based on the US_DPOC data

Similar to the instrumental models presented in the previous examples, based on the US domestic price of copper data set, namely the US_DPOC data, some selected instrumental models are now presented. For general purposes the X- and Y-variables, namely $Y1$, $Y2$, $X1$, $X2$ and $X3$, will again be used, which have been defined and used in the previous chapter.

Note that most of the illustrative models are presented without testing the correlation between an exogenous variable with the corresponding residual series. However, it is assumed that at least one of the exogenous variables of the model is highly correlated with its residual series, although it was found that it is very difficult to develop an acceptable time series model that has an exogenous variable significantly correlated with its residual series.

Dependent Variable: Y1
Method: Least Squares
Date: 12/31/07 Time: 09:06
Sample: 1951 1980
Included observations: 30

	Coefficient	Std. Error	t-Statistic	Prob.
T	0.428428	0.384174	1.115193	0.2742
X1	0.021247	0.005855	3.628652	0.0011

R-squared	0.528201	Mean dependent var	30.87700
Adjusted R-squared	0.511352	S.D. dependent var	12.71732
S.E. of regression	8.889838	Akaike info criterion	7.272035
Sum squared resid	2212.818	Schwarz criterion	7.365448
Log likelihood	-107.0805	Hannan-Quinn criter.	7.301919
Durbin-Watson stat	0.125389		

Covariance Analysis: Ordinary
Date: 12/31/07 Time: 09:17
Sample: 1951 1980
Included observations: 30

Correlation t-Statistic Probability	RESID21
RESID21	1.000000 ----- -----
X1	-0.717681 -5.453424 0.0000
T	-0.872737 -9.459623 0.0000

Figure 7.14 Statistical results based on a regression of $Y1$ on the time t and $X1$, as a mean model for developing instrumental models

Example 7.8. (An AR(1) model with trend) After conducting experime- ntation, a simple first-order autoregressive model with trend was finally found, which has a significant correlated exogenous variable with the error term. The equation of the model is as follows:

$$Y1_t = C(1)^*X1_t + C(2)^*t + [AR(1) = C(3)] + \varepsilon_t \tag{7.3}$$

Note that this model is a model through the origin. If a model with an intercept is used, then its error term and $X1$ are insignificantly correlated. Figure 7.14 presents the statistical results of this model and the significant correlation of $\rho(Resid21, X1)$ with a p-value $= 0.0000$. As a result this model should be improved or modified to be an instrumental model, which will be presented in the following example. □

Example 7.9. (Simple instrumental models with trend) Corresponding to the basic regression in (7.3), Table 7.4 presents simple instrumental models with trend based on only three variables $Y1$, $X1$ and the time t. This table presents only the summary of the Probability(t-statistic) for the null hypothesis $\rho(t, Resid) = 0$ and $\rho(X1, Resid) = 0$. Based on this summary the following notes are presented:

(1) The set instrumental variables C, $Y1(-1)$ and $X1(-1)$ is not sufficient or effective enough to improve the basic model considered. In fact, other sets of instrumental variables have been tried, but acceptable estimates could not be obtained. For this reason the basic regressions have been modified, as presented in Table 7.4.
(2) By using the five variables $X1$, $X2$, $X3$, $Y1$ and $Y2$, as well as the time t, many more instrumental models could easily be developed. Any of the models presented in the

Table 7.4 Selected simple instrumental models based on the variables $Y1$, $X1$ and the time t

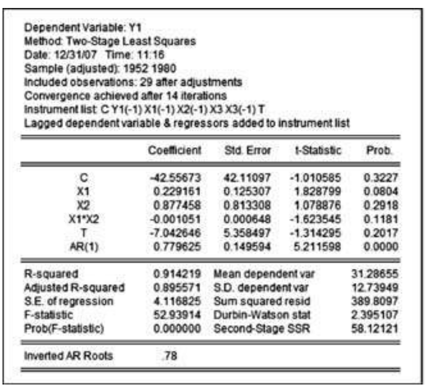
			Probability(t-statistic) of	
Number	Equation specification	Inst. variables	$\rho(t, Resid)$	$\rho(x1, Resid)$
1	$y1\ t\ x1$	$C\ y1(-1)\ x1(-1)$	0.0000	0.0013
2	$y1\ c\ t\ x1$	$C\ y1(-1)\ x1(-1)$	0.1962	0.8197
3	$y1\ c\ t\ x1\ ar(1)$	[a]	0.3661	0.9007
4	$y1\ c\ t\ x1$	$C\ y1(-1)\ x1(-1)\ t$	1.0000	0.7096
5	$\log(y1)\ t\ x1$	$\log(y1(-1))\ x1(-1)$	0.0076	0.6583
6	$\log(y1)\ c\ t\ x1$	$\log(y1(-1))\ x1(-1)$	0.0503	0.9206
7	$\log(y1)\ c\ t\ x1\ ar(1)$	[a]	0.2967	0.7556
8	$\log(y1)\ c\ t\ x1$	$\log(y1(-1))\ x1(-1)\ t$	1.0000	0.8085

[a]Instrument list: lagged dependent variable and regressors added to the instrument list.

previous chapters could also be used as a base model. Do this as an exercise, since the data analysis could be done in a short time. The only problem is how to define a base model and then select the best possible set of instrumental variables. □

Example 7.10. (Instrumental interaction models with trend) Under the assumption that the effect of X_2 on Y_1 depends on X_1 or the effect of X_1 on Y_1 depends on X_2, then in a mathematical sense the two-way interaction $X1^*X2$ has to be used as an independent variable of a defined model, which have been presented in the previous chapters. However, here the instrumental interaction models are considered.

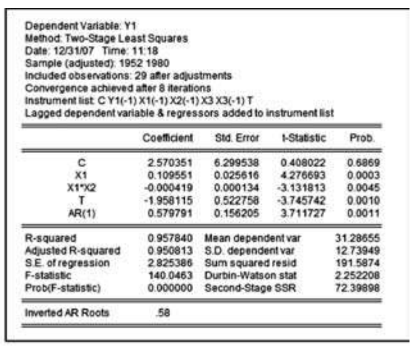
As an illustration, Figure 7.15 presents the statistical results based on an AR(1) instrumental two-way interaction model with trend and its reduced model, with the lagged dependent variable and regressors added to the instrument list. For a


Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 11:16
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 14 iterations
Instrument list: C Y1(-1) X1(-1) X2(-1) X3 X3(-1) T
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-42.55673	42.11097	-1.010585	0.3227
X1	0.229161	0.125307	1.828799	0.0804
X2	0.877458	0.813308	1.078876	0.2918
X1*X2	-0.001051	0.000648	-1.623545	0.1181
T	-7.042646	5.358497	-1.314295	0.2017
AR(1)	0.779625	0.149594	5.211598	0.0000

R-squared	0.914219	Mean dependent var	31.28655
Adjusted R-squared	0.895571	S.D. dependent var	12.73949
S.E. of regression	4.116825	Sum squared resid	389.8097
F-statistic	52.93914	Durbin-Watson stat	2.395107
Prob(F-statistic)	0.000000	Second-Stage SSR	58.12121

Inverted AR Roots .78


Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 11:18
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 8 iterations
Instrument list: C Y1(-1) X1(-1) X2(-1) X3 X3(-1) T
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.570351	6.299538	0.408022	0.6869
X1	0.109551	0.025616	4.276693	0.0003
X1*X2	-0.000419	0.000134	-3.131813	0.0045
T	-1.958115	0.522758	-3.745742	0.0010
AR(1)	0.579791	0.156205	3.711727	0.0011

R-squared	0.957840	Mean dependent var	31.28655
Adjusted R-squared	0.950813	S.D. dependent var	12.73949
S.E. of regression	2.825386	Sum squared resid	191.5874
F-statistic	140.0463	Durbin-Watson stat	2.252208
Prob(F-statistic)	0.000000	Second-Stage SSR	72.39898

Inverted AR Roots .58

Figure 7.15 Statistical results based on an AR(1) instrumental interaction model with trend and its reduced model, where all of the lagged variables are added to the instrument list

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 11:21
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 2 iterations
Instrument list: C Y1(-1) X1(-1) X2(-1) X3 X3(-1) T
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	31.31306	20.81133	1.504616	0.1460
X1	0.049199	0.046341	1.061674	0.2994
X2	-0.483037	0.316385	-1.526737	0.1405
X1*X2	-8.77E-05	0.000259	-0.338025	0.7384
T	0.458322	1.481858	0.309289	0.7599
AR(1)	0.330155	0.223436	1.477626	0.1531

R-squared	0.961664	Mean dependent var	31.28655
Adjusted R-squared	0.953330	S.D. dependent var	12.73949
S.E. of regression	2.752134	Sum squared resid	174.2076
F-statistic	117.6654	Durbin-Watson stat	1.980093
Prob(F-statistic)	0.000000	Second-Stage SSR	88.11549
Inverted AR Roots	.33		

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 11:20
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 2 iterations
Instrument list: C Y1(-1) X1(-1) X2(-1) X3 X3(-1) T
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.217034	6.401235	-0.033905	0.9732
X1	0.111072	0.027476	4.042437	0.0005
X1*X2	-0.000440	0.000146	-3.023530	0.0059
T	-1.748819	0.462108	-3.784441	0.0009
AR(1)	0.543893	0.176044	3.089534	0.0050

R-squared	0.959866	Mean dependent var	31.28655
Adjusted R-squared	0.953177	S.D. dependent var	12.73949
S.E. of regression	2.756636	Sum squared resid	182.3770
F-statistic	145.9691	Durbin-Watson stat	2.169674
Prob(F-statistic)	0.000000	Second-Stage SSR	107.3467
Inverted AR Roots	.54		

Figure 7.16 Statistical results based on an AR(1) instrumental interaction model with trend and its reduced model, where no lagged variables are added to the instrument list

comparison, Figure 7.16 presents statistical results based on the same models, but the lagged dependent variable and regressors are not added to the instrument list.

Based on the statistical results in both figures, the following notes and conclusions are presented:

(1) Compared to the reduced model in Figure 7.15, the reduced model in Figure 7.16 has a smaller set of instrumental variables, so has less parameters. For this reason, the second reduced model is preferred.
(2) Furthermore, it has been found that each of the exogenous and instrumental variables is insignificantly correlated with its residual, with a minimal p-value of 0.6656 for $X2(-1)$ and a sufficient value of the DW-statistic.
(3) Note that $X1$ and $X2$ are not in the instrument list since it is assumed that they are highly correlated with the residual of the basic model. Refer to the basic model in Figure 7.15.

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 12:15
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 22 iterations
Instrument list:
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	44.85821	1170057.	3.83E-05	1.0000
X1	-0.018290	4389.929	-4.17E-06	1.0000
X2	-0.403167	4560.558	-8.84E-05	0.9999
X1*X2	0.000289	24.29906	1.19E-05	1.0000
T	0.635274	52323.60	1.21E-05	1.0000
AR(1)	0.350113	6806.020	5.14E-05	1.0000

R-squared	0.946626	Mean dependent var	31.28655
Adjusted R-squared	0.935023	S.D. dependent var	12.73949
S.E. of regression	3.247367	Sum squared resid	242.5441
F-statistic	84.57797	Durbin-Watson stat	1.809216
Prob(F-statistic)	0.000000	Second-Stage SSR	84.70903
Inverted AR Roots	.35		

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 12:16
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 5 iterations
Instrument list:
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	27.98541	5.047195	5.544746	0.0000
X1*X2	0.000193	3.48E-05	5.549491	0.0000
T	-1.131024	0.498160	-2.270405	0.0321
AR(1)	0.495560	0.180413	2.746804	0.0110

R-squared	0.909225	Mean dependent var	31.28655
Adjusted R-squared	0.898332	S.D. dependent var	12.73949
S.E. of regression	4.062040	Sum squared resid	412.5042
F-statistic	90.06210	Durbin-Watson stat	1.758511
Prob(F-statistic)	0.000000	Second-Stage SSR	86.12932
Inverted AR Roots	.50		

Figure 7.17 Unexpected statistical results based on an instrumental model and its reduced model

(4) Figure 7.17 presents unexpected statistical results based on the full AR(1) basic model but using only the first lagged variable of all variables as the instrumental variable. This instrumental model should be considered as the worst model, since each of the exogenous variables has a p-value $= 1.0000$ and would never be presented as an empirical model, in practice. On the other hand, its reduced model is a good fit AR(1) instrumental interaction model.

(5) In this case, a problem should be considered that is related to the option 'Lagged dependent . . . ,' since the first lagged variables $Y1(-1)$, $X1^*X2(-1)$ and $t(-1)$ will be additional instruments, where $X1^*X2(-1)$ and $t(-1)$ can be considered as uncommon lagged variables in the time series data analysis, especially the lagged variable $t(-1)$. □

Example 7.11. (Instrumental translog linear models with trend) Figure 7.18(a) presents the statistical results based on an AR(1) translog linear model with trend, under the assumption that log(x1) and log(x2) are correlated with the residual. Therefore, they cannot be used as instrumental variables. Since log(x2), t and AR(1) are insignificant, several possible reduced models could be obtained by deleting either one or two of these variables.

Figure 7.18(b) presents the statistical results based on a reduced model that is an acceptable instrumental model, in a statistical sense. Try to apply other possible reduced models as an exercise.

Note that both models have constant elasticity with respect to the exogenous or input variable X_1, which is equal to $C(2)$ with positive values of 2.054 674 based on the full model and 2.466 828 based on the reduced model, both of which are significant with p-values of 0.0230 and 0.0017 respectively. □

Dependent Variable: LOG(Y1)
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 20:56
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 4 iterations
Instrument list: C LOG(Y1(-1)) LOG(Y1(-2)) LOG(X3) LOG(X3(-1)) T
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.045253	9.177186	-0.331829	0.7430
LOG(X1)	2.054674	0.843558	2.435725	0.0230
LOG(X2)	-1.411948	1.270575	-1.111267	0.2779
T	-0.061058	0.117011	-0.521814	0.6068
AR(1)	0.266134	1.004339	0.264984	0.7934

R-squared	0.936624	Mean dependent var	3.397764
Adjusted R-squared	0.925602	S.D. dependent var	0.326101
S.E. of regression	0.088947	Sum squared resid	0.181967
F-statistic	88.37460	Durbin-Watson stat	1.477533
Prob(F-statistic)	0.000000	Second-Stage SSR	0.074493
Inverted AR Roots	.27		

(a)

Dependent Variable: LOG(Y1)
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 20:52
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 2 iterations
Instrument list: C LOG(Y1(-1)) LOG(Y1(-2)) LOG(X3) LOG(X3(-1)) T
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-10.69792	3.780158	-2.830020	0.0093
LOG(X1)	2.466828	0.698266	3.532791	0.0017
T	-0.151047	0.055396	-2.726669	0.0118
AR(1)	0.603540	0.217251	2.778080	0.0104

R-squared	0.901403	Mean dependent var	3.397764
Adjusted R-squared	0.889078	S.D. dependent var	0.326101
S.E. of regression	0.108608	Sum squared resid	0.283095
F-statistic	75.86872	Durbin-Watson stat	1.940057
Prob(F-statistic)	0.000000	Second-Stage SSR	0.186477
Inverted AR Roots	.60		

(b)

Figure 7.18 Statistical results based on (a) an AR(1) instrumental translog linear model and (b) its reduced model

7.6 Instrumental models with time-related effects

In this section, examples of the GLMs with time-related effects will be presented, under the assumption that the time t is uncorrelated with the disturbance term.

Example 7.12. (AR Instrumental models with time-related effects) Figure 7.19(a) presents the statistical results based on an AR(2) instrumental model with time-related effects, which is indicated by the interaction $t^*\log(X1)$. Note that the set of instrumental variables are selected using the trial-and-error methods in order to obtain acceptable parameter estimates. Since $\log(X1)$ is insignificant with a large p-value $= 0.5378$, this may be a reduced model. In general, a reduced model will be obtained by deleting $\log(x1)$.

However, here a special or unexpected reduced model is presented, as shown in Figure 7.19(b). This instrumental model is a good fit model with a time-related effect, in a statistical sense, since it has a DW-statistic of 1.944 560, and each of the independent variables, as well as the indicator AR(1), is significant, at the 0.05 significant level.

Furthermore, based on the statistical results in this figure the following notes apply:

(1) The interaction $t^*\log(X1)$ has a significant effect on $\log(Y1)$ based on both models.
(2) Based on the reduced model, the marginal elasticity of Y_1 with respect to X_1 is a linear function of the time t, as follows:

$$\frac{\partial \log(Y_1)}{\partial \log(X_1)} = \frac{\partial Y_1}{\partial X_1} * \frac{X_1}{Y_1} = c(2) + c(3)^* t = 3.927 + 0.015^* t \tag{7.4}$$

□

Dependent Variable: LOG(Y1)
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 21:27
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 2 iterations
Instrument list: C LOG(Y1(-1)) LOG(X1(-1)) LOG(X2(-1)) LOG(X3) LOG(X3(-1)) T
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	7.318354	6.278420	1.165636	0.2562
LOG(X1)	-0.683527	1.091914	-0.625989	0.5378
T	-0.248084	0.072659	-3.414374	0.0025
T*LOG(X1)	0.041205	0.016077	2.563025	0.0177
AR(1)	1.234855	0.292266	4.225108	0.0003
AR(2)	-0.632964	0.299812	-2.111203	0.0463

R-squared	0.956545	Mean dependent var	3.397764
Adjusted R-squared	0.946669	S.D. dependent var	0.326101
S.E. of regression	0.075308	Sum squared resid	0.124769
F-statistic	98.45622	Durbin-Watson stat	2.127030
Prob(F-statistic)	0.000000	Second-Stage SSR	0.079351
Inverted AR Roots	.62+.50i	.62-.50i	

(a)

Dependent Variable: LOG(Y1)
Method: Two-Stage Least Squares
Date: 12/31/07 Time: 21:29
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 4 iterations
Instrument list: C LOG(Y1(-1)) LOG(X1(-1)) LOG(X2(-1)) LOG(X3) LOG(X3(-1)) T
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-19.32934	9.380847	-2.060511	0.0499
LOG(X1)	3.926951	1.660330	2.365163	0.0261
T*LOG(X1)	-0.031764	0.015473	-2.052857	0.0507
AR(1)	0.626766	0.164789	3.803451	0.0008

R-squared	0.860754	Mean dependent var	3.382868
Adjusted R-squared	0.844045	S.D. dependent var	0.330119
S.E. of regression	0.130368	Sum squared resid	0.424895
F-statistic	56.35444	Durbin-Watson stat	1.944560
Prob(F-statistic)	0.000000	Second-Stage SSR	0.178039
Inverted AR Roots	.63		

(b)

Figure 7.19 Statistical results based on (a) an AR(2) instrumental model with a time-related effect and (b) its special or unexpected reduced model

Dependent Variable: LOG(Y1)
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 07:05
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 16 iterations
Instrument list: C LOG(Y2) LOG(X3) LOG(Y2(-1)) LOG(X3(-1))
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-15.21672	15.10067	-1.007685	0.3246
LOG(X1)	0.753107	2.746992	0.274157	0.7865
LOG(X2)	3.555650	2.537673	1.401146	0.1751
T	0.283716	0.206001	1.377256	0.1823
T*LOG(X1)	0.112866	0.067132	1.681258	0.1069
T*LOG(X2)	-0.262429	0.116885	-2.245181	0.0351
AR(1)	0.251171	0.243059	1.033375	0.3127

R-squared	0.895561	Mean dependent var	3.382868
Adjusted R-squared	0.867078	S.D. dependent var	0.330119
S.E. of regression	0.120356	Sum squared resid	0.318685
F-statistic	33.97440	Durbin-Watson stat	1.741495
Prob(F-statistic)	0.000000	Second-Stage SSR	0.098555

Inverted AR Roots	.25

(a)

Dependent Variable: LOG(Y1)
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 07:08
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 14 iterations
Instrument list: C LOG(Y2) LOG(X3) LOG(Y2(-1)) LOG(X3(-1))
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-11.16128	8.659829	-1.288857	0.2103
LOG(X2)	3.631012	2.209861	1.643095	0.1140
T	0.239883	0.157124	1.526714	0.1405
T*LOG(X1)	0.123107	0.042705	2.882711	0.0084
T*LOG(X2)	-0.259734	0.110411	-2.352439	0.0276
AR(1)	0.292136	0.198599	1.470981	0.1548

R-squared	0.904656	Mean dependent var	3.382868
Adjusted R-squared	0.883928	S.D. dependent var	0.330119
S.E. of regression	0.112469	Sum squared resid	0.290935
F-statistic	46.66539	Durbin-Watson stat	1.695129
Prob(F-statistic)	0.000000	Second-Stage SSR	0.099976

Inverted AR Roots	.29

(b)

Figure 7.20 Statistical results based on (a) an AR(1) instrumental model with time-related effects and (b) its reduced model

Example 7.13. (Other AR instrumental models with time-related effects) In this example an attempt is made to develop an AR instrumental model with time-related effects by using the five defined variables $X1$, $X2$, $X3$, $Y1$ and $Y2$, and the time t, either as independent or instrumental variables. By using the trial-and-error methods the AR(1) instrumental model with time-related effects presented in Figure 7.20(a) is obtained, as a full model, and its reduced model in Figure 7.20(b).

At the 0.10 significant level, each of the independent variables, as well as the indicator AR(1), is significant based on a one-sided hypothesis, and $DW = 1.695$. This model can therefore be considered as a good fit model, in a statistical sense.

Based on the reduced model, the following regression function is obtained:

$$\begin{aligned} \log(Y1) = \; & 3.0862 + 0.0188{*}\log(X3) \\ & + \{-0.0208 + 0.0530\log(X1) - 0.0724\log(X2)\}{*}t \\ & + [AR(1) = 0.507516495207] \end{aligned} \tag{7.5}$$

Note that this function shows that the effect of the time t is dependent on the function $\{-0.0208 + 0.0530 \log(X1) - 0.0724 \log(X2)\}$. This effect can be presented as the partial derivative:

$$\frac{\partial \log(y1)}{\partial t} = -0.0208 + 0.0530\log(x1) - 0.0724\log(x2) \tag{7.6}$$

Furthermore, in a two-dimensional space with t and $\log(y1)$ axes, this regression function represents a set of lines with various slopes and intercepts. □

7.7 Instrumental seemingly causal models

This section will present examples of instrumental models without using the time t as an independent variable, which will be called the *instrumental seemingly causal model*(*s*), namely ISCM.

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 08:03
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 5 iterations
Instrument list: C Y1(-1) X1(-1)
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.228413	14.29728	0.295749	0.7698
X1	0.025629	0.007241	3.539394	0.0015
AR(1)	0.879576	0.145550	6.043111	0.0000

R-squared	0.937116	Mean dependent var	31.28655
Adjusted R-squared	0.932279	S.D. dependent var	12.73949
S.E. of regression	3.315230	Sum squared resid	285.7594
F-statistic	201.1148	Durbin-Watson stat	1.713960
Prob(F-statistic)	0.000000	Second-Stage SSR	123.4444
Inverted AR Roots	.88		

(a)

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 08:07
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 5 iterations
Instrument list:
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.228413	14.29728	0.295749	0.7698
X1	0.025629	0.007241	3.539394	0.0015
AR(1)	0.879576	0.145550	6.043111	0.0000

R-squared	0.937116	Mean dependent var	31.28655
Adjusted R-squared	0.932279	S.D. dependent var	12.73949
S.E. of regression	3.315230	Sum squared resid	285.7594
F-statistic	201.1148	Durbin-Watson stat	1.713960
Prob(F-statistic)	0.000000	Second-Stage SSR	123.4444
Inverted AR Roots	.88		

(b)

Figure 7.21 Statistical results based on an equation specification, using different methods for inserting the instrument list

Example 7.14. (ISCM with an exogenous variable) Figure 7.21 presents two statistical results based on a basic model or an equation specification, but using different methods of inserting the instrument list. However, Figure 7.21(a) and (b) present equal statistical values.

Note that Figure 7.21(a) presents the statement 'Instrument list $C\ y1(-1)\ x1(-1)$ and Lagged dependent variables and regressors added to instrument list,' but Figure 7.21(b) presents only the statement 'Lagged dependent variables and regressors added to instrument list.' For these findings, it could be said that the instrument list C, $y1(-1)$ and $x1(-1)$ should be considered as a useless list. In other words, if the instrument list '$C\ y1(-1)\ x1(-1)$' is used, the statement 'Lagged dependent variables and regressors added to instrument list' does not operate.

On the other hand, by using the 'Instrument list $C\ y1(-1)\ x1(-1)$ but Lagged dependent variables and regressors not added to instrument list', exactly the same statistical output would be obtained. □

Example 7.15. (ISCM with two exogenous variables) Figure 7.22 presents statistical results based on an AR(2) additive ISCM with two exogenous variable $X1$ and $X2$, and its reduced model. In this model, it is assumed that $X2$ is uncorrelated with the residual, so it can be in the instrument list.

As an extension, Figure 7.23(a) and (b) presents statistical results based on an AR(1) interaction ISCM with exogenous variables $X1$, $X2$ and $X1^*X2$ and its reduced model respectively, using the same instrument list as the model in Figure 7.22. This reduced model is a nonhierarchical model, since it has an interaction $X1^*X2$ as an independent variable, but the main factor $X2$ is not in the model.

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 09:09
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 2 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	31.48428	3.459601	9.100552	0.0000
X1	0.034740	0.003173	10.94829	0.0000
X2	-0.360178	0.062780	-5.737146	0.0000
AR(1)	0.518801	0.276786	1.874378	0.0736
AR(2)	-0.295809	0.336224	-0.879799	0.3881

R-squared	0.958645	Mean dependent var	31.71071
Adjusted R-squared	0.951453	S.D. dependent var	12.76302
S.E. of regression	2.812121	Sum squared resid	181.8845
F-statistic	136.4362	Durbin-Watson stat	2.008732
Prob(F-statistic)	0.000000	Second-Stage SSR	82.39601

Inverted AR Roots	.26+.48i	.26-.48i

(a)

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 09:10
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 4 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	32.64874	3.734522	8.742416	0.0000
X1	0.034758	0.003003	11.57272	0.0000
X2	-0.371503	0.064675	-5.744190	0.0000
AR(1)	0.342233	0.184334	1.856590	0.0757

R-squared	0.961369	Mean dependent var	31.71071
Adjusted R-squared	0.956540	S.D. dependent var	12.76302
S.E. of regression	2.660702	Sum squared resid	169.9040
F-statistic	203.6626	Durbin-Watson stat	1.983292
Prob(F-statistic)	0.000000	Second-Stage SSR	72.77166

Inverted AR Roots	.34

(b)

Figure 7.22 Statistical results based on (a) an AR(2) additive ISCM of $Y1$ on ($X1$, X2) and (b) its reduced model

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 09:13
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 2 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	26.56511	6.088621	4.363075	0.0002
X1	0.054616	0.019443	2.809042	0.0100
X2	-0.372407	0.057979	-6.423172	0.0000
X1*X2	-0.000120	0.000114	-1.054135	0.3028
AR(1)	0.276791	0.195290	1.417335	0.1698

R-squared	0.963844	Mean dependent var	31.71071
Adjusted R-squared	0.957556	S.D. dependent var	12.76302
S.E. of regression	2.629434	Sum squared resid	159.0203
F-statistic	156.2545	Durbin-Watson stat	1.931283
Prob(F-statistic)	0.000000	Second-Stage SSR	76.83044

Inverted AR Roots	.28

(a)

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 09:14
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 7 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	40.95399	5.293805	7.736210	0.0000
X2	-0.335054	0.076132	-4.400970	0.0002
X1*X2	0.000200	2.11E-05	9.471616	0.0000
AR(1)	0.320026	0.195251	1.639048	0.1142

R-squared	0.945132	Mean dependent var	31.71071
Adjusted R-squared	0.938273	S.D. dependent var	12.76302
S.E. of regression	3.170955	Sum squared resid	241.3189
F-statistic	142.3043	Durbin-Watson stat	1.788968
Prob(F-statistic)	0.000000	Second-Stage SSR	105.5699

Inverted AR Roots	.32

(b)

Figure 7.23 Statistical results based on (a) an AR(2) interaction ISCM of $Y1$ on ($X1$, $X2$, $X1^*X2$) and (b) its reduced model

For further illustration, Figure 7.24 presents statistical results based on two alternative reduced models. Figure 7.24(a) presents a note 'Estimated AR process is nonstationary,' so this model is not an appropriate time series model.

Since, based on the full model, $X1^*X2$ is insignificant, then it may be deleted to obtain a reduced model. In this case, an additive ISCM is obtained, as presented in Figure 7.24(b), which is exactly the same as Figure 7.22(b). However, this additive model cannot represent either the effect of $X1$ on $Y1$ which is dependent on $X2$ or the effect of $X2$ on $Y1$ which is dependent on $X1$, which have been theoretically defined. Therefore, the statistical results in Figure 7.24(a) should be considered as an unacceptable estimate and the results in Figure 7.24(b) should be considered as inappropriate to present the theoretical relationships between the three variables $X1$, $X2$ and $Y1$. □

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 09:15
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 4 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-4.679826	33.28800	-0.140586	0.8894
X1	0.073324	0.029528	2.483177	0.0204
X1*X2	-0.000307	0.000117	-2.618054	0.0151
AR(1)	1.023883	0.108801	9.410571	0.0000

R-squared	0.957338	Mean dependent var	31.71071
Adjusted R-squared	0.952005	S.D. dependent var	12.76302
S.E. of regression	2.796101	Sum squared resid	187.6364
F-statistic	177.4832	Durbin-Watson stat	1.810134
Prob(F-statistic)	0.000000	Second-Stage SSR	235.3718

Inverted AR Roots 1.02
Estimated AR process is nonstationary

(a)

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 09:34
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 4 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	32.64874	3.734522	8.742416	0.0000
X1	0.034758	0.003003	11.57272	0.0000
X2	-0.371503	0.064675	-5.744190	0.0000
AR(1)	0.342233	0.184334	1.856590	0.0757

R-squared	0.961369	Mean dependent var	31.71071
Adjusted R-squared	0.956540	S.D. dependent var	12.76302
S.E. of regression	2.660702	Sum squared resid	169.9040
F-statistic	203.6626	Durbin-Watson stat	1.983292
Prob(F-statistic)	0.000000	Second-Stage SSR	72.77166

Inverted AR Roots .34

(b)

Figure 7.24 Alternative reduced models of the model in Figure 7.23(a): (a) an AR(1) interaction model and (b) an AR(1) additive model

Example 7.16. (Other two-way interaction ISCMs) Figure 7.25(a) presents an LV(2) interactions ISCM and its three alternative reduced models in Figure 7.25 (b), (c) and (d). Based on these statistical results, it could be said that the two-way

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 10:51
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Instrument list: C Y1(-1) Y2(-1) X1(-1) X2 X2(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C	-11.52103	210.3713	-0.054765	0.9568
X1	0.112433	0.599781	0.187458	0.8530
X2	-0.330128	0.810459	-0.407335	0.6877
X1*X2	-0.000536	0.003524	-0.151937	0.8806
Y1(-1)	-0.789021	11.78510	-0.066951	0.9472
Y1(-2)	1.677230	15.66671	0.107057	0.9157

R-squared	0.910164	Mean dependent var	31.71071
Adjusted R-squared	0.889746	S.D. dependent var	12.76302
S.E. of regression	4.237894	Sum squared resid	395.1144
F-statistic	48.16054	Durbin-Watson stat	0.758354
Prob(F-statistic)	0.000000	Second-Stage SSR	73.40408

(a) A Base Interaction LV (2) ISCM

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 10:58
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Instrument list: C Y1(-1) Y2(-1) X1(-1) X2 X2(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C	72.30498	83.70071	0.863851	0.3966
X1	-0.129609	0.156372	-0.828846	0.4157
X1*X2	0.000883	0.001033	0.855287	0.4012
Y1(-1)	3.965093	3.132718	1.265704	0.2183
Y1(-2)	-4.619157	4.891954	-0.944236	0.3549

R-squared	0.654770	Mean dependent var	31.71071
Adjusted R-squared	0.594730	S.D. dependent var	12.76302
S.E. of regression	8.125043	Sum squared resid	1518.375
F-statistic	16.36631	Durbin-Watson stat	0.956394
Prob(F-statistic)	0.000002	Second-Stage SSR	76.38398

(b) An Interaction Reduced ISCM

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 10:30
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Instrument list: C Y1(-1) Y2(-1) X1(-1) X2 X2(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C	27.79880	10.31164	2.695865	0.0129
X2	-0.179613	0.070524	-2.546836	0.0180
X1*X2	0.000125	4.07E-05	3.072055	0.0054
Y1(-1)	1.417180	0.393237	3.603879	0.0015
Y1(-2)	-1.255335	0.540954	-2.320595	0.0295

R-squared	0.961542	Mean dependent var	31.71071
Adjusted R-squared	0.954854	S.D. dependent var	12.76302
S.E. of regression	2.711833	Sum squared resid	169.1429
F-statistic	146.9983	Durbin-Watson stat	1.794716
Prob(F-statistic)	0.000000	Second-Stage SSR	74.03519

(c) Another Interaction Reduced Model

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 10:55
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Instrument list: C Y1(-1) Y2(-1) X1(-1) X2 X2(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C	20.38606	6.475224	3.148317	0.0045
X1	0.021319	0.005625	3.789821	0.0009
X2	-0.208436	0.064349	-3.239119	0.0036
Y1(-1)	0.998645	0.350081	2.852614	0.0090
Y1(-2)	-0.700326	0.394718	-1.774245	0.0893

R-squared	0.974651	Mean dependent var	31.71071
Adjusted R-squared	0.970243	S.D. dependent var	12.76302
S.E. of regression	2.201657	Sum squared resid	111.4877
F-statistic	223.0286	Durbin-Watson stat	2.078935
Prob(F-statistic)	0.000000	Second-Stage SSR	73.81867

(d) An. Additive Reduced Model

Figure 7.25 Statistical results based on an LV(2) interaction ISCM of *Y*1 and its three alternative reduced models interaction model in Figure 7.25(c) and the additive model in Figure 7.25(d) are the acceptable models, in a statistical sense. □

Example 7.17. (Three-way interaction ISCM) Figure 7.26(a) presents a hierarchical three-way interaction ISCM of $Y1$ with exogenous variables $X1$, $X2$ and $X3$, with the instrument list the same as the model in the previous Example 7.16. In order to operate the option 'Lagged dependent variables and regressors added to instrument list,' the model should be at least an AR(1) model. Without the indicator AR(1), the error message 'Insufficient instruments' would be as presented in Figure 7.6.

After doing experimentation, an AR(1) nonhierarchical three-way interaction ISCM is obtained, as presented in Figure 7.26(b), where the three-way interaction $X1^*X2^*X3$ has a significant negative adjusted effect on $Y1$ with a p-value = 0.0152. Note that the three-way interaction $X1^*X2^*X3$ can be used as an independent variable if and only if the three main factors or variables $X1$, $X2$ and $X3$ have a complete association or correlation. In practice, however, it is very difficult to evaluate or identify whether or not a set of three variables have a complete association. For this reason, it should be highly dependent on the statistical test. □

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 11:34
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 66 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	53.45746	50.08153	1.067409	0.2992
X1	-0.133898	0.110684	-1.209730	0.2412
X2	0.164488	0.506642	0.324663	0.7490
X3	-0.052646	0.096366	-0.546310	0.5912
X1*X2	0.000918	0.000693	1.323958	0.2012
X1*X3	0.000261	0.000164	1.589801	0.1284
X2*X3	-0.000655	0.000841	-0.778668	0.4458
X1*X2*X3	-1.42E-06	1.06E-06	-1.335549	0.1975
AR(1)	0.368268	0.831391	0.442954	0.6628

R-squared	0.958977	Mean dependent var	31.71071
Adjusted R-squared	0.941704	S.D. dependent var	12.76302
S.E. of regression	3.081590	Sum squared resid	180.4277
F-statistic	57.00385	Durbin-Watson stat	1.859004
Prob(F-statistic)	0.000000	Second-Stage SSR	67.60201

Inverted AR Roots	.37

(a) AR(1) Hierarchical ISCM

Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/01/08 Time: 12:49
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 96 iterations
Instrument list: C Y1(-1) Y1(-2) X1(-1) X2 X2(-1)
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	43.95652	5.443153	8.075562	0.0000
X2	-0.502229	0.099472	-5.048965	0.0000
X1*X2	0.000339	5.66E-05	5.997551	0.0000
X1*X3	5.09E-05	2.94E-05	1.731823	0.0973
X1*X2*X3	-4.41E-07	1.67E-07	-2.632928	0.0152
AR(1)	0.463454	0.265021	1.748743	0.0943

R-squared	0.965225	Mean dependent var	31.71071
Adjusted R-squared	0.957321	S.D. dependent var	12.76302
S.E. of regression	2.636692	Sum squared resid	152.9472
F-statistic	123.3346	Durbin-Watson stat	1.987248
Prob(F-statistic)	0.000000	Second-Stage SSR	110.9562

Inverted AR Roots	.46

(b) AR(1) Nonhierarchical ISCM

Figure 7.26 Statistical results based on (a) an AR(1) hierarchical three-way ISCM of $Y1$ on ($X1$, $X2$, $X3$) and (b) its reduced ISCM

7.7.1 Special notes and comments

In fact, many other alternative instrumental two-way or three-way interaction models could be developed or defined based on only three variables $X1$, $X2$ and $Y1$, or on models based on five variables $X1$, $X2$, $X3$, $Y1$ and $Y2$. In addition to these variables using the time t-variable could also be considered.

Since their lags could be used, with or without AR indicator(s), as well as many of the alternative sets of instrument variables demonstrated in Table 7.1, then it is possible to have countless infinite alternative instrumental models based on only a set of three or five variables. By having more variables, many more problems would be faced in defining a model, either with or without instrumental variables, since the use of selected two-way or higher interaction exogenous variables in the model would need to be considered. Three-way or higher interaction should be used in a model if there is confidence that there is a complete association.

In practice, however, it is very difficult or almost impossible to identify a complete association between three or more variables. For this reason, there should be great dependence on the statistical tests, as presented in Example 7.17 and other examples in the previous chapters.

It has been found that many alternative exogenous variables have to be tried, as well as the sets of instrumental variables, either with or without AR indicator(s), in order to obtain one or two acceptable models. For this reason, it could be said that some of the findings can be unpredictable or unexpected models, since the impact of a set of instrument variables cannot be predicted, as well as the impact of multicollinearity between the exogenous variables (see Section 2.14.2). In other words, an acceptable or a good model is in fact the result of experimentation by using the trial-and-error methods.

7.8 Multivariate instrumental models based on the US_DPOC

7.8.1 Simple multivariate instrumental models

In this subsection simple multivariate instrumental models are presented, such as the bivariate instrumental models, which are associated with the Cobb–Douglas (CD) and constant elasticity of substitution (CES) models.

Example 7.18. (Bivariate translog linear instrumental models) Figure 7.27 presents statistical results based on a bivariate translog linear instrumental model, using the following AR(1) model as a base model:

$$\begin{aligned} \log(y1) &= c(11) + c(12)^{*}\log(x1) + [ar(1) = c(13)] \\ \log(y2) &= c(21) + c(22)^{*}\log(x1) + [ar(1) = c(23)] \end{aligned} \tag{7.7}$$

For illustration purposes, alternative instrumental variables are presented as follows:

(a) Instrument C, with the option 'Lagged dependent variable and regressors added to instrument list.'
(b) Instrument C log($x2$), with the option 'Lagged dependent variable . . . .'

Therefore, the data analysis will use the first lags of the dependent and regressors of each regression as additional instrumental variables. Find their outputs in Figure 7.27.

Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 06:04
Sample: 1952 1980
Included observations: 30
Total system (balanced) observations 58
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-7.185591	39.58661	-0.181516	0.8567
C(12)	1.247570	2.284177	0.546179	0.5873
C(13)	0.980528	0.140982	6.954997	0.0000
C(21)	-0.793485	0.433365	-1.830988	0.0728
C(22)	0.677740	0.063946	10.59864	0.0000
C(23)	0.424215	0.179098	2.368620	0.0216
Determinant residual covariance		0.000100		

Equation: LOG(Y1)=C(11)+C(12)*LOG(X1)+[AR(1)=C(13)]
Instruments: C LOG(Y1(-1)) LOG(X1(-1))
Observations: 29

R-squared	0.924651	Mean dependent var	3.382868
Adjusted R-squared	0.918855	S.D. dependent var	0.330119
S.E. of regression	0.094038	Sum squared resid	0.229921
Durbin-Watson stat	1.570922		

Equation: LOG(Y2)=C(21)+C(22)*LOG(X1)+[AR(1)=C(23)]
Instruments: C LOG(Y2(-1)) LOG(X1(-1))
Observations: 29

R-squared	0.926333	Mean dependent var	3.743045
Adjusted R-squared	0.920666	S.D. dependent var	0.438383
S.E. of regression	0.123476	Sum squared resid	0.396405
Durbin-Watson stat	1.612872		

(a)

Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 06:07
Sample: 1952 1980
Included observations: 30
Total system (balanced) observations 58
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	14.46752	5.952176	2.430626	0.0186
C(12)	-2.334882	1.249823	-1.868171	0.0674
C(13)	1.048947	0.013931	75.29653	0.0000
C(21)	-0.795756	0.433276	-1.836603	0.0720
C(22)	0.678076	0.063933	10.60605	0.0000
C(23)	0.424288	0.179103	2.368960	0.0216
Determinant residual covariance		0.000118		

Equation: LOG(Y1)=C(11)+C(12)*LOG(X1)+[AR(1)=C(13)]
Instruments: C LOG(X2) LOG(Y1(-1)) LOG(X1(-1))
Observations: 29

R-squared	0.903800	Mean dependent var	3.382868
Adjusted R-squared	0.896400	S.D. dependent var	0.330119
S.E. of regression	0.106255	Sum squared resid	0.293546
Durbin-Watson stat	2.010534		

Equation: LOG(Y2)=C(21)+C(22)*LOG(X1)+[AR(1)=C(23)]
Instruments: C LOG(X2) LOG(Y2(-1)) LOG(X1(-1))
Observations: 29

R-squared	0.926335	Mean dependent var	3.743045
Adjusted R-squared	0.920668	S.D. dependent var	0.438383
S.E. of regression	0.123475	Sum squared resid	0.396397
Durbin-Watson stat	1.612891		

(b)

Figure 7.27 Statistical results based on the model in (7.7) with (a) instrument C and (b) instrument C log(x2), with the option 'lagged ....'

Furthermore, note that the output in Figure 7.27(a) can also be obtained by using the following equation specification, without the option 'Lagged dependent ...:'

$$\begin{aligned} \log(y1) &= c(11) + c(12)^*\log(x1) + [ar(1) = c(13)] \ @\, c \log(y1(-1) \log(x1(-1)) \\ \log(y2) &= c(21) + c(22)^*\log(x1) + [ar(1) = c(23)] \ @\, c \log(y2(-1) \log(x1(-1)) \end{aligned} \tag{7.8}$$

This equation specification can easily be modified by using various sets of instrumental variables. For illustration purposes, Figure 7.28 presents statistical results by using the following equation specifications:

$$\begin{aligned} \log(y1) &= c(11) + c(12)^*\log(x1) + [ar(1) = c(13)] \ @\, c \log(y1(-1) \log(x2) \\ \log(y2) &= c(21) + c(22)^*\log(x1) + [ar(1) = c(23)] \ @\, c \log(y2(-1) \log(x3) \end{aligned} \tag{7.9}$$

with and without the option 'Lagged ....' However, by not using the option, the error message 'Near singular matrix' is obtained. For this reason, the instruments are modified, giving the instrumental model in Figure 7.28(b), p. 408. □

Example 7.19. (Another bivariate translog linear instrumental Model) Figure 7.29(a) presents statistical results based on a bivariate translog linear instrumental model having two exogenous variables, with the following equation

Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 06:46
Sample: 1952 1980
Included observations: 30
Total system (balanced) observations 58
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	14.46751	5.952172	2.430627	0.0186
C(12)	-2.334881	1.249822	-1.868170	0.0674
C(13)	1.048947	0.013931	75.29724	0.0000
C(21)	-0.813453	0.434292	-1.873054	0.0667
C(22)	0.680699	0.064085	10.62180	0.0000
C(23)	0.424962	0.179156	2.372021	0.0214

Determinant residual covariance 0.000118

Equation: LOG(Y1)=C(11)+C(12)*LOG(X1)+[AR(1)=C(13)]
Instruments: C LOG(Y1(-1)) LOG(X2) LOG(X1(-1))
Observations: 29

R-squared	0.903800	Mean dependent var	3.382868
Adjusted R-squared	0.896400	S.D. dependent var	0.330119
S.E. of regression	0.106255	Sum squared resid	0.293546
Durbin-Watson stat	2.010534		

Equation: LOG(Y2)=C(21)+C(22)*LOG(X1)+[AR(1)=C(23)]
Instruments: C LOG(Y2(-1)) LOG(X3) LOG(X1(-1))
Observations: 29

R-squared	0.926341	Mean dependent var	3.743045
Adjusted R-squared	0.920675	S.D. dependent var	0.438383
S.E. of regression	0.123469	Sum squared resid	0.396362
Durbin-Watson stat	1.613026		

(a)

Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 06:53
Sample: 1952 1980
Included observations: 30
Total system (balanced) observations 58
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	8.190707	3.930490	2.083890	0.0421
C(12)	-1.042386	0.800465	-1.302226	0.1986
C(13)	1.058422	0.021525	49.17279	0.0000
C(21)	-0.985920	0.276355	-3.567580	0.0008
C(22)	0.705957	0.041143	17.15874	0.0000
C(23)	-0.108746	0.308232	-0.352807	0.7257

Determinant residual covariance 9.62E-05

Equation: LOG(Y1)=C(11)+C(12)*LOG(X1)+[AR(1)=C(13)]
Instruments: C LOG(Y1(-1)) LOG(X2) LOG(X2(-1))
Observations: 29

R-squared	0.943572	Mean dependent var	3.382868
Adjusted R-squared	0.939231	S.D. dependent var	0.330119
S.E. of regression	0.081379	Sum squared resid	0.172186
Durbin-Watson stat	1.570093		

Equation: LOG(Y2)=C(21)+C(22)*LOG(X1)+[AR(1)=C(23)]
Instruments: C LOG(Y2(-1)) LOG(X3)
Observations: 29

R-squared	0.898379	Mean dependent var	3.743045
Adjusted R-squared	0.890562	S.D. dependent var	0.438383
S.E. of regression	0.145023	Sum squared resid	0.546825
Durbin-Watson stat	1.030269		

(b)

Figure 7.28 Statistical results based on (a) the model in (7.9) with the option 'lagged . . .' and (b) modified instruments without the option

Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 07:04
Sample: 1952 1980
Included observations: 30
Total system (balanced) observations 58
Convergence achieved after 4 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2.002853	0.546647	3.663887	0.0006
C(12)	1.589665	0.233135	6.818651	0.0000
C(13)	-2.059922	0.432997	-4.757360	0.0000
C(14)	0.483828	0.132365	3.655265	0.0006
C(21)	-0.869641	0.731197	-1.189340	0.2399
C(22)	0.633607	0.339216	1.867855	0.0676
C(23)	0.082744	0.624663	0.132461	0.8952
C(24)	0.430569	0.192103	2.241345	0.0295

Determinant residual covariance 8.07E-05

Equation: LOG(Y1)=C(11)+C(12)*LOG(X1)+C(13)*LOG(X2)+[AR(1)=C(14)]
Instruments: C LOG(Y1(-1)) LOG(X1(-1)) LOG(X2(-1))
Observations: 29

R-squared	0.940737	Mean dependent var	3.382868
Adjusted R-squared	0.933626	S.D. dependent var	0.330119
S.E. of regression	0.085049	Sum squared resid	0.180834
Durbin-Watson stat	2.096057		

Equation: LOG(Y2)=C(21)+C(22)*LOG(X1)+C(23)*LOG(X2)+[AR(1)=C(24)]
Instruments: C LOG(Y2(-1)) LOG(X1(-1)) LOG(X2(-1))
Observations: 29

R-squared	0.926231	Mean dependent var	3.743045
Adjusted R-squared	0.917379	S.D. dependent var	0.438383
S.E. of regression	0.126008	Sum squared resid	0.396952
Durbin-Watson stat	1.598201		

(a)

Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 07:08
Sample: 1952 1980
Included observations: 30
Total system (balanced) observations 58
Convergence achieved after 3 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	2.002853	0.546647	3.663884	0.0006
C(12)	1.589665	0.233136	6.818615	0.0000
C(13)	-2.059922	0.432999	-4.757337	0.0000
C(14)	0.483828	0.132365	3.655244	0.0006
C(21)	-0.793485	0.433365	-1.830988	0.0729
C(22)	0.677740	0.063946	10.59864	0.0000
C(24)	0.424215	0.179098	2.368620	0.0217

Determinant residual covariance 7.99E-05

Equation: LOG(Y1)=C(11)+C(12)*LOG(X1)+C(13)*LOG(X2)+[AR(1)=C(14)]
Instruments: C LOG(Y1(-1)) LOG(X1(-1)) LOG(X2(-1))
Observations: 29

R-squared	0.940737	Mean dependent var	3.382868
Adjusted R-squared	0.933626	S.D. dependent var	0.330119
S.E. of regression	0.085049	Sum squared resid	0.180834
Durbin-Watson stat	2.096057		

Equation: LOG(Y2)=C(21)+C(22)*LOG(X1)+[AR(1)=C(24)]
Instruments: C LOG(Y2(-1)) LOG(X1(-1))
Observations: 29

R-squared	0.926333	Mean dependent var	3.743045
Adjusted R-squared	0.920666	S.D. dependent var	0.438383
S.E. of regression	0.123476	Sum squared resid	0.396405
Durbin-Watson stat	1.612872		

(b)

Figure 7.29 Statistical results based on (a) the instrumental model in (7.10) and (b) its reduced model specification:

$$\begin{aligned}&\log(y1) = c(11) + c(12)*\log(x1) + c(13)*\log(x2) + [ar(1) = c(14)]\\ &\log(y2) = c(21) + c(22)*\log(x1) + c(13)*\log(x2) + [ar(1) = c(23)]\\ &\quad \textit{Instrument } C\end{aligned} \qquad (7.10)$$

Since $\log(x2)$ has an insignificant effect on $\log(y2)$, then a reduced model is obtained, as presented in Figure 7.29(b).

For an extension of this instrumental model, a CES model and its modification, presented in Chapter 2, can be considered as the base model. Furthermore, using various sets of instrumental variables,many translog instrumental models could be obtained. Do this as an exercise. □

7.8.2 Multivariate instrumental models

Corresponding to the path diagram in Figure 2.89, which has been modified to the path diagram in Figure 6.31 for the VAR model, it would be good to study a causal relationship or effects between the five variables $X1$, $X2$, $X3$, $Y1$ and $Y2$, by using instrumental variables, without using the time t. For illustration purposes, the path diagram presented in Figure 7.30 should be considered.

Corresponding to this path diagram, alternative multivariate models could be defined, such as additive, two-way and three-way interaction models, which have been presented in the previous chapters. By using instrumental variables, various additive, two-way or three-way interaction multivariate instrumental seemingly causal models (ISCMs) could be obtained.

Example 7.20. (Additive multivariate ISCMs) Corresponding to the path diagram in Figure 7.30, the following AR(1) additive ISCM is presented, with an instrument list '$y1(-1)$ $y2(-1)$ $x1(-1)$ $x2(-1)$ $x3(-1)$:'

$$\begin{aligned}y1 &= c(11)+c(12)*y1(-1)+c(13)*y2(-1)+c(14)*x1+c(15)*x2+[ar(1)=c(16)]\\ y2 &= c(21)+c(22)*y1(-1)+c(23)*y2(-1)+c(24)*x1+[ar(1)=c(25)]\\ x1 &= c(31)+c(32)*x2+c(33)*x3+[ar(1)=c(34)]\\ x2 &= c(41)+c(44)*x3+[ar(1)=c(43)]\\ &\qquad\qquad \textit{Instrument } y1(-1)y2(-1)x1(-1)x2(-1)x3(-1)\end{aligned} \qquad (7.11)$$

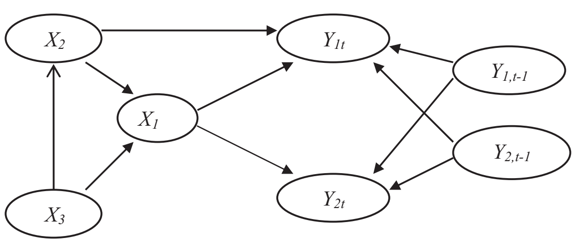


Figure 7.30 A hypothetical path diagram of seemingly causal models

System: SYS01
Estimation Method: Iterative Two-Stage Least Squares
Date: 01/01/08 Time: 17:42
Sample: 1952 1980
Included observations: 30
Total system (unbalanced) observations 114
Convergence not achieved after 500 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	39.14892	23.14004	1.691826	0.0939
C(12)	-0.178586	0.586398	-0.304548	0.7614
C(13)	0.133982	0.078387	1.709230	0.0906
C(14)	0.038008	0.019080	1.992007	0.0492
C(15)	-0.478132	0.265582	-1.800317	0.0750
C(16)	0.488319	0.231352	2.110718	0.0374
C(21)	30.61224	12.73157	2.404436	0.0181
C(22)	-1.006414	0.556039	-1.809969	0.0734
C(23)	0.028308	0.324356	0.087274	0.9306
C(24)	0.045504	0.016344	2.784090	0.0065
C(25)	0.555365	0.515451	1.077436	0.2840
C(31)	-59.77568	276.4176	-0.216251	0.8293
C(32)	7.887895	8.046817	0.980250	0.3294
C(33)	-0.035113	0.267830	-0.131102	0.8960
C(34)	1.163865	0.063132	18.43555	0.0000
C(41)	267.5679	416.3707	0.642619	0.5220
C(44)	0.029417	0.008350	3.523123	0.0007
C(43)	0.985289	0.031717	31.06498	0.0000
Determinant residual covariance		5574227.		

Equation: Y1=C(11)+C(12)*Y1(-1)+C(13)*Y2(-1)+C(14)*X1+C(15)*X2 +[AR(1)=C(16)]
Instruments: Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1) C Y1(-2) Y2(-2)
Observations: 28

R-squared	0.958687	Mean dependent var	31.71071
Adjusted R-squared	0.949297	S.D. dependent var	12.76302
S.E. of regression	2.873887	Sum squared resid	181.7030
Durbin-Watson stat	2.053331		

Equation: Y2=C(21)+C(22)*Y1(-1)+C(23)*Y2(-1)+C(24)*X1+[AR(1)=C(25)]
Instruments: Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1) C Y1(-2) Y2(-2)
Observations: 28

R-squared	0.941025	Mean dependent var	47.32179
Adjusted R-squared	0.930769	S.D. dependent var	21.53564
S.E. of regression	5.666429	Sum squared resid	738.4936
Durbin-Watson stat	1.768560		

Equation: X1=C(31) + C(32)*X2+C(33)*X3 + [AR(1)=C(34)]
Instruments: Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1) C
Observations: 29

R-squared	0.988284	Mean dependent var	985.4655
Adjusted R-squared	0.986878	S.D. dependent var	666.2947
S.E. of regression	76.32587	Sum squared resid	145640.9
Durbin-Watson stat	2.313827		

Equation: X2=C(41)+C(44)*X3+[AR(1)=C(43)]
Instruments: Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1) C
Observations: 29

R-squared	0.983682	Mean dependent var	94.90000
Adjusted R-squared	0.982427	S.D. dependent var	32.78583
S.E. of regression	4.346253	Sum squared resid	491.1379
Durbin-Watson stat	1.620852		

Figure 7.31 Statistical results based on the additive ISCM in (7.11)

The data analysis can be done by selecting *Object/New Object .../System ... OK*; then insert or copy (7.11) to the equation specification window. However, if an equation specification needs to be copied to the system, then the equation should be produced or typed by using Microsoft Word, instead of the Object/ Microsoft Equation 3.0.

Then by selecting the TSLS estimation method and clicking *OK*, the statistical results in Figure 7.31 are obtained. By using the AR(1) model and the option 'Lagged dependent variable ...,' then additional instrumental variables will be obtained for each regression.

It has been found that by using the option 'Lagged dependent variable ...,' any set of instrument list can be used to replaced the instrument list of the model in (7.11), which has been demonstrated in Example 7.18 based on the simplest multivariate instrumental model using only '*C*' or '*C* log(x3)' in the instrument list.

In this example, in fact, it was found that by using only 'C' as an instrument for the model in (7.11) with the option 'Lagged ...,' the statistical results could also be obtained, but almost all of the independent variables, as well as the indicator AR(1), are insignificant. Based on this experimentation, it is certain that various or any sets of instrumental variables could be used in the instrument list if the option 'Lagged ...' is used. Do this as an exercise using your own data set, with 'C' or one external variable in the instrumental list with the option 'Lagged ....' However, in some cases, the error messages 'Near singular matrix' may be obtained.

The problem is that the true or the best instrumental variables are never known for a particular basic model. For further illustration purposes, the following

System: SYS02
Estimation Method: Iterative Two-Stage Least Squares
Date: 01/01/08 Time: 17:38
Sample: 1952 1980
Included observations: 30
Total system (unbalanced) observations 114
Convergence not achieved after 500 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	31.79040	22.29963	1.425602	0.1572
C(12)	0.002531	0.583729	0.004336	0.9965
C(13)	0.142780	0.120254	1.187318	0.2380
C(14)	0.031647	0.019597	1.614920	0.1096
C(15)	-0.396434	0.242647	-1.633792	0.1056
C(16)	0.436052	0.258513	1.686766	0.0949
C(21)	-135.1372	517.2904	-0.261240	0.7945
C(22)	1.895037	11.27539	0.168068	0.8669
C(23)	1.538365	3.703237	0.415411	0.6788
C(24)	0.340138	0.751392	0.452677	0.6518
C(25)	1.125838	0.030420	37.01024	0.0000
C(31)	-32.21642	289.0040	-0.111474	0.9115
C(32)	8.020626	9.082817	0.883055	0.3794
C(33)	-0.109416	0.290675	-0.376419	0.7074
C(34)	1.159565	0.062759	18.47644	0.0000
C(41)	195.5147	212.9405	0.918166	0.3608
C(44)	0.036944	0.011171	3.306998	0.0013
C(43)	0.978471	0.036160	27.05913	0.0000

Determinant residual covariance 47710927

Equation: Y1=C(11)+C(12)*Y1(-1)+C(13)*Y2(-1)+C(14)*X1+C(15)*X2
+[AR(1)=C(16)]
Instruments: C Y1(-1) Y1(-2) Y2(-2) X1(-1) X2(-1)
Observations: 28

R-squared	0.968863	Mean dependent var	31.71071
Adjusted R-squared	0.961787	S.D. dependent var	12.76302
S.E. of regression	2.494949	Sum squared resid	136.9450
Durbin-Watson stat	2.048809		

Equation: Y2=C(21)+C(22)*Y1(-1)+C(23)*Y2(-1)+C(24)*X1+[AR(1)=C(25)]
Instruments: C X2(-1) Y2(-1) Y1(-2) Y2(-2) X1(-1)
Observations: 28

R-squared	0.125226	Mean dependent var	47.32179
Adjusted R-squared	-0.026909	S.D. dependent var	21.53564
S.E. of regression	21.82347	Sum squared resid	10954.07
Durbin-Watson stat	2.473017		

Equation: X1=C(31) + C(32)*X2+C(33)*X3 + [AR(1)=C(34)]
Instruments: C Y1(-1) X1(-1) X2(-1) X3(-1)
Observations: 29

R-squared	0.987752	Mean dependent var	985.4655
Adjusted R-squared	0.986283	S.D. dependent var	666.2947
S.E. of regression	78.03706	Sum squared resid	152244.6
Durbin-Watson stat	2.313337		

Equation: X2=C(41)+C(44)*X3+[AR(1)=C(43)]
Instruments: C Y1(-1) X2(-1) X3(-1)
Observations: 29

R-squared	0.982148	Mean dependent var	94.90000
Adjusted R-squared	0.980775	S.D. dependent var	32.78583
S.E. of regression	4.545861	Sum squared resid	537.2861
Durbin-Watson stat	1.693860		

Figure 7.32 Statistical results based on the additive ISCM in (7.12)

model presents a multivariate AR(1) model, where each of the regressions has one or two instrumental variables:

$$
\begin{aligned}
y1 &= c(11) + c(12)^{*}y1(-1) + c(13)^{*}y2(-1) + c(14)^{*}x1 + c(15)^{*}x2 \\
&\quad + [ar(1) = c(16)]\ @\, c \\
y2 &= c(21) + c(22)^{*}y1(-1) + c(23)^{*}y2(-1) + c(24)^{*}x1 \\
&\quad + [ar(1) = c(25)]\ @\, c\, x2(-1) \\
x1 &= c(31) + c(32)^{*}x2 + c(33)^{*}x3 + [ar(1) = c(34)]\ @\, c\, y1(-1) \\
x2 &= c(41) + c(44)^{*}x3 + [ar(1) = c(43)]\ @\, c\, y1(-1)
\end{aligned}
\tag{7.12}
$$

By using the option 'Lagged dependent variables ...,' the statistical results in Figure 7.32 are obtained. However, without the option the error message 'Insufficient instrument' would appear. It has been found that it is not easy to select the sets of instrument lists if the option is not being used. In many cases several error messages have been obtained, either 'Near singular matrix' or 'Insufficient instrument' or 'Convergence not achieved after 500 or 1000 interactions.'

Since some of the independent variables are insignificant with large p-values, alternative reduced models may be produced, as presented in the previous examples. Do this as an exercise. □

Example 7.21. (An AR(1) two-way interaction model with instruments) Corresponding to the path diagram in Figure 7.30, the following full or complete multivariate AR(1) two-way interaction model with instruments can be defined; its statistical results are presented in Figure 7.33.

System: UNTITLED
Estimation Method: Iterative Two-Stage Least Squares
Date: 01/01/08 Time: 19:33
Sample: 1952 1980
Included observations: 30
Total system (unbalanced) observations 114
Convergence achieved after 39 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-4.523561	14.49792	-0.312014	0.7558
C(12)	1.456292	0.685944	2.123047	0.0365
C(13)	0.028799	0.132914	0.216677	0.8290
C(14)	0.038789	0.034533	1.123258	0.2643
C(15)	-0.380865	0.248988	-1.529650	0.1296
C(16)	-0.000285	0.000276	-1.034701	0.3036
C(17)	-8.57E-06	2.24E-05	-0.383299	0.7024
C(18)	0.000519	0.000461	1.125020	0.2636
C(19)	-0.139392	0.342863	-0.406553	0.6853
C(21)	72.19178	34.32242	2.103342	0.0382
C(22)	-1.221654	0.565067	-2.161962	0.0333
C(23)	0.781275	0.458437	1.704213	0.0918
C(24)	-0.140332	0.128898	-1.088709	0.2792
C(25)	0.001034	0.000912	1.133659	0.2599
C(26)	-2.90E-06	3.27E-05	-0.088781	0.9295
C(27)	0.345308	0.256315	1.347204	0.1813
C(31)	1197.631	265.4644	4.511454	0.0000
C(32)	-2.359180	3.649352	-0.646465	0.5196
C(33)	-5.091636	1.022799	-4.978140	0.0000
C(34)	0.046766	0.006317	7.402887	0.0000
C(35)	0.022610	0.208246	0.108576	0.9138
C(41)	267.5679	416.3707	0.642619	0.5221
C(42)	0.029417	0.008350	3.523123	0.0007
C(43)	0.985289	0.031717	31.06498	0.0000
Determinant residual covariance		29069799		

Equation: Y1=C(11)+C(12)*Y1(-1)+C(13)*Y2(-1)+C(14)*X1+C(15)*X2+C(16)*X1*X2+C(17)*X1*X3+C(18)*X2*X3+[AR(1)=C(19)]
Instruments: C Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1) Y1(-2) Y2(-2) X1(-1)*X2(-1) X1(-1)*X3(-1) X2(-1)*X3(-1)
Observations: 28

R-squared	0.959939	Mean dependent var	31.71071
Adjusted R-squared	0.943071	S.D. dependent var	12.76302
S.E. of regression	3.045238	Sum squared resid	176.1960
Durbin-Watson stat	2.125519		

Equation: Y2=C(21)+C(22)*Y1(-1)+C(23)*Y2(-1)+C(24)*X1+C(25)*X1*X2+C(26)*X1*X3+[AR(1)=C(27)]
Instruments: C Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1) Y1(-2) Y2(-2) X1(-1)*X2(-1) X1(-1)*X3(-1)
Observations: 28

R-squared	0.931061	Mean dependent var	47.32179
Adjusted R-squared	0.911365	S.D. dependent var	21.53564
S.E. of regression	6.411522	Sum squared resid	863.2599
Durbin-Watson stat	1.862388		

Equation: X1=C(31) + C(32)*X2+C(33)*X3 +C(34)*X2*X3+ [AR(1)=C(35)]
Instruments: C Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1) X2(-1)*X3(-1)
Observations: 29

R-squared	0.958551	Mean dependent var	985.4655
Adjusted R-squared	0.951643	S.D. dependent var	666.2947
S.E. of regression	146.5202	Sum squared resid	515235.8
Durbin-Watson stat	1.971407		

Equation: X2=C(41)+C(42)*X3+[AR(1)=C(43)]
Instruments: C Y1(-1) Y2(-1) X1(-1) X2(-1) X3(-1)
Observations: 29

R-squared	0.983682	Mean dependent var	94.90000
Adjusted R-squared	0.982427	S.D. dependent var	32.78583
S.E. of regression	4.346253	Sum squared resid	491.1379
Durbin-Watson stat	1.620852		

Figure 7.33 Statistical results based on a multivariate AR(1) two-way interaction model with instruments

$$\begin{aligned}
y1 &= c(11)+c(12)^*y1(-1)+c(13)^*y2(-1)+c(14)^*x1+c(15)^*x2+c(16)^*x1^*x2 \\
&\quad +c(17)^*x1^*x3+c(18)^*x2^*x3+[ar(1)=c(19)] \\
y2 &= c(21)+c(22)^*y1(-1)+c(23)^*y2(-1)+c(24)^*x1+c(25)^*x1^*x2 \\
&\quad +c(26)^*x1^*x3+[ar(1)=c(27)] \\
x1 &= c(31)+c(32)^*x2+c(33)^*x3+c(34)^*x2^*x3+[ar(1)=c(35)] \\
x2 &= c(41)+c(42)^*x3+[ar(1)=c(43)] \\
&\qquad \mathit{Instrument}\ \ c\ \ y1(-1)\,y2(-1)\,x1(-1)\,x2(-1)\,x3(-1)
\end{aligned} \tag{7.13}$$

This model has the following characteristics:

(1) The interactions $X1^*X2$ and $X1^*X3$ in the first regression indicate that the effect of $X1$ on $Y1$ is dependent on $X2$ and $X3$. In other words, $X2$ and $X3$ have indirect effects on $Y1$, through $X1$. Similarly, the interaction X2*X3 indicates that $X3$ also has an indirect effect on $Y1$, through $X2$.
(2) The interactions $X1^*X2$ and $X1^*X3$ in the second regression indicate that the effect of $X1$ on $Y2$ is dependent on $X2$ and $X3$.
(3) The interactions $X2^*X3$ in the third regression indicate that the effect of $X2$ on $X1$ is dependent on $X3$.
(4) Since many independent variables are insignificant, alternative reduced models could be developed, which can easily be done. Do this as an exercise. □

System: SYS07
Estimation Method: Iterative Two-Stage Least Squares
Date: 01/01/08 Time: 20:38
Sample: 1952 1979
Included observations: 29
Total system (unbalanced) observations 105
Convergence achieved after 19 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	23.35394	3.229516	7.231405	0.0000
C(12)	-0.729402	0.281978	-2.586738	0.0113
C(13)	-0.287148	0.204454	-1.404461	0.1637
C(14)	0.033399	0.005771	5.787073	0.0000
C(16)	0.127420	0.226704	0.562055	0.5755
C(21)	3.930634	9.931319	0.395782	0.6932
C(22)	0.010081	0.012271	0.821499	0.4136
C(23)	0.326517	0.209013	1.562186	0.1218
C(24)	0.476480	0.233954	2.036644	0.0447
C(31)	787.4453	2323.082	0.338966	0.7354
C(32)	-47.33921	103.2505	-0.458489	0.6477
C(33)	9.858541	16.16005	0.610056	0.5434
C(34)	0.196096	0.226304	0.866517	0.3885
C(41)	-4451.967	5263.117	-0.845880	0.3999
C(42)	26.32573	7.671634	3.431567	0.0009
C(43)	0.974800	0.042010	23.20373	0.0000
Determinant residual covariance		5.41E+11		

Equation: Y1=C(11)+C(12)*T +C(13)*Y2+C(14)*X1+[AR(2)=C(15)]
Instruments: C(1) Y1(-1) Y2(-1) X1(-1) Y1(-2) T(-2) Y2(-2) X1(-2)
Observations: 27

R-squared	0.902892	Mean dependent var	30.26037
Adjusted R-squared	0.885235	S.D. dependent var	10.39215
S.E. of regression	3.520538	Sum squared resid	272.6722
Durbin-Watson stat	1.065166		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+[AR(1)=C(24)]
Instruments: C(2) Y2(-1) X1(-1) X2 X2(-1)
Observations: 27

R-squared	0.897693	Mean dependent var	42.50370
Adjusted R-squared	0.884348	S.D. dependent var	16.40678
S.E. of regression	5.579556	Sum squared resid	716.0234
Durbin-Watson stat	1.675102		

Equation: X1=C(31) +C(32)*X2+ C(33)*X3 + [AR(1)=C(34)]
Instruments: C(3) X1(-1) X2(-1) X3(-1)
Observations: 26

R-squared	-2.404709	Mean dependent var	814.9615
Adjusted R-squared	-2.868988	S.D. dependent var	445.1957
S.E. of regression	875.6884	Sum squared resid	16870265
Durbin-Watson stat	1.904800		

Equation: X3=C(41) + C(42)*X2+[AR(1)=C(43)]
Instruments: C(4) X2(-1) X3(-1)
Observations: 25

R-squared	0.703917	Mean dependent var	416.7240
Adjusted R-squared	0.677000	S.D. dependent var	193.2920
S.E. of regression	109.8538	Sum squared resid	265492.8
Durbin-Watson stat	2.014795		

Figure 7.34 Statistical results based on an MAR growth model with instruments

Example 7.22. (An extension of the additive growth model in (2.83)) Corresponding to the additive growth model in (2.83), an instrumental model needs to be found using this model as a base model. Therefore, the following equation specification has been tried as the first trial model:

$$\begin{aligned} &y1 = c(11) + c(12)*t + c(13)*y2 + c(14)*x1 \ @\, c(1)\, y1(-1)\, y2(-1)\, x1(-1)\, t \\ &y2 = c(21) + c(22)*x1 + c(23)*x2 \ @\, c(2)\, y2(-1)\, x1(-1)\, x2\, x2(-1) \\ &x1 = c(31) + c(32)*x2 + c(33)*x3 \ @\, c(3)\, x1(-1)\, x2(-1)\, x3(-1) \\ &x3 = c(41) + c(42)*x2 \ @\, c(4)\, x2(-1)\, x3(-1) \end{aligned} \tag{7.14}$$

Note that each of the regressions is a basic multiple regression. Since time series data are being used, these models would have small values of the DW-statistic (the statistical results are not presented). Therefore, multivariate autoregressive models and the same sets of instrumental variables are used, finally giving the unexpected AR model in Figure 7.34, since the first regression uses the indicator AR(2), which is insignificant, instead of AR(1). The reasons for this are as follows:

(1) By using the indicator AR(1) in the first regression, an output would be obtained with the statement 'Convergence not achieved after 500 iterations.'
(2) By using both indicators AR(1) and AR(2), an output would be obtained where all independent variables of the first regression are insignificant.
(3) Since the output presents so many insignificant independent variables, it is suggested that the model should be modified, as well as using other sets of instrumental variables. Do this as an exercise.

(4) On the other hand, you may try to use 'C' or one external variable in the instrument list with the option 'Lagged ....' □

Example 7.23. (An extension of the interaction growth model in (2.84)) The two-way interaction growth model in (2.84) could be used as a base model and alternative multivariate autoregressive models could be applied directly. Finally, an acceptable autoregressive instrumental model is obtained as follows:

$$
\begin{aligned}
y1 &= c(11) + c(12)^{*}t + c(13)^{*}y2 + c(14)^{*}x1 + c(15)^{*}y2^{*}x1 + [ar(2) = c(16)] \\
&\quad @\ c(1)\ y1(-1)y2(-1)\ x1(-1)t \\
y2 &= c(21) + c(22)^{*}x1 + c(23)^{*}x2 + c(24)x1^{*}x2 + [ar(1) = c(25)] \\
&\quad @\ c(2)\ y2(-1)\ x1(-1)\ x2\ x2(-1)) \\
x1 &= c(31) + c(32)^{*}x2 + c(33)^{*}x3 + c(34)^{*}x2^{*}x3 + [ar(1) = c(35)] \\
&\quad @\ c(23)\ x1(-1)\ x2(-1)\ x3(-1) \\
x3 &= c(41) + c(42)^{*}x2 + [ar(1) = c(43)]\ @\ c(4)x1(-1)\ x3(-1)
\end{aligned}
\tag{7.15}
$$

However, the statistical results of this model are not presented. Only the statistical results based on an autoregressive instrumental model in Figure 7.35 are presented. Corresponding to this model, the following notes are given:

(1) In this experimentation, several statistical results are obtained, but the convergence was not achieved after 1000 or 500 iterations, as presented in Figure 7.35. On the other hand, in some cases the '*Near singular matrix*'

Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 08:15
Sample: 1952 1979
Included observations: 29
Total system (unbalanced) observations 105
Convergence not achieved after 500 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	7.437329	8.541823	0.870696	0.3863
C(12)	-2.600686	1.353779	-1.921057	0.0580
C(13)	0.333196	0.370742	0.898728	0.3713
C(14)	0.077642	0.028191	2.754189	0.0072
C(15)	-0.000440	0.000251	-1.753159	0.0831
C(16)	0.425771	0.247087	1.723163	0.0885
C(21)	-3.762096	16.90479	-0.222546	0.8244
C(22)	0.048038	0.047302	1.015568	0.3127
C(23)	0.263817	0.245070	1.076495	0.2847
C(24)	-0.000209	0.000264	-0.792905	0.4300
C(25)	0.354770	0.279302	1.270200	0.2074
C(31)	1092.943	1333.487	0.819613	0.4147
C(32)	-22.78942	77.76863	-0.293041	0.7702
C(33)	1.189495	24.63281	0.048289	0.9616
C(34)	0.028646	0.080641	0.355232	0.7233
C(35)	0.093811	0.641032	0.146344	0.8840
C(41)	-4451.967	5263.117	-0.845880	0.4000
C(42)	26.32573	7.671634	3.431567	0.0009
C(43)	0.974800	0.042010	23.20373	0.0000
Determinant residual covariance		1.43E+11		

Equation: Y1=C(11)+C(12)*T +C(13)*Y2+C(14)*X1+C(15)*X1*Y2+[AR(2)=C(16)]
Instruments: C(1) Y1(-1) Y2(-1) X1(-1) T Y1(-2) Y2(-2) X1(-2) X1(-2)*Y2(-2)
Observations: 27

R-squared	0.894047	Mean dependent var	30.26037
Adjusted R-squared	0.868820	S.D. dependent var	10.39215
S.E. of regression	3.763915	Sum squared resid	297.5082
Durbin-Watson stat	1.542320		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X2+[AR(1)=C(25)]
Instruments: C(2) Y2(-1) X1(-1) X2 X2(-1) X1(-1)*X2(-1)
Observations: 27

R-squared	0.896594	Mean dependent var	42.50370
Adjusted R-squared	0.877793	S.D. dependent var	16.40678
S.E. of regression	5.735507	Sum squared resid	723.7129
Durbin-Watson stat	1.755069		

Equation: X1=C(31) +C(32)*X2+ C(33)*X3+C(34)*X2*X3 + [AR(1)=C(35)]
Instruments: C(3) X1(-1) X2(-1) X3(-1) X2(-1)*X3(-1)
Observations: 26

R-squared	0.233895	Mean dependent var	814.9615
Adjusted R-squared	0.087970	S.D. dependent var	445.1957
S.E. of regression	425.1631	Sum squared resid	3796037
Durbin-Watson stat	1.963451		

Equation: X3=C(41) + C(42)*X2+[AR(1)=C(43)]
Instruments: C(4) X2(-1) X3(-1)
Observations: 25

R-squared	0.703917	Mean dependent var	416.7240
Adjusted R-squared	0.677000	S.D. dependent var	193.2920
S.E. of regression	109.8538	Sum squared resid	265492.8
Durbin-Watson stat	2.014795		

Figure 7.35 Statistical results based on an instrumental two-way interaction model, as an extension of the multivariate growth model in (2.84)

System: UNTITLED
Estimation Method: Iterative Two-Stage Least Squares
Date: 01/02/08 Time: 12:46
Sample: 1952 1980
Included observations: 30
Total system (unbalanced) observations 115
Convergence achieved after 45 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	48.54830	20.33028	2.387979	0.0189
C(12)	2.300735	1.078085	2.134094	0.0354
C(13)	0.352222	0.324840	1.084293	0.2810
C(14)	-0.116923	0.057925	-2.018513	0.0463
C(15)	-0.000189	0.000237	-0.796870	0.4275
C(16)	1.500765	0.092425	16.23756	0.0000
C(21)	-3.074409	584.6852	-0.005258	0.9958
C(22)	1.637700	1.753296	0.934069	0.3526
C(23)	7.986901	9.313800	0.857534	0.3933
C(24)	-0.007648	0.008658	-0.883286	0.3793
C(25)	1.040828	0.021028	49.49808	0.0000
C(31)	1189.987	252.7421	4.708306	0.0000
C(32)	-4.220662	3.937663	-1.071870	0.2865
C(33)	-4.550155	1.048939	-4.337866	0.0000
C(34)	0.045358	0.006051	7.496348	0.0000
C(35)	-0.060578	0.210121	-0.288302	0.7737
C(41)	-174.0467	76.31056	-2.280768	0.0248
C(42)	6.835458	0.756835	9.031632	0.0000
C(43)	0.285634	0.201042	1.420764	0.1586

Determinant residual covariance 1.28E+12

Equation: Y1=C(11)+C(12)*T +C(13)*Y2+C(14)*X1+C(15)*X1*Y2+[AR(2)=C(16)]
Instruments: C Y1(-1) Y1(-2) T(-2) Y2(-2) X1(-2) X1(-2)*Y2(-2)
Observations: 28

R-squared	0.766130	Mean dependent var	31.71071
Adjusted R-squared	0.712978	S.D. dependent var	12.76302
S.E. of regression	6.837717	Sum squared resid	1028.596
Durbin-Watson stat	2.540813		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X2+[AR(1)=C(25)]
Instruments: C Y1(-1) Y2(-1) X1(-1) X2(-1) X1(-1)*X2(-1)
Observations: 29

R-squared	-0.318335	Mean dependent var	46.45862
Adjusted R-squared	-0.538058	S.D. dependent var	21.65240
S.E. of regression	26.85298	Sum squared resid	17305.98
Durbin-Watson stat	2.625455		

Equation: X1=C(31) +C(32)*X2+ C(33)*X3+C(34)*X2*X3 + [AR(1)=C(35)]
Instruments: C Y1(-1) X1(-1) X2(-1) X3(-1) X2(-1)*X3(-1)
Observations: 29

R-squared	0.955132	Mean dependent var	985.4655
Adjusted R-squared	0.947654	S.D. dependent var	666.2947
S.E. of regression	152.4431	Sum squared resid	557733.7
Durbin-Watson stat	2.060590		

Equation: X3=C(41) + C(42)*X2+[AR(1)=C(43)]
Instruments: C Y1(-1) X3(-1) X2(-1)
Observations: 29

R-squared	0.868864	Mean dependent var	474.3172
Adjusted R-squared	0.858777	S.D. dependent var	234.6746
S.E. of regression	88.18987	Sum squared resid	202213.8
Durbin-Watson stat	1.852139		

Figure 7.36 Statistical results based on the ISCM in (7.16)

error message was obtained. Therefore, it could be said that the statistical result in this figure should be considered as an unacceptable result.

(2) Then an attempt was made to use or enter 'C' and one or two variables in the instrument list and to use the option 'Lagged dependent ....' Several alternative instruments were found where convergence was achieved after less than 100 iterations. One of the models with the least number of instruments is presented in Figure 7.36, using the following equation:

$$
\begin{aligned}
y1 &= c(11) + c(12)*t + c(13)*y2 + c(14)*x1 + c(15)*x1*y2 + [ar(2) = c(16)] \\
y2 &= c(21) + c(22)*x1 + c(23)*x2 + c(24)*x1*x2 + [ar(1) = c(25)] \\
x1 &= c(31) + c(32)*x2 + c(33)*x3 + c(34)*x2*x3 + [ar(1) = c(35)] \\
x3 &= c(41) + c(42)*x2 + [ar(1) = c(43)] \\
&\quad \textit{Instrument} \quad C \quad y1(-1)
\end{aligned}
\tag{7.16}
$$

(3) Corresponding to this result, the following notes are presented:
- The first regression uses the indicator AR(2) instead of AR(1), since by using AR(1), the error message 'Near singular matrix' is obtained.
- On the other hand, by using both AR(1) and AR(2), the indicator AR(1) is insignificant with a large p-value = 0.7241.
- Convergence is achieved after 45 iterations. For this reason, the estimates of parameters are acceptable statistical results, which can be considered as unexpected estimates.
- The instrument list is very simple. By using 'C' only in the instrument list, statistical results would be obtained where the convergence was not achieved after 500 iterations and many parameters are insignificant with very large p-values. □

Example 7.24. (An extension of the three-way interaction model in (2.89)) Corresponding to the two-way interaction model in (2.89), it would be desirable to develop an instrumental three-way interaction growth model. By using the trial-and-error methods, the acceptable statistical results based on the ISCM in the following model were found, using the generalized method of moment (GMM) estimation method instead of the TSLS or WTSLS estimation methods:

$$
\begin{aligned}
y1 &= c(11) + c(12)*t + c(13)*y2 + c(14)*x1 + c(15)*y2*x1 \\
&\quad + c(16)*y2*x2 + c(17)*x1 + c(17)*x1*x2 + c(18)*x1*x3 \\
&\quad + c(19)*y2*x1*x2 + c(100)*x1*x2*x3 + [ar(2) = c(101)] \\
y2 &= c(21) + c(22)*x1 + c(23)*x2 + c(24)*x1*x2 \\
&\quad + c(25)*x1*x3 + c(26)*x1*x2*x3 + [ar(1) = c(27)] \\
x1 &= c(31) + c(32)*x2 + c(33)*x3 + c(34)*x2*x3 + [ar(1) = c(35)] \\
x3 &= c(41) + c(42)*x2 + [ar(1) = c(43)] \\
&\quad \mathit{Instrument}\ c\ y1(-1)\ x1(-1)
\end{aligned} \tag{7.17}
$$

Corresponding to the statistical results in Figure 7.37, the following notes are presented:

(1) By using theTSLS estimation method, convergence is not achieved after 500 iterations and by using the WTSLS estimation method, convergence is not achieved after one weight matrix and 1000 total coefficient iterations.

Estimation Method: Generalized Method of Moments
Date: 01/02/08 Time: 13:40
Sample: 1952 1980
Included observations: 30
Total system (unbalanced) observations 115
White Covariance
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 508 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	45.09743	19.98185	2.256920	0.0265
C(12)	1.634631	0.579656	2.820001	0.0059
C(13)	1.395020	0.451798	3.087709	0.0027
C(14)	-0.216654	0.074038	-2.926263	0.0044
C(15)	0.002956	0.001028	2.875002	0.0051
C(16)	-0.022216	0.003096	-7.174935	0.0000
C(17)	0.001443	0.000406	3.549832	0.0006
C(18)	-2.66E-05	3.61E-05	-0.736751	0.4632
C(19)	-1.69E-05	6.53E-06	-2.578902	0.0116
C(100)	3.19E-07	2.96E-07	1.078853	0.2836
C(101)	-0.378969	0.124510	-3.043675	0.0031
C(21)	-30.74685	17.34635	-1.772526	0.0797
C(22)	0.144494	0.051624	2.798952	0.0063
C(23)	0.378137	0.150271	2.516369	0.0136
C(24)	-0.001147	0.000406	-2.827430	0.0058
C(25)	-3.32E-05	5.64E-05	-0.588364	0.5578
C(26)	6.21E-07	4.21E-07	1.476819	0.1433
C(27)	0.444681	0.127202	3.495865	0.0007
C(31)	1054.934	127.7473	8.257980	0.0000
C(32)	-3.763319	1.858129	-2.025327	0.0458
C(33)	-3.999531	0.325639	-12.28211	0.0000
C(34)	0.041932	0.002356	17.79704	0.0000
C(35)	-0.126587	0.253884	-0.498601	0.6193
C(41)	-171.0879	66.55170	-2.570752	0.0118
C(42)	6.698434	0.726428	9.221056	0.0000
C(43)	0.344681	0.172354	1.999845	0.0486

Determinant residual covariance	5.38E+09
J-statistic	0.441257

Equation: Y1=C(11)+C(12)*T +C(13)*Y2+C(14)*X1+C(15)*X1*Y2+C(16)*X2
*Y2+C(17)*X1*X2+C(18)*X1*X3+C(19)*Y2*X1*X2+C(100)*X1*X2*X3+[
AR(2)=C(101)]
Instruments: C Y1(-1) X1(-1) Y1(-2) T(-2) Y2(-2) X1(-2) X1(-2)*Y2(-2) X2(-2)
*Y2(-2) X1(-2)*X2(-2) X1(-2)*X3(-2) X1(-2)*X2(-2)*Y2(-2) X1(-2)*X2(-2)
*X3(-2)
Observations: 28

R-squared	0.966665	Mean dependent var	31.71071
Adjusted R-squared	0.947057	S.D. dependent var	12.76302
S.E. of regression	2.936699	Sum squared resid	146.6115
Durbin-Watson stat	1.554699		

Equation: Y2=C(21)+C(22)*X1+C(23)*X2+C(24)*X1*X2+C(25)*X1*X3+C(26)
*X1*X2*X3+[AR(1)=C(27)]
Instruments: C Y1(-1) X1(-1) Y2(-1) X2(-1) X1(-1)*X2(-1) X1(-1)*X3(-1) X1(-1)
*X2(-1)*X3(-1)
Observations: 29

R-squared	0.964693	Mean dependent var	46.45862
Adjusted R-squared	0.955064	S.D. dependent var	21.65240
S.E. of regression	4.589900	Sum squared resid	463.4780
Durbin-Watson stat	1.427279		

Equation: X1=C(31) +C(32)*X2+ C(33)*X3+C(34)*X2*X3 + [AR(1)=C(35)]
Instruments: C Y1(-1) X1(-1) X2(-1) X3(-1) X2(-1)*X3(-1)
Observations: 29

R-squared	0.955205	Mean dependent var	985.4655
Adjusted R-squared	0.947739	S.D. dependent var	666.2947
S.E. of regression	152.3190	Sum squared resid	556826.2
Durbin-Watson stat	2.028355		

Equation: X3=C(41) + C(42)*X2+[AR(1)=C(43)]
Instruments: C Y1(-1) X1(-1) X3(-1) X2(-1)
Observations: 29

R-squared	0.867067	Mean dependent var	474.3172
Adjusted R-squared	0.856841	S.D. dependent var	234.6746
S.E. of regression	88.79216	Sum squared resid	204985.3
Durbin-Watson stat	1.931826		

Figure 7.37 Statistical results based on the ISCM in (7.16), using the generalized method of moments

(2) The first regression uses the indicator AR(2) instead of AR(1). By using AR(1) the 'Near singular matrix' error message is obtained.
(3) Only three out of 23 parameters are insignificant with large p-values, so these estimates can be considered as acceptable or good statistics. On the other hand, it is very easy to derive alternative reduced models.
(4) Convergence is achieved after one weight matrix and 508 total coefficient iterations. By using only the instrument '$C\ y1(-1)$', convergence is achieved after one weight matrix and 127 total coefficient iterations, but eight parameters are insignificant with large p-values. □

7.9 Further extension of the instrumental models

Each of the instrumental models presented in the previous examples using the (original) variables $X1$, $X2$, $X3$, $Y1$ and $Y2$ can easily be extended to the following types of instrumental models. However, additional examples will not be presented. Refer to Chapters 2, 3 and 4 for the equation specification of each model.

(1) Semilog instrumental models, with or without lower and upper bounds, as well as with or without trend and time-related effects.
(2) Translog linear instrumental models, namely the Cobb–Douglas models, with or without lower and upper bounds.
(3) Translog quadratic instrumental models, namely the constant elasticity of substitution (CES) models, with or without lower and upper bounds.
(4) By using the first- or higher-order lagged independent or dependent variables, as well as the AR indicators, and other forms of transformation, such as the exponential transformations, e.g. the Box–Cox model, using $(Y^{\lambda} - 1)/\lambda$ as an endogenous variable.
(5) Finally, instrumental models with dummy variables should also be considered in order to represent different patterns of the multiple associations of the components of a multivariate time series between defined time periods, as the impact of an external or environmental factor(s) should be observed or known in advance by a researcher.

8

ARCH models

8.1 Introduction

Autoregressive conditional heteroskedasticity (ARCH) models are specifically designed to model and forecast variance. The variance of a dependent variable is defined as a function of exogenous variables, which consists of the lagged dependent and independent variables and other pure exogenous variables. In the first stage, ARCH models were introduced by Engle (1982) and then generalized as GARCH (Generalized ARCH) by Bollerslev (1986) (see EViews 4 User's Guide, 2001, p. 385, or EViews 6 User's Guide II, p. 185).

In presenting an ARCH model, there are two distinct equations or specifications, the first for the conditional mean and the second for the conditional variance. A more detail explanation of the ARCH model will be presented in the following sections by using examples.

8.2 Options of ARCH models

After opening the workfile by selecting *Quick/Estimate Equation* . . . , the options or window on the left-hand side appear as shown in Figure 8.1. Then by selecting the estimation setting '*ARCH-* . . . ,' the window on the right-hand side appears.

This window presents four alternative ARCH models, namely the GARCH/ TARCH (General/Threshold ARCH), EGARCH (Exponential GARCH), PARCH and Component ARCH(1,1) models, or CGARCH, and other options, such as three options of restriction, four options for ARCH-M, various alternative variance regressors and five options for error distributions. In addition to these options, there are various or many alternative selections for the orders of ARCH, GARCH and Threshold. Since EViews 6 provides three types of orders, then the symbol TGARCH (a, b, c) will be used to the indicate the model where the first integer indicates the ARCH order, the second indicates the GARCH order and the third indicates the Threshold order (refer to Example 8.1).

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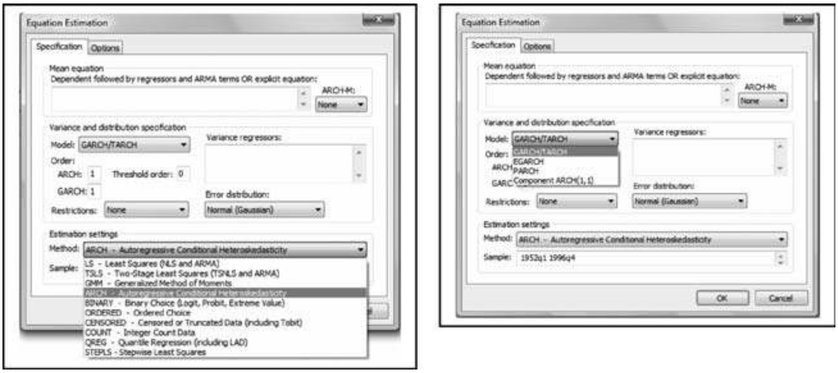

Figure 8.1 The windows and options for the ARCH estimation method

8.3 Simple ARCH models

Corresponding to the classical growth model in (2.3), this section will presents examples of simple ARCH models, namely ARCH(1), GARCH(1), TGARCH(1), and GARCH(1,1) models, as well as special notes on ARCH models.

8.3.1 Simple ARCH models

Example 8.1. (The simplest ARCH classical growth models) Corresponding to the classical growth model of *M*1 in Example 2.1, here the two simplest alternative ARCH CGM (classical growth models) are considered, with the following AR(1) _GCM as a base model or the mean model;

$$\log(m1)\ c\ t\ ar(1) \tag{8.1}$$

By using the default options presented in Figure 8.1 and entering the orders of ARCH = 1, GARCH = 0 and Threshold = 0, the statistical results based on an ARCH (1) or TGARCH(1,0,0) are obtained, as presented in Figure 8.2(a). Then by entering orders of ARCH = 0, GARCH = 1 and Threshold = 0, the statistical results based on a GARCH(1) or TGARCH(0,1,0) are obtained, as presented in Figure 8.2(b). Based on these results, the following notes and conclusions are made:

(1) Both models are acceptable models, corresponding to their values of the DW-statistic, as well as other statistics, including the *Z*-statistics.
(2) The equation of the ARCH(1) model is

$$\begin{aligned} \log(m1_t) &= c(1) + c(2)t + c(3)\mu_{t-1} + \varepsilon_t \\ \sigma_t^2 &= c(4) + c(5)\varepsilon_{t-1}^2 \end{aligned} \tag{8.2}$$

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/04/08 Time: 05:34
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 28 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.042246	0.270034	14.96938	0.0000
T	0.017182	0.001456	11.79945	0.0000
AR(1)	0.978185	0.009688	100.9654	0.0000
	Variance Equation			
C	0.000176	1.95E-05	9.017395	0.0000
RESID(-1)^2	0.186351	0.102213	1.823158	0.0683

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999606	S.D. dependent var	0.753241
S.E. of regression	0.014954	Akaike info criterion	-5.585296
Sum squared resid	0.038912	Schwarz criterion	-5.496263
Log likelihood	504.8840	Hannan-Quinn criter.	-5.549194
F-statistic	112857.9	Durbin-Watson stat	2.173891
Prob(F-statistic)	0.000000		
Inverted AR Roots	.98		

(a) ARCH(1) Model

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/04/08 Time: 05:35
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 54 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	3.964284	0.403059	9.835485	0.0000
T	0.017453	0.002107	8.281419	0.0000
AR(1)	0.982146	0.007897	124.3666	0.0000
	Variance Equation			
C	2.74E-06	2.22E-06	1.234173	0.2171
GARCH(-1)	0.992968	0.011790	84.22061	0.0000

R-squared	0.999614	Mean dependent var	5.816642
Adjusted R-squared	0.999605	S.D. dependent var	0.753241
S.E. of regression	0.014976	Akaike info criterion	-5.598666
Sum squared resid	0.039024	Schwarz criterion	-5.509633
Log likelihood	506.0806	Hannan-Quinn criter.	-5.562564
F-statistic	112532.9	Durbin-Watson stat	2.176147
Prob(F-statistic)	0.000000		
Inverted AR Roots	.98		

(b) GARCH(1) Model

Figure 8.2 Statistical results based on (a) ARCH(1) and (b) GARCH(1) models

This model shows that the variance or volatility model is a simple linear regression of σ_t^2 on ε_{t-1}^2.

(3) The equation of the GARCH(1) model is

$$\begin{aligned}\log(m1_t) &= c(1) + c(2)t + c(3)\mu_{t-1} + \varepsilon_t \\ \sigma_t^2 &= c(4) + c(5)\sigma_{t-1}^2\end{aligned} \tag{8.3}$$

This model shows that the variance or volatility model is a simple linear regression of σ_t^2 on σ_{t-1}^2.

(4) Since the GARCH(1) model has smaller values of the AIC and SC statistics than the ARCH(1) model, the GARCH(1) model is preferred.

(5) For a comparison, by entering orders of ARCH = 0, GARCH = 0 and Threshold = 1, the TARCH(1) or TGARCH(0,0,1) model is obtained as follows:

$$\begin{aligned}\log(m1_t) &= c(1) + c(2)t + c(3)\mu_{t-1} + \varepsilon_t \\ \sigma_t^2 &= c(4) + c(5)\varepsilon_{t-1}^2 * (\varepsilon_{t-1} < 0)\end{aligned} \tag{8.4}$$

This model shows that the variance or volatility model is a simple linear regression of σ_t^2 on $\varepsilon_{t-1}^2*(\varepsilon_{t-1} < 0)$. Note that the special interaction factor is $\varepsilon_{t-1}^2*(\varepsilon_{t-1} < 0)$, where $(\varepsilon_{t-1} < 0)$ is a dummy variable with $\varepsilon_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $\varepsilon_{t-1} = 0$ otherwise.

(6) For other types of the simplest ARCH models, conduct the analysis using the models EGARCH, PARCH and Component ARCH(1,1), using the same orders as above. Based on each output, the equation of the model can easily be written. For GARCH variance series, in general, refer to Section 8.5. □

Example 8.2. (The TGARCH(1,1,0) classical growth models) Figure 8.3 presents statistical results based on four alternative ARCH models using the default options, namely GARCH/TARCH, EGARCH, PARCH and Component ARCH(1,1), with the orders of ARCH = 1, GARCH = 1 and Threshold = 1, which is associated with the TGARCH(1,1,0) model. Based on these statistical results, the following notes and conclusions are presented:

(1) In mathematical statistics, the four models should be good statistical models. However, corresponding to each data used, the best fit model should be selected out of the four ARCH models, based on specific criteria.

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/03/08 Time: 08:07
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 32 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.102179	0.220593	18.59613	0.0000
T	0.016754	0.001289	13.00272	0.0000
AR(1)	0.976671	0.008692	112.3584	0.0000
	Variance Equation			
C	2.93E-05	2.06E-05	1.423724	0.1545
RESID(-1)^2	0.168296	0.074736	2.251887	0.0243
GARCH(-1)	0.706114	0.136373	5.177826	0.0000

R-squared	0.999615	Mean dependent var	5.816642
Adjusted R-squared	0.999604	S.D. dependent var	0.753241
S.E. of regression	0.014995	Akaike info criterion	-5.619050
Sum squared resid	0.038897	Schwarz criterion	-5.512210
Log likelihood	508.9049	Hannan-Quinn criter.	-5.575727
F-statistic	89800.35	Durbin-Watson stat	2.171418
Prob(F-statistic)	0.000000		
Inverted AR Roots	.98		

(a) The GARCH/TARCH Model

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/03/08 Time: 08:08
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 41 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(4) + C(5)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(6)
*RESID(-1)/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.255291	0.151222	28.13933	0.0000
T	0.015776	0.001020	15.47189	0.0000
AR(1)	0.972568	0.009808	99.15919	0.0000
	Variance Equation			
C(4)	-2.130165	0.954438	-2.231854	0.0256
C(5)	0.290062	0.132731	2.185338	0.0289
C(6)	0.131359	0.077777	1.688921	0.0912
C(7)	0.776406	0.110521	7.024972	0.0000

R-squared	0.999613	Mean dependent var	5.816642
Adjusted R-squared	0.999600	S.D. dependent var	0.753241
S.E. of regression	0.015066	Akaike info criterion	-5.616237
Sum squared resid	0.039040	Schwarz criterion	-5.491591
Log likelihood	509.6532	Hannan-Quinn criter.	-5.565694
F-statistic	74129.03	Durbin-Watson stat	2.154673
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97		

(b) The EGARCH Model

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/03/08 Time: 08:05
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 157 iterations
Presample variance: backcast (parameter = 0.7)
@SQRT(GARCH)^C(8) = C(4) + C(5)*(ABS(RESID(-1)) - C(6)*RESID(
-1))^C(8) + C(7)*@SQRT(GARCH(-1))^C(8)

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.253876	0.151187	28.13651	0.0000
T	0.015702	0.000968	16.21724	0.0000
AR(1)	0.973475	0.009254	105.2001	0.0000
	Variance Equation			
C(4)	5.72E-06	5.45E-05	0.105009	0.9164
C(5)	0.128735	0.149612	0.860461	0.3895
C(6)	-0.295303	0.389768	-0.757640	0.4487
C(7)	0.600820	0.183324	3.277366	0.0010
C(8)	2.487765	2.168619	1.147166	0.2513

R-squared	0.999613	Mean dependent var	5.816642
Adjusted R-squared	0.999597	S.D. dependent var	0.753241
S.E. of regression	0.015125	Akaike info criterion	-5.609331
Sum squared resid	0.039119	Schwarz criterion	-5.466878
Log likelihood	510.0351	Hannan-Quinn criter.	-5.551567
F-statistic	63041.44	Durbin-Watson stat	2.152225
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97		

(c). The PARCH Model

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/03/08 Time: 08:04
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 22 iterations
Presample variance: backcast (parameter = 0.7)
Q = C(4) + C(5)*(Q(-1) - C(4)) + C(6)*(RESID(-1)^2 - GARCH(-1))
GARCH = Q + C(7) * (RESID(-1)^2 - Q(-1)) + C(8)*(GARCH(-1) - Q(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.287089	0.096835	44.27189	0.0000
T	0.015530	0.000725	21.42854	0.0000
AR(1)	0.968454	0.008252	117.3545	0.0000
	Variance Equation			
C(4)	0.135312	0.071739	1.886161	0.0593
C(5)	0.999838	6.45E-05	15506.42	0.0000
C(6)	0.371512	0.087254	4.257838	0.0000
C(7)	-0.168805	0.082387	-2.048920	0.0405
C(8)	0.729957	0.249794	2.922238	0.0035

R-squared	0.999611	Mean dependent var	5.816642
Adjusted R-squared	0.999595	S.D. dependent var	0.753241
S.E. of regression	0.015151	Akaike info criterion	-5.573625
Sum squared resid	0.039254	Schwarz criterion	-5.431172
Log likelihood	506.8395	Hannan-Quinn criter.	-5.515862
F-statistic	62825.32	Durbin-Watson stat	2.134208
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97		

(d). The Component ARCH(1,1) Model

Figure 8.3 Statistical results based on four alternative GARCH(1,1) models using the AR(1) classical growth model in (8.2) and the default options

(2) Based on the largest adjusted R-squared, the GARCH/TARCH model would be chosen as the best fit model, as presented in Figure 8.2(a). Compare them using other measures or statistics (refer to Section 11.3). Furthermore, based on this model, the following findings are made:
- The exponential growth rate of $M1$ is 1.6754, which is also the largest growth rate.
- Each of the dependent variables, as well as the indicator AR(1), has a significant effect on its corresponding dependent variable, based on the Z-statistic.

(3) The equation of the model in Figure 8.3(a) is

$$\begin{aligned}\log(m1_t) &= c(1)+c(2)t+c(3)\mu_{t-1}+\varepsilon_t \\ \sigma_t^2 &= c(4)+c(5)\varepsilon_{t-1}^2+c(6)\sigma_{t-1}^2\end{aligned} \tag{8.5}$$

Compare this with the conditional variance equations of the ARCH(1) model in (8.2) and the GARCH(1) model in (8.3). Note that, in a three-dimensional space, the variance model in (8.5) could be considered as a simple linear regression of σ_1^2 on two independent variables ε_{t-1}^2 and σ_{t-1}^2.

(4) The equation of the EGARCH model in Figure 8.3(b) is

$$\begin{aligned}\log(m1_t) &= c(1)+c(2)t+c(3)\mu_{t-1}+\varepsilon_t \\ \log(\sigma_t^2) &= c(4)+c(5)\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right|+c(6)\frac{\varepsilon_{t-1}}{\sigma_{t-1}}+c(7)\log(\sigma_{t-1}^2)\end{aligned} \tag{8.6}$$

Compared to the previous models, this model is a complex model, in a theoretical aspect, as well as various alternative EGARCH models. It would be interesting to know its real advantages, and likewise for the following PARCH and Component ARCH(1,1) models. Refer to the special notes presented in Section 8.3.2.

(5) The equation of the PARCH model in Figure 8.3(c) is

$$\begin{aligned}\log(m1_t) &= c(1)+c(2)t+c(3)\mu_{t-1}+\varepsilon_t \\ (\sigma_t)^{c(8)} &= c(4)+c(5)[|\varepsilon_{t-1}|-c(6)\varepsilon_{t-1}]^{c(8)}+c(7)(\sigma_{t-1})^{c(8)}\end{aligned} \tag{8.7}$$

(6) The equation of the Component ARCH(1,1) model in Figure 8.3(d) is

$$\begin{aligned}\log(m1_t) &= c(1)+c(2)t+c(3)\mu_{t-1}+\varepsilon_t \\ Q &= c(4)+c(5)[Q(-1)-c(4)]+c(6)[\varepsilon_{t-1}^2-\sigma_{t-1}] \\ \sigma_t^2 &= Q+c(7)[\varepsilon_{t-1}^2-Q(-1)]+c(8)[\sigma_{t-1}^2-Q(-1)]\end{aligned} \tag{8.8}$$

(7) For the equations of additional simple ARCH models, conduct an analysis using the orders of ARCH = 1, GARCH = 1 and Threshold = 1, which will be called TGARCH(1,1,1) for each model GARCH/TARCH, EGARCH and PARCH. This gives the models E_TGARCH(1,1,1) and P_TGARCH(1,1,1). Then, based on each output, the equation of each model can easily be written. □

8.3.2 Special notes on the ARCH models

Corresponding to the simple ARCH models in the previous examples, the following special notes apply:

(1) In statistical theory, all of the simple ARCH models presented in Example 8.1, as well as more advanced ARCH models, namely theTGARCH(a, b, c) model, are acceptable or good models, in a theoretical sense. However, their statistical results are highly dependent on the data available for a researcher. An error message, such as 'Near singular matrix' or 'Convergence not achieved . . . ,' could be obtained, based on any of those models.
(2) Moreover, for a more advanced or complex ARCH model with variance regressors, refer to the GARCH variance series presented in Section 8.5.
(3) Corresponding to various options available for the ARCH models, as presented in Figure 8.1, it would be very difficult or almost impossible to select or define the best combination of such a large number of possible options, since the true population model is never known, and nor is the true population TGARCH(a, b, c) model (refer to Section 2.14.1).
(4) Furthermore, since there is only a single observation at one time t, then the variance or volatility of the observation would be unrealistic, particularly when testing residual tests (refer to the special notes in Section 2.14.3). Tsay (2002, p. 86) presents several statements on the weaknesses of ARCH models. Some of those weaknesses are as follows:

The ARCH model does not provide any new insight for understanding the source of variation of a financial time series. They only provide a mechanical way to describe the behavior of conditional variance. It gives no indication about what causes such behavior to occur.

(5) Even though, a good fit ARCH model or an acceptable estimate has been obtained, it is suggested that various residual analyses should be conducted in order to explore the limitation of the model. Refer to various analyses that have been illustrated in previous examples. For this reason, the following examples will not present the residual analysis.

8.4 ARCH models with exogenous variables

8.4.1 ARCH models with one exogenous variable

The ARCH growth models presented in the previous examples can be generalized to the ARCH model, where the mean model has one exogenous variable, with the equation specification of the mean model as follows:

$$y\ c\ x\ ar(1)\ ar(2) \cdots ar(p) \tag{8.9}$$

Since this model is an AR(p) model, then the ARCH model will be named the AR(p)_TGARCH(a, b, c) model with one exogenous variable, where the AR(p) indicates the pth order autoregressive mean model of the TGARCH(a, b, c) model.

Example 8.3. (AR(2)_TGARCH(1,0,0) and AR(2)_TGARCH(0,1,0) models) Corresponding to the two simplest models in Example 8.1, Figure 8.4 presents statistical results based on the two simplest ARCH models in (8.8) for $p = 2$, namely the AR(2)_TGARCH(1,0,0) and AR(2)_TGARCH(0,1,0) models.

Note that both models present different coefficients, which indicates the different impacts of the conditional variance models. Furthermore, it could be said that the impact of a conditional variance model on the coefficient of the corresponding mean model is unpredictable. □

Dependent Variable: LOG(Y1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/04/08 Time: 07:54
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 346 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	181.9723	185.8609	0.979078	0.3275
X1	0.666706	0.313148	2.129044	0.0333
AR(1)	0.882049	0.073475	12.00471	0.0000
AR(2)	0.117895	0.073515	1.603693	0.1088
	Variance Equation			
C	0.000189	2.57E-05	7.373053	0.0000
RESID(-1)^2	0.183500	0.123593	1.484716	0.1376

R-squared	0.999588	Mean dependent var	5.822083
Adjusted R-squared	0.999576	S.D. dependent var	0.751831
S.E. of regression	0.015472	Akaike info criterion	-5.497295
Sum squared resid	0.041176	Schwarz criterion	-5.390044
Log likelihood	495.2593	Hannan-Quinn criter.	-5.453802
F-statistic	83551.16	Durbin-Watson stat	1.831723
Prob(F-statistic)	0.000000		
Inverted AR Roots	1.00	-.12	

Dependent Variable: LOG(Y1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/04/08 Time: 07:55
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 65 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	19.64994	35.65468	0.551118	0.5816
X1	0.989145	0.420775	2.350771	0.0187
AR(1)	0.899685	0.073760	12.19742	0.0000
AR(2)	0.099785	0.073098	1.365087	0.1722
	Variance Equation			
C	2.55E-06	2.46E-06	1.033459	0.3014
GARCH(-1)	0.994738	0.012544	79.29704	0.0000

R-squared	0.999585	Mean dependent var	5.822083
Adjusted R-squared	0.999573	S.D. dependent var	0.751831
S.E. of regression	0.015537	Akaike info criterion	-5.524041
Sum squared resid	0.041523	Schwarz criterion	-5.416790
Log likelihood	497.6397	Hannan-Quinn criter.	-5.480548
F-statistic	82851.68	Durbin-Watson stat	1.862631
Prob(F-statistic)	0.000000		
Inverted AR Roots	1.00	-.10	

Figure 8.4 Statistical results based on the AR(2)_TGARCH(1,0,0) and AR(2)_GARCH (0,1,0) models of log($Y1$) on $X1$

8.4.2 ARCH models with two exogenous variables

The models presented in the previous examples can be generalized to the AR(p)_GARCH(a, b, c) model with two exogenous variables, with the equation specification as follows:

$$y\ c\ x1\ x2\ ar(1)\ ar(2) \cdots\ ar(p) \tag{8.10}$$

Example 8.4. (AR(2)_TGARCH(1,1,0) model) In fact, here the use is explored of AR(2) indicators as an extension of the AR(1)_TGARCH(1,0,0) model presented in Example 8.2. For this reason, the statistical results in Figure 8.5 have been obtained based on the following AR(2)_TGARCH(1,1,0) model:

$$\begin{aligned} Y_{1,t} &= c(1) + c(2) * X_{1,t} + c(3) * X_{2,t} + u_t \\ u_t &= \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \end{aligned} \tag{8.11}$$

Dependent Variable: Y1
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/04/08 Time: 08:16
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 66 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	98.04030	23.25236	4.216359	0.0000
X1	272.0363	130.6877	2.081575	0.0374
X2	-0.312175	0.251774	-1.239903	0.2150
AR(1)	1.235873	0.036896	33.49573	0.0000
AR(2)	-0.223536	0.038256	-5.843186	0.0000
	Variance Equation			
C	218.2416	19.80559	11.01919	0.0000
RESID(-1)^2	0.123739	0.013966	8.860201	0.0000
GARCH(-1)	-1.001632	0.000545	-1839.254	0.0000

R-squared	0.999324	Mean dependent var	448.5793
Adjusted R-squared	0.999297	S.D. dependent var	345.1043
S.E. of regression	9.152907	Akaike info criterion	6.725109
Sum squared resid	14241.87	Schwarz criterion	6.868110
Log likelihood	-590.5347	Hannan-Quinn criter.	6.783100
F-statistic	35922.33	Durbin-Watson stat	2.012907
Prob(F-statistic)	0.000000		

Inverted AR Roots 1.02 .22
Estimated AR process is nonstationary

(a) AR(2)_TGARCH(1,1,0) Model Using Data_1

Dependent Variable: Y2
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/05/08 Time: 10:05
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Failure to improve Likelihood after 41 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	12.25446	9.170803	1.336247	0.1815
X1	0.032088	0.008532	3.760703	0.0002
X2	0.022383	0.170092	0.131595	0.8953
AR(1)	0.653591	0.252316	2.590370	0.0096
AR(2)	-0.328168	0.367980	-0.891809	0.3725
	Variance Equation			
C	17.69952	9.805940	1.804979	0.0711
RESID(-1)^2	0.996666	0.723482	1.377596	0.1683
GARCH(-1)	-0.282386	0.497890	-0.567167	0.5706

R-squared	0.928856	Mean dependent var	47.32179
Adjusted R-squared	0.903956	S.D. dependent var	21.53564
S.E. of regression	6.674098	Akaike info criterion	6.569263
Sum squared resid	890.8717	Schwarz criterion	6.949893
Log likelihood	-83.96968	Hannan-Quinn criter.	6.685625
F-statistic	37.30308	Durbin-Watson stat	2.296340
Prob(F-statistic)	0.000000		

Inverted AR Roots 33-.47i .33+.47i

(b) AR(2)_TGARCH(1,1,0) Using Data_2

Figure 8.5 Statistical results based on the A_GARCH(2,1,1) model using two data sets

Based on the results in Figure 8.5(a) and (b), the following notes and conclusions are presented:

(1) Figure 8.5(a) presents a note 'Estimated AR process is nonstationary,' so the data does not support the model as a good time series model. Note that the results are highly dependent on the data, whether or not the model is a good model.
(2) For a comparison, the same model is run using another data set, with the statistical results presented in Figure 8.5(b). Based on the results, the following notes and conclusions are presented:
  - The data support the model as a good fit time series model, even though *X*2 and the indicator AR(2) are insignificant.
  - The two statistical results in Figure 8.5 have demonstrated that the good fit model is highly dependent on the data.
  - In a statistical sense, this model should be reduced. By deleting *X*2 and the indicator AR(2), the AR(1)_GARCH(1,1,0) model with one exogenous variable should have a good fit. Do this as an exercise.
  - On the other hand, the $Resid(1)^2 = \varepsilon_{t-1}^2$ and $\text{GARCH}(-1) = \sigma_{t-1}^2$ are insignificant. Therefore, the variance model should also be modified. By deleting either one of these variables, the statistical results in Figure 8.6 are obtained.
(3) The equation of the first reduced model, namely the AR(2)_TGARCH(1,0,0) model, is

$$\begin{aligned} Y_{1,t} &= c(1) + c(2)^*X_{1,t} + c(3)^*X_{2,t} + u_t \\ u_t &= \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \end{aligned} \tag{8.12}$$

Dependent Variable: Y2
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/05/08 Time: 10:27
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 108 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	17.51454	2.682870	6.528284	0.0000
X1	0.035963	0.004750	7.571406	0.0000
X2	-0.081101	0.065350	-1.241026	0.2146
AR(1)	0.220573	0.140760	1.567013	0.1171
AR(2)	-0.164108	0.075280	-2.179977	0.0293
	Variance Equation			
C	0.166510	1.080961	0.154039	0.8776
RESID(-1)^2	2.720064	1.369701	1.985881	0.0470

R-squared	0.915021	Mean dependent var	47.32179
Adjusted R-squared	0.890742	S.D. dependent var	21.53564
S.E. of regression	7.118437	Akaike info criterion	6.196012
Sum squared resid	1064.115	Schwarz criterion	6.529063
Log likelihood	-79.74416	Hannan-Quinn criter.	6.297829
F-statistic	37.68686	Durbin-Watson stat	1.333755
Prob(F-statistic)	0.000000		
Inverted AR Roots	.11-.39i	.11+.39i	

(a). Ar(2)_TGARCH(1,0,0) Model

Dependent Variable: Y2
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/05/08 Time: 10:30
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 41 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	8.583312	6.888152	1.246098	0.2127
X1	0.023902	0.006271	3.811679	0.0001
X2	0.151293	0.124574	1.214484	0.2246
AR(1)	0.433880	0.219239	1.979026	0.0478
AR(2)	-0.405493	0.258807	-1.566779	0.1172
	Variance Equation			
C	47.04278	88.78141	0.529872	0.5962
GARCH(-1)	-0.672509	3.093798	-0.217373	0.8279

R-squared	0.937119	Mean dependent var	47.32179
Adjusted R-squared	0.919153	S.D. dependent var	21.53564
S.E. of regression	6.123364	Akaike info criterion	6.671842
Sum squared resid	787.4073	Schwarz criterion	7.004893
Log likelihood	-86.40579	Hannan-Quinn criter.	6.773659
F-statistic	52.16061	Durbin-Watson stat	2.053521
Prob(F-statistic)	0.000000		
Inverted AR Roots	.22-.60i	.22+.60i	

(b). AR(2)_TGARCH(0,1,0) Model

Figure 8.6 Statistical results based on two ARCH models, namely (a). AR(2)_GARCH (1,0,0) and (b) AR(2)_TGARCH(0,1,0) models

and the equation of the second reduced model, namely the AR(2)_TGARCH (0,1,0) model, is

$$\begin{aligned} Y_{1,t} &= c(1) + c(2)^* X_{1,t} + c(3)^* X_{2,t} + u_t \\ u_t &= \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \sigma_{t-1}^2 \end{aligned} \tag{8.13}$$

(4) So far there have been three alternative acceptable statistical results having endogenous $Y1$ and two exogenous variables $X1$ and $X2$. Considering only these three results, which one do you think is the best model? Use your judgment to select one, by using the values of any statistics available in the output. □

Example 8.5. (Alternative AR(2)_TGARCH(1,1,0) models) By using any AR(2) time series models presented in the previous chapters, it is easy to derive or define various alternative AR(2)_TGARCH(1,1,0) models. In this example, alternative special models with the endogenous variable log($Y1$) and two exogenous variables $X1$ and $X2$ are presented, as follows:

(i) *Semilogarithmic (Semilog) AR(2)_TGARCH(1,1,0) Model*

$$\begin{aligned} \log(Y_t) &= c(1) + c(2) * X_{1,t} + c(3) * X_{2,t} + \mu_t \\ \mu_t &= \rho_1 \mu_{t-1} + \rho_2 \mu_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \end{aligned} \tag{8.14}$$

(ii) *Cobb–Douglas (CD) or Translog Linear AR(2)_TGARCH(1,1,0) Model*

$$\begin{aligned} \log(Y_t) &= c(1) + c(2) * \log(X_{1,t}) + c(3) * \log(X_{3,t}) + \mu_t \\ \mu_t &= \rho_1\mu_{t-1} + \rho_2\mu_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\sigma_{t-1}^2 \end{aligned} \tag{8.15}$$

(iii) *Mixed AR(2)_TGARCH(1,1,0) Model*

$$\begin{aligned} \log(Y_t) &= c(1) + c(2) * \log(X_{1,t}) + c(3) * X_{2,t} + \mu_t \\ \mu_t &= \rho_1\mu_{t-1} + \rho_2\mu_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\sigma_{t-1}^2 \end{aligned} \tag{8.16}$$

(iv) *CES or Translog Quadratic AR(2)_TGARCH(1,1,0) Model*

$$\begin{aligned} \log(Y_t) &= c(1) + c(2)*\log(X_{1,t}) + c(3)*\log(X_{2,t}) \\ &\quad + c(4)\log(X_{1,t})^2 + c(5)\log(X_{1,t})\log(X_{2t}) + c(6)\log(X_{2,t})^2 + \mu_t \\ \mu_t &= \rho_1\mu_{t-1} + \rho_2\mu_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\sigma_{t-1}^2 \end{aligned} \tag{8.17}$$

Note that this model is in fact an extension of the CES (constant elasticity of substitution) production model in (4.103), which is a Taylor approximation of a nonlinear production function:

$$Q = Q(K, L) = A[aK^{-\tau} + (1-\alpha)L^{-\tau}]^{1/\tau} \tag{8.18}$$

where $A > 0$ is an efficiency parameter, α is a distribution parameter with $0 < \alpha < 1$ and τ is a substitution parameter with $\tau > -1$ of the CES model.

Under the null hypothesis H_0: $c(4) = c(5) = c(6) = 0$, the model in (8.15) will become the CD model.

(v) *A Modified Translog Quadratic AR(2)_TGARCH(1,1,0) Model*

$$\begin{aligned} \log(Y_t) &= c(1) + c(2)*\log(X_{1,t}) + c(3)\log(X_{2,t\log}) \\ &\quad + c(4)(\log(X_{1,t}) - \log(X_{2,t}))^2 + \mu_t \\ \mu_t &= \rho_1\mu_{t-1} + \rho_2\mu_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\sigma_{t-1}^2 \end{aligned} \tag{8.19}$$

This model is an extension of the modified CES model in (4.104). □

Example 8.6. (An application of the AR(2)_TGARCH(1,1,0) model in (8.19)) Figure 8.7 presents statistical results based on the model in (8.19) and a correlation matrix of its residual, namely *Resid*01, with the three exogenous

Dependent Variable: LOG(Y1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/05/08 Time: 17:27
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 474 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(7) + C(8)*RESID(-1)^2 + C(9)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	5.936775	1.223157	4.853648	0.0000
LOG(X1)	-3.404939	1.025937	-3.318860	0.0009
LOG(X2)	3.523009	1.040766	3.385014	0.0007
(LOG(X1)-LOG(X2))^2	0.896587	0.226452	3.959286	0.0001
AR(1)	1.054971	0.223972	4.710283	0.0000
AR(2)	-0.365118	0.204640	-1.784195	0.0744
	Variance Equation			
C	0.000311	0.000886	0.350475	0.7260
RESID(-1)^2	0.951911	1.224788	0.777205	0.4370
GARCH(-1)	0.335172	0.604081	0.554846	0.5790

R-squared	0.953481	Mean dependent var	3.397764
Adjusted R-squared	0.933894	S.D. dependent var	0.326101
S.E. of regression	0.083844	Akaike info criterion	-2.330348
Sum squared resid	0.133566	Schwarz criterion	-1.902139
Log likelihood	41.62487	Hannan-Quinn criter.	-2.199440
F-statistic	48.67971	Durbin-Watson stat	2.336484
Prob(F-statistic)	0.000000		
Inverted AR Roots	.53-.29i	.53+.29i	

Covariance Analysis: Ordinary
Date: 01/05/08 Time: 17:39
Sample: 1951 1980
Included observations: 30

Correlation t-Statistic Probability	RESID01	LOG(X1)	LOG(X2)
RESID01	1.000000		

LOG(X1)	0.316220	1.000000	
	1.763787	-----	
	0.0887	-----	
LOG(X2)	0.328502	0.970320	1.000000
	1.840408	21.23218	-----
	0.0763	0.0000	-----
(LOG(X1)-LOG(X2)...	0.268644	0.935361	0.822593
	1.475781	13.99364	7.654806
	0.1512	0.0000	0.0000

Figure 8.7 Statistical results based on the A_GARCH(2,1,1) model in (8.19) and its covariance analysis

variables log($X1$), log($X2$) and $(\log(X1) - \log(X2))^2$. Based on this figure the following notes are presented:

(1) Each of the independent variables of the variance model is insignificant, so this model should be modified. It has been found that the A_GARCH(2,0,1) model is an acceptable model.
(2) On the other hand, each of the exogenous variables log($X1$) and log($X2$) is significantly correlated with *Resid*01. For this reason, the instrumental mean model should be used. However, since there is no ARCH model with instruments, the mean model should be modified by using other additional exogenous variables or other types of mean model. Do this as an exercise. □

8.4.3 Advanced ARCH models

By using a similar process or method presented in the previous examples, it is easy to define various alternative ARCH models based on any models with multivariate exogenous variables presented in the previous chapters, as the mean models. In general, corresponding to the LVAR(p, q) models for various integers (p, q), including the LVAR(p, q)_GM in (2.26) and LVAR(p, q)_SCM in (4.37), there can be advanced ARCH models, namely LVAR(p, q)_TGARCH(a, b, c) models, with various *multivariate variance regressors*. For illustrative purposes, note the following examples.

Example 8.7. (Extension of the model in (2.38)) Based on the translog linear AR (1) model in (2.38) presented in the previous Example 8.16, as shown by the following equation, it is easy to derive various alternative ARCH models, as presented in the previous examples:

$$\begin{aligned}\log(m1) = C(1) + C(2)^{*}t + C(3)^{*}\log(gdp) \\ + C(4)^{*}\log(pr) + [AR(1) = C(5)]\end{aligned} \tag{8.20}$$

However, here two alternative simple models, with and without a variance regressor, will be presented, as follows.

(1) *Models without a Variance Regressor*
Figure 8.8 presents statistical results based on two models, namely AR(1)_TGARCH(1,1,0) and AR(1)_TGARCH(1,1,1) models. Both variance models are acceptable models, even though log(*pr*) is insignificant with large *p*-values in both mean models. Therefore, the mean models should be modified. Do this as an exercise and compare with the results in Figure 8.9.

(2) *Models with a Variance Regressor*
Figure 8.9 presents statistical results based on two models, namely AR(1)_TGARCH(1,1,0) and LV(1)_TGARCH(1,1,0) models with a variance regressor log(*RS*). Based on this figure the following notes are presented:
   (a) The variance model of the AR(1)_TGARCH(1,1,0) is an acceptable model, in a statistical sense, since each of the independent variables is significant. However, the variance model of the LV(1)_TGARCH(1,1,0) is an unacceptable model, since all independent variables are insignificant. Therefore, this model should be modified.
   (b) Compared to the models without a variance regressor in Figure 8.9, where log (*PR*) is insignificant, based on the model with the variance regressor log(*RS*), log(*PR*) is significant in both models.

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/05/08 Time: 18:33
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 67 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	3.479699	0.657376	5.293320	0.0000
T	0.008942	0.002264	3.949101	0.0001
LOG(GDP)	0.252617	0.111100	2.273789	0.0230
LOG(PR)	0.112574	0.147238	0.764574	0.4445
AR(1)	0.968783	0.010869	89.13633	0.0000
	Variance Equation			
C	3.35E-05	2.08E-05	1.607757	0.1079
RESID(-1)^2	0.253277	0.098510	2.571077	0.0101
GARCH(-1)	0.605590	0.146946	4.121185	0.0000

R-squared	0.999621	Mean dependent var	5.816642
Adjusted R-squared	0.999606	S.D. dependent var	0.753241
S.E. of regression	0.014956	Akaike info criterion	-5.652395
Sum squared resid	0.038249	Schwarz criterion	-5.509942
Log likelihood	513.8894	Hannan-Quinn criter.	-5.594632
F-statistic	64475.96	Durbin-Watson stat	2.104325
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97		

(a). AR(1)_TGARCH(1,1,0) Model

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/05/08 Time: 18:30
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 39 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*RESID(-1)^2*(RESID(-1)<0) +
C(9)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	3.341052	0.686074	4.869809	0.0000
T	0.007655	0.002151	3.558944	0.0004
LOG(GDP)	0.300121	0.113259	2.649853	0.0081
LOG(PR)	0.110080	0.154474	0.712614	0.4761
AR(1)	0.968917	0.011658	83.10902	0.0000
	Variance Equation			
C	5.39E-05	2.64E-05	2.038465	0.0415
RESID(-1)^2	0.429989	0.213482	2.014169	0.0440
RESID(-1)^2*(RESID(-1)<0)	-0.428229	0.229272	-1.867776	0.0618
GARCH(-1)	0.504327	0.203419	2.479249	0.0132

R-squared	0.999619	Mean dependent var	5.816642
Adjusted R-squared	0.999601	S.D. dependent var	0.753241
S.E. of regression	0.015045	Akaike info criterion	-5.676700
Sum squared resid	0.038481	Schwarz criterion	-5.516441
Log likelihood	517.0647	Hannan-Quinn criter.	-5.611716
F-statistic	55748.85	Durbin-Watson stat	2.084789
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97		

(b). AR(1)_TGARCH(1,1,1) Model

Figure 8.8 Statistical results based on (a) AR(1)_TGARCH(1,1,0) and (b) AR(1)_TGARCH (1,1,1) models

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/06/08 Time: 07:27
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 33 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1) + C(9)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	4.134499	0.722796	5.720149	0.0000
T	0.011780	0.002860	4.118457	0.0000
LOG(GDP)	0.109441	0.081249	1.346982	0.1780
LOG(PR)	0.275707	0.179060	1.539749	0.1236
AR(1)	0.985012	0.006153	160.0984	0.0000
		Variance Equation		
C	0.000106	7.57E-05	1.406354	0.1596
RESID(-1)^2	0.115620	0.026529	4.358264	0.0000
GARCH(-1)	-1.000462	0.010582	-94.54229	0.0000
LOG(RS)	0.000200	4.85E-05	4.119195	0.0000

R-squared	0.999619	Mean dependent var	5.816642
Adjusted R-squared	0.999601	S.D. dependent var	0.753241
S.E. of regression	0.015050	Akaike info criterion	-5.631946
Sum squared resid	0.038505	Schwarz criterion	-5.471686
Log likelihood	513.0592	Hannan-Quinn criter.	-5.566962
F-statistic	55714.07	Durbin-Watson stat	2.163501
Prob(F-statistic)	0.000000		
Inverted AR Roots	.99		

(a) An AR(1)_TGARch(1,1,0) Model

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/06/08 Time: 07:28
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 1 iteration
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1) + C(9)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.043281	0.007172	6.034925	0.0000
LOG(M1-1)	1.001378	0.000111	9035.768	0.0000
T	2.88E-05	1.45E-05	1.987974	0.0468
LOG(GDP)	-0.007689	0.001272	-6.045424	0.0000
LOG(PR)	0.004722	0.001033	4.569463	0.0000
		Variance Equation		
C	3.96E-08	6.46E-08	0.612566	0.5402
RESID(-1)^2	0.150000	0.411022	0.364944	0.7152
GARCH(-1)	0.600000	0.668901	0.896994	0.3697
LOG(RS)	-5.61E-10	1.51E-08	-0.037176	0.9703

R-squared	1.000000	Mean dependent var	5.811220
Adjusted R-squared	1.000000	S.D. dependent var	0.754650
S.E. of regression	0.000207	Akaike info criterion	-13.75697
Sum squared resid	7.29E-06	Schwarz criterion	-13.59732
Log likelihood	1247.127	Hannan-Quinn criter.	-13.69224
F-statistic	2.99E+08	Durbin-Watson stat	0.212666
Prob(F-statistic)	0.000000		

(b) A LV(1)_TGARCH(1,1) Model

Figure 8.9 Statistical results based on (a) AR(1)_TGARCH(1,1,0) and (b) LV(1)_TGARCH (1,1,0) models with a variance regressor

(c) The statistical results in Figures 8.8 and 8.9 have demonstrated unpredictable impact(s) of the variance model on the parameter estimates of a certain or specific mean model, which is highly dependent on the data set as well as the variance model. □

Example 8.8. (Student's *t* and GED error distributions) Figure 8.10 presents statistical results based on the AR(1)_TGARCH(1,1,0) model in Figure 8.9(a), by using the assumptions that the error terms have Student's *t* or generalized error (GED) distributions, instead of the normal (Gaussian) distribution. Based on these results the following notes are derived:

1. Based on the results in Figure 8.10(a), the following notes and conclusions are presented:
   (a) It has been recognized that using Student's *t* error distribution a better parameter estimate can be obtained than the normal error distribution. Note that Figure 8.10(a) shows that the T_DIST CDF is accepted based on the *Z*-statistic, where $Z_0 = 1.212\ 689$ with a *p*-value $= 0.2252$.
   (b) The *R*-squared, as well as the adjusted *R*-squared, are greater than using the normal error distribution.
   (c) The inverted AR roots = 0.98 compared with 0.99 based on the results using the normal error distribution.
   (d) On the other hand, log(*pr*) has an insignificant effect on log($m1$) with a large *p*-value $= 0.3297 > 0.20$. For this reason, there may be a reduced model. However, the results of the reduced model are not presented. Do this as an exercise.

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/05/08 Time: 16:52
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 28 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1) + C(9)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	3.846004	0.684730	5.616814	0.0000
T	0.012000	0.002761	4.345404	0.0000
LOG(GDP)	0.144266	0.084531	1.706674	0.0879
LOG(PR)	0.181612	0.185168	0.980796	0.3267
AR(1)	0.983052	0.007040	139.6468	0.0000
	Variance Equation			
C	0.000115	9.13E-05	1.254451	0.2097
RESID(-1)^2	0.106352	0.030283	3.511938	0.0004
GARCH(-1)	-0.994519	0.012991	-76.55741	0.0000
LOG(RS)	0.000189	6.33E-05	2.989701	0.0028
T-DIST. DOF	10.04056	8.279584	1.212689	0.2252

R-squared	0.999623	Mean dependent var	5.816642
Adjusted R-squared	0.999603	S.D. dependent var	0.753241
S.E. of regression	0.015009	Akaike info criterion	-5.641259
Sum squared resid	0.038069	Schwarz criterion	-5.463193
Log likelihood	514.8927	Hannan-Quinn criter.	-5.569055
F-statistic	49796.04	Durbin-Watson stat	2.171672
Prob(F-statistic)	0.000000		
Inverted AR Roots	.98		

(a)

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Date: 08/05/08 Time: 16:54
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 42 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1) + C(9)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	7.678806	18791.64	0.000409	0.9997
T	0.005675	0.689783	0.008228	0.9934
LOG(GDP)	0.144690	0.110508	1.309320	0.1904
LOG(PR)	0.421630	0.231667	1.819985	0.0688
AR(1)	1.000035	0.012252	81.61953	0.0000
	Variance Equation			
C	0.000189	0.000115	1.640252	0.1010
RESID(-1)^2	0.095839	0.026230	3.653849	0.0003
GARCH(-1)	-1.008810	0.014172	-71.18147	0.0000
LOG(RS)	0.000159	7.30E-05	2.185758	0.0288
GED PARAMETER	1.595593	0.268903	5.933707	0.0000

R-squared	0.999600	Mean dependent var	5.816642
Adjusted R-squared	0.999579	S.D. dependent var	0.753241
S.E. of regression	0.015457	Akaike info criterion	-5.584126
Sum squared resid	0.040375	Schwarz criterion	-5.406060
Log likelihood	509.7793	Hannan-Quinn criter.	-5.511921
F-statistic	46951.39	Durbin-Watson stat	2.085690
Prob(F-statistic)	0.000000		
Inverted AR Roots	1.00		
	Estimated AR process is nonstationary		

(b)

Figure 8.10 Statistical results based on the AR(1)_TGARCH(1,1,0) model in Figure 8.9(a) using (a) the Student's *t* and (b) the GED error distributions

2. Based on the GED error distribution in Figure 8.10(b), the following notes are presented:
   (a) Since the inverted AR roots = 1.00 with a message 'Estimated AR process is nonstationary,' the results are not acceptable or good estimates, in a statistical sense. Furthermore, note that the GED parameter is rejected based on the Z-statistic, where $Z_0 = 5.933\ 707$ with a p-value = 0.0000.
   (b) Observing that the independent variable t has such a large p-value = 0.9934, an attempt was made to apply the reduced model by deleting the independent variable t. The inverted AR roots = 1.00 was obtained, but without the message 'Estimated AR process is nonstationary.' Therefore, this reduced model should be considered as an acceptable model, based on the data set used.
3. In fact, there are two other alternative assumptions of the error distributions, namely the Student's t with fixed *df* and the GED with fixed parameter. Do this to provide a comparison. □

Example 8.9. (Extensions of the model in (2.59)) For more advanced ARCH models, the TGARCH(a, b, c) model can be derived using the AR(1) model with time-related effects in (2.59) as the mean model. In this case, the equation specification of the mean model is

$$\begin{aligned} &\log(m1)\ c \log(gdp) \log(pr) \log(gdp) * \log(pr) \\ &t\ t * \log(gdp)\ t * \log(pr)\ t * \log(gdp) * \log(pr)\ ar(1) \end{aligned} \tag{8.21}$$

As an illustration, statistical results are only presented based on two models, namely the AR(1)_TGARCH(0,1,0) and AR(1)_TGARCH(0,1,1) models, with the variance regressor log(*RS*) (see Figure 8.11). □

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/06/08 Time: 13:13
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 38 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(10) + C(11)*GARCH(-1) + C(12)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	3.474046	1.852788	1.875037	0.0608
LOG(GDP)	0.083887	0.312868	0.268124	0.7886
LOG(PR)	-0.668927	1.109100	-0.603126	0.5464
LOG(GDP)*LOG(PR)	-0.002962	0.192142	-0.015413	0.9877
T	0.018348	0.020115	0.912147	0.3617
T*LOG(GDP)	-0.000147	0.002577	-0.057227	0.9544
T*LOG(PR)	0.027054	0.009748	2.775206	0.0055
T*LOG(GDP)*LOG(PR)	-0.003558	0.001108	-3.210254	0.0013
AR(1)	0.912019	0.037093	24.58712	0.0000
Variance Equation				
C	-1.17E-06	3.17E-06	-0.369182	0.7120
GARCH(-1)	0.938720	0.051070	18.38095	0.0000
LOG(RS)	9.03E-06	6.52E-06	1.383898	0.1664

R-squared	0.999650	Mean dependent var	5.816642
Adjusted R-squared	0.999627	S.D. dependent var	0.753241
S.E. of regression	0.014543	Akaike info criterion	-5.657390
Sum squared resid	0.035318	Schwarz criterion	-5.443711
Log likelihood	518.3364	Hannan-Quinn criter.	-5.570745
F-statistic	43397.43	Durbin-Watson stat	2.122257
Prob(F-statistic)	0.000000		
Inverted AR Roots	.91		

(a). AR(1)_TGARCH(01,0) Model

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/06/08 Time: 13:15
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 19 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(10) + C(11)*RESID(-1)^2*(RESID(-1)<0) + C(12)*GARCH(-1) + C(13)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	3.282576	1.079922	3.039641	0.0024
LOG(GDP)	0.143400	0.208531	0.687667	0.4917
LOG(PR)	-1.362027	0.747072	-1.823154	0.0683
LOG(GDP)*LOG(PR)	0.160861	0.145076	1.108809	0.2675
T	0.006884	0.004746	1.450694	0.1469
T*LOG(GDP)	0.001211	0.000167	7.252623	0.0000
T*LOG(PR)	0.024762	0.007070	3.502462	0.0005
T*LOG(GDP)*LOG(PR)	-0.003972	0.001002	-3.965526	0.0001
AR(1)	0.908069	0.039555	22.95704	0.0000
Variance Equation				
C	6.20E-07	1.88E-06	0.329979	0.7414
RESID(-1)^2*(RESID(-1)<0)	-0.030410	0.026153	-1.162794	0.2449
GARCH(-1)	0.999257	0.020376	49.04111	0.0000
LOG(RS)	2.34E-06	9.22E-07	2.538171	0.0111

R-squared	0.999648	Mean dependent var	5.816642
Adjusted R-squared	0.999622	S.D. dependent var	0.753241
S.E. of regression	0.014641	Akaike info criterion	-5.636191
Sum squared resid	0.035584	Schwarz criterion	-5.404705
Log likelihood	517.4391	Hannan-Quinn criter.	-5.542326
F-statistic	39246.74	Durbin-Watson stat	2.117128
Prob(F-statistic)	0.000000		
Inverted AR Roots	.91		

(b). AR(1)_TGARCH(01,1) Model

Figure 8.11 Statistical results based on (a) AR(1)_TGARCH(0,1,0) and (b) AR(1)_TGARCH(0,1,1) models with a variance regressor

Example 8.10. (Extensions of the LVAR(2,1)_SCM in (4.39)) As the extension of the LVAR(2,1)_SCM in (4.39), the following mean model and TGARCH(*a*, *b*, *c*) models will be presented:

$$\begin{aligned}\log(m1) = c(10) + c(11) * \log(m1(-1)) + c(12) * \log(m1(-2)) \\ + c(20) * \log(gdp) + c(21) * \log(gdp(-1)) + [ar(1) = c(1)]\end{aligned} \quad (8.22)$$

By using trial-and-error methods, two acceptable statistical results or estimates have finally been found, as presented in Figure 8.12. Figure 8.12(a) presents the estimates based on an LV(2)_TGARCH(1,0,1) model with variance regressors log(*PR*) and log(*PR*)*log(*RS*), under the assumption of the Student's *t* error distribution, and Figure 8.12(b) presents the estimates based on an LVAR(1,1)_TGARCH(1,0,1) model with variance regressors log(*PR*) and log(*RS*), under the assumption of the GED error distribution.

Based on the results in this figure, the following notes are given:

(1) Figure 8.12(a) shows an interaction LV(2)_GARCH (1,0,1) model and Figure 8.12(b) shows an interaction LVAR(1,1)_GARCH(1,0,1) model, with the the variance model as follows:

$$\begin{aligned}\sigma_t^2 = c(6) + c(7)\varepsilon_{t-1}^2 + c(8)\varepsilon_{t-1}^2 * (\varepsilon_{t-1} < 0) \\ + c(9)\log(PR) + c(10)\log(PR) * \log(RS)\end{aligned} \quad (8.23)$$

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 01/06/08 Time: 13:57
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 201 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*RESID(-1)^2*(RESID(-1)<0) + C(9)*LOG(PR) + C(10)*LOG(PR)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.073392	0.017924	4.094558	0.0000
LOG(M1(-1))	0.679595	0.074438	9.129650	0.0000
LOG(M1(-2))	0.269096	0.071118	3.783823	0.0002
LOG(GDP)	0.326522	0.098354	3.319854	0.0009
LOG(GDP(-1))	-0.287592	0.102442	-2.807374	0.0050
Variance Equation				
C	0.000130	5.03E-05	2.592890	0.0095
RESID(-1)^2	0.948862	0.312372	3.037600	0.0024
RESID(-1)^2*(RESID(-1)<0)	-0.959163	0.324318	-2.957475	0.0031
LOG(PR)	5.67E-05	3.30E-05	1.718517	0.0857
LOG(PR)*LOG(RS)	-3.54E-05	1.76E-05	-2.012682	0.0441
T-DIST. DOF	181.6877	3164.389	0.057416	0.9542

R-squared	0.999606	Mean dependent var	5.822083
Adjusted R-squared	0.999582	S.D. dependent var	0.751831
S.E. of regression	0.015362	Akaike info criterion	-5.679380
Sum squared resid	0.039413	Schwarz criterion	-5.482753
Log likelihood	516.4648	Hannan-Quinn criter.	-5.599642
F-statistic	42375.98	Durbin-Watson stat	1.492933
Prob(F-statistic)	0.000000		

(a) Student's t Error Distribution TGARCH

Dependent Variable: LOG(M1)
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Date: 01/06/08 Time: 14:24
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 62 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*RESID(-1)^2*(RESID(-1)<0) + C(9)*LOG(PR) + C(10)*LOG(PR)*LOG(RS)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.057775	0.007489	7.714197	0.0000
LOG(M1(-1))	0.958825	0.003255	294.5920	0.0000
LOG(GDP)	0.250206	0.017639	14.18446	0.0000
LOG(GDP(-1))	-0.218786	0.019411	-11.27138	0.0000
AR(1)	-0.262909	0.078509	-3.348761	0.0008
Variance Equation				
C	0.000134	5.15E-05	2.594289	0.0095
RESID(-1)^2	0.896185	0.313474	2.858880	0.0043
RESID(-1)^2*(RESID(-1)<0)	-0.869412	0.342285	-2.540025	0.0111
LOG(PR)	5.62E-05	3.62E-05	1.551947	0.1207
LOG(PR)*LOG(RS)	-3.15E-05	2.11E-05	-1.492323	0.1356
GED PARAMETER	1.791731	0.360516	4.969912	0.0000

R-squared	0.999609	Mean dependent var	5.822083
Adjusted R-squared	0.999585	S.D. dependent var	0.751831
S.E. of regression	0.015311	Akaike info criterion	-5.669194
Sum squared resid	0.039148	Schwarz criterion	-5.472567
Log likelihood	515.5583	Hannan-Quinn criter.	-5.589457
F-statistic	42662.89	Durbin-Watson stat	1.534752
Prob(F-statistic)	0.000000		
Inverted AR Roots	-.26		

(b) GED Error Distribution TGARCH

Figure 8.12 Statistical results based on two LVAR(p, q)_TGARCH(a, b, c) models, under the assumptions of (a) Student's t and (b) GED error distributions

(2) The development of this variance model is under the assumption that the conditional variance σ_t^2 is dependent on log(PR) and log(RS). Furthermore, it is known that the effect of log(PR) on the variance σ_t^2 is dependent on log(RS), so that the model has the two-way interaction log(PR)*log(RS). Note that RS has a positive growth rate for $t < 119$ and is negative otherwise.
(3) It has been found that it is not easy to obtain acceptable or good estimates as presented in Figure 8.12. Trial-and-error methods have been used to select the best fit for both the mean model in (8.22) and the variance model with the exogenous variables log(PR), log(RS) and log(PR)*log(RS), as well as alternative orders of the ARCH, GARCH and Threshold models and the error distribution. Therefore, these statistical results should be considered as unexpected results, which are highly dependent on the data, and they cannot be generalized. In some cases, after having a large number of trials, there may not be success in getting acceptable statistical results or estimates.
(4) Best judgment should be used to select a set of variance regressors, since the true set of variance regressors may never be known, as well as the true (population) variance model. □

Example 8.11. (Graphical representation of the GARCH variance series) This example presents additional analyses based on the model in Figure 8.12(a). By selecting *Proc/Make GARCH variance series ...*, an additional variable, namely

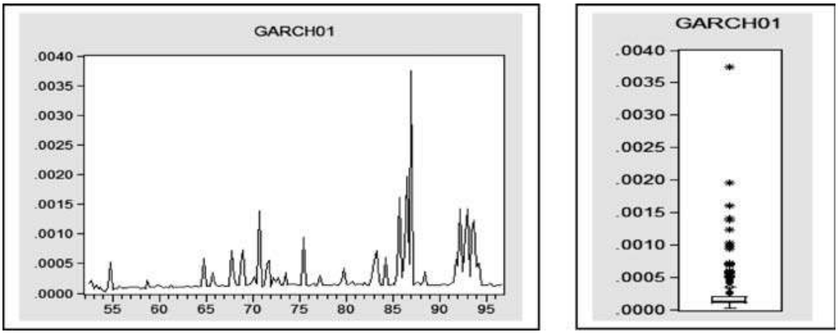

Figure 8.13 Growth curve and box plot of the *GARCH* variance series of the LV(2)_TGARCH(1,0,1) model in Figure 8.12(a)

*GARCH*01, can be placed in the workfile. Then further data analysis can be conducted on this series, such as its graphical representation.

Figure 8.13 presents two graphical representations of *GARCH*01, which is the variance series of the LV(2)_TGARCH(1,0,1) model in Figure 8.12(a). This figure clearly shows that there are several near and far outliers. Therefore, it should be mentioned that this is a limitation of the defined model.

For a comparison, it was found that a *GARCH* variance series of the LV(2)_TGARCH(0,0,1) model, namely the *GARCH*02 series, as presented in Figure 8.14, shows no outlier. However, compared to the previous model, this variance model has two insignificant independent variables, with the following regression function and the t-statistics shown in $[\cdot]$, but the variance series tends to increase with time:

$$\begin{aligned}\hat{\sigma}_t^2 &= \underset{[4.11]}{0.0003} + \underset{[4.11]}{0.0358}\varepsilon_t^2 * (\varepsilon_{t-1} < 0) \\ &\quad + \underset{[2.72]}{0.0001}\log(PR) - \underset{[-0.6196]}{0.0001}\log(PR) * \log(RS)\end{aligned} \tag{8.24}$$

Which model do you prefer?

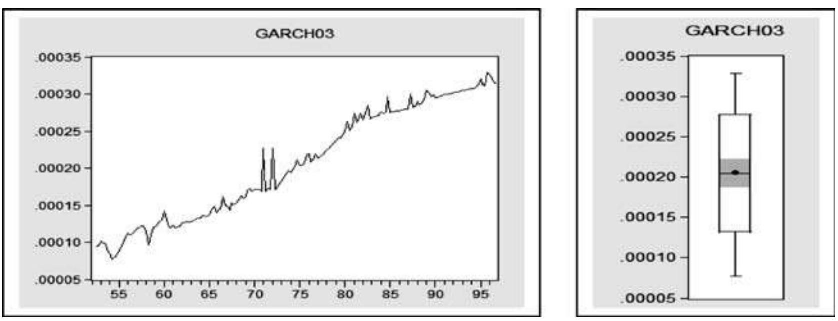

Figure 8.14 Growth curve and box plot of the *GARCH* variance series of the LV(2)_TGARCH(0,0,1) model as a modified model in Figure 8.12(a)

Figure 8.15 presents an additional comparison between the empirical CDF (cumulative distribution function) of the GARCH variance series. Since the conditional variance model of the LV(2)_TGARCH(0,0,1) model has two insignificant independent variables, then the LV(2)_TGARCH(1,0,1) model is preferred, in a statistical sense.

However, at the significant level of 0.10, Figure 8.12(b) shows that log(*PR*) has a significant positive adjusted effect on σ_t^2 with a *p*-value = 0.1207/2 = 0.06035 and log (*PR*)*log(*RS*) has a significant negative adjusted effect on σ_t^2 with a *p*-value = 0.1356/2 = 0.0678. Based on these conclusions, and since the *GARCH*03 series does not have an outlier, this model should be considered as a good or best fit model. □

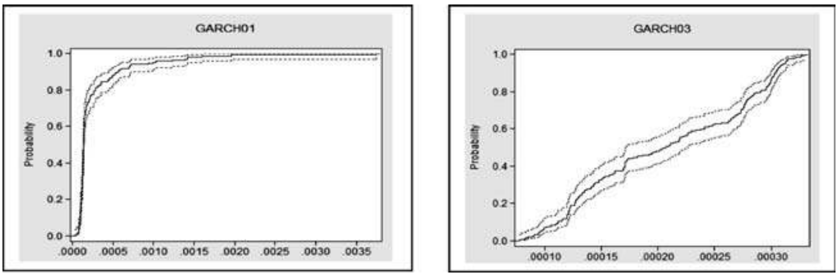

Figure 8.15 Empirical CDF of the *GARCH*01 and *GARCH*03 variance series

8.5 Alternative GARCH variance series

Corresponding to options of the orders ARCH = *a*, GARCH = *b* and Threshold/Asymmetric = *c*, and alternative models GARCH/TARCH, EGARCH and PARCH, other terminologies should be used, as follows.

8.5.1 General GARCH variance series for the GARCH/TARCH model

In this case, corresponding to the orders of ARCH = *a*, GARCH = *b* and TARCH = *c*, the conditional variance model, namely TGARCH(*a*, *b*, *c*), has the following general equation:

$$\sigma_t^2 = \omega + \sum_{i=1}^{a} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{b} \beta_j \sigma_{t-j}^2 \sum_{k=1}^{c} \gamma_k \varepsilon_{t-k}^2 * (\varepsilon_{t-k} < 0) + \sum_{l=1}^{K} \lambda_1 X_{l,t} \qquad (8.25)$$

where the $X_{l,t}$'s are the variance regressors. However, the output of EViews 6 presents different ordering of the independent variables. Then for selected integers of *a*, *b* and *c*, the following special TGARCH models are obtained:

(1) *For* $a \neq 0$, $b = 0$ *and* $c = 0$
In this case, the conditional variance model, namely TGARCH(*a*,0,0), has the following general equation, which has been known as the ARCH(*a*) model with

variance regressors:

$$\sigma_t^2 = \omega + \sum_{i=1}^{a} \alpha_i \varepsilon_{t-i}^2 + \sum_{l=1}^{K} \lambda_1 X_{l,t} \tag{8.26}$$

(2) *For* $a = 0$, $b \neq 0$ *and* $c = 0$
In this case, the conditional variance model, namely TGARCH(0,b,0), has the following general equation, which has been known as the GARCH(b) model with variance regressors:

$$\sigma_t^2 = \omega + \sum_{j=1}^{b} \beta_j \sigma_{t-j}^2 + \sum_{l=1}^{K} \lambda_1 X_{l,t} \tag{8.27}$$

(3) *For* $a = 0$, $b = 0$ *and* $c \neq 0$
In this case, the conditional variance model, namely TGARCH(0,0,c), has the following general equation, which has been known as the TARCH(c) model with variance regressors:

$$\sigma_t^2 = \omega + \sum_{k=1}^{c} \gamma_k \varepsilon_{t-k}^2 * (\varepsilon_{t-k} < 0) + \sum_{l=1}^{K} \lambda_1 X_{l,t} \tag{8.28}$$

8.5.2 General GARCH variance series for the EGARCH model

In this case, corresponding to the orders of ARCH $= a$, GARCH $= b$ and Asymmetric $= c$, the conditional variance model, namely EGARCH(a, b, c), has the following general equation:

$$\begin{aligned}\log(\sigma_t^2) = \omega + \sum_{i=1}^{a} \alpha_i \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \sum_{j=1}^{b} \beta_i \log(\sigma_{t-j}^2) \\ + \sum_{k=1}^{c} \gamma_k \left| \frac{\varepsilon_{t-k}}{\sigma_{t-1}} \right| + \sum_{l=1}^{K} \lambda_1 X_{l,t}\end{aligned} \tag{8.29}$$

where the $X_{l,t}$'s are the variance regressors. However, the output of EViews 6 presents different ordering of the independent variables. For selected integers of a, b and c, the following special EGARCH models are obtained, but if $c \neq 0$, then $a \neq 0$ or $b \neq 0$:

(1) *For* $a \neq 0$, $b = 0$ *and* $c = 0$
In this case, the conditional variance model, namely EGARCH(a,0,0), has the following general equation, which has been known as the EARCH(a) model with variance regressors

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{a} \alpha_i \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \sum_{l=1}^{K} \lambda_1 X_{l,t} \tag{8.30}$$

(2) *For* $a=0$, $b\neq 0$ *and* $c=0$
In this case, the conditional variance model, namely EGARCH(0,*b*,0), has the following general equation, which has been known as the EGARCH(*b*) model with variance regressors:

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^{b} \beta_j \log(\sigma_{t-j}^2) + \sum_{l=1}^{K} \lambda_1 X_{l,t} \quad (8.31)$$

8.5.3 General GARCH variance series for the PARCH model

In this case, corresponding to the orders of ARCH = *a*, GARCH = *b* and Asymmetric = *c*, the conditional variance model, namely PARCH(*a*, *b*, *c*), has the following general equation. However, note that the following error message in Figure 8.16 can be obtained for some selected integers *a*, *b* and *c*, which indicates that the model has been using a variable having negative values to a noninteger power. Refer to the power to *C*(8) in the variance model in (8.7), where *C*(8) will, in general, be a noninteger power.

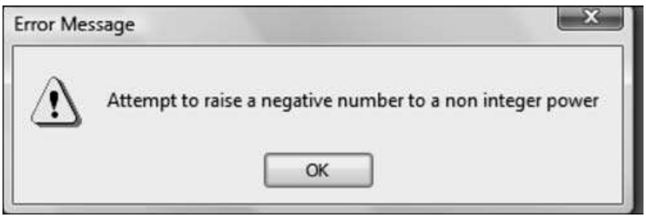

Figure 8.16 An error message for selected integers *a*, *b* and *c*

For this reason, only selected PARCH(*a*, *b*, *c*) models are presented, which are estimable models, as follows:

(1) *PARCH*(*a*,0,0) *Models*
This model has the following general equation:

$$(\sigma_t)^\theta = \omega + \sum_{j=1}^{a} \alpha_i |\varepsilon_{t-i}|^\theta + \sum_{l=1}^{K} \lambda_1 X_{l,t} \quad (8.32)$$

(2) *PARCH*(0,*b*,0) *Models*
This model has the following general equation:

$$(\sigma_t)^\theta = \omega + \sum_{j=1}^{b} \beta_j |\sigma_{t-i}|^\theta + \sum_{l=1}^{K} \lambda_1 X_{l,t} \quad (8.33)$$

(3) *A PARCH*(3,1,2) *Model*

Based on the output, this model has the following equation, where the mean model has five parameters, namely $C(1)$ up to $C(5)$:

$$
\begin{aligned}
@SQRT(GARCH)^{C(15)} &= (\sigma_t)^{C(15)} \\
&= C(6) + C(7) * \{ABS(RESID(-1)) - C(8) * RESID(-1)\}^{C(15)} \\
&\quad + C(9) * \{ABS(RESID(-2)) - C(10) * RESID(-2)\}^{C(15)} \\
&\quad + C(11) * ABS(RESID(-3))^{C(15)} \\
&\quad + C(12) * \ @SQRT(GARCH(-1))^{C(15)}
\end{aligned}
\tag{8.34}
$$

Note that the first integer in PARCH(3,1,2) indicates that the variance model has three independent variables $ABS(RESID(-1)) = |\varepsilon_{t-1}|$, $ABS(RESID(-2)) = |\varepsilon_{t-2}|$ and $ABS(RESID(-3)) = |\varepsilon_{t-3}|$, which in general can be presented as $|\varepsilon_{t-i}|$, $i = 1,2$ and 3; the second integer indicates an independent variable SQRT(GARCH $(-1)) = \sigma_{t-1}$ and the third integer indicates the two independent variables $Resid(1) = \varepsilon_{t-1}$ and $Resid(-2) = \varepsilon_{t-1}$, which in general can be presented as ε_{t-j}, $j = 1$ and 2. This model can easily be extended by using the variance regressors.

8.5.4 General GARCH variance series for the component ARCH(1,1) model

This model has the options in Figure 8.17, so there are two alternative conditional variance models corresponding to the threshold term.

Based on the default options, namely the model without the threshold term, the conditional variance model has the following equation, if and only if the mean model

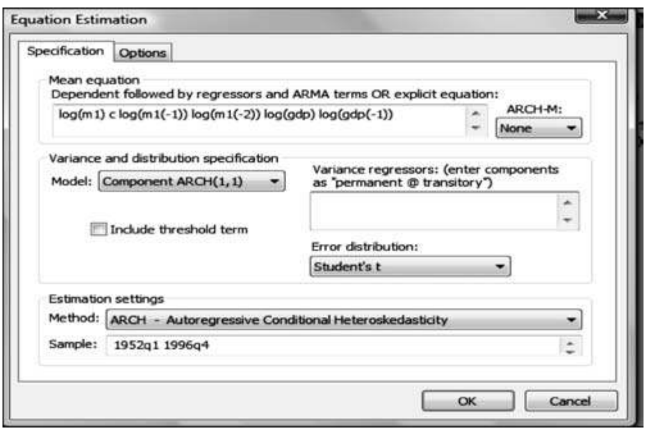

Figure 8.17 The options for the component ARCH(1,1) model

has five parameters $C(1)$ to $C(5)$:

$$Q_t = C(6) + C(7) * \{Q_{t-1} - C(6)\} + C(8) * (\varepsilon_{t-1}^2 - \sigma_{t-1}^2)$$
$$\sigma_t^2 = Qt + C(9) * (\varepsilon_{t-1}^2 - \sigma_{t-1}^2) + C(10) * (\varepsilon_{t-1}^2 - \sigma_{t-1}^2) \quad (8.35)$$

The equation of the conditional variance model with the threshold term is as follows:

$$Q_t = C(6) + C(7) * \{Q_{t-1} - C(6)\} + C(8) * (\varepsilon_{t-1}^2 - \sigma_{t-1}^2)$$
$$\sigma_t^2 = Q_t + \{C(9) + C(10) * (\varepsilon_{t-1} < 0)\} * (\varepsilon_{t-1}^2 - Q_{t-1}) + C(11) * (\sigma_{t-1}^2 - Q_{t-1}) \quad (8.36)$$

The extension of these models are the condition variance Component ARCH(1,1) models with variance regressors.

8.5.5 *Special notes on the GARCH variance series*

Corresponding to the *GARCH* variance series with general equations in (8.25) up to (8.36), there is every confidence that an infinite number of alternative ARCH models could be obtained by using any univariate time series models presented in the previous chapters, as well as other univariate models or multiple regressions, as the mean models. However, in this experimentation, many of the conditional variance models have been found to have insignificant independent variables with large p-values. Refer to the model in (8.24) and conduct additional data analysis using various integers a, b and c.

Based on the *rule of thumb*, if a conditional variance model has an insignificant independent variable with a p-value ≥ 0.20, then the conditional variance model should be modified. Corresponding to a p-value < 0.20, a conclusion can be made that the corresponding independent variable has a significant effect, either positive or negative, on the dependent variable at the 0.10 significant level. In other words, if all independent variables of any model have p-values < 0.20, then the models should be considered as acceptable or good models.

9

Additional testing hypotheses

9.1 Introduction

This chapter presents specific testing hypotheses, such as the unit root test, omitted and redundant variable tests and Ramsey's RESET test, in addition to the testing hypotheses, which have been presented in the previous chapters.

Previous chapters show that an analyst can have a very large number of alternative linear models, even ones based on only three or four time series. Hence there is uncertainty regarding the appropriateness or goodness of fit of all models presented or specified by a researcher. EViews provides an excellent interactive procedure or process for evaluating the equation specifications. However, it should be remembered or realized that any statistic, including the conclusion of a testing hypothesis, that is based only on sampled data should be used empirically with care. Considering the truth of any population model, as well as the true set of instrumental variables, the true mean and variance equations, note the statement 'In data analysis we must look on a very heavy emphasis on judgment' (Tukey, 1962, quoted by Gifi, 1990, p.23). Corresponding to this statement, it is suggested that there should be a good or strong theoretical and substantial base for any proposed model specification.

Furthermore, also note that the conclusion of a testing hypothesis to omit or delete an exogenous variable from the model cannot be taken absolutely or for granted. Corresponding to a testing hypothesis, Hample (1973, quoted by Gifi, 1990, p. 27) stated: 'Often in statistics one is using parametric models. . . . Classical (parametric) statistics derives results under the assumption that these models are strictly true. However, apart from simple discrete models perhaps, such models are never exactly true.' Therefore, it could be said that the conclusion of a testing hypothesis based on a model could not represent the true value(s) of the population parameters, especially if the model has a large number of independent variables.

Time Series Data Analysis Using EViews IGN Agung

Corresponding to the simple linear models, Agung (2006) has presented the application of linear models, either univariate or multivariate, starting from the simplest linear model, i.e. the cell-means models, based on either a single factor or multifactors. Refer to the cell-means models presented in Section 4.2.2. Even though this cell-means model could easily be justified as the true population model, the corresponding estimated regression function is highly dependent on the sampled data.

9.2 The unit root tests

9.2.1 Simple unit root test

Note that the simple unit root test described in this subsection is valid only if the series $\{Y_t\}$ is an AR(1) process. If the process $\{Y_t\}$ has a unit root, then the following first difference model should be applied, which can be considered as the simplest first difference time series model.

$$Y_t = Y_{t-1} + u_t \tag{9.1}$$

or

$$d(Y_t) = Y_t - Y_{t-1} = u_t \tag{9.2}$$

In practice, in order to test the unit root of a stochastic process $\{Y_t\}$, the following equation should be considered:

$$d(Y_t) = \delta Y_{t-1} + u_t \tag{9.3}$$

If the null hypothesis H_0: $\delta = 0$ (or the first autocorrelation $\rho_1 = 1$) is true, then a unit root is obtained, which indicates that the time series under consideration is nonstationary. Dickey and Fuller have computed the critical values of the t-statistic on the basis of Monte Carlo simulations. This t-statistic or test is known as the *Dickey–Fuller* (*DF*) *test*, which does not follow the usual t-distribution. The DF test is estimated by using three different equations, as presented in EViews. The three test equations are

$$d(Y_t) = c(1)Y_{t-1} + c(2) + u_t \tag{9.4}$$

$$d(Y_t) = c(1)Y_{t-1} + c(2) + c(3)\,@\text{TREND} + u_t \tag{9.5}$$

$$d(Y_t) = c(1)Y_{t-1} + u_t \tag{9.6}$$

The model in (9.4) with an intercept, indicated by the parameter $c(2)$, represents a random walk with drift, the model in (9.5) with a trend and an intercept represents a random walk with drift around a stochastic trend and the model in (9.6) represents a random walk.

In each case, the null hypothesis is $c(1) = \delta = 0$, which indicates that there is a unit root or the time series is nonstationary. The alternative hypothesis is $c(1) = \delta < 0$, which indicates that the time series is stationary. Therefore, if the null hypothesis is rejected, it means that Y_t is a stationary time series with a mean of $c(2)/c(1)$ in the case of a random walk with drift (9.4), that Y_t is a stationary time series around a deterministic trend in the case of a random walk with drift around a stochastic trend (9.5) and that Y_t is a stationary time series with zero mean in the case of a random walk (9.6).

The data analysis for testing the unit root can be done as follows:

(1) Click *View/Show* . . . and then enter the name of a defined or selected variable. By clicking *OK*, the values of the variables will be seen on the screen.
(2) Click *View/Unit Root Test*. . . and the window or options, as presented in Figure 9.1 will be seen. By clicking *OK*, it is easy to obtain the statistical results using the default option.

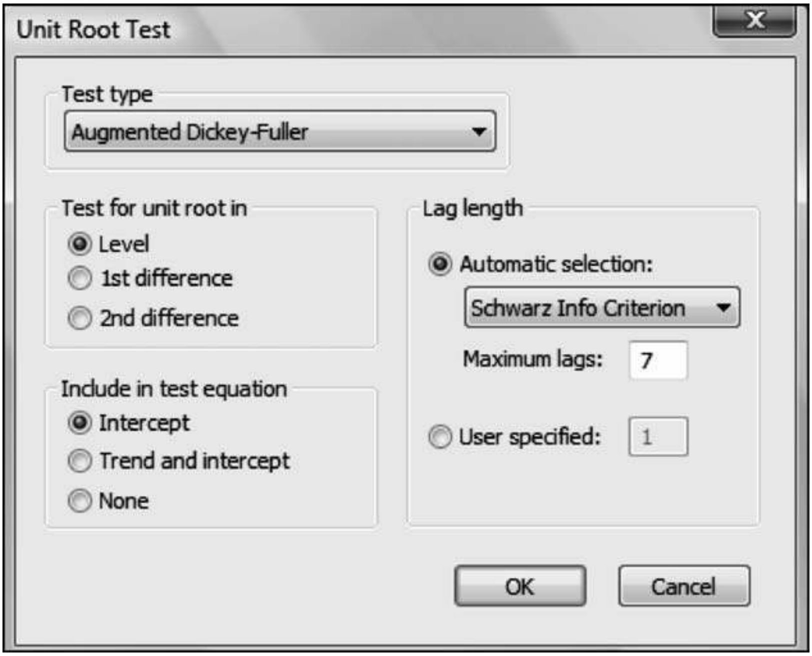

Figure 9.1 The default options for the unit root test

(3) Figure 9.2 presents the type test and criterion alternative options, so that alternative unit root tests can easily be conducted. Find the following examples.

Example 9.1. (Regression with a unit root) Figure 9.3 presents statistical results for testing that log(p) has a unit root. Based on this figure, the following notes and conclusions can be obtained:

(1) The null hypothesis of log(p) that has a unit root is accepted, that is H_0: $\delta = 0$, either using the *intercept* model in (9.4) or the *trend and intercept* model

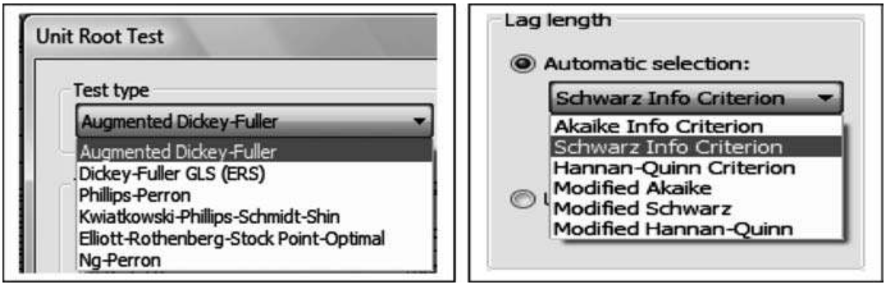

Figure 9.2 Test type and criterion options for the unit root test

Null Hypothesis: LOG(P) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=7)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.307405	0.9121
Test critical values:	1% level	-3.679322	
	5% level	-2.967767	
	10% level	-2.622989	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(P))
Method: Least Squares
Date: 01/08/08 Time: 12:34
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
LOG(P(-1))	-0.020952	0.068157	-0.307405	0.7609
C	0.130179	0.253087	0.514366	0.6112

R-squared	0.003488	Mean dependent var	0.052864
Adjusted R-squared	-0.033420	S.D. dependent var	0.149382
S.E. of regression	0.151857	Akaike info criterion	-0.865279
Sum squared resid	0.622637	Schwarz criterion	-0.770983
Log likelihood	14.54655	Hannan-Quinn criter.	-0.835747
F-statistic	0.094498	Durbin-Watson stat	1.949559
Prob(F-statistic)	0.760894		

(a) Model With Intercept In (9.4)

Null Hypothesis: LOG(P) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic based on SIC, MAXLAG=7)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.757701	0.2229
Test critical values:	1% level	-4.309824	
	5% level	-3.574244	
	10% level	-3.221728	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(P))
Method: Least Squares
Date: 01/08/08 Time: 12:36
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
LOG(P(-1))	-0.486623	0.176460	-2.757701	0.0105
C	1.480621	0.530818	2.789322	0.0098
@TREND(1951)	0.024531	0.008726	2.811247	0.0093

R-squared	0.235783	Mean dependent var	0.052864
Adjusted R-squared	0.176997	S.D. dependent var	0.149382
S.E. of regression	0.135518	Akaike info criterion	-1.061724
Sum squared resid	0.477495	Schwarz criterion	-0.920280
Log likelihood	18.39500	Hannan-Quinn criter.	-1.017425
F-statistic	4.010884	Durbin-Watson stat	1.641741
Prob(F-statistic)	0.030326		

(b) Model With Trend And Intercept In (9.5)

Figure 9.3 The unit root tests of log(P), using the models in (9.4) and (9.5)

in (9.5). In fact, H_0: $\delta = 0$ is also accepted based on the *random walk* model in (9.6).

(2) Based on the intercept model, log($p(-1)$) has an insignificant adjusted effect on the first difference, $d\log(p)$, at a significant level of $\alpha = 0.05$. Hence, using the usual or common t-test, the null hypothesis, H_0: $\delta = 0$, is also accepted, based on $t_0 = -0.307\,405$ with a p-value $= 0.7609$. However, based on the trend and intercept model, the null hypothesis is rejected, based on the usual t-statistic of $t_0 = -2.757\,71$ with a p-value $= 0.0105$. Based on these findings, the following notes are presented:
- The contradictory results occur because of the very high correlation between the trend variable, t, and log($p(-1)$). Figure 9.4 presents the statistical results based on a simple linear regression of log($p(-1)$) on the time t, with its scatter graph in Figure 9.5.

Dependent Variable: LOG(P(-1))
Method: Least Squares
Date: 11/24/06 Time: 16:27
Sample(adjusted): 1952 1980
Included observations: 29 after adjusting endpoints
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.947448	0.067622	43.58730	0.0000
T	0.046421	0.003499	13.26866	0.0000

R-squared	0.881191	Mean dependent var	3.690182
Adjusted R-squared	0.876790	S.D. dependent var	0.421063
S.E. of regression	0.147798	Akaike info criterion	-0.919462
Sum squared resid	0.589798	Schwarz criterion	-0.825166
Log likelihood	15.33221	F-statistic	200.2547
Durbin-Watson stat	1.061013	Prob(F-statistic)	0.000000

Figure 9.4 Simple linear regression of log($P(-1)$) on the time t

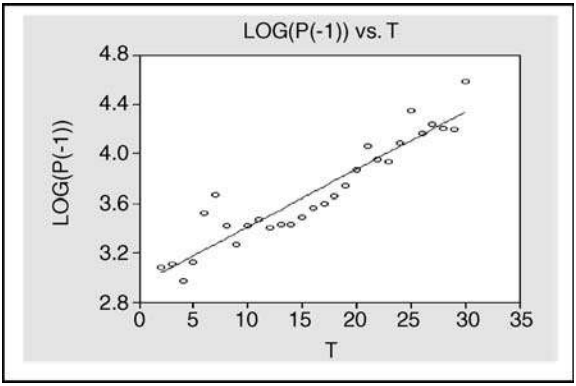

Figure 9.5 Scatter graph of (t, log($P(-1)$)) with the regression line

- The results can raise some questions, such as (i) should only the augmented Dickey–Fuller (ADF) test be obeyed and (ii) could the unit root problem be ignored when doing further data analysis, because the H_0: $\delta = 0$ is rejected based on the usual t-test? Note the following example.

(3) Considering the contradictory conclusions, based on the two linear models above, the problem can be generalized to a multiple regression with three or more exogenous variables. The adjusted effect of each independent or exogenous variable on the dependent or endogenous variable is unpredictable, because of the multicollinearity between the exogenous variables. Note that even though a pair of variables is not correlated substantively, the coefficient of correlation always has a quantitative value, and it is counted in the estimation of the model parameters. Refer to the special notes in Section 2.14. □

Example 9.2. (The White estimation methods) For a comparison, Figure 9.6 presents the statistical results based on two models of the first difference of log(p), that is $d$log(p), by using the Newey–West HAC... estimation method. Note the following conclusions and comments:

Dependent Variable: DLOG(P)
Method: Least Squares
Date: 01/08/08 Time: 13:06
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
LOG(P(-1))	-0.486623	0.131149	-3.710470	0.0010
C	1.480621	0.391254	3.784299	0.0008
@TREND	0.024531	0.007328	3.347853	0.0025

R-squared	0.235783	Mean dependent var	0.052864
Adjusted R-squared	0.176997	S.D. dependent var	0.149382
S.E. of regression	0.135518	Akaike info criterion	-1.061724
Sum squared resid	0.477495	Schwarz criterion	-0.920280
Log likelihood	18.39500	Hannan-Quinn criter.	-1.017425
F-statistic	4.010884	Durbin-Watson stat	1.641741
Prob(F-statistic)	0.030326		

(a) Result Based On The Model In (9.5)

Dependent Variable: DLOG(P)
Method: Least Squares
Date: 01/08/08 Time: 13:07
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
LOG(P(-1))	0.013888	0.005039	2.755986	0.0102

R-squared	-0.006277	Mean dependent var	0.052864
Adjusted R-squared	-0.006277	S.D. dependent var	0.149382
S.E. of regression	0.149850	Akaike info criterion	-0.924494
Sum squared resid	0.628738	Schwarz criterion	-0.877346
Log likelihood	14.40516	Hannan-Quinn criter.	-0.909727
Durbin-Watson stat	1.999012		

(b) Result Based On The Model In (9.6)

Figure 9.6 Statistical results based on two models in (a) (9.5) and (b) (9.6) of d log(P)

(1) Both models show that log($P(-1)$) has a significant effect on Dlog($P1$) with the p-values of 0.0010 and 0.0102 respectively. Based on these findings, it may be concluded that log($P1$) does not have a unit root, which is contradictory to the DF test presented in Example 9.1.
(2) Based on the model in (9.5), log($P(-1)$) has a significant negative effect on Dlog($P1$), but it has a significant positive effect based on the model in (9.6). □

Example 9.3. (Additional alternative models for *D*log(*p*)) Figure 9.7 presents statistical results based on the US domestic price of copper data using two alternative models, which show that log($p(-1)$) has a significant adjusted effect on $d$log($p(-1)$) using the standard t-test or the null hypothesis H_0: $\delta = 0$ is rejected. In addition to this conclusion, both models have lower values of AIC and SC statistics compared to the previous models, and the AR model, on the right-hand side in Figure 9.7, should be the best statistical results or estimates with a DW-statistic of 2.031. However, this model is an unusual AR model, since it has the indicator AR(2) without the indicator AR(1). □

9.2.2 Unit root test for higher-order serial correlation

The ADF approach controls for higher-order correlation by adding lagged difference terms of the dependent variable Y to the right-hand side of the

Dependent Variable: DLOG(P)
Method: Least Squares
Date: 01/08/08 Time: 13:26
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
LOG(P(-1))	-0.524300	0.124418	-4.214011	0.0003
LOG(G)	0.263846	0.175431	1.503987	0.1456
LOG(I)	-0.257034	0.241813	-1.062942	0.2984
LOG(L)	0.344993	0.131409	2.625326	0.0148
C	-0.709984	0.298931	-2.375076	0.0259

R-squared	0.557675	Mean dependent var	0.052864
Adjusted R-squared	0.483954	S.D. dependent var	0.149382
S.E. of regression	0.107310	Akaike info criterion	-1.470600
Sum squared resid	0.276372	Schwarz criterion	-1.234859
Log likelihood	26.32369	Hannan-Quinn criter.	-1.396768
F-statistic	7.564688	Durbin-Watson stat	1.530288
Prob(F-statistic)	0.000431		

Dependent Variable: DLOG(P)
Method: Least Squares
Date: 01/08/08 Time: 13:30
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 7 iterations
Newey-West HAC Standard Errors & Covariance (lag truncation=2)

	Coefficient	Std. Error	t-Statistic	Prob.
LOG(P(-1))	-0.331987	0.100187	-3.313688	0.0030
LOG(L)	0.293647	0.080333	3.655390	0.0013
C	-0.492156	0.165308	-2.977196	0.0067
AR(2)	-0.460806	0.201416	-2.287832	0.0317

R-squared	0.652569	Mean dependent var	0.060815
Adjusted R-squared	0.607252	S.D. dependent var	0.150579
S.E. of regression	0.094367	Akaike info criterion	-1.747296
Sum squared resid	0.204818	Schwarz criterion	-1.555320
Log likelihood	27.58850	Hannan-Quinn criter.	-1.690212
F-statistic	14.40006	Durbin-Watson stat	2.031354
Prob(F-statistic)	0.000017		

Figure 9.7 Statistical results based on two multiple linear regressions of $D(\log(p))$, based on the US_DPOC

regression. The general form of the equation can be written as

$$DY_t = \mu + \delta Y_{t-1} + \beta_1 DY_{t-1} + \beta_2 DY_{t-2} + \cdots + \beta_p DY_{t-p} + \varepsilon_t \qquad (9.7)$$

The null hypothesis of the series $\{Y_t\}$ has a unit root and will be the same as above, that is H_0: $\delta = 0$. On the other hand, other alternative tests for higher serial correlations also exist, as presented in Figures 9.1 and 9.2.

Example 9.4. (An application of a model in (9.7) and alternatives) Figure 9.8(a) presents statistical results based on the model in (9.7) for $p = 2$, which show that the null hypothesis H_0: $\delta = 0$ is accepted, based on the MacKenon critical criteria. Therefore, these results show that the series $\{Y_t\}$ is nonstationary.

For a comparison, Figure 9.8(b) presents an alternative test, namely the Phillips–Perron (PP) test with bandwidth 2 (fixed using the Barlett kernel), which also shows that the series is nonstationary. Other alternative tests could be conducted easily. If at least three test statistics show that a series is nonstationary, then the conclusions could be given with confidence. □

9.2.3 *Comments on the unit root tests*

If the unit root test is conducted for any single endogenous variables, say Y, in the previous examples, as well as the previous chapters, the conclusion may be assumed that the series Y_t is nonstationary. Based on those findings, should the models presented in this chapter, as well as all the models presented in the previous chapters, be modified? This would be the same for other single time series, such as the first- or higher-order differences of any endogenous variables Y and $\log(Y)$. However, note that the first difference $d\log(y_t)$ has quite a

Null Hypothesis: Y has a unit root
Exogenous: Constant
Lag Length: 2 (Fixed)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		1.139303	0.9968
Test critical values:	1% level	-3.699871	
	5% level	-2.976263	
	10% level	-2.627420	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 01/08/08 Time: 13:39
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
Y(-1)	0.115126	0.101050	1.139303	0.2663
D(Y(-1))	-0.312770	0.227710	-1.373542	0.1828
D(Y(-2))	-0.536452	0.268033	-2.001442	0.0573
C	-0.422027	4.467434	-0.094467	0.9256

R-squared	0.173652	Mean dependent var	3.028519
Adjusted R-squared	0.065867	S.D. dependent var	8.440537
S.E. of regression	8.157826	Akaike info criterion	7.171786
Sum squared resid	1530.653	Schwarz criterion	7.363762
Log likelihood	-92.81911	Hannan-Quinn criter.	7.228871
F-statistic	1.611098	Durbin-Watson stat	2.178053
Prob(F-statistic)	0.214157		

(a) An ADF Unit Root Test

Null Hypothesis: Y has a unit root
Exogenous: Constant
Bandwidth: 2 (Fixed using Bartlett kernel)

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		1.097560	0.9965
Test critical values:	1% level	-3.679322	
	5% level	-2.967767	
	10% level	-2.622989	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	64.69017
HAC corrected variance (Bartlett kernel)	40.24475

Phillips-Perron Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 01/08/08 Time: 13:56
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
Y(-1)	0.035436	0.081369	0.435492	0.6667
C	1.192584	3.879400	0.307415	0.7609

R-squared	0.006975	Mean dependent var	2.741724
Adjusted R-squared	-0.029804	S.D. dependent var	8.214083
S.E. of regression	8.335588	Akaike info criterion	7.145417
Sum squared resid	1876.015	Schwarz criterion	7.239714
Log likelihood	-101.6086	Hannan-Quinn criter.	7.174950
F-statistic	0.189653	Durbin-Watson stat	2.313200
Prob(F-statistic)	0.666667		

(b) A PP Unit Root Test

Figure 9.8 The unit root tests based on (a) ADF and (b) PP test statistics

different meaning from $\log(y_t)$. Refer to the return rate models presented in Section 5.6.

If it is absolutely certain that stationary variables should always be used in a time series model, then all other variables should also be tested before developing a model. Afterwards, a defined model could use only stationary variables, either dependent or independent variables. If this is the case, then the original variables will not be modeled, only other types of variables.

Furthermore, in many cases it has been recognized that researchers were not following this process, but kept using the original time series variables. On the other hand, a defined (population) model could never represent what really happens in the corresponding population. Hence, it is suggested that best knowledge and judgment should be used to define several alternative models, not only one.

In this experimentation, it was found that EViews will directly provide a statement of the nonstationary condition if a model should be modified. For this reason, it could be said that all models presented in this book should be acceptable time series models, as long as there are no '*Nonstationary*' or '*Convergence not achieved after ... iterations*' messages.

9.3 The omitted variables tests

Suppose the following initial regression is given:

$$y\ c\ x1\ t\ ar(1) \tag{9.8}$$

As an example to test for the omitted variables $x2$ and $x3$, two nested models are in fact compared, the model in (9.8) and the following model:

$$y\ c\ x1\ x2\ x3\ t\ ar(1) \tag{9.9}$$

In this test, in fact, two regressions are considered with the following explicit equations:

$$\begin{aligned}&\text{Full model}: Y = c(11) + c(12)X_1 + c(13)X_2 + c(14)X_3 + c(15)^{*}t + [ar(1) = c(16)]\\&\text{Reduced model}: Y = c(21) + c(22)X_1 + c(23)^{*}t + [ar(1) = c(24)]\end{aligned} \tag{9.10}$$

The hypothesis can be written as

$$\begin{aligned}&H_0 : \text{Reduced model or } H_0 : C(13) = C(14) = 0\\&H_1 : \text{Full model or } H_1 : \text{Otherwise}\end{aligned} \tag{9.11}$$

After obtaining the result of the reduced model on the screen, this hypothesis can be tested by selecting *View/Coefficient/Omitted Variables – Likelihood Ratio…* and then entering the list '$x2$ $x3$' in the dialog. Note the following example.

Example 9.5. (Omitted variables and joint effects tests) By using the six variables *P*, *A*, *G*, *H*, *I* and *L* in the US_DPOC data, consider the equation specification

$$\log(p)\ c\ \log(a)\ \log(l)\ ar(1)\ ar(2) \tag{9.12}$$

for conducting an omitted variable test of the three variables log(*g*), log(*h*) and log(*i*). The process of the analysis is as follows:

(1) Conduct the regression analysis by using the equation specification in (9.12).
(2) Having the statistical results on the screen, select *View/Coefficient Tests/ Omitted Variables –Likelihood Ratio….*
(3) Then enter the variables list, log(*g*) log(*h*) log(*i*), and by clicking *OK*, the statistical results in Figure 9.9 will be obtained. Based on the LR chi-squared-statistic of 6.152 392 with $df = 3$ and a p-value $= 0.1044$, it can be concluded that the three omitted variables have an insignificant effect on log(*p*), at the 0.10 significant level. Therefore, in a statistical sense, there is no need to use all of the three variables as additional independent variables of the model. However, one or two of these variables may be used.
(4) In fact, by testing each of these variables using the omitted variables test, it was found that log(*g*) and log(*i*) are significant, based on the chi-squared-statistic with p-values of 0.0274 and 0.0131 respectively.

Omitted Variables: LOG(G) LOG(H) LOG(I)

F-statistic	1.638255	Prob. F(3,20)	0.2123
Log likelihood ratio	6.152392	Prob. Chi-Square(3)	0.1044

Test Equation:
Dependent Variable: LOG(P)
Method: Least Squares
Date: 01/09/08 Time: 20:51
Sample: 1953 1980
Included observations: 28
Convergence achieved after 14 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.529450	0.533238	-2.868231	0.0095
LOG(L)	0.306178	0.116329	2.632006	0.0160
LOG(A)	0.412663	0.247910	1.664568	0.1116
LOG(G)	-0.003661	0.345754	-0.010589	0.9917
LOG(H)	0.000389	0.011962	0.032558	0.9743
LOG(I)	0.456581	0.447053	1.021313	0.3193
AR(1)	0.777510	0.198783	3.911359	0.0009
AR(2)	-0.477150	0.201219	-2.371295	0.0279

R-squared	0.959118	Mean dependent var	3.765864
Adjusted R-squared	0.944809	S.D. dependent var	0.428531
S.E. of regression	0.100674	Akaike info criterion	-1.518909
Sum squared resid	0.202704	Schwarz criterion	-1.138279
Log likelihood	29.26472	Hannan-Quinn criter.	-1.402547
F-statistic	67.03028	Durbin-Watson stat	1.871026
Prob(F-statistic)	0.000000		

Inverted AR Roots	.39-.57i	.39+.57i

Figure 9.9 An omitted variables test based on the model in (9.12)

(5) For a comparison, the joint effects will be found of the three variables log(g), log(h) and log(i) on log(p), based on an AR(2) translog linear model as follows:

$$\begin{aligned}\log(P) &= c(1) + c(2)\log(A) + c(3)\log(L) + c(4)\log(G) + c(5)\log(H)\\ &\quad + c(6)\log(I) + [ar(1) = c(7),\ ar(2) = c(8)]\end{aligned} \qquad (9.13)$$

In this case, the following hypothesis using the Wald test will be tested, which has been demonstrated in the previous examples:

$$\begin{aligned}H_0 &: C(4) = C(5) = C(6) = 0\\ H_1 &: \text{Otherwise}\end{aligned} \qquad (9.14)$$

It was found that, at a significant level of 0.10, the null hypothesis is accepted based on the F-statistic of 2.295 127 with $df = (3, 20)$ and a p-value = 0.1088, but it is rejected based on the chi-squared-statistic of 6.885 382 with $df = 3$ and a p-value = 0.0756, as presented in Figure 9.10.

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	2.295124	(3, 20)	0.1088
Chi-square	6.885373	3	0.0756

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(4)	-0.003661	0.345754
C(5)	0.000389	0.011962
C(6)	0.456581	0.447053

Restrictions are linear in coefficients.

Figure 9.10 A joint effects test based on the model in (9.13)

(6) Looking at the contradictory conclusions based on the chi-squared test presented in Figures 9.9 and 9.10 raises a question: What are their cause factors? From the present point of view, the hypothesis considered represents different statuses, based on the omitted variables test and the Wald test. Corresponding to the omitted variables test, the hypothesis could be considered as an external hypothesis, since the tested variables are not in the model. This questions whether all external variables considered should be used as additional independent variables of the model or not.

On the other hand, corresponding to the Wald test, the tested variables are in the model, so the hypothesis could be considered as an internal hypothesis, which indicates whether or not all tested variables can be deleted in order to obtain a reduced model. In this case, the conclusion of the Wald test indicates that at most two out of the three tested variables may be deleted. Therefore, the trial-and-error methods should be used to obtain alternative reduced models, as well as the best reduced model. Do this as an exercise. □

Example 9.6. (Omitted variables test in the instrumental models) For any instrumental models, with or without trend, the omitted variables tests can also be conducted. As an illustration, the following instrumental model with trend is considered, which has been presented in Figure 7.15:

$$\begin{aligned} Y_1 = \; & c(1) + c(2)X_1 + c(3)*t + c(4)*X_2 + c(4)X_1*X_2 + [ar(1) = c(6)] \\ & \text{Instrument list } c\; y1(-1)x1(-1)x2(-1)x3\; x3(-1)\; t \end{aligned} \tag{9.15}$$

Omitted Variables: X2 X1*X2

F-statistic	2.389302	Prob. F(2,23)	0.1141

Test Equation:
Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/10/08 Time: 08:55
Sample: 1952 1980
Included observations: 29
Convergence achieved after 14 iterations
Instrument list: C Y1(-1) X1(-1) X2(-1) X3 X3(-1) T
Lagged dependent variable & regressors added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	-42.55673	42.11097	-1.010585	0.3227
X1	0.229161	0.125307	1.828799	0.0804
T	-7.042646	5.358497	-1.314295	0.2017
X2	0.877458	0.813308	1.078876	0.2918
X1*X2	-0.001051	0.000648	-1.623545	0.1181
AR(1)	0.779625	0.149594	5.211598	0.0000

R-squared	0.914219	Mean dependent var	31.28655
Adjusted R-squared	0.895571	S.D. dependent var	12.73949
S.E. of regression	4.116825	Sum squared resid	389.8097
F-statistic	52.93914	Durbin-Watson stat	2.395107
Prob(F-statistic)	0.000000	Second-Stage SSR	58.12121

Inverted AR Roots .78

(a) With The Option "Lagged..."

Omitted Variables: X2 X1*X2

F-statistic	3.366321	Prob. F(2,23)	0.0522

Test Equation:
Dependent Variable: Y1
Method: Two-Stage Least Squares
Date: 01/10/08 Time: 08:57
Sample: 1952 1980
Included observations: 29
Convergence achieved after 2 iterations
Instrument list: C Y1(-1) X1(-1) X2(-1) X3 X3(-1) T
Lagged dependent variable & regressors not added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
C	31.31306	20.81133	1.504616	0.1460
X1	0.049199	0.046341	1.061674	0.2994
T	0.458322	1.481858	0.309289	0.7599
X2	-0.483037	0.316385	-1.526737	0.1405
X1*X2	-8.77E-05	0.000259	-0.338025	0.7384
AR(1)	0.330155	0.223436	1.477626	0.1531

R-squared	0.961664	Mean dependent var	31.28655
Adjusted R-squared	0.953330	S.D. dependent var	12.73949
S.E. of regression	2.752134	Sum squared resid	174.2076
F-statistic	117.6654	Durbin-Watson stat	1.980093
Prob(F-statistic)	0.000000	Second-Stage SSR	88.11549

Inverted AR Roots .33

(b) Without The Option "Lagged ..."

Figure 9.11 The omitted variables tests based on the models in (9.16), (a) with and (b) without the option 'lagged …'

In this case, the list of omitted variables considered is 'X_2 X_1*X_2.' Hence, using the omitted variables test, the following steps should be applied:

(1) Select *Quick/Estimates Equation/TSLS Method…* and then enter the following equation specifications:

$$\begin{aligned} &y1\ c\ x1\ t\ ar(1) \\ &\text{Instrument list}: c\, y1(-1)x1(-1)x2(-1)x3\ x3(-1)t \end{aligned} \tag{9.16}$$

(2) With the statistical results on the screen, select *View/Coefficient Tests/ Omitted Variables –Likelihood Ratio…*
(3) Then enter the two variables $x2$, $x1*x2$ in the window and click *OK*. The results with the option 'Include lagged …' appear as in Figure 9.11(a) and without the option as in Figure 9.11(b).
(4) Corresponding to the number of instrumental variables, the model in Figure 9.11(b) has less instrumental variables, so is a simple model. For this reason this model is preferred.
(5) Corresponding to the other variables in US_DPOC, the omitted variables tests could also be conducted for each of the main factors, as well as the two-way and three-way interaction factors. Table 9.1 presents the p-values of the omitted variables tests (or the F-test) for each of the variables considered. Based on this table the following notes and conclusions are made:
   - Each of the variables with a p-value < 0.20 (by rule of thumb) should be considered as a candidate for an additional variable of the main model

Table 9.1 The p-values of the omitted variable tests (F-statistic) for the instrumental model in Figure 9.11(b)

Number	Omitted variable	p-value	Number	Omitted variable	p-value
1	$x2$	0.0141	13	$x3^*y2$	0.1496
2	$x3$	0.5575	14	t^*x1^*x2	0.3751
3	$y2$	1.0000	15	t^*x1^*x3	1.0000
4	t^*x1	0.0145	16	t^*x1^*y2	0.5575
5	t^*x2	0.0264	17	t^*x2^*x3	0.2789
6	t^*x3	0.1632	18	t^*x2^*y2	0.1396
7	t^*y2	0.3119	19	t^*x3^*y2	0.3852
8	$x1^*x2$	0.0520	20	$x1^*x2^*x3$	1.0000
9	$x1^*x3$	0.3322	21	$x1^*x2^*y2$	0.3503
10	$x1^*y2$	0.4068	22	$x1^*x3^*y2$	0.3914
11	$x2^*x3$	0.0989	23	$x2^*x3^*y2$	0.2414
12	$x2^*y2$	0.1473			

in (9.16), since the corresponding variable has a significant positive or negative effect on the independent variable, at the 0.10 significant level.

- Therefore, in this case, there are one main factor, seven two-way interaction factors and one three-way interaction factors, as presented using bold italic in Table 9.1. Either one or two of these variables may be used as additional independent variable(s) of the main model in (9.16). However, the choice is a matter of personal judgment.
- By using lagged variables, as well as transformed variables, many more alternative models and omitted variables tests could be obtained. □

Example 9.7. (Omitted variables test and correlation analysis) Corresponding to the results of the omitted tests presented in Table 9.2, as an illustration Table 9.2 presents a correlation matrix between the independent variable $y1$ with selected omitted variables in Table 9.1. Based on the p-values of the correlations in Table 9.2, it can be concluded that each of the variables $x2$, $x3$, t^*x2, t^*x3, t^*x2^*x3 and $x2^*x3^*y2$ is significantly positive correlated with the dependent variable $y1$.

In fact, it was found that all of the omitted variables in Figure 9.11 are significantly correlated with $y1$. This indicates that there could be an acceptable

Table 9.2 The p-values of the correlation tests between $Y1$ in model (9.16) with selected omitted variables in Figure 9.11

	$Y1$	$X2$	$X3$	T^*X2	T^*X3	T^*X2^*Y2	$X2^*X3^*Y2$
Correlation	1	0.768 719	0.782 616	0.844 775	0.868 539	0.928 601	0.903 109
p-value	—	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 13:56
Sample: 1951 1980
Included observations: 30

	Coefficient	Std. Error	t-Statistic	Prob.
C	20.73049	1.164479	17.80237	0.0000
T*X2*Y2	9.54E-05	7.21E-06	13.24160	0.0000

R-squared	0.862300	Mean dependent var	30.87700
Adjusted R-squared	0.857382	S.D. dependent var	12.71732
S.E. of regression	4.802676	Akaike info criterion	6.040564
Sum squared resid	645.8394	Schwarz criterion	6.133977
Log likelihood	-88.60846	Hannan-Quinn criter.	6.070448
F-statistic	175.3399	Durbin-Watson stat	0.642911
Prob(F-statistic)	0.000000		

(a) A Simple Linear Regression

Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 13:54
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 55 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	805.2997	38308.95	0.021021	0.9834
T*X2*Y2	2.76E-05	1.93E-05	1.431720	0.1641
AR(1)	0.998267	0.085133	11.72597	0.0000

R-squared	0.943442	Mean dependent var	31.28655
Adjusted R-squared	0.939091	S.D. dependent var	12.73949
S.E. of regression	3.144061	Akaike info criterion	5.226805
Sum squared resid	257.0132	Schwarz criterion	5.368049
Log likelihood	-72.78577	Hannan-Quinn criter.	5.270904
F-statistic	216.8530	Durbin-Watson stat	0.972005
Prob(F-statistic)	0.000000		
Inverted AR Roots	1.00		

(b) An AR(1) Model

Figure 9.12 Statistical results based on (a) an SLR of $Y1$ on $t^* X2^* Y2$ and (b) an AR(1) model of $Y1$ on $t^* X2^* Y2$

simple linear regression (SLR) of $Y1$ on each of the omitted variables. However, the SLR is not an acceptable model for the time series data set. As an illustration, Figure 9.12 presents statistical results based on an SLR of $Y1$ on $T^* X2^* X3$ and its corresponding AR(1) model.

Note that $T^* X2^* Y2$ has a significant effect on $Y1$, based on the standard t-test in Figure 9.12(a), as presented in Table 9.2, but it is insignificant when based on the AR(1) model in Figure 9.12(b). However, at a significant level of 0.10, $T^* X2^* Y2$ has a significant positive effect on $Y1$, with a p-value $= 0.1641/2 = 0.082\,05$.

Furthermore, also note that the standard t-test is valid under the assumption that the sample is a random sample. For this reason, it could be said that the omitted variables test is quite different from the correlation test based on any time series data.

As an exploration study, it was found that the residual of the model in Figure 9.11(b), namely RESID01, has an insignificant correlation with each of the omitted variables in Table 9.1, with such a large p-value. Table 9.3 presents some of the correlation tests. Based on this finding, each of the omitted variables could be considered as a candidate for an instrumental variable of the mean model in (9.16). □

Table 9.3 The p-values of the correlation tests between $Y1$ in model (9.16) with selected omitted variables in Figure 9.11

	RESID01	$X2$	$X3$	$T^* X2$	$T^* X3$	$T^* X2^* Y2$	$X2^* X3^* Y2$
Correlation	1	0.061 492	0.071 344	0.064 717	0.085 968	0.050 197	0.042 194
Probability	—	0.7468	0.7079	0.734	0.6515	0.7922	0.8248

9.4 Redundant variables test (RV-test)

This test can be done by selecting *View/Coefficient Tests/Redundant Variables-Likelihood Ratio ....* In this test, a full model and its reduced model are considered. Hence, they are nested models. Note the following examples, which are associated with the models presented in the previous examples.

Example 9.8. (An RV-test for an AR(2) translog linear model) Corresponding to the model in Example 9.5, the following list of variables is used as the initial equation specification, which is the same as the model in (9.13):

$$\log(p)\ c\,\log(l)\ \log(a)\ \log(g)\ \log(h)\ \log(i)\ ar(1)\ ar(2) \tag{9.17}$$

Then by entering the list

$$\log(g)\ \log(h)\ \log(i)$$

in the dialog, the output in Figure 9.13 is obtained.

Redundant Variables: LOG(G) LOG(H) LOG(I)

F-statistic	1.638255	Prob. F(3,20)	0.2123
Log likelihood ratio	6.152392	Prob. Chi-Square(3)	0.1044

Test Equation:
Dependent Variable: LOG(P)
Method: Least Squares
Date: 01/11/08 Time: 14:27
Sample: 1953 1980
Included observations: 28
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.998851	0.491150	-2.033697	0.0537
LOG(A)	0.560148	0.145450	3.851146	0.0008
LOG(L)	0.472732	0.084427	5.599275	0.0000
AR(1)	0.841159	0.195408	4.304634	0.0003
AR(2)	-0.297911	0.195773	-1.521716	0.1417

R-squared	0.949072	Mean dependent var	3.765864
Adjusted R-squared	0.940215	S.D. dependent var	0.428531
S.E. of regression	0.104780	Akaike info criterion	-1.513466
Sum squared resid	0.252516	Schwarz criterion	-1.275573
Log likelihood	26.18853	Hannan-Quinn criter.	-1.440740
F-statistic	107.1537	Durbin-Watson stat	1.942907
Prob(F-statistic)	0.000000		

Inverted AR Roots	.42+.35i	.42-.35i

Figure 9.13 A redundant variable test based on the model in (9.17)

Note that this output, the *F*-statistic and LR chi-squared-statistic of the redundant variables test in particular, is exactly the same as the output of the same model in Example 9.5, specifically the p-values of the F-statistic, as well as the log likelihood ratio and chi-squared-statistic in Figure 9.9. Furthermore, here a full model is used as the initial regression, but in the previous example a reduced model was used as the initial regression. □

9.5 Nonnested test (NN-test)

All of the testing hypotheses based on the F-statistic, t-statistic or Wald statistic are related to nested models, namely a full model and its reduced model. In this section a pair of nonnested models having the same dependent variable are considered. For the testing, Davidson and MacKenon (1993, in EViews 6 User's Guide II, p. 179) proposed the J-test. In this case, an hypothesis is considered with a general form as follows:

$$\begin{aligned} &H_1 : \text{Model-1} : y = f(x1, x2, \ldots, xp) \\ &H_2 : \text{Model-2} : y = g(z1, z2, \ldots, zq) \end{aligned} \tag{9.18}$$

where both models are nonnested and some or all of the X-variables should be unequal to the Z-variable.

To test the hypothesis, the statistical results are studied or observed based on the following two models:

$$\begin{aligned} &\text{Model-1}a : y = f(x1, x2, \ldots, xp, \hat{g}) \\ &\text{Model-2}a : y = g(z1, z2, \ldots, zq, \hat{f}) \end{aligned} \tag{9.19}$$

where $\hat{f}$ and $\hat{g}$ are the fitted values variables of the model-1 and model-2 respectively. Note that each of the fitted values $\hat{f}$ and $\hat{g}$ become independent variables of the new models, namely the model-2a and model-1a in (9.19) respectively.

The conclusion of the testing hypothesis is completely dependent on whether the fitted values $\hat{f}$ or $\hat{g}$ have an insignificant effect or not. If $\hat{f}$ has a significant adjusted effect on the dependent variable of model-2a, then model-2 is accepted or model-1 is rejected, and if $\hat{g}$ has a significant adjusted effect on the dependent variable of model-1a, then model-1 is accepted or model-2 is rejected. For illustrative purposes, find the following examples.

Example 9.9. (Nonnested basic regression models) Here, the following pair of nonnested basic regression models with an endogenous variable, Y_1, or hypotheses are considered:

$$\begin{aligned} &H_1 : y1_t = c(11) + c(12)^*x1_t + c(13)^*x1_{t-1} + \mu 1_t \\ &H_2 : y1_t = c(21) + c(22)^*x1_t + c3(23)^*x2_t + \mu 2_t \end{aligned} \tag{9.20}$$

The processes for selecting one of the two models are as follows:

(1) By applying each of the models, the corresponding fitted value variables could be produced or generated, namely *F_head* = *Fh* = *y*1*h*1 and *G_head* = *Gh* = *y*1*h*2 respectively. For example, to generate *Fh* = *y*1*h*1, the dialogs are as follows:
   - Select *Quick/Estimation Equation . . . OK*; then enter the list '*y*1 *c x*1 *x*1 (−1) . . .' and click *OK*. . ., giving the output of the regression analysis.
   - Click *Generate Series . . . or Genr* and then enter the equation

$$Fh = c(1) + c(2)^*x1) + c(3)^*x1(-1) \tag{9.21}$$

   - Similarly, the fitted values *Gh* can be obtained.

(2) Do a data analysis using each of the nonnested models with the additional independent variable *Gh* for the first model and *Fh* for the second model.
(3) As usual, by selecting *Quick/Estimation Equation* . . . and entering the list

$$y1\ c\ x1\ x1(-1)\ Gh \tag{9.22}$$

the result in Figure 9.14 is obtained.
(4) By selecting *Quick/Estimation Equation*. . . and entering the list.

$$y1\ c\ x1\ x2\ Fh \tag{9.23}$$

the result in Figure 9.15 is obtained.
(5) Since *Gh* = *y*1*h*2 has a significant effect on the dependent variable of model-1, then model-1 is rejected. Similarly, since *Fh* = *y*1*h*1 has a significant effect on the dependent variable of model-2, then model-2 is rejected.

Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 14:56
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.061541	2.134051	-0.497430	0.6232
X1	-0.017632	0.006772	-2.603809	0.0153
X1(-1)	0.017954	0.007098	2.529531	0.0181
GH	1.075772	0.148361	7.251066	0.0000

R-squared	0.964215	Mean dependent var	31.28655
Adjusted R-squared	0.959921	S.D. dependent var	12.73949
S.E. of regression	2.550421	Akaike info criterion	4.837836
Sum squared resid	162.6162	Schwarz criterion	5.026429
Log likelihood	-66.14863	Hannan-Quinn criter.	4.896901
F-statistic	224.5385	Durbin-Watson stat	0.706464
Prob(F-statistic)	0.000000		

Figure 9.14 Statistical results based on the model in (9.22)

Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 15:04
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	15.90662	5.236511	3.037638	0.0055
X1	0.015207	0.006526	2.330249	0.0282
X2	-0.282137	0.038910	-7.251066	0.0000
FH	0.868371	0.343294	2.529531	0.0181

R-squared	0.964215	Mean dependent var	31.28655
Adjusted R-squared	0.959921	S.D. dependent var	12.73949
S.E. of regression	2.550421	Akaike info criterion	4.837836
Sum squared resid	162.6162	Schwarz criterion	5.026429
Log likelihood	-66.14863	Hannan-Quinn criter.	4.896901
F-statistic	224.5385	Durbin-Watson stat	0.706464
Prob(F-statistic)	0.000000		

Figure 9.15 Statistical results based on the model in (9.23)

(6) Since both model-1 and model-2 are rejected, it can be concluded that the data does not support both models in the hypothesis. □

Example 9.10. (Nonnested AR(1) models) As a modification of the nonnested models in (9.20), here nonnested AR(1) models are considered in the following hypothesis:

$$\begin{aligned} H_1 &: y1_t = c(11) + c(12)^* x1_t + c(13)^* x1_{t-1} + c(14)^* \mu 1_{t-1} + \varepsilon 1_t \\ H_2 &: y1_t = c(21) + c(22)^* x1_t + c(23)^* x2_t + c(14)^* \mu 2_{t-1} + \varepsilon 2_t \end{aligned} \tag{9.24}$$

By using the same process as above new variables of their fitted values can be generated, namely $GH\_AR = y1h1$ and $FH\_AR = y1h2$. Finally, the statistical results based on two AR(1) models are obtained, as in Figure 9.16. Based on these results, we can conclude that the data does not support both models. □

Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 15:29
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 10 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2.477780	3.227385	0.767736	0.4505
X1	-0.016408	0.005307	-3.091962	0.0051
X1(-1)	0.020849	0.004014	5.194501	0.0000
GH_AR	0.843179	0.213854	3.942781	0.0006
AR(1)	0.652618	0.152201	4.287875	0.0003

R-squared	0.981458	Mean dependent var	31.71071
Adjusted R-squared	0.978234	S.D. dependent var	12.76302
S.E. of regression	1.882988	Akaike info criterion	4.264029
Sum squared resid	81.54977	Schwarz criterion	4.501923
Log likelihood	-54.69640	Hannan-Quinn criter.	4.336755
F-statistic	304.3602	Durbin-Watson stat	1.510744
Prob(F-statistic)	0.000000		
Inverted AR Roots	.65		

Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 15:30
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	18.23170	4.567583	3.991543	0.0006
X1	0.009291	0.004874	1.906182	0.0692
X2	-0.245947	0.062377	-3.942888	0.0006
FH_AR	0.909922	0.175172	5.194461	0.0000
AR(1)	0.652616	0.152201	4.287861	0.0003

R-squared	0.981458	Mean dependent var	31.71071
Adjusted R-squared	0.978234	S.D. dependent var	12.76302
S.E. of regression	1.882988	Akaike info criterion	4.264029
Sum squared resid	81.54977	Schwarz criterion	4.501923
Log likelihood	-54.69640	Hannan-Quinn criter.	4.336755
F-statistic	304.3602	Durbin-Watson stat	1.510749
Prob(F-statistic)	0.000000		
Inverted AR Roots	.65		

Figure 9.16 Statistical results for testing the hypothesis in (9.24)

Dependent Variable: LOG(Y1)
Method: Least Squares
Date: 01/11/08 Time: 16:04
Sample: 1951 1980
Included observations: 30

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.989780	0.272356	3.634140	0.0012
LOG(X1)	1.100876	0.161845	6.802058	0.0000
LOG(X2)	-1.337836	0.216141	-6.189651	0.0000
LY1H2	0.301855	0.194279	1.553717	0.1323

R-squared	0.912392	Mean dependent var	3.368254
Adjusted R-squared	0.902283	S.D. dependent var	0.334108
S.E. of regression	0.104441	Akaike info criterion	-1.556818
Sum squared resid	0.283608	Schwarz criterion	-1.369991
Log likelihood	27.35226	Hannan-Quinn criter.	-1.497050
F-statistic	90.25844	Durbin-Watson stat	0.685343
Prob(F-statistic)	0.000000		

Dependent Variable: LOG(Y1)
Method: Least Squares
Date: 01/11/08 Time: 16:09
Sample: 1951 1980
Included observations: 30

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.289992	0.316428	0.916455	0.3679
LOG(Y2)	0.200388	0.145769	1.374700	0.1810
LOG(X3)	-0.107498	0.098761	-1.088464	0.2864
LY1H1	0.884707	0.130832	6.762165	0.0000

R-squared	0.910749	Mean dependent var	3.368254
Adjusted R-squared	0.900451	S.D. dependent var	0.334108
S.E. of regression	0.105416	Akaike info criterion	-1.538242
Sum squared resid	0.288925	Schwarz criterion	-1.351416
Log likelihood	27.07363	Hannan-Quinn criter.	-1.478475
F-statistic	88.43783	Durbin-Watson stat	0.663702
Prob(F-statistic)	0.000000		

Figure 9.17 Statistical results for testing the hypothesis (9.25)

Example 9.11. (Nonnested translog linear model) In this example the following hypothesis is considered:

$$\begin{aligned} H_1 &: \log(y1_t) = c(11) + c(12)^*\log(x1_t) + c(13)^*\log(x2_t) + \mu 1_t \\ H_2 &: \log(y1_t) = c(21) + c(22)^*\log(y2_t) + c(23)^*\log(x3_t) + \mu 2_t \end{aligned} \tag{9.25}$$

By using the same process as in Example 9.9, finally the statistical Figure 9.17 is obtained, where the independent variables $ly1h1$ and $ly1h2$ are the variables of the fitted values of model-1 and model-2 (H_1 and H_2) respectively.

Since $ly1h2$ has an insignificant adjusted effect, but $ly1h1$, then model-1 (or H_1) is accepted with a p-value $= 0.1323$, at a significant level of $\alpha = 0.10$. Therefore, it can be concluded that the data supports model-1 in (9.25). □

9.6 The Ramsey RESET test

The main objective of the Ramsey RESET test is to test or select an additive model and a multiplicative model, such as follows:

$$\begin{aligned} &\text{Additive model}: y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \\ &\text{Multiplicative model}: y = \lambda_0 X_1^{\lambda 1} X_2^{\lambda 2} + \varepsilon \end{aligned} \tag{9.26}$$

By using a Taylor approximation, the multiplicative function will yield an expression involving powers and cross-products of the explanatory variables. Ramsey suggested including powers of the predicted values of the dependent variable as additional independent variables of the model. A set of the predicted values could be presented as

$$\{\hat{y}_2, \hat{y}_3, \ldots, \hat{y}_k, \ldots\} \tag{9.27}$$

To apply the test, select *View/Stability Tests/Ramsey Test* ... and specify the number of fitted terms to include in the test regression. The number of fitted terms represents the powers of the fitted values from the original regression, starting with the square or the second power. The first power is not included because it is perfectly collinear with the X matrix. On the other hand, if a large number of fitted terms are specified, EViews may report a near singular matrix error message.

Note that the Ramsey RESET test is applicable only to an equation estimated by least squares.

Example 9.12. (Ramsey's test on a seemingly causal model) The list of variables entered as an initial regression or at the first stage of the data analysis is

$$y1\ c\ x1\ x2\ ar(1)\ ar(2) \tag{9.28}$$

with the statistical results in Figure 9.18(a). Note that the X and Y variables are derived from Demo.wf1, in order to present a more general model presentation. Therefore, a researcher could apply similar models by using his/her own data sets.

After having the output on the screen, click *View/Stability Tests/Ramsey Test* .... Then by using the default option, which is an integer '1' in the window, and clicking *OK*, the statistical results in Figure 9.18(b) are obtained.

Based on these results, the following notes and conclusions are produced:

(1) The additive model is rejected based on the F-statistic, with a p-value 0.0002. Hence, the data support the multiplicative model.

Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 16:50
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence not achieved after 500 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	91848.07	15684963	0.005856	0.9953
X1	543.2794	266.3504	2.039717	0.0429
X2	-2.170746	0.890151	-2.438625	0.0158
AR(1)	1.292931	0.078233	16.52662	0.0000
AR(2)	-0.292956	0.079493	-3.685314	0.0003

R-squared	0.999329	Mean dependent var	448.5793
Adjusted R-squared	0.999314	S.D. dependent var	345.1043
S.E. of regression	9.040810	Akaike info criterion	7.269063
Sum squared resid	14140.37	Schwarz criterion	7.358438
Log likelihood	-641.9466	Hannan-Quinn criter.	7.305307
F-statistic	64432.94	Durbin-Watson stat	2.158348
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	.29

(a)

Ramsey RESET Test:

F-statistic	14.53996	Prob. F(1,172)	0.0002
Log likelihood ratio	14.44488	Prob. Chi-Square(1)	0.0001

Test Equation:
Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 16:52
Sample: 1952Q3 1996Q4
Included observations: 178
Convergence achieved after 95 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	6181.846	157726.9	0.039193	0.9688
X1	380.7762	151.6763	2.510452	0.0130
X2	-2.095951	0.710702	-2.949126	0.0036
FITTED^2	0.000308	3.87E-05	7.959215	0.0000
AR(1)	0.513569	0.084713	6.062452	0.0000
AR(2)	0.486037	0.081982	5.928565	0.0000

R-squared	0.999381	Mean dependent var	448.5793
Adjusted R-squared	0.999364	S.D. dependent var	345.1043
S.E. of regression	8.706517	Akaike info criterion	7.199147
Sum squared resid	13038.19	Schwarz criterion	7.306399
Log likelihood	-634.7241	Hannan-Quinn criter.	7.242641
F-statistic	55583.57	Durbin-Watson stat	1.999292
Prob(F-statistic)	0.000000		

Inverted AR Roots	1.00	-.49

(b)

Figure 9.18 Statistical results of a Ramsey RESET test based on the AR(2) model in (9.28), using the default option

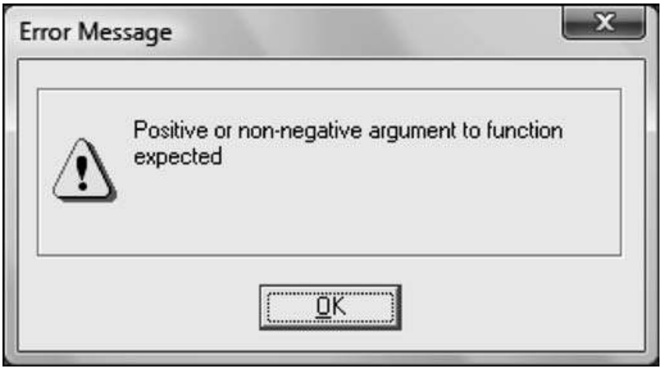

Figure 9.19 An error message in conducting the Ramsey test

(2) In the process of experimentation, the following cases are found:
- Without the AR indicators, a very small value of the DW-statistic is obtained, since the time series data are used. For this reason, statistical results based on an autoregressive model are presented directly.
- By entering a '2' in the window, the error message in Figure 9.19 is obtained. This indicates that the corresponding series has at least one negative argument. □

Example 9.13. (Unexpected results of the Ramsey tests) The list of variables used in the initial regression or at the first stage of the data analysis is '$y\ c\ x1\ x2\ y(-1)$.' After some experimentation, the statistical results in Figure 9.20(a) are obtained, with each of the independent variables, specifically the fitted terms, having a significant adjusted effect.

It was found that by using a lower power of the fitted terms, models were obtained where each of the fitted terms are insignificant, as presented in Figure 9.20(b). For this reason, the models presented in Figure 9.20 should be considered as unexpected models. It could be said that these statistical results also show or demonstrate the unpredictable impact of the multicollinearity of the independent variables of a model. □

9.7 Illustrative examples based on the Demo.wf1

The main objective of this section is to demonstrate that, based on a data set, namely Demo.wf1, many more alternative models could be developed, or it could be an infinite number of models, as well as the testing hypotheses, by only using the four variables in the workfile. Find some selected models presented in the following examples, which can easily be extended to additional time series models, aside from the models presented in the previous chapters.

Ramsey RESET Test

F-statistic	15.13319	Prob. F(5,170)	0.0000
Log likelihood ratio	65.90321	Prob. Chi-Square(5)	0.0000

Test Equation:
Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 17:14
Sample: 1952Q2 1996Q4
Included observations: 179

	Coefficient	Std. Error	t-Statistic	Prob.
C	-108.0345	28.37083	-3.807944	0.0002
X1	211.8471	57.19794	3.703754	0.0003
X2	-4.360646	0.773680	-5.636238	0.0000
Y1(-1)	2.340409	0.334923	6.987908	0.0000
FITTED^2	-0.008224	0.001970	-4.174598	0.0000
FITTED^3	2.23E-05	4.95E-06	4.512956	0.0000
FITTED^4	-3.13E-08	6.40E-09	-4.881873	0.0000
FITTED^5	2.15E-11	4.11E-12	5.233637	0.0000
FITTED^6	-5.75E-15	1.03E-15	-5.554762	0.0000

R-squared	0.999632	Mean dependent var	446.7856
Adjusted R-squared	0.999615	S.D. dependent var	344.9693
S.E. of regression	6.768030	Akaike info criterion	6.711268
Sum squared resid	7787.058	Schwarz criterion	6.871528
Log likelihood	-591.6585	Hannan-Quinn criter.	6.776252
F-statistic	57783.89	Durbin-Watson stat	2.438163
Prob(F-statistic)	0.000000		

(a)

Ramsey RESET Test

F-statistic	3.920789	Prob. F(2,173)	0.0216
Log likelihood ratio	7.935033	Prob. Chi-Square(2)	0.0189

Test Equation:
Dependent Variable: Y1
Method: Least Squares
Date: 01/11/08 Time: 17:19
Sample: 1952Q2 1996Q4
Included observations: 179

	Coefficient	Std. Error	t-Statistic	Prob.
C	-10.05517	4.065823	-2.473095	0.0144
X1	66.27393	23.26641	2.848481	0.0049
X2	-2.217640	0.438688	-5.055164	0.0000
Y1(-1)	1.031475	0.044156	23.35975	0.0000
FITTED^2	-8.16E-05	5.60E-05	-1.455659	0.1473
FITTED^3	2.08E-08	2.61E-08	0.793994	0.4283

R-squared	0.999492	Mean dependent var	446.7856
Adjusted R-squared	0.999477	S.D. dependent var	344.9693
S.E. of regression	7.888340	Akaike info criterion	7.001593
Sum squared resid	10765.08	Schwarz criterion	7.108433
Log likelihood	-620.6426	Hannan-Quinn criter.	7.044916
F-statistic	68048.56	Durbin-Watson stat	1.833318
Prob(F-statistic)	0.000000		

(b)

Figure 9.20 Unexpected statistical results of two Ramsey tests based on an additive LV(1)_SCM of $Y1$ on $X1$ and $X2$

Example 9.14. (Omitted variables test, based on the model in (4.39)) Corresponding to the LVAR(2,1)_SCM in (4.39) presented in Example 4.12, an omitted variables test will be conducted. The base model considered is an additive model as follows:

$$\begin{aligned}\log(m1) &= c(1) + c(2)^*\log(m1(-1)) + c(3)^*\log(m1(-2)) \\ &\quad + c(4)^*\log(gdp) + c(5)^*\log(gdp(-1)) + [ar(1) = c(6)]\end{aligned} \tag{9.29}$$

Figure 9.21(a) presents the statistical results for testing the omitted variables log(rs) and log($rs(-1)$). These results show that the joint effects of log(rs) and log($rs(-1)$) are significant based on the F-statistic, as well as the LR chi-squared-statistic, with a p-value $= 0.0000$. As a result, both variables should be used in the model, in a statistical sense.

However, note that log(RS) has an insignificant adjusted effect on log($m1$) with a p-value $= 0.2361$. As a comparison, Figure 9.21(b) presents the statistical results for testing the omitted variables with a dummy variable $Drs1$, which has been defined as $Drs1 = 1$ if $t \leq 119$ and $Drs1 = 0$ otherwise. This figure shows that log(RS) has a significant negative effect on log($m1$) for $t \leq 119$. □

Example 9.15. (Redundant variables tests) Based on the full model presented in the previous example, namely the model in Figure 9.21(a), several redundant

Omitted Variables: LOG(RS) LOG(RS(-1))

F-statistic	19.11887	Prob. F(2,169)	0.0000
Log likelihood ratio	36.10232	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/11/08 Time: 18:24
Sample: 1952Q4 1996Q4
Included observations: 177
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.246621	0.046359	5.319765	0.0000
LOG(M1(-1))	0.355254	0.105596	3.364289	0.0009
LOG(M1(-2))	0.431184	0.083839	5.143019	0.0000
LOG(GDP)	0.137739	0.112910	1.219906	0.2242
LOG(GDP(-1))	0.040375	0.124369	0.324635	0.7459
LOG(RS)	-0.010010	0.008419	-1.189014	0.2361
LOG(RS(-1))	-0.027336	0.010131	-2.698259	0.0077
AR(1)	0.358820	0.109648	3.272478	0.0013

R-squared	0.999712	Mean dependent var	5.827503
Adjusted R-squared	0.999700	S.D. dependent var	0.750468
S.E. of regression	0.013005	Akaike info criterion	-5.802749
Sum squared resid	0.028585	Schwarz criterion	-5.659194
Log likelihood	521.5433	Hannan-Quinn criter.	-5.744529
F-statistic	83695.47	Durbin-Watson stat	1.996170
Prob(F-statistic)	0.000000		
Inverted AR Roots	.36		

(a)

Omitted Variables: LOG(PR) LOG(RS) DRS1 DRS1*LOG(RS)

F-statistic	16.28648	Prob. F(4,167)	0.0000
Log likelihood ratio	58.29891	Prob. Chi-Square(4)	0.0000

Test Equation:
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/11/08 Time: 18:26
Sample: 1952Q4 1996Q4
Included observations: 177
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.398496	0.123817	3.218418	0.0015
LOG(M1(-1))	0.667442	0.254113	2.626555	0.0094
LOG(M1(-2))	0.158263	0.220011	0.719344	0.4729
LOG(GDP)	0.317387	0.100126	3.169876	0.0018
LOG(GDP(-1))	-0.194689	0.097293	-2.001051	0.0470
LOG(PR)	0.007278	0.019302	0.377066	0.7066
LOG(RS)	-0.050755	0.011866	-4.277346	0.0000
DRS1	-0.095185	0.025901	-3.674893	0.0003
DRS1*LOG(RS)	0.038007	0.010663	3.564293	0.0005
AR(1)	-0.074339	0.276200	-0.269148	0.7881

R-squared	0.999746	Mean dependent var	5.827503
Adjusted R-squared	0.999732	S.D. dependent var	0.750468
S.E. of regression	0.012288	Akaike info criterion	-5.905555
Sum squared resid	0.025216	Schwarz criterion	-5.726111
Log likelihood	532.6416	Hannan-Quinn criter.	-5.832779
F-statistic	72923.00	Durbin-Watson stat	1.972651
Prob(F-statistic)	0.000000		
Inverted AR Roots	-.07		

(b)

Figure 9.21 Two omitted variables tests based on the models in (a) (9.29) and (b) (4.39)

variables tests can be conducted, since there are three insignificant independent variables with p-values > 0.2, namely log(gdp), log($gdp(-1)$) and log(rs).

Figure 9.22 presents two alternative redundant variables test of three and two independent variables of the model in Figure 9.21(a). □

Redundant Variables: LOG(GDP) LOG(GDP(-1)) LOG(RS)

F-statistic	21.35970	Prob. F(3,169)	0.0000
Log likelihood ratio	56.90182	Prob. Chi-Square(3)	0.0000

Test Equation:
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/11/08 Time: 19:04
Sample: 1952Q4 1996Q4
Included observations: 177
Convergence achieved after 11 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.008025	0.015486	-0.518213	0.6050
LOG(M1(-1))	0.540397	0.093791	5.761706	0.0000
LOG(M1(-2))	0.463934	0.094174	4.926370	0.0000
LOG(RS(-1))	0.000778	0.003863	0.201306	0.8407
AR(1)	0.408089	0.097585	4.181870	0.0000

R-squared	0.999602	Mean dependent var	5.827503
Adjusted R-squared	0.999593	S.D. dependent var	0.750468
S.E. of regression	0.015140	Akaike info criterion	-5.515168
Sum squared resid	0.039424	Schwarz criterion	-5.425446
Log likelihood	493.0924	Hannan-Quinn criter.	-5.478781
F-statistic	108073.1	Durbin-Watson stat	2.003084
Prob(F-statistic)	0.000000		
Inverted AR Roots	.41		

Redundant Variables: LOG(RS) LOG(RS(-1))

F-statistic	19.11887	Prob. F(2,169)	0.0000
Log likelihood ratio	36.10232	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/11/08 Time: 19:06
Sample: 1952Q4 1996Q4
Included observations: 177
Convergence achieved after 8 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.064997	0.027230	2.386949	0.0181
LOG(M1(-1))	0.560354	0.117835	4.755391	0.0000
LOG(M1(-2))	0.381471	0.108779	3.506839	0.0006
LOG(GDP)	0.224623	0.116404	1.929688	0.0553
LOG(GDP(-1))	-0.176831	0.121774	-1.452128	0.1483
AR(1)	0.295772	0.123823	2.388676	0.0180

R-squared	0.999646	Mean dependent var	5.827503
Adjusted R-squared	0.999636	S.D. dependent var	0.750468
S.E. of regression	0.014317	Akaike info criterion	-5.621380
Sum squared resid	0.035053	Schwarz criterion	-5.513714
Log likelihood	503.4921	Hannan-Quinn criter.	-5.577715
F-statistic	96678.28	Durbin-Watson stat	1.978124
Prob(F-statistic)	0.000000		
Inverted AR Roots	.30		

Figure 9.22 Two redundant variables tests based on the full model in Figure 9.21(a)

Example 9.16. (Nonnested LVAR(2,1)_SCMs) In this example, the following nonnested LVAR(2,1)_SCMs are considered, where the first model is the full model in Figure 9.21(a):

$$\begin{aligned}\log(m1)=&c(11)+c(12)\log(m1(-1))+c(13)\log(m1(-2))+c(14)\log(gdp)\\&+c(15)\log(gdp(-1))+c(16)\log(rs)+c(17)\log(rs(-1))+[ar(1)=c(18)]\end{aligned} \tag{9.30}$$

$$\begin{aligned}\log(m1)=&c(21)+c(22)\log(m1(-1))+c(23)\log(m1(-2))+c(24)\log(gdp)\\&+c(25)\log(gdp(-1))+c(26)\log(pr)+c(27)\log(pr(-1))+[ar(1)=c(28)]\end{aligned} \tag{9.31}$$

This gives an hypothesis as follows:

$$\begin{aligned}H_1 &: \text{Model-1} = \text{Model in (9.30)}\\ H_2 &: \text{Model-2} = \text{Model in (9.31)}\end{aligned} \tag{9.32}$$

By using the same process as in Example 9.9, the variables *Fh* and *Gh* of the fitted values of the models in (9.30) and (9.31) respectively can be generated. Finally, the statistical results for testing the hypothesis (9.32) in Figure 9.23 is obtained. Since *Gh* is insignificant and *Fh* is significant, it can be concluded that the data supports model-1 or the LVAR(2,1)_SCM in (9.30). □

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/11/08 Time: 20:46
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 10 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.249926	0.064043	3.902469	0.0001
LOG(M1(-1))	0.379450	0.330145	1.149344	0.2520
LOG(M1(-2))	0.446922	0.222838	2.005588	0.0465
LOG(GDP)	0.147328	0.168194	0.875939	0.3823
LOG(GDP(-1))	0.033222	0.154889	0.214491	0.8304
LOG(RS)	-0.010055	0.008461	-1.188435	0.2363
LOG(RS(-1))	-0.027425	0.010254	-2.674514	0.0082
GH	-0.042882	0.558299	-0.076808	0.9389
AR(1)	0.358172	0.110160	3.251372	0.0014

R-squared	0.999712	Mean dependent var	5.827503
Adjusted R-squared	0.999698	S.D. dependent var	0.750468
S.E. of regression	0.013044	Akaike info criterion	-5.791485
Sum squared resid	0.028584	Schwarz criterion	-5.629986
Log likelihood	521.5464	Hannan-Quinn criter.	-5.725987
F-statistic	72802.76	Durbin-Watson stat	1.996059
Prob(F-statistic)	0.000000		
Inverted AR Roots	.36		

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/11/08 Time: 20:48
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 9 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.016550	0.142102	-0.116468	0.9074
LOG(M1(-1))	-0.000610	0.151684	-0.004022	0.9968
LOG(M1(-2))	-0.001213	0.093888	-0.012920	0.9897
LOG(GDP)	-0.002660	0.115708	-0.022993	0.9817
LOG(GDP(-1))	0.003143	0.115137	0.027294	0.9783
LOG(PR)	0.013188	0.295918	0.044568	0.9645
LOG(PR(-1))	-0.016408	0.292508	-0.056095	0.9553
FH	1.003683	0.197723	5.076207	0.0000
AR(1)	0.357760	0.108631	3.293360	0.0012

R-squared	0.999712	Mean dependent var	5.827503
Adjusted R-squared	0.999698	S.D. dependent var	0.750468
S.E. of regression	0.013043	Akaike info criterion	-5.791547
Sum squared resid	0.028582	Schwarz criterion	-5.630048
Log likelihood	521.5519	Hannan-Quinn criter.	-5.726050
F-statistic	72807.33	Durbin-Watson stat	1.995998
Prob(F-statistic)	0.000000		
Inverted AR Roots	.36		

Figure 9.23 Statistical results for testing the hypothesis in (9.32)

Example 9.17. (Ramsey RESET tests) The basic model considered is this example has the following equation specification:

$$m1\ c\ gdp\ p \tag{9.33}$$

Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 10:28
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	158.0953	10.01963	15.77856	0.0000
GDP	0.903752	0.029113	31.04260	0.0000
PR	-553.6573	54.12820	-10.22863	0.0000

R-squared	0.993977	Mean dependent var	445.0064
Adjusted R-squared	0.993909	S.D. dependent var	344.8315
S.E. of regression	26.91168	Akaike info criterion	9.439524
Sum squared resid	128190.2	Schwarz criterion	9.492740
Log likelihood	-846.5572	Hannan-Quinn criter.	9.461101
F-statistic	14606.02	Durbin-Watson stat	0.141430
Prob(F-statistic)	0.000000		

Ramsey RESET Test

F-statistic	1.499691	Prob. F(2,175)	0.2261
Log likelihood ratio	3.058940	Prob. Chi-Square(2)	0.2167

Test Equation:
Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 10:15
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	149.3226	24.16767	6.178611	0.0000
GDP	0.766272	0.159621	4.800565	0.0000
PR	-449.9389	188.1062	-2.391941	0.0178
FITTED^2	0.000242	0.000156	1.547270	0.1236
FITTED^3	-1.19E-07	6.92E-08	-1.718298	0.0875

R-squared	0.994079	Mean dependent var	445.0064
Adjusted R-squared	0.993943	S.D. dependent var	344.8315
S.E. of regression	26.83602	Akaike info criterion	9.444752
Sum squared resid	126030.1	Schwarz criterion	9.533445
Log likelihood	-845.0277	Hannan-Quinn criter.	9.480713
F-statistic	7344.994	Durbin-Watson stat	0.139148
Prob(F-statistic)	0.000000		

Figure 9.24 A Ramsey RESET test based on the model in (9.33)

Figure 9.24 presents the statistical results of a Ramsey RESET test based on this model. At a significant level of 0.10, it can be concluded that the data supports the additive model based on the *F*-statistic, as well as the LR chi-squared-statistic, with p-values > 0.20.

However, since the test equation has a small value of $DW = 0.139\,148$, then an attempt is made to apply autoregressive models. Finally, an acceptable statistical result was found with $DW = 1.950\,819$, as presented in Figure 9.25, based on an AR(2) model with the following equation specification:

$$m1\ c\ gdp\ pr\ ar(1)\ ar(2) \tag{9.34}$$

Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 10:25
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 28 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.511622	72.36584	0.007070	0.9944
GDP	0.344474	0.115058	2.993910	0.0032
PR	446.2211	237.6602	1.877560	0.0621
AR(1)	1.232830	0.076812	16.04992	0.0000
AR(2)	-0.255670	0.076339	-3.349145	0.0010

R-squared	0.999325	Mean dependent var	448.5793
Adjusted R-squared	0.999309	S.D. dependent var	345.1043
S.E. of regression	9.068506	Akaike info criterion	7.275180
Sum squared resid	14227.14	Schwarz criterion	7.364556
Log likelihood	-642.4910	Hannan-Quinn criter.	7.311424
F-statistic	64039.72	Durbin-Watson stat	2.111674
Prob(F-statistic)	0.000000		
Inverted AR Roots	.97	.26	

Ramsey RESET Test

F-statistic	8.176797	Prob. F(1,172)	0.0048
Log likelihood ratio	8.267049	Prob. Chi-Square(1)	0.0040

Test Equation:
Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 10:24
Sample: 1952Q3 1996Q4
Included observations: 178
Convergence achieved after 80 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	2940.991	53230.59	0.055250	0.9560
GDP	0.073780	0.102530	0.719593	0.4728
PR	269.6959	169.8582	1.587771	0.1142
FITTED^2	0.000297	4.13E-05	7.194001	0.0000
AR(1)	0.541337	0.088755	6.099229	0.0000
AR(2)	0.457910	0.085972	5.326284	0.0000

R-squared	0.999356	Mean dependent var	448.5793
Adjusted R-squared	0.999337	S.D. dependent var	345.1043
S.E. of regression	8.886062	Akaike info criterion	7.239972
Sum squared resid	13581.48	Schwarz criterion	7.347223
Log likelihood	-638.3575	Hannan-Quinn criter.	7.283465
F-statistic	53358.73	Durbin-Watson stat	1.950819
Prob(F-statistic)	0.000000		
Inverted AR Roots	1.00	-.46	

Figure 9.25 A Ramsey RESET test based on the AR(2) model in (9.34)

Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 11:09
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	-11.77693	5.913825	-1.991423	0.0480
M1(-1)	1.143715	0.076896	14.87350	0.0000
M1(-2)	-0.138905	0.081637	-1.701501	0.0906
GDP	-0.035812	0.027922	-1.282583	0.2014
PR	72.82841	26.05880	2.794772	0.0058

R-squared	0.999402	Mean dependent var	448.5793
Adjusted R-squared	0.999388	S.D. dependent var	345.1043
S.E. of regression	8.535487	Akaike info criterion	7.154030
Sum squared resid	12603.84	Schwarz criterion	7.243406
Log likelihood	-631.7087	Hannan-Quinn criter.	7.190274
F-statistic	72293.25	Durbin-Watson stat	2.039986
Prob(F-statistic)	0.000000		

Ramsey RESET Test

F-statistic	4.134905	Prob. F(2,171)	0.0176
Log likelihood ratio	8.406661	Prob. Chi-Square(2)	0.0149

Test Equation:
Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 11:16
Sample: 1952Q3 1996Q4
Included observations: 178

	Coefficient	Std. Error	t-Statistic	Prob.
C	-6.955668	6.050748	-1.149555	0.2519
M1(-1)	1.044689	0.090888	11.49420	0.0000
M1(-2)	-0.073746	0.083336	-0.884921	0.3774
GDP	-0.052638	0.029654	-1.775028	0.0777
PR	76.32348	39.42252	1.936037	0.0545
FITTED^2	0.000120	5.09E-05	2.359788	0.0194
FITTED^3	-6.53E-08	2.37E-08	-2.756497	0.0065

R-squared	0.999430	Mean dependent var	448.5793
Adjusted R-squared	0.999410	S.D. dependent var	345.1043
S.E. of regression	8.384898	Akaike info criterion	7.129273
Sum squared resid	12022.41	Schwarz criterion	7.254400
Log likelihood	-627.5053	Hannan-Quinn criter.	7.180015
F-statistic	49943.56	Durbin-Watson stat	2.032578
Prob(F-statistic)	0.000000		

Figure 9.26 A Ramsey RESET test based on the LV(2) model in (9.35)

Based on the F-statistic with a p-value = 0.0048, as well as the LR chi-squared-statistic with a p-value = 0.0040, it can be concluded that the data supports the multiplicative model.

As a comparison, Figure 9.26 presents the statistical results of a Ramsey RESET test, based on an LV(2) model as follows:

$$m1\ c\ m1(-1)\ m1(-2)\ gdp\ pr \tag{9.35}$$

Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 11:12
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C	-13.57829	6.384208	-2.126857	0.0348
M1(-1)	1.005921	0.031228	32.21206	0.0000
GDP	-0.041335	0.030806	-1.341814	0.1814
PR	83.77841	27.49393	3.047160	0.0027
AR(1)	0.133717	0.082868	1.613625	0.1084

R-squared	0.999401	Mean dependent var	448.5793
Adjusted R-squared	0.999388	S.D. dependent var	345.1043
S.E. of regression	8.540531	Akaike info criterion	7.155211
Sum squared resid	12618.74	Schwarz criterion	7.244587
Log likelihood	-631.8138	Hannan-Quinn criter.	7.191456
F-statistic	72207.84	Durbin-Watson stat	2.028882
Prob(F-statistic)	0.000000		
Inverted AR Roots	.13		

Ramsey RESET Test

F-statistic	3.850871	Prob. F(2,171)	0.0231
Log likelihood ratio	7.841721	Prob. Chi-Square(2)	0.0198

Test Equation:
Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 11:14
Sample: 1952Q3 1996Q4
Included observations: 178
Convergence achieved after 1 iteration

	Coefficient	Std. Error	t-Statistic	Prob.
C	-8.541845	5.910582	-1.445178	0.1502
M1(-1)	0.976052	0.047805	20.41716	0.0000
GDP	-0.062953	0.028152	-2.236164	0.0266
PR	85.57631	38.34540	2.231723	0.0269
FITTED^2	0.000130	4.99E-05	2.604170	0.0100
FITTED^3	-7.07E-08	2.30E-08	-3.075571	0.0024
AR(1)	0.002501	0.080711	0.030990	0.9753

R-squared	0.999427	Mean dependent var	448.5793
Adjusted R-squared	0.999407	S.D. dependent var	345.1043
S.E. of regression	8.403177	Akaike info criterion	7.133629
Sum squared resid	12074.89	Schwarz criterion	7.258755
Log likelihood	-627.8929	Hannan-Quinn criter.	7.184371
F-statistic	49726.40	Durbin-Watson stat	1.879957
Prob(F-statistic)	0.000000		
Inverted AR Roots	.00		

Figure 9.27 A Ramsey RESET test based on the LVAR(1,1) model in (9.36)

These statistical results also show that the data support the multiplicative model based on the *F*-statistic, as well as the LR chi-squared-statistic, with p-values < 0.02.

Finally, Figure 9.27 presents the statistical results of a Ramsey RESET test, based on an LVAR(1,1) model as follows:

$$m1\ c\ m1(-1)\ gdp\ pr\ ar(1) \tag{9.36}$$

These statistical results also show that the data support the multiplicative model. □

10

Nonlinear least squares models

10.1 Introduction

The nonlinear least squares (NLS) model could be presented as

$$Y_t = f(X_t, t, \theta) + \mu_t \tag{10.1}$$

where Y_t is an endogenous variable and X_t is a vector exogenous variable, t is the time variable, θ is a vector or a finite set of *nonlinear* parameters and u_t is a vector of the error terms. As usual, the least squares estimation chooses the parameter values that minimize the sum of the squared residuals:

$$S(\theta) = \sum (Y_t - f(X_t, t, \theta))^2 \tag{10.2}$$

Note that the function $f(X_t,\ t,\ \theta)$ can be all types of models presented in the previous chapters. As a review, note the following general equations, which are included in (10.1):

(a) *Model with a Trend*

$$Y_t = f(X_t, \theta) + \delta^* t + u_t \tag{10.3}$$

note that, for a multivariate model, δ is a vector of trend parameters.

(b) *Model with Time-Related Effects*

$$Y_t = f_1(X_t, \theta) + f_2(X_t, \delta)^* t + u_t \tag{10.4}$$

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Note that the effect of each X-variable in $f_2(X_t, \delta)$ depends on t. Hence this model is called the model with time-related effects. For example, the following equation presents a general univariate model:

$$Y_t = \sum_{i=0} \beta_i X_{i,t} + \sum_{j=0} \delta_j X_{j,t} {}^* t + u_t \tag{10.5}$$

(c) *Model with Dummy Variables*

$$Y_t = f_1(X_t, t, \theta)^* D_1 + f_2(X_t, t, \theta)^* D_2 + u_t \tag{10.6}$$

where D_1 and D_2 are the zero-one or dummy variables of a defined dichotomous variable. However, for the time series data, the dichotomous variable (dummy variables) should probably be defined based on the time-variable, as presented in Chapter 3. This model could be presented as the following two alternative general models:

$$Y_t = f_1(X_t, t, \theta)^* D_1 + f_2(X_t, t, \theta) + u_t \tag{10.7}$$

$$Y_t = f_1(X_t, t, \theta) + f_2(X_t, t, \theta)^* D_2 + u_t \tag{10.8}$$

(d) *Model without the Time t-Variable*

The model without the time t-variable could be written easily based on the model in (9.11), as follows:

$$Y_t = f(X_t, \theta) + u_t \tag{10.9}$$

Furthermore, note that the components of the exogenous variables (or the X-variables) in all models presented above could include some of the endogenous variables, the lags of independent as well as dependent variables and their selected interaction factors and powers. Each of the models presented above should be extended to the AR models, ARCH and GARCH models, as well as the system equation and instrumental variable models.

At the first stage, examples based on the three basic NLS models are presented, namely the classical growth models, translog linear models or Cobb–Douglas production functions and the quadratic translog models or the CES (constant elasticity of substitution) production functions.

By using the same process as presented in the previous chapters, in fact it is expected that the statistical results could easily be obtained based on any nonlinear models. However, in many cases, the '*Overflow*' error message or '*Warning: Singular covariance – coefficients are not unique*' have been found. On the other hand, it was also found that EViews 5 and EViews 6 do not give consistent statistical results. For this reason, some of the examples using EViews 5 are presented. Corresponding to these problems, refer to the notes presented in Section 10.5.

Finally, an attempt should be made to develop alternative NLS models. By using the trial-and-error methods, uncommon or unexpected NLS models have been found, as presented in Section 10.6.

10.2 Classical growth models

This section presents statistical results based on NLS models compared to their corresponding (linear) LS models, based on the Demo.wf1, starting with the classical growth model in (2.3).

Example 10.1. (The classical exponential growth model) Corresponding to the classical growth model in (2.3), Figure 10.1 presents statistical results based on LS and NLS growth models of the endogenous variable $M1$, using the following equation specifications:

$$\log(m1)\ c\ t \tag{10.10}$$

$$m1 = C(1)^{*}\exp(C(2)^{*}t) \tag{10.11}$$

□

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/13/08 Time: 14:15
Sample: 1952Q1 1996Q4
Included observations: 180

	Coefficient	Std. Error	t-Statistic	Prob.
C	4.517962	0.018429	245.1609	0.0000
T	0.014290	0.000177	80.92125	0.0000

R-squared	0.973537	Mean dependent var	5.811220
Adjusted R-squared	0.973388	S.D. dependent var	0.754650
S.E. of regression	0.123108	Akaike info criterion	-1.340464
Sum squared resid	2.697683	Schwarz criterion	-1.304987
Log likelihood	122.6418	Hannan-Quinn criter.	-1.326080
F-statistic	6548.249	Durbin-Watson stat	0.015856
Prob(F-statistic)	0.000000		

(a) LS Model In (10.1)

Dependent Variable: M1
Method: Least Squares
Date: 01/13/08 Time: 14:17
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 13 iterations
M1 =C(1)*EXP(C(2)*T)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	75.28172	1.812596	41.53255	0.0000
C(2)	0.015987	0.000158	101.3822	0.0000

R-squared	0.989683	Mean dependent var	445.0064
Adjusted R-squared	0.989625	S.D. dependent var	344.8315
S.E. of regression	35.12395	Akaike info criterion	9.966693
Sum squared resid	219597.2	Schwarz criterion	10.00217
Log likelihood	-895.0023	Hannan-Quinn criter.	9.981077
Durbin-Watson stat	0.069847		

(b) NLS Model In (10.2)

Figure 10.1 Statistical results based on the classical growth model of $M1$, using (a) LS and (b) NLS models

Example 10.2. (NLS growth model with intercept) Figure 10.2 presents statistical results based on the following NLS growth model, with its residual graph presented in Figure 10.3:

$$m1 = c(1) + c(2)^{*}\exp(c(3)^{*}t) \tag{10.12}$$

Compare these statistical results and residual graphs with Figures 2.2 and 2.3. Based on these results, especially the residual graph and a very small DW-statistic, it could be stated that the NLS model is a poor time series model, in a statistical sense, and similarly for the NLS model in (10.12). Therefore, an autoregressive model or a lagged-variable model should be found. For this reason experimentation is carried out as presented in the following example. □

Dependent Variable: M1
Method: Least Squares
Date: 01/13/08 Time: 14:26
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 38 iterations
M1 =C(1)+C(2)*EXP(C(3)*T)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	37.29318	8.844310	4.216629	0.0000
C(2)	58.19883	4.097117	14.20482	0.0000
C(3)	0.017341	0.000385	44.98879	0.0000

R-squared	0.990587	Mean dependent var	445.0064
Adjusted R-squared	0.990481	S.D. dependent var	344.8315
S.E. of regression	33.64409	Akaike info criterion	9.886078
Sum squared resid	200350.6	Schwarz criterion	9.939294
Log likelihood	-886.7470	Hannan-Quinn criter.	9.907655
F-statistic	9313.484	Durbin-Watson stat	0.078695
Prob(F-statistic)	0.000000		

Figure 10.2 Statistical results based on the model in (10.3)

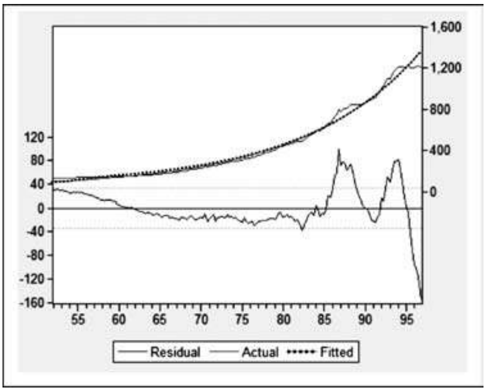

Figure 10.3 Residual graphs of the model in (10.3)

Example 10.3. (Experimentation on autoregressive NLS models) Figure 10.4 presents statistical results based on the following AR(1) NLS model:

$$\log(m1) = c(1)^*\exp(c(2)^*t) + [ar(1) = c(3)] \tag{10.13}$$

In fact, analyses have been conducted based on the following NLS models, but the 'Near singular matrix' error messages have been obtained:

$$m1 = c(1) + c(2)^*\exp(c(3)^*t) + [ar(1) = c(4)] \tag{10.14}$$

$$m1 = c(1)^*\exp(c(2)^*t) + [ar(1) = c(4)] \tag{10.15}$$

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 01/13/08 Time: 15:21
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 9 iterations
LOG(M1)=C(1)*EXP(C(2)*T) + [AR(1)=C(3)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	4.494304	0.092550	48.56066	0.0000
C(2)	0.002591	0.000129	20.15810	0.0000
C(3)	0.971160	0.012578	77.21294	0.0000

R-squared	0.999606	Mean dependent var	5.816642
Adjusted R-squared	0.999602	S.D. dependent var	0.753241
S.E. of regression	0.015033	Akaike info criterion	-5.540548
Sum squared resid	0.039773	Schwarz criterion	-5.487128
Log likelihood	498.8790	Hannan-Quinn criter.	-5.518886
Durbin-Watson stat	2.112171		

Inverted AR Roots	.97

Figure 10.4 Statistical results based on the NLS AR(1) model in (10.4)

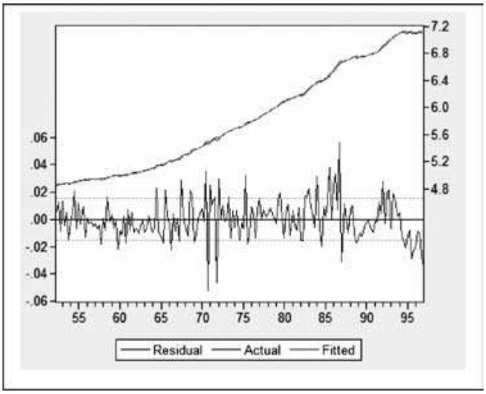

Figure 10.5 Residual graph of the model in Figure 10.4

Note that the model in (10.15) can be transformed to the following model, with the statistical results presented in Figure 2.4:

$$\log(m1) = c(1) + c(2)^{*}t + [ar(1) = c(3)] \tag{10.16}$$

This raises the question 'Why do we have an error message based on the NLS model in (10.15).' In order to answer this question, refer to the notes presented in Section 10.5. □

10.3 Generalized Cobb–Douglas models

The basic Cobb–Douglas production function can be presented as

$$Q = AK^{\alpha}L^{\beta} \tag{10.17}$$

where Q is an output variable or factor, and K and L are two input variables or factors, *Capital* and *Labor*. The generalized CD (GCD) model, in EViews, can be presented as follows:

$$Y_t = c(1)^*X_1^{c(2)}{}^*X_2^{c(3)} \cdots {}^*X_k^{c(k+1)} + u_t \qquad (10.18)$$

where Y is an endogenous variable and X_1, X_2, ..., X_k are the exogenous variables. Note that this model is without the time t as an independent variable.

10.3.1 Cases based on the Demo.wf1

Example 10.4. (NLS corresponding to the model in (4.39)) Corresponding to the additive model in (4.39), namely the following model:

$$\begin{aligned} \log(m1) = {} & c(10) + c(11)^*\log(m1(-1)) + c(12)^*\log(m1(-2)) \\ & + c(20)^*\log(gdp) + c(21)^*\log(gdp(-1)) + [ar(1) = c(31)] \end{aligned} \qquad (10.19)$$

an NLS model needs to be considered with the following equation:

$$m1 = c(1)^*m1(-1)^{c(2)*}m1(-2)^{c(3)*}gdp^{c(4)*}gdp(-1)^{c(5)} \qquad (10.20)$$

By using EViews 5, the statistical results in Figure 10.6 are obtained, together with its reduced model, since by using EViews 6 the 'Overflow' error message is found. Based on $DW = 2.062\,611$, it can be concluded that this reduced model is an acceptable NLS model. In fact, if $gdp(-1)$ is deleted from the full model, instead of gdp, another acceptable model would be obtained. Which one would you prefer?

Dependent Variable: M1
Method: Least Squares
Date: 07/26/07 Time: 07:45
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 9 iterations
M1=C(1)*M1(-1)^C(2)*M1(-2)^C(3)*GDP^C(4)*GDP(-1)^C(5)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.063904	0.021007	50.64642	0.0000
C(2)	1.146199	0.075748	15.13179	0.0000
C(3)	-0.194620	0.073350	-2.653302	0.0087
C(4)	-0.001992	0.168109	-0.011847	0.9906
C(5)	0.040391	0.170086	0.237472	0.8126

R-squared	0.999386	Mean dependent var	448.5793
Adjusted R-squared	0.999371	S.D. dependent var	345.1043
S.E. of regression	8.652923	Akaike info criterion	7.181359
Sum squared resid	12953.04	Schwarz criterion	7.270735
Log likelihood	-634.1410	Durbin-Watson stat	2.062868

Dependent Variable: M1
Method: Least Squares
Date: 07/26/07 Time: 07:47
Sample (adjusted): 1952Q3 1996Q4
Included observations: 178 after adjustments
Convergence achieved after 5 iterations
M1=C(1)*M1(-1)^C(2)*M1(-2)^C(3)*GDP(-1)^C(5)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.063783	0.018301	58.12707	0.0000
C(2)	1.146194	0.075529	15.17558	0.0000
C(3)	-0.194584	0.073076	-2.662749	0.0085
C(5)	0.038381	0.012552	3.057667	0.0026

R-squared	0.999386	Mean dependent var	448.5793
Adjusted R-squared	0.999375	S.D. dependent var	345.1043
S.E. of regression	8.628026	Akaike info criterion	7.170124
Sum squared resid	12953.05	Schwarz criterion	7.241625
Log likelihood	-634.1410	Durbin-Watson stat	2.062611

Figure 10.6 Statistical results based on the NLS model in (10.9), and its reduced model, by using EViews 5

Dependent Variable: M1
Method: Least Squares
Date: 07/26/07 Time: 08:01
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 6 iterations
M1=C(1)*M1(-1)^C(2)*M1(-2)^C(3)*GDP(-1)^C(5) + [AR(1)=C(6)]

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.036056	0.011533	89.83643	0.0000
C(2)	1.588322	0.067375	23.57443	0.0000
C(3)	-0.620148	0.065106	-9.525234	0.0000
C(5)	0.025673	0.008014	3.203606	0.0016
C(6)	-0.518869	0.077059	-6.733437	0.0000

R-squared	0.999449	Mean dependent var	450.3826
Adjusted R-squared	0.999436	S.D. dependent var	345.2413
S.E. of regression	8.200514	Akaike info criterion	7.074113
Sum squared resid	11566.73	Schwarz criterion	7.163834
Log likelihood	-621.0590	Durbin-Watson stat	2.127954
Inverted AR Roots	-.52		

Dependent Variable: M1
Method: Least Squares
Date: 07/26/07 Time: 08:04
Sample (adjusted): 1952Q4 1996Q4
Included observations: 177 after adjustments
Convergence achieved after 3 iterations
M1=C(1)*M1(-1)^C(2)*M1(-2)^C(3)*GDP^C(4) + [AR(1)=C(6)]

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.034351	0.011389	90.82213	0.0000
C(2)	1.588036	0.067708	23.45433	0.0000
C(3)	-0.618799	0.065422	-9.458517	0.0000
C(4)	0.024850	0.007948	3.126584	0.0021
C(6)	-0.516776	0.077327	-6.682951	0.0000

R-squared	0.999447	Mean dependent var	450.3826
Adjusted R-squared	0.999434	S.D. dependent var	345.2413
S.E. of regression	8.211302	Akaike info criterion	7.076742
Sum squared resid	11597.18	Schwarz criterion	7.166464
Log likelihood	-621.2917	Durbin-Watson stat	2.123792
Inverted AR Roots	-.52		

Figure 10.7 Statistical results based on two AR(1) NLS models, using EViews 5

Furthermore, by using the trial-and-error methods, the statistical results based on two AR(1) NLS models presented in Figure 10.7 are found, using EViews 5, since EViews 6 also presents the 'Overflow' error messages. □

Example 10.5. (NLS interaction models) Corresponding to the interaction model presented in Example 4.18, an NLS interaction model will be considered, as follows:

$$m1 = c(1)*gdp^{c(2)}*pr^{c(3)}*\exp(c(4)*\log(gdp)*\log(pr)) \tag{10.21}$$

Figure 10.8(a) presents the statistical results using EViews 5 and Figure 10.8(b) presents the results using EViews 6, which demonstrates that EViews 5 and 6 do not give consistent statistical results. For this reason, experimentation based on simple models are presented in the following example. □

Dependent Variable: M1
Method: Least Squares
Date: 07/26/07 Time: 14:48
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 2 iterations
M1=C(1)*GDP^C(2)*PR^C(3)*EXP(C(4)*LOG(GDP)*LOG(PR))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-2.57E-25	7.86E-24	-0.032750	0.9739
C(2)	10.77207	0.000124	87067.17	0.0000
C(3)	-45.83148	0.000298	-153857.1	0.0000
C(4)	5.798206	2.59E-05	223656.9	0.0000

R-squared	-872720...	Mean dependent var	445.0064
Adjusted R-squared	-887596...	S.D. dependent var	344.8315
S.E. of regression	1.03E+10	Akaike info criterion	48.96550
Sum squared resid	1.86E+22	Schwarz criterion	49.03645
Log likelihood	-4402.895	Durbin-Watson stat	0.014651

(a) Statistical Result Using EViews 5

Dependent Variable: M1
Method: Least Squares
Date: 01/13/08 Time: 19:58
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 1 iteration
WARNING: Singular covariance - coefficients are not unique
White Heteroskedasticity-Consistent Standard Errors & Covariance
M1=C(1)*(GDP^C(2))*(PR^C(3))*EXP(C(4)*LOG(GDP)*LOG(PR))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.075381	NA	NA	NA
C(2)	-226.2387	NA	NA	NA
C(3)	380.5161	NA	NA	NA
C(4)	0.995377	NA	NA	NA

R-squared	-1.674704	Mean dependent var	445.0064
Adjusted R-squared	-1.720296	S.D. dependent var	344.8315
S.E. of regression	568.7416	Akaike info criterion	15.54670
Sum squared resid	56930201	Schwarz criterion	15.61766
Log likelihood	-1395.203	Hannan-Quinn criter.	15.57547
Durbin-Watson stat	0.000415		

(b) Statistical Result Using EViews 6

Figure 10.8 Statistical results based on the NLS model in (10.21) using (a) EViews 5 and (b) EViews 6

Table 10.1 Status of simple NLS models using EViews 5 and 6.

			Output using	
Number	Dependent variable	Independent variable	EViews 6	EViews 5
1	*M*1	*GDP*	Warning	Warning
2	*M*1	*GDP*(−1)	Warning	Warning
3	*M*1	*PR*	(*)	Overflow
4	*M*1	*PR*(−1)	(*)	Overflow
5	*M*1	*RS*	Warning	Warning
6	*M*1	*RS*(−1)	Warning	Warning
7	*M*1	*M1*(−1)	Warning	Warning
8	*M*1	*T*	Estimable	Estimable
9	*GDP*	*T*	Estimable	Estimable
10	*PR*	*T*	Estimable	Estimable
11	*RS*	*T*	Estimable	Estimable

Example 10.6. (Experimentation with simple NLS models) Table 10.1 presents the status or outputs of selected simple NLS models, with the following general equation, using EViews 5 and 6:

$$Y = c(1)^{*}X^{c(2)} + \varepsilon \qquad (10.22)$$

Note that for the NLS models with independent variables *PR* and *PR*(−1), EViews 6 presents another type of error message, as in Figure 10.9. For the other models, EViews 5 and 6 present consistent results. In fact, simple NLS models have also been applied using log(*m*1) and *d* log(*m*1). Do this as an exercise.

It is really unexpected error messages that are obtained based on a very simple NLS model, since based on its corresponding translog linear models, namely log(*y*) = *c*(1) + *c*(2)log(*x*) + ε, in general acceptable statistical results would be obtained.

Compared to the LS models, error messages based on many alternative NLS models have been found or obtained, specifically the Cobb–Douglas (CD) and the constant elasticity of substitution (CES) models, some of which will be presented as illustrations in the following examples.

Figure 10.9 An error message in EViews 6 for estimating NLS models

Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 13:41
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 290 iterations
M1=C(1)*(GDP^C(2))*(PR^C(3))*(RS^C(4))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.719406	0.375693	1.914879	0.0571
C(2)	1.025359	0.070882	14.46562	0.0000
C(3)	-0.286489	0.114008	-2.512881	0.0129
C(4)	-0.176250	0.008011	-22.00070	0.0000

R-squared	0.995670	Mean dependent var	445.0064
Adjusted R-squared	0.995596	S.D. dependent var	344.8315
S.E. of regression	22.88289	Akaike info criterion	9.120628
Sum squared resid	92158.32	Schwarz criterion	9.191583
Log likelihood	-816.8565	Hannan-Quinn criter.	9.149397
Durbin-Watson stat	0.191128		

(a)

Dependent Variable: M1
Method: Least Squares
Date: 01/12/08 Time: 13:46
Sample (adjusted): 1952Q2 1996Q4
Included observations: 179 after adjustments
Convergence achieved after 31 iterations
M1=C(1)*(GDP^C(2))*(PR^C(3))*(RS^C(4)) +[AR(1)=C(5)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-2.71E-20	5.33E-19	-0.050925	0.9594
C(2)	7.377519	5.60E-10	1.32E+10	0.0000
C(3)	-1.553119	1.09E-09	-1.43E+09	0.0000
C(4)	-0.896403	8.27E-11	-1.08E+10	0.0000
C(5)	1.007853	6.83E-12	1.48E+11	0.0000

R-squared	0.695072	Mean dependent var	446.7856
Adjusted R-squared	0.688062	S.D. dependent var	344.9693
S.E. of regression	192.6702	Akaike info criterion	13.38737
Sum squared resid	6459192.	Schwarz criterion	13.47641
Log likelihood	-1193.170	Hannan-Quinn criter.	13.42347
Durbin-Watson stat	0.471180		

Inverted AR Roots 1.01
Estimated AR process is nonstationary

(b)

Figure 10.10 Statistical results based on (a) NLS model in (10.23) and (b) AR(1) NLS model in (10.24), using EViews 5

For a more advanced NLS model, Figure 10.10(a) presents statistical results, using EViews 5, based on a GCD model using the equation specification in (9.14), and Figure 10.10(b) presents the statistical results based on the AR(1) GCD model in (9.15):

$$m1 = c(1)*(gdp^{c(2)})*(pr^{c(3)})*(rs^{c(4)}) \tag{10.23}$$

$$m1 = c(1)*(gdp^{c(2)})*(pr^{c(3)})*(rs^{c(4)}) + [ar(1) = c(5)] \tag{10.24}$$

□

10.3.2 Cases based on the BASIC.wf1

Example 10.7. (Simple NLS models) Figure 10.11 presents statistical results based on an NLS model of the endogenous variable Y on X, in BASIC.wf1, and

Dependent Variable: Y
Method: Least Squares
Date: 01/14/08 Time: 06:52
Sample: 1959M01 1989M12
Included observations: 340
Convergence achieved after 30 iterations
Y=C(1)*(X^C(2))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.708444	20.19686	0.183615	0.8544
C(2)	-0.849179	1.343380	-0.632121	0.5277

R-squared	0.000757	Mean dependent var	0.108655
Adjusted R-squared	-0.002200	S.D. dependent var	0.883724
S.E. of regression	0.884696	Akaike info criterion	2.598719
Sum squared resid	264.5480	Schwarz criterion	2.621242
Log likelihood	-439.7822	Hannan-Quinn criter.	2.607693
Durbin-Watson stat	1.966090		

Figure 10.11 Statistical results based on an NLS model, using EViews 6

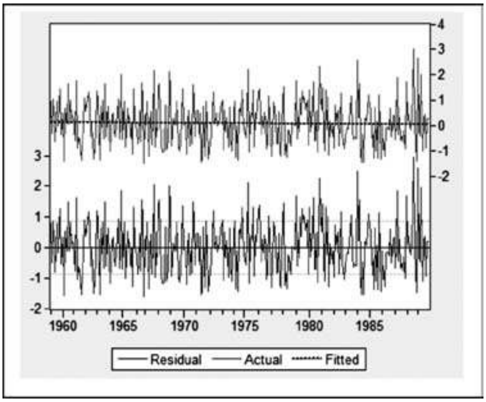

Figure 10.12 Residual graphs of the model in Figure 10.11

Figure 10.12 presents its residual graphs. These results should be acceptable estimates, since $DW = 1.966\,090$. Compared to the results in Table 10.1, this finding has demonstrated that the statistical results are highly dependent on the data used, and are not dependent on the model(s). □

Example 10.8. (Unexpected statistical results based on an NLS model) By using the trial-and-error methods, finally the statistical results are obtained based on an NLS model having three independent variables, as presented in Figure 10.13, with its residual graphs in Figure 10.14. However, each of the t-statistics has a very large p-value, and the DW- statistic is very close to zero. This also demonstrates that the statistical results are highly dependent on the data used; there is nothing wrong with the model.

Dependent Variable: M1
Method: Least Squares
Date: 01/14/08 Time: 07:13
Sample: 1959M01 1989M12
Included observations: 372
Failure to improve SSR after 1 iteration
M1=C(1)*(IP^C(2))*(FF^C(3))*(URATE^C(4))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.708444	2981.380	0.001244	0.9990
C(2)	-0.849179	233.2546	-0.003641	0.9971
C(3)	0.966817	111.2156	0.008693	0.9931
C(4)	-0.114895	147.7882	-0.000777	0.9994

R-squared	-2.887176	Mean dependent var	337.5911
Adjusted R-squared	-2.918865	S.D. dependent var	198.6367
S.E. of regression	393.2237	Akaike info criterion	14.79733
Sum squared resid	56901950	Schwarz criterion	14.83947
Log likelihood	-2748.303	Hannan-Quinn criter.	14.81406
Durbin-Watson stat	6.26E-05		

Figure 10.13 Unexpected results based on an NLS model

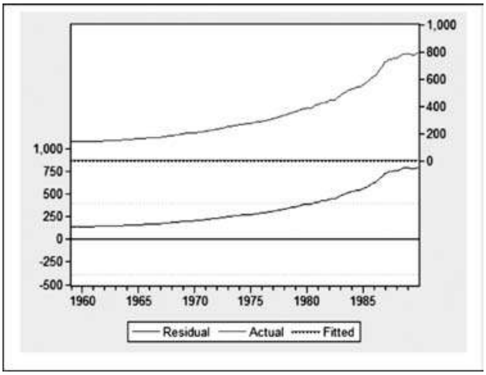

Figure 10.14 Residual graphs of the model in Figure 10.13

Furthermore, it has been recognized that many other NLS models with an alternative dependent variable and two or three independent variables present the 'Overflow' error messages. Once again there is nothing wrong with the NLS models, but the estimation process cannot provide estimates of the parameters. □

10.3.3 Cases based on the US_DPOC data

As presented in the previous chapters, in this section limited examples of NLS models will be given using selected subsets of the five variables $X1$, $X2$, $X3$, $Y1$ and $Y2$. Since several statistical results using EViews 5 have been found, those results will be presented if the results using EViews 6 does not present output with statistically acceptable estimates, such as those given in the previous examples. Otherwise, only the statistical results using EViews 6 will be presented.

Example 10.9. (GCD model with one input variable and trend) Figure 10.15 presents statistical results based on the following GCD model with trend, using EViews 5 and 6 with the default options:

$$Y = c(1) + c(2)^{*}X1^{c(3)} + c(4)^{*}t \tag{10.25}$$

Dependent Variable: Y
Method: Least Squares
Date: 12/08/06 Time: 09:32
Sample: 1951 1980
Included observations: 30
Convergence achieved after 10 iterations
Y= C(1) + C(2)*(X1^C(3)) + C(4)*T

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	9429.342	2.23E+20	4.22E-17	1.0000
C(2)	-1.18E-15	1.23E-13	-0.009630	0.9924
C(3)	10.59045	1.64E-06	6452756.	0.0000
C(4)	125.6853	1.47E+19	8.54E-18	1.0000

R-squared	-56886...	Mean dependent var	45.63967
Adjusted R-squared	-63450...	S.D. dependent var	21.74352
S.E. of regression	5.48E+20	Akaike info criterion	98.46598
Sum squared resid	7.80E+42	Schwarz criterion	98.65281
Log likelihood	-1472.990	Durbin-Watson stat	0.393186

Dependent Variable: Y
Method: Least Squares
Date: 01/14/08 Time: 14:12
Sample: 1951 1980
Included observations: 30
Convergence achieved after 4 iterations
WARNING: Singular covariance - coefficients are not unique
Y=C(1)+C(2)*(X1^C(3))+C(4)*T

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	11.17136	NA	NA	NA
C(2)	346.8278	NA	NA	NA
C(3)	-218.0497	NA	NA	NA
C(4)	2.223762	NA	NA	NA

R-squared	0.810622	Mean dependent var	45.63967
Adjusted R-squared	0.788771	S.D. dependent var	21.74352
S.E. of regression	9.993250	Akaike info criterion	7.565263
Sum squared resid	2596.491	Schwarz criterion	7.752089
Log likelihood	-109.4789	Hannan-Quinn criter.	7.625030
F-statistic	37.09724	Durbin-Watson stat	0.730591
Prob(F-statistic)	0.000000		

Figure 10.15 Statistical results based on the model in (10.25)

where Y is in fact equal to $Y2$. Based on the results using EViews 6, the following notes are given:

(1) By replacing $X1$ with $X2$ and $X3$, similar results are obtained, and by replacing it with $X4$, EViews 6 presents the 'Overflow' error message.
(2) Data analysis has been based on alternative NLS models with one or two exogenous variables, giving the 'Overflow' or the 'Warning' error messages. □

Example 10.10. (GCD models with trend) Figure 10.16(a) and (b) presents statistical results based on GCD models with trend, using EViews 5 with the default options, since EViews 6 presents the 'Overflow' error message. The results are obtained by using or entering the following equation specifications respectively:

$$Y = c(1)^* X1^{c(2)} {}^* X3^{c(3)} {}^* X4^{c(4)} + c(5)^* t \tag{10.26}$$

$$Y = c(1)^* X1^{c(2)} {}^* X3^{c(3)} {}^* X4^{c(4)} + c(5)^* t + [ar(1) = c(6)] \tag{10.27}$$

□

Method: Least Squares
Date: 12/06/06 Time: 10:19
Sample: 1951 1980
Included observations: 30
Convergence achieved after 26 iterations
Y=C(1)*(X1^C(2))*(X3^C(3))*(X4^C(4)) + C(5)*T

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.046994	1.010495	1.036120	0.3101
C(2)	1.076842	0.440455	2.444839	0.0219
C(3)	1.005676	0.493845	2.036422	0.0524
C(4)	2.259685	1.511002	1.495487	0.1473
C(5)	1.243154	0.610678	2.035693	0.0525

R-squared	0.955876	Mean dependent var	45.63967
Adjusted R-squared	0.948816	S.D. dependent var	21.74352
S.E. of regression	4.919216	Akaike info criterion	6.175187
Sum squared resid	604.9671	Schwarz criterion	6.408720
Log likelihood	-87.62781	Durbin-Watson stat	1.183943

(a) GCD Model In (9.17)

Date: 12/06/06 Time: 10:21
Sample(adjusted): 1952 1980
Included observations: 29 after adjusting endpoints
Convergence achieved after 10 iterations
Y=C(1)*(X1^C(2))*(X3^C(3))*(X4^C(4)) + C(5)*T +[AR(1)=C(6)]

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.570717	0.456703	1.249646	0.2240
C(2)	0.859463	0.287984	2.984411	0.0066
C(3)	0.725106	0.345117	2.101044	0.0468
C(4)	1.355232	0.987731	1.372066	0.1833
C(5)	0.643204	0.821053	0.783390	0.4414
C(6)	0.492781	0.207996	2.369182	0.0266

R-squared	0.963287	Mean dependent var	46.45862
Adjusted R-squared	0.955306	S.D. dependent var	21.65240
S.E. of regression	4.577526	Akaike info criterion	6.062186
Sum squared resid	481.9361	Schwarz criterion	6.345075
Log likelihood	-81.90170	Durbin-Watson stat	1.873286

(b) GCD Model In (9.18)

Figure 10.16 Statistical results using EViews 5, based on the NLS models in (10.26) and (10.27)

Example 10.11. (GCD models with the time *t* as an input variable) Figure 10.17(a) and (b) presents statistical results based on two GCD models, where the time t is considered as one of the input variables, using EViews 5. The equation specifications are as follows:

$$\begin{aligned} Y &= c(1)^* X1^{c(2)} {}^* X3^{c(3)} {}^* X4^{c(4)} {}^* t^{c(5)} \\ Y &= c(1)^* X1^{c(2)} {}^* X3^{c(3)} {}^* X4^{c(4)} {}^* t^{c(5)} + [ar(1) = c(6)] \end{aligned} \tag{10.28}$$

□

Dependent Variable: Y
Method: Least Squares
Date: 12/06/06 Time: 10:13
Sample: 1951 1980
Included observations: 30
Convergence achieved after 14 iterations
Y=C(1)*(X1^C(2))*(X3^C(3))*(X4^C(4))*(T^C(5))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.383833	1.032044	1.340865	0.1920
C(2)	0.659196	0.112205	5.874928	0.0000
C(3)	0.533465	0.126312	4.223392	0.0003
C(4)	1.054275	0.402636	2.618434	0.0148
C(5)	0.202450	0.115125	1.758526	0.0909

R-squared	0.955638	Mean dependent var	45.63967
Adjusted R-squared	0.940540	S.D. dependent var	21.74352
S.E. of regression	4.932474	Akaike info criterion	6.180570
Sum squared resid	608.2326	Schwarz criterion	6.414103
Log likelihood	-87.70856	Durbin-Watson stat	1.134412

(a)

Date: 12/06/06 Time: 10:16
Sample(adjusted): 1952 1980
Included observations: 29 after adjusting endpoints
Convergence achieved after 8 iterations
Y=C(1)*(X1^C(2))*(X3^C(3))*(X4^C(4))*(T^C(5))+[AR(1)=C(6)]

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.967503	1.139905	0.848757	0.4048
C(2)	0.695333	0.150491	4.620441	0.0001
C(3)	0.546242	0.119533	4.569794	0.0001
C(4)	1.019275	0.508739	2.003531	0.0570
C(5)	0.152794	0.254695	0.599910	0.5544
C(6)	0.474723	0.211268	2.247023	0.0345

R-squared	0.962855	Mean dependent var	46.45862
Adjusted R-squared	0.954780	S.D. dependent var	21.65240
S.E. of regression	4.604376	Akaike info criterion	6.073883
Sum squared resid	487.6064	Schwarz criterion	6.356772
Log likelihood	-82.07130	Durbin-Watson stat	1.826987

(b)

Figure 10.17 Statistical results based on the two GCD models in (10.28)

Example 10.12. (GCD model with dummy variables) For illustration purposes, Figure 10.18 presents statistical results based on a GCD model, with its residual graph presented in Figure 10.20, using the following equation specification:

$$Y = (c(11)^*X1^{c(12)}{*}X3^{c(13)})^*DV1 + (c(21)^*X1^{c(22)}{*}X3^{c(23)})^*DV2 \qquad (10.29)$$

where $DV1$ and $DV2$ are the two dummy variables defined for two time periods, namely for $t \leq 15$ and $t > 15$.

Based on these statistical results, the following notes and conclusions are presented:

(1) The model in (10.30) in fact represents a pair of models, as follows:

$$\begin{aligned} Y_t &= (c(11)^*X1^{c(12)}{*}X3^{c(13)}) + u_t \quad \text{for } t \leq 15 \\ Y_t &= (c(21)^*X1^{c(22)}{*}X3^{c(23)}) + u_t \quad \text{for } t > 15 \end{aligned} \qquad (10.30)$$

Dependent Variable: Y
Method: Least Squares
Date: 01/16/08 Time: 06:39
Sample: 1951 1980
Included observations: 30
Convergence achieved after 10 iterations
Y=(C(11)*X1^C(12)*X3^C(13))*DV1+(C(21)*X1^C(22)*X3^C(23))*DV2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.746749	1.049523	0.711513	0.4836
C(12)	0.372866	0.237670	1.568838	0.1298
C(13)	0.245787	0.222037	1.106963	0.2793
C(21)	0.128061	0.075684	1.692042	0.1036
C(22)	0.441343	0.092622	4.765011	0.0001
C(23)	0.459341	0.149134	3.080043	0.0051

R-squared	0.948500	Mean dependent var	45.63967
Adjusted R-squared	0.937771	S.D. dependent var	21.74352
S.E. of regression	5.424073	Akaike info criterion	6.396427
Sum squared resid	706.0935	Schwarz criterion	6.676667
Log likelihood	-89.94641	Hannan-Quinn criter.	6.486078
Durbin-Watson stat	1.119416		

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	0.653579	(2, 24)	0.5292
Chi-square	1.307159	2	0.5202

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(12) - C(22)	-0.068477	0.255080
C(13) - C(23)	-0.213553	0.267473

Restrictions are linear in coefficients.

Figure 10.18 Statistical results based on the GCD model in (10.29) and the Wald test, using EViews 6 and the output represents a pair of NLS regression functions as follows:

$$Y_t = 0.746\,749{*}X1^{0.372\,866}\,{*}X3^{0.245\,787} \quad \text{for} \quad t \le 15$$
$$Y_t = 0.128\,061{*}X1^{0.441\,343}\,{*}X3^{0.459\,341} \quad \text{for} \quad t > 15 \qquad (10.31)$$

(2) Based on the Wald test, namely the chi-squared-statistics, the null hypothesis H_0: $c(12) = c(22)$, $c(13) = c(23)$ is accepted, with a p-value = 0.5202.
(3) Furthermore, the hypotheses on the 'return to scales of the function' can also be tested, in each time period, by entering $c(12) + c(13) = 1$ or $c(22) + c(23) = 1$. Do this as an exercise. □

Dependent Variable: Y
Method: Least Squares
Date: 01/16/08 Time: 06:44
Sample (adjusted): 1953 1980
Included observations: 28 after adjustments
Convergence achieved after 12 iterations
Y=(C(11)*X1^C(12)*X3^C(13))*DV1+(C(21)*X1^C(22)*X3^C(23))*DV2
+[AR(1)=C(1), AR(2)=C(2)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.685755	1.169504	0.586363	0.5642
C(12)	0.283693	0.335698	0.845083	0.4081
C(13)	0.363269	0.242338	1.499016	0.1495
C(21)	0.105560	0.067163	1.571702	0.1317
C(22)	0.465847	0.093341	4.990801	0.0001
C(23)	0.461004	0.121798	3.784999	0.0012
C(1)	0.642336	0.219729	2.923313	0.0084
C(2)	-0.377790	0.224911	-1.679733	0.1086

R-squared	0.962747	Mean dependent var	47.32179
Adjusted R-squared	0.949708	S.D. dependent var	21.53564
S.E. of regression	4.829556	Akaike info criterion	6.222342
Sum squared resid	466.4922	Schwarz criterion	6.602972
Log likelihood	-79.11279	Hannan-Quinn criter.	6.338705
Durbin-Watson stat	2.019488		

Inverted AR Roots	.32+.52i	.32-.52i

Figure 10.19 Statistical results based on the AR(2) GCD model in (10.32)

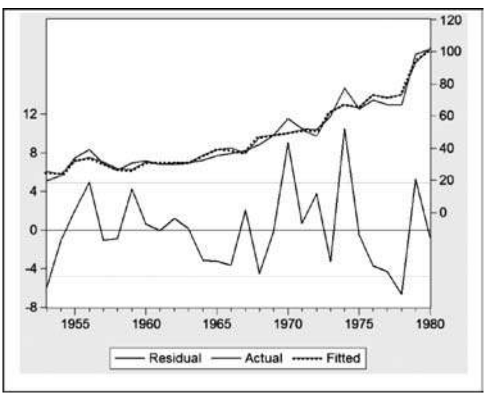

Figure 10.20 Residual graph of the AR GCD model in (10.30)

Example 10.13. (AR(2) GCD model with dummy variables) As an extension of the GCD model in (10.29), Figure 10.19 presents statistical results, and Figure 10.20 shows the residual graph, based on an AR(2) GCD model, using the following equation specification:

$$\begin{aligned} Y = &\ (c(11)^*X1^{c(12)}{}^*X3^{c(13)})^*DV1 + (c(21)^*X1^{c(22)}{}^*X3^{c(23)})^*DV2 \\ &+ [ar(1) = c(1), ar(2) = c(2)] \end{aligned} \tag{10.32}$$

□

Example 10.14. (Autoregressive bivariate GCD models) The following equation specifications represent a bivariate AR(1) GCD model, but the '*Overflow*' error message was obtained:

$$\begin{aligned} Y1 &= c(11)^*X1^{c(12)}{}^*X3^{c(13)}{}^*X4^{c(14)} + [ar(1) = c(15)] \\ Y2 &= c(21)^*X1^{c(22)}{}^*X3^{c(23)} + [ar(1) = c(24)] \end{aligned} \tag{10.33}$$

After doing experimentation, the statistical results based on a modified model are finally obtained, as presented in Figure 10.21, with its residual graphs in Figure 10.22, using EViews 6 with the default options. The equation specifications used are as follows:

$$\begin{aligned} Y1 &= C(11)^*X1^{C(12)}{}^*X3^{C(13)}{}^*Y1(-1)^{C(14)} \\ Y2 &= C(21)^*X1^{C(22)}{}^*X2^{C(23)}{}^*Y2(-1)^{C(24)} \end{aligned} \tag{10.34}$$

System: UNTITLED
Estimation Method: Iterative Least Squares
Date: 01/14/08 Time: 16:36
Sample: 1952 1980
Included observations: 29
Total system (balanced) observations 58
Convergence achieved after 6 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.498449	0.079585	6.263120	0.0000
C(12)	-0.132685	0.088015	-1.507520	0.1380
C(13)	0.209227	0.076707	2.727616	0.0088
C(14)	1.109961	0.090150	12.31238	0.0000
C(21)	0.299560	0.087650	3.417677	0.0013
C(22)	0.226358	0.122244	1.851695	0.0700
C(23)	0.370872	0.104469	3.550081	0.0008
C(24)	0.321680	0.135689	2.370708	0.0216

Determinant residual covariance 78.26632

Equation: Y1=C(11)*X1^C(12)*X3^C(13)*Y1(-1)^C(14)
Observations: 29

R-squared	0.976713	Mean dependent var	31.28655
Adjusted R-squared	0.973918	S.D. dependent var	12.73949
S.E. of regression	2.057408	Sum squared resid	105.8232
Durbin-Watson stat	1.671710		

Equation: Y2=C(21)*X1^C(22)*X3^C(23)*Y2(-1)^C(24)
Observations: 29

R-squared	0.951178	Mean dependent var	46.45862
Adjusted R-squared	0.945319	S.D. dependent var	21.65240
S.E. of regression	5.063170	Sum squared resid	640.8922
Durbin-Watson stat	1.505400		

Figure 10.21 Statistical results based on the GCD model in (10.34)

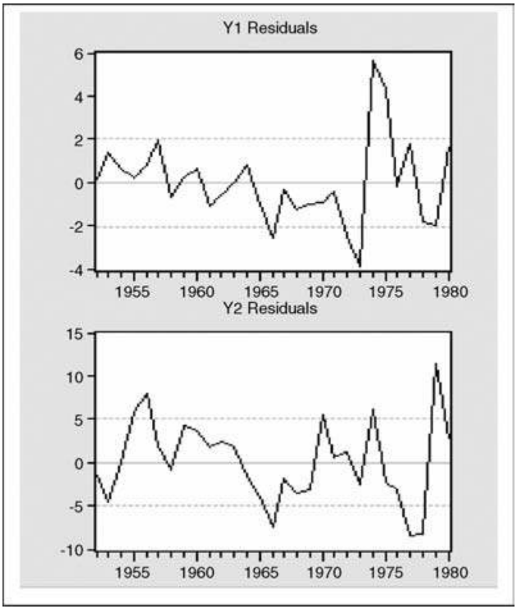

Figure 10.22 Residual graphs of the GCD model in (10.34)

Corresponding to these results the following notes and additional findings are observed:

(1) Since both regressions in the model (10.34) have $DW < 1.70$, the trial-and-error methods are used in order to obtain better models. Finally, a full GCD model is obtained, as given in Figure 10.23(a), with the following equation:

$$\begin{aligned} y1 &= c(11)*x1^{c(12)}*x3^{c(13)}*y1(-1)^{c(14)}*y1(-2)^{c(15)} \\ y2 &= c(21)*x1^{c(22)}*x3^{c(23)}*y2(-1)^{c(24)} + [ar(1) = c(25), ar(2) = C(26)] \end{aligned} \quad (10.35)$$

(2) Furthermore, since $y2(-1)$ is insignificant with a large p-value $= 0.7716$, the reduced model in Figure 10.23(b) is obtained, which could be considered as the best fit GCD model and therefore the final acceptable model.
(3) The first regression in (10.35) can be presented as follows:

$$\begin{aligned} \log(y1) = {} & \log(c(11)) + c(12)\log(X1) + c(13)\log(x3) \\ & + c(14)\log(y1(-1)) + c(15)\log(y1(-2)) \end{aligned} \quad (10.36)$$

Estimation Method: Iterative Least Squares
Date: 01/14/08 Time: 17:27
Sample: 1953 1980
Included observations: 29
Total system (unbalanced) observations 55
Convergence achieved after 12 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.541475	0.087947	6.156800	0.0000
C(12)	-0.136569	0.085703	-1.593515	0.1182
C(13)	0.214288	0.074880	2.861753	0.0064
C(14)	1.389846	0.174343	7.971929	0.0000
C(15)	-0.310256	0.162936	-1.904166	0.0634
C(21)	0.271174	0.103274	2.625766	0.0118
C(22)	0.368691	0.161964	2.276374	0.0277
C(23)	0.392680	0.111209	3.531012	0.0010
C(24)	0.056530	0.193570	0.292038	0.7716
C(25)	0.565864	0.302567	1.870209	0.0681
C(26)	-0.341691	0.255973	-1.334868	0.1888

Determinant residual covariance 51.76652

Equation: Y1=C(11)*X1^C(12)*X3^C(13)*Y1(-1)^C(14)*Y1(-2)^C(15)
Observations: 28

R-squared	0.979085	Mean dependent var	31.71071
Adjusted R-squared	0.975448	S.D. dependent var	12.76302
S.E. of regression	1.999847	Sum squared resid	91.98593
Durbin-Watson stat	2.002016		

Equation: Y2=C(21)*X1^C(22)*X3^C(23)*Y2(-1)^C(24)+[AR(1)=C(25),AR(2)=C(26)]
Observations: 27

R-squared	0.956158	Mean dependent var	48.34741
Adjusted R-squared	0.945719	S.D. dependent var	21.23760
S.E. of regression	4.947989	Sum squared resid	514.1344
Durbin-Watson stat	2.031563		

(a) The Full GCD Model

Estimation Method: Iterative Least Squares
Date: 01/14/08 Time: 17:32
Sample: 1953 1980
Included observations: 30
Total system (balanced) observations 56
Convergence achieved after 7 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.541475	0.087947	6.156800	0.0000
C(12)	-0.136569	0.085703	-1.593515	0.1179
C(13)	0.214288	0.074880	2.861753	0.0063
C(14)	1.389846	0.174343	7.971929	0.0000
C(15)	-0.310256	0.162936	-1.904166	0.0632
C(21)	0.243919	0.090897	2.683484	0.0101
C(22)	0.419756	0.082984	5.058272	0.0000
C(23)	0.387068	0.104384	3.708104	0.0006
C(25)	0.637540	0.211951	3.007964	0.0043
C(26)	-0.372686	0.211070	-1.765701	0.0841

Determinant residual covariance 51.76638

Equation: Y1=C(11)*X1^C(12)*X3^C(13)*Y1(-1)^C(14)*Y1(-2)^C(15)
Observations: 28

R-squared	0.979085	Mean dependent var	31.71071
Adjusted R-squared	0.975448	S.D. dependent var	12.76302
S.E. of regression	1.999847	Sum squared resid	91.98593
Durbin-Watson stat	2.002016		

Equation: Y2=C(21)*X1^C(22)*X3^C(23)+[AR(1)=C(25),AR(2)=C(26)]
Observations: 28

R-squared	0.957307	Mean dependent var	47.32179
Adjusted R-squared	0.949883	S.D. dependent var	21.53564
S.E. of regression	4.821161	Sum squared resid	534.6027
Durbin-Watson stat	2.021896		

(b) It's Reduced Model

Figure 10.23 Statistical results based on (a) the LVAR GCD model in (10.35) and (b) its reduced model

Since this model is an LV(2) translog linear model, then the first regression in (10.35) will be named using the terminology the LV(2)_GCD model. Furthermore, the second regression in (10.35) will be called the LVAR(1,2)_GCD model. Transform the second model to a translog model.

(4) By selecting *Quick/Estimate Equations* and then entering the first regression in (10.35) as the equation specification, the 'Overflow' error message is obtained. However, by selecting *Object/New Object/System* and entering the same equation, the results presented in Figure 10.24(a) are obtained. These are really unexpected findings. They suggest that the system estimation method may need to be tried if the quick estimation method presents an error message. For this reason, it is proposed that readers should apply the system estimation method for all the GCD models presented in the previous examples as an exercise.
(5) For the second GCD model in (10.35), however, the results in Figure 10.24(b) are obtained by using the *Quick/Estimate Equations*. Compare the estimates in this figure with the estimates in Figure 10.23(a).
(6) Based on each GCD model in the multivariate GCD model in (10.35), various alternative GCD models could be derived. Some selected models are presented in the following examples. □

System: SYS15 Workfile: GUJARATI_12.7::Untitled\

View | Proc | Object | Print | Name | Freeze | MergeText | Estimate | Spec | Stats | Resids

System: SYS15
Estimation Method: Iterative Least Squares
Date: 01/15/08 Time: 05:35
Sample: 1953 1980
Included observations: 28
Total system (balanced) observations 28
Convergence achieved after 2 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.541475	0.087947	6.156800	0.0000
C(12)	-0.136569	0.085703	-1.593515	0.1247
C(13)	0.214288	0.074880	2.861753	0.0088
C(14)	1.389846	0.174343	7.971929	0.0000
C(15)	-0.310256	0.162936	-1.904166	0.0695

Determinant residual covariance 3.285212

(a) The System Equation Estimates

Dependent Variable: Y2
Method: Least Squares
Date: 01/15/08 Time: 06:06
Sample (adjusted): 1954 1980
Included observations: 27 after adjustments
Convergence achieved after 12 iterations
Y2=C(21)*X1^C(22)*X3^C(23)*Y2(-1)^C(24)+[AR(1)=C(25),AR(2)=C(26)]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(21)	0.271174	0.103274	2.625766	0.0158
C(22)	0.368691	0.161964	2.276374	0.0334
C(23)	0.392680	0.111209	3.531012	0.0020
C(24)	0.056530	0.193570	0.292038	0.7731
C(25)	0.565864	0.302567	1.870209	0.0755
C(26)	-0.341691	0.255973	-1.334868	0.1962

R-squared	0.956158	Mean dependent var	48.34741
Adjusted R-squared	0.945719	S.D. dependent var	21.23760
S.E. of regression	4.947989	Akaike info criterion	6.228969
Sum squared resid	514.1344	Schwarz criterion	6.516933
Log likelihood	-78.09109	Hannan-Quinn criter.	6.314596
Durbin-Watson stat	2.031563		

Inverted AR Roots .28+.51i .28-.51i

(b) The Quick Equation Estimates

Figure 10.24 Statistical results based on each GCD model in (10.35), using (a) the system equation estimates and (b) the quick equation estimates

Example 10.15. (VAR GCD models) Figure 10.25 presents the statistical results based on the GCD model, which should be considered as a VAR GCD model, since the GCD model

$$\begin{aligned} y1 &= c(11)*x1^{c(12)}*x3^{c(13)}*y1(-1)^{c(14)}*y2(-1)^{c(15)} \\ y2 &= c(21)*x1^{c(22)}*x3^{c(23)}*y1(-1)^{c(24)}*y2(-1)^{c(25)} \end{aligned} \tag{10.37}$$

System: UNTITLED
Estimation Method: Iterative Least Squares
Date: 01/16/08 Time: 17:55
Sample: 1952 1980
Included observations: 29
Total system (balanced) observations 58
Convergence achieved after 5 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.526594	0.075286	6.994618	0.0000
C(12)	-0.258339	0.089919	-2.873020	0.0060
C(13)	0.226409	0.068130	3.323193	0.0017
C(14)	1.096778	0.078884	13.90372	0.0000
C(15)	0.196549	0.070264	2.797298	0.0074
C(21)	0.291287	0.087854	3.315587	0.0017
C(22)	0.150732	0.181814	0.829046	0.4112
C(23)	0.411561	0.129139	3.186963	0.0025
C(24)	0.078535	0.140306	0.559743	0.5783
C(25)	0.329547	0.137874	2.390200	0.0208

Determinant residual covariance 57.54364

Equation: Y1=C(11)*X1^C(12)*X3^C(13)*Y1(-1)^C(14)*Y2(-1)^C(15)
Observations: 29

R-squared	0.982356	Mean dependent var	31.28655
Adjusted R-squared	0.979416	S.D. dependent var	12.73949
S.E. of regression	1.827761	Sum squared resid	80.17706
Durbin-Watson stat	1.689802		

Equation: Y2=C(21)*X1^C(22)*X3^C(23)*Y1(-1)^C(24)*Y2(-1)^C(25)
Observations: 29

R-squared	0.951779	Mean dependent var	46.45862
Adjusted R-squared	0.943742	S.D. dependent var	21.65240
S.E. of regression	5.135668	Sum squared resid	633.0021
Durbin-Watson stat	1.570503		

Figure 10.25 Statistical results based on the VAR GCD model in (10.37)

in this figure can be presented as the following translog LS VAR model:

$$\begin{aligned}\log(Y1) &= \log(c(11)) + c(12)\log(X1) + c(13)\log(X3) \\ &\quad + c(14)\log(Y1(-1)) + c(15)\log(Y2(-1)) \\ \log(Y2) &= \log(c(21)) + c(22)\log(X1) + c(23)\log(X3) \\ &\quad + c(24)\log(Y1(-1)) + c(25)\log(Y2(-1))\end{aligned} \tag{10.38}$$

As a further study, an attempt has been made to apply a similar NLS model based on the Demo.wf1, namely

$$\begin{aligned} m1 &= c(11)*pr^{c(12)}*rs^{c(13)}*m1(-1)^{c(14)}*gdp(-1)^{c(15)} \\ gdp &= c(21)*pr^{c(22)}*rs^{c(23)}*m1(-1)^{c(24)}*gdp(-1)^{c(25)}\end{aligned} \tag{10.39}$$

However, the error message presented in Figure 10.26 is obtained. This finding again proves the statement that the statistical result, using the defaults options, is highly dependent on the data that happen to be selected by the researchers, as well as the starting values of parameters. Refer to the special notes in Section 10.5. □

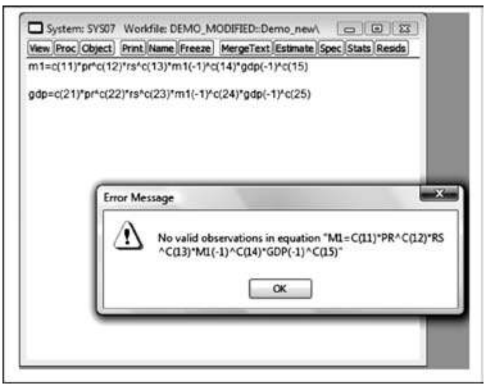

Figure 10.26 An error message using Demo.wf1

Example 10.16. (GCD models with trend) Corresponding to the first model in (10.35), Figure 10.27 presents the statistical results based on the GCD model with trend, as follows:

$$y1 = c(11)*x1^{c(12)}*x3^{c(13)}*y1(-1)^{c(14)}*y2(-1)^{c(15)} + (16)*t) \tag{10.40}$$

However, by using the following two modified models:

$$\begin{aligned} y1 &= c(11)*x1^{c(12)}*x3^{c(13)}*y1(-1)^{c(14)}*y2(-1)^{c(15)}*\exp(c(16)*t) \\ y1 &= c(11)*x1^{c(12)}*x3^{c(13)}*y1(-1)^{c(14)}*y2(-1)^{c(15)}*t^{c(16)}\end{aligned} \tag{10.41}$$

System: UNTITLED
Estimation Method: Iterative Least Squares
Date: 01/16/08 Time: 17:42
Sample: 1953 1980
Included observations: 28
Total system (balanced) observations 28
Convergence achieved after 6 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	0.596098	0.091241	6.533237	0.0000
C(12)	0.078265	0.146062	0.535834	0.5974
C(13)	0.147977	0.075203	1.967692	0.0618
C(14)	1.043190	0.246286	4.235681	0.0003
C(15)	-0.256744	0.136769	-1.877209	0.0738
C(16)	-0.325578	0.227512	-1.431039	0.1665

Determinant residual covariance 3.052975

Equation: Y1=C(11)*X1^C(12)*X3^C(13)*Y1(-1)^C(14)*Y1(-2)^C(15)+C(16)*T
Observations: 28

R-squared	0.980564	Mean dependent var	31.71071
Adjusted R-squared	0.976147	S.D. dependent var	12.76302
S.E. of regression	1.971194	Sum squared resid	85.48331
Durbin-Watson stat	2.091834		

Figure 10.27 Statistical results based on the GCD model in (10.40)

Figure 10.28 An error message using the second model in (10.35) with trend

the 'overflow'error message is obtained. On the other hand, by using the second model in (10.35) with trend, the error message presented in Figure 10.28 is obtained. □

Example 10.17. (A VAR GCD model with trend) Based on the VAR GCD model in (10.37), a VAR model with trend as presented in Figure 10.29 is applied, with an error message. □

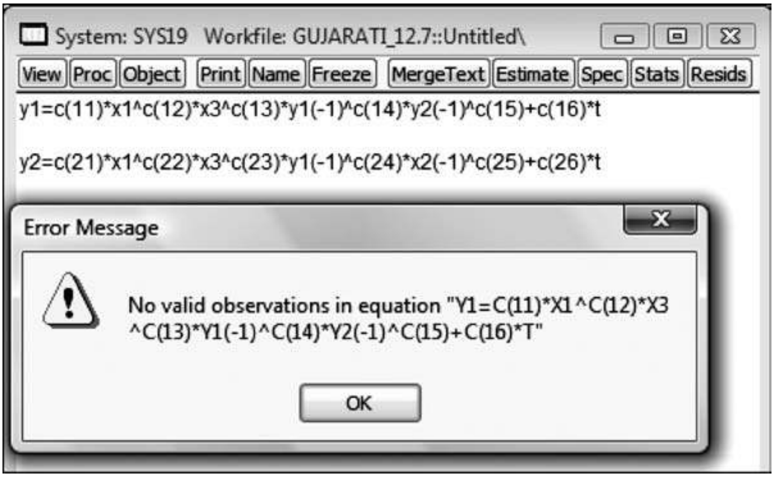

Figure 10.29 A VAR GCD model with trend based on the model in (10.37)

Example 10.18. (Instrumental variable GCD models) As an extension of the model in (10.37), Figure 10.30 presents the statistical results based on an instrumental GCD model in (10.42), using the *Quick/Estimate Equations* in EViews 5, since EViews 6 presents unexpected results, as in Figure 10.31:

$$\begin{array}{l} Y = c(1)^* X1^{c(2)} {}^* X3^{c(3)} {}^* X4^{c(4)} \\ Instrument \quad Y(-1)\, X1\, X1(-1)\, X3(-1) \end{array} \tag{10.42}$$

Note that no good guide exists on how to select a set of instrumental variables, as already mentioned in Chapter 7. In this case, it is assumed that $X4$ is correlated with the residual series, so $X4$ is not in the instrumental list.

Dependent Variable: Y
Method: Two-Stage Least Squares
Date: 12/08/06 Time: 15:31
Sample(adjusted): 1952 1980
Included observations: 29 after adjusting endpoints
Convergence achieved after 1 iteration
Y=C(1)*X1^C(2)*X3^C(3)*X4^C(4)
Instrument list: Y(-1) X1 X1(-1) X3(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	5.14E-15	3.15E-13	0.016325	0.9871
C(2)	-0.377802	6.914936	-0.054636	0.9569
C(3)	21.82117	37.88019	0.576057	0.5697
C(4)	21.82302	49.57646	0.440189	0.6636

R-squared	-17.212733	Mean dependent var	46.45862
Adjusted R-squared	-19.398261	S.D. dependent var	21.65240
S.E. of regression	97.79186	Sum squared resid	239081.2
Durbin-Watson stat	2.159574		

Figure 10.30 Statistical results using EViews 5, based on the model in (10.42)

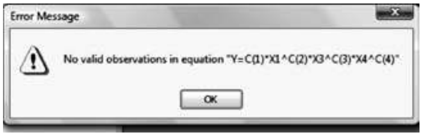

Figure 10.31 An error message in the system equation method of EViews 6

Based on this model, further analyses have been done, producing the statistical results presented in Figure 10.32. Based on these results, the following notes and conclusions are presented:

(1) The null hypothesis H_0: $C(2) + C(3) + C(4) = 1$ is accepted based on the chi-squared-statistic with $df = 1$ and a p–value $= 0.6513$. Note that the results show a negative estimate of $C(2)$. Hence, this model does not meet the basic requirement of the CD production function.
(2) The null hypothesis of no serial correlation of the error terms is accepted, based on the Breusch–Godfrey serial correlation test with a p-value $=$ 0.286 780, even though the residual graph represents something different.
(3) The null hypothesis H_0: $C(2) = C(3) = C(4) = 0$ is accepted based on the chi-squared-statistic with $df = 3$ and a p–value $= 0.6651$. The null hypothesis H_0: $C(1) = 0$ is also accepted, based on the t-statistic with a large p-value $=$ 0.9871. Based on these findings, in addition to the residual graph, it could be said that this model is an unacceptable or a bad model, in a statistical sense. However, how could this model be improved? Experimentation should be done to find several or as many as possible acceptable models and then

Wald Test:
Equation: EQ02_INST

Test Statistic	Value	df	Probability
F-statistic	0.204249	(1, 25)	0.6562
Chi-square	0.204249	1	0.6513

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2) + C(3) + C(4)	42.26639	93.62227

Restrictions are linear in coefficients.

Wald Test:
Equation: EQ02_INST

Test Statistic	Value	df	Probability
F-statistic	0.524934	(3, 25)	0.6692
Chi-square	1.574803	3	0.6651

Breusch-Godfrey Serial Correlation LM Test:

Obs*R-squared	2.498078	Probability	0.286780

Test Equation:
Dependent Variable: RESID
Method: Two-Stage Least Squares
Date: 12/08/06 Time: 15:43
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	4.58E-13	3.19E-13	1.432364	0.1655
C(2)	-3.903693	6.946395	-0.561960	0.5796
C(3)	-50.58775	37.95802	-1.332729	0.1957
C(4)	-57.84198	49.20144	-1.175616	0.2518
RESID(-1)	-0.255798	0.241846	-1.057690	0.3012
RESID(-2)	-0.039002	0.211684	-0.184247	0.8554

R-squared	0.086141	Mean dependent var	14.49702
Adjusted R-squared	-0.112524	S.D. dependent var	91.21921
S.E. of regression	96.21463	Akaike info criterion	12.15303
Sum squared resid	212916.9	Schwarz criterion	12.43692
Log likelihood	-170.2190	Durbin-Watson stat	1.849134

Figure 10.32 Statistical results for testing selected hypotheses

some of them could be presented for a comparative study. Finally, one should be selected that could be judged or considered as the best. Do not worry too much on your findings, since the true population model is never known (refer to Section 2.14.1).

(4) On the other hand, using the *Quick/Estimate Equations* of EViews 6, unexpected estimates were obtained, as presented in Figure 10.33, where all probabilities of the t-statistic are equal to one. By using the system estimation method, the error message in Figure 10.31 is obtained. Refer to the special notes in Section 10.5. □

Dependent Variable: Y
Method: Two-Stage Least Squares
Date: 01/16/08 Time: 07:04
Sample (adjusted): 1952 1980
Included observations: 29 after adjustments
Convergence achieved after 1 iteration
Y=C(1)*X1^C(2)*X3^C(3)*X4^C(4)
Instrument list: Y2(-1) X1 X1(-1) X3(-1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.010791	3.70E+53	2.92E-56	1.0000
C(2)	1.728886	2.30E+54	7.53E-55	1.0000
C(3)	-6.282216	4.16E+54	-1.51E-54	1.0000
C(4)	21.75734	4.18E+54	5.20E-54	1.0000

R-squared	-4.768264	Mean dependent var	46.45862
Adjusted R-squared	-5.460456	S.D. dependent var	21.65240
S.E. of regression	55.03484	Sum squared resid	75720.85
Durbin-Watson stat	0.027826	Second-Stage SSR	75720.85

Figure 10.33 Statistical results using EViews 6

10.4 Generalized CES models

The basic *constant elasticity of substitution* (CES) production function has the following form:

$$Q = A[\alpha K^{-\tau} + (1-\alpha)L^{-\tau}]^{-r/\tau} \tag{10.43}$$

where Q is an output factor, K and L are two input variables or factors, A, α and τ are model parameters and $r > 0$ is the scale of production (homogeneity degree). This CES function is considered as an homogenous function with r degrees. For data analysis using EViews, the model in (10.38) would have the following form:

$$Y = c(1)^*(c(2)^*X_1^{-c(3)} + (1-c(2)^*X_2^{-c(3)})^{-r/c(3)} + u \tag{10.44}$$

Note that the coefficients of the exogenous variables have a total of one. A more general model with multivariate exogenous variables could be considered, as follows:

$$Y = c(1)^*(c(2)^*X_1^{-c(3)} + c(4)^*X_2^{-c(3)} + \cdots)^{-r/c(3)} + u \tag{10.45}$$

Then the following hypothesis could be tested:

$$\begin{array}{l} H_0: \ C(2) + C(4) + \cdots + C(k+2) = 1 \\ H_0: \ \text{Otherwise} \end{array} \tag{10.46}$$

Dependent Variable: Y
Method: Least Squares
Date: 12/06/06 Time: 10:40
Sample: 1951 1980
Included observations: 30
Convergence not achieved after 500 iterations
Y=C(1)*(C(2)*((1/X1)^C(3)) + (1-C(2))*((1/X4)^C(3)))^(-1/C(3))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.209569	0.429688	0.487723	0.6297
C(2)	0.937551	0.232457	4.033224	0.0004
C(3)	0.122707	0.340551	0.360320	0.7214

R-squared	0.924191	Mean dependent var	45.63967
Adjusted R-squared	0.918576	S.D. dependent var	21.74352
S.E. of regression	6.204505	Akaike info criterion	6.583068
Sum squared resid	1039.389	Schwarz criterion	6.723188
Log likelihood	-95.74602	Durbin-Watson stat	1.408972

Dependent Variable: Y
Method: Least Squares
Date: 12/06/06 Time: 10:43
Sample: 1951 1980
Included observations: 30
Convergence achieved after 35 iterations
Y=C(1)*(C(2)*((1/X1)^C(3)) + (1-C(2))*((1/X4)^C(3)))^(-2/C(3))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.002178	0.002934	0.742324	0.4643
C(2)	0.974660	0.035948	27.11284	0.0000
C(3)	0.295472	0.125458	2.355145	0.0260

R-squared	0.917242	Mean dependent var	45.63967
Adjusted R-squared	0.911111	S.D. dependent var	21.74352
S.E. of regression	6.482652	Akaike info criterion	6.670776
Sum squared resid	1134.669	Schwarz criterion	6.810896
Log likelihood	-97.06164	Durbin-Watson stat	1.499363

Figure 10.34 Statistical results based on two basic CES models in (10.38), for $r = 1$ and $r = 2$ respectively

Example 10.19. (Simple CES models) Figure 10.34 presents statistical results using EViews 5 based on the models in (10.44) with $r = 1$ and $r = 2$ respectively, since EViews 6 presents the 'Overflow' error message. Note that the message '*Convergence not achieved after* 500 *iterations*' is given for the first model ($r = 1$), but for the second model ($r = 2$) the message is '*Convergence achieved after* 35 *iterations*.' For the other cases, using the same model may give the '*Overflow*' or other error massages, which is highly dependent on the data sets. □

Example 10.20. (CES models having three exogenous variables) By entering the following equation specification, an error message using EViews 5 and the 'Warning' error message using EViews 6 are obtained:

$$Y1 = c(1)*(c(2)*X1^{(-c(5))} + c(3)*X2^{(-c(5))} + c(4)*X3^{(-c(5))})\left(\frac{-1}{c(5)}\right) \tag{10.47}$$

Then by replacing $C(4)$ with $(1 - C(2) - C(3))$, the statistical results in Figure 10.35(a) are obtained using EViews 5 and using EViews 6 in Figure 10.35(b) with the 'Warning' error message. Since both results present very small DW-statistics, an attempt should be made to apply autoregressive models.

However, by using the corresponding AR(1) model the 'Near singular matrix' error message using EViews 6 and 'Attempt to raise a negative number to a non integer power' using EViews 5 will be obtained. □

Dependent Variable: Y1
Method: Least Squares
Date: 07/28/07 Time: 06:37
Sample: 1951 1980
Included observations: 30
Failure to improve SSR after 2 iterations
Y1=C(1)*(C(2)*X1^(-C(5))+C(3)*X2^(-C(5))+(1-C(2)-C(3))*X3^(-C(5)))^(-1
/C(5))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.000878	644.5155	1.36E-06	1.0000
C(2)	-441.9494	1.67E+09	-2.65E-07	1.0000
C(5)	5.101033	20556.14	0.000248	0.9998
C(3)	0.020932	78208.37	2.68E-07	1.0000

R-squared	-6.044187	Mean dependent var	30.87700
Adjusted R-squared	-6.856978	S.D. dependent var	12.71732
S.E. of regression	35.64703	Akaike info criterion	10.10877
Sum squared resid	33038.48	Schwarz criterion	10.29560
Log likelihood	-147.6316	Durbin-Watson stat	0.011271

(a) Using EViews 5

Dependent Variable: Y1
Method: Least Squares
Date: 01/16/08 Time: 16:34
Sample: 1951 1980
Included observations: 30
Convergence achieved after 3 iterations
WARNING: Singular covariance - coefficients are not unique
Y1=C(1)*(C(2)*X1^(-C(5))+C(3)*X2^(-C(5))+(1-C(2)-C(3))*X3^(-C(5)))^(-1
/C(5))

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.416310	NA	NA	NA
C(2)	142.2687	NA	NA	NA
C(5)	63.22380	NA	NA	NA
C(3)	4416089.	NA	NA	NA

R-squared	0.581507	Mean dependent var	30.87700
Adjusted R-squared	0.533220	S.D. dependent var	12.71732
S.E. of regression	8.688641	Akaike info criterion	7.285476
Sum squared resid	1962.804	Schwarz criterion	7.472302
Log likelihood	-105.2821	Hannan-Quinn criter.	7.345243
Durbin-Watson stat	0.237962		

(b) Using EViews 6

Figure 10.35 Statistical results using (a) EViews 5 and (b) EViews 6, based on a CES model with three exogenous variables

10.5 Special notes and comments

(1) Experimentation with LS and NLS models proved that their statistical results are highly dependent on the data set that happens to be selected by the researchers. Therefore, in fact, models have been found that fit the data. In other words, these are models that can give acceptable results in a statistical sense. Refer to the VAR models in Examples 10.15 and 10.17.

(2) However, corresponding to the NLS models, the statistical results also greatly depend on the starting coefficient values of the iterative estimation. Corresponding to the starting values, Eviews 6 User's Guide II (2007, pp. 625–627) presents several notes. Some of them are as follows:
- There are no general rules for selecting starting values for parameters.
- For nonlinear least squares type problems, EViews uses the values in the coefficient vector at the time the estimation procedure begins as starting values.
- For system estimators and ARCH, EViews uses the starting values based upon the preliminary single equation OLS or TSLS estimation.
- A poor choice of starting values may cause the nonlinear least squares algorithm to fail. EViews begins nonlinear estimation by taking derivatives of the objective function with respect to the parameters, evaluated at these values.

(3) Since the starting coefficient values are highly dependent on the coefficient vector at the time the estimation procedure begins, different estimates could be obtained by using or entering the same objective function at several time points. In some cases, there could also be an error message.

(4) The starting values can be changed, but it is not easy to select a good set of starting values. For this reason, the default starting values have been used for all GCD and GCES models presented in the previous examples.

10.6 Other NLS models

Since we have so many error messages based on the GCD and GCES models, then we have been doing experimentation with quadratic and higher degree polynomial objective functions. The main reason in using a quadratic objective function, since we know that a quadratic function should have either a minimum or maximum value. So that we can expect that the iterative procedure gives a global minimum or maximum value. However, we have found unexpected results or error messages based on selected quadratic NLS models. In the following examples, we only present the estimable NLS models, even though most of the regressors are insignificant, based on selected data sets.

10.6.1 Cases based on Demo.wf1

Example 10.21. (Quadratic NLS models) Figure 10.36 presents statistical results based on a quadratic NLS objective function as follows:

$$M1 = c(1) + (1 + c(3)*GDP + c(4)*PR^{c(5)})^2 \tag{10.48}$$

By using the following objective function, the 'Overflow' error message is obtained:

$$M1 = c(1) + c(2)*(1 + c(3)*GDP + c(4)*PR^{c(5)})^2 \tag{10.49}$$

In fact, error messages based on several other objective functions are also obtained. □

Dependent Variable: M1
Method: Least Squares
Date: 01/18/08 Time: 06:10
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 14 iterations
M1 =C(1)+(1+ C(3)*GDP+C(4)*PR^C(5))^2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	144.4052	4.704982	30.69197	0.0000
C(3)	0.029188	0.001709	17.07717	0.0000
C(4)	-17.36194	2.697872	-6.435421	0.0000
C(5)	2.889541	0.200197	14.43351	0.0000

R-squared	0.989277	Mean dependent var	445.0064
Adjusted R-squared	0.989094	S.D. dependent var	344.8315
S.E. of regression	36.01110	Akaike info criterion	10.02750
Sum squared resid	228236.7	Schwarz criterion	10.09846
Log likelihood	-898.4753	Hannan-Quinn criter.	10.05627
F-statistic	5412.412	Durbin-Watson stat	0.109425
Prob(F-statistic)	0.000000		

Figure 10.36 Statistical results based on the NLS model in (10.38)

Dependent Variable: M1B
Method: Least Squares
Date: 01/18/08 Time: 14:43
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 24 iterations
M1B= C(1)+(1+C(2)*GDP^C(3)+C(4)*PR)^3

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-5.626746	5.005207	-1.124179	0.2625
C(2)	12.90229	58.60632	0.220152	0.8260
C(3)	-0.667466	1.164425	-0.573216	0.5672
C(4)	1.167955	0.066365	17.59893	0.0000

R-squared	0.651560	Mean dependent var	1.415360
Adjusted R-squared	0.645621	S.D. dependent var	3.138281
S.E. of regression	1.868211	Akaike info criterion	4.109811
Sum squared resid	614.2771	Schwarz criterion	4.180765
Log likelihood	-365.8830	Hannan-Quinn criter.	4.138580
F-statistic	109.7029	Durbin-Watson stat	0.061487
Prob(F-statistic)	0.000000		

Figure 10.37 Statistical results based on the NLS model in (10.50)

Example 10.22. (Third-degree objective function) Figure 10.37 presents statistical results based on the following third-degree objective function:

$$M1B = c(1) + (1 + c(2)*GDP^{c(3)} + c(4)*PR)^3 \tag{10.50}$$

where $M1B = (M1 - 100)/(1300 - M1) > 0$ for all observed values of $M1$, with the lower and upper bounds subjectively selected for illustration purposes. This model corresponds to the bounded objective function as follows:

$$\log\left(\frac{m1-100}{1300-m1}\right) = \log\left[c(1) + \{1 + c(2)*gdp^{c(3)} + c(4)*pr\}^3\right] \tag{10.51}$$

□

Example 10.23. (Other NLS models) Figure 10.38, p. 496, presents statistical results based on the following NLS model and its reduced model:

$$M1 = c(1) + (1 + C92)*PR + C(3)*RS^{C(4)}) + C(5)*RS \tag{10.52}$$

□

Example 10.24. (Other NLS models based on Demo.wf1) By making an error in typing a parameter, the statistical results in Figure 10.39, with its residual graph in Figure 10.40, are obtained using the following objective function:

$$M1 = c(1) + (1 + c(2)*PR + c(3)*RS^{c(4)})^2 + RS^{c(3)} \tag{10.53}$$

Note the position of the parameter $C(3)$, which is the coefficient of $RS^{c(4)}$ and the power of RS. Furthermore, based on the following objective functions, the statistical results in Figure 10.41 are obtained:

$$M1 = c(1) + (1 + c(2)*PR + c(3)*RS)^2 + RS^{c(4)}) \tag{10.54}$$

□

Dependent Variable: M1
Method: Least Squares
Date: 01/18/08 Time: 17:36
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 10 iterations
M1=C(1)+(1+C(2)*PR+C(3)*RS^C(4))^2+C(5)*RS

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-591.0526	7.47E+11	-7.91E-10	1.0000
C(2)	-23.33042	2.09E+10	-1.11E-09	1.0000
C(3)	-11.34941	3.89E+10	-2.91E-10	1.0000
C(4)	-9.572267	1.70E+10	-5.62E-10	1.0000
C(5)	7.22E+11	1.22E+11	5.902456	0.0000

R-squared	-166376...	Mean dependent var	445.0064
Adjusted R-squared	-170179...	S.D. dependent var	344.8315
S.E. of regression	4.50E+12	Akaike info criterion	61.13476
Sum squared resid	3.54E+27	Schwarz criterion	61.22345
Log likelihood	-5497.128	Hannan-Quinn criter.	61.17072
Durbin-Watson stat	0.016102		

Dependent Variable: M1
Method: Least Squares
Date: 01/18/08 Time: 17:38
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 35 iterations
M1=C(1)+(1+C(2)*PR+C(3)*RS^C(4))^2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	132.1897	4.276065	30.91388	0.0000
C(2)	-31.14393	0.122754	-253.7102	0.0000
C(3)	3.857228	2.373477	1.625138	0.1059
C(4)	-4.015625	2.794933	-1.436752	0.1526

R-squared	0.988582	Mean dependent var	445.0064
Adjusted R-squared	0.988388	S.D. dependent var	344.8315
S.E. of regression	37.15928	Akaike info criterion	10.09028
Sum squared resid	243023.0	Schwarz criterion	10.16123
Log likelihood	-904.1248	Hannan-Quinn criter.	10.11904
F-statistic	5079.534	Durbin-Watson stat	0.075487
Prob(F-statistic)	0.000000		

Figure 10.38 Statistical results based on the NLS model in (10.52) and its reduced model

Dependent Variable: M1
Method: Least Squares
Date: 01/18/08 Time: 19:28
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 27 iterations
M1=C(1)+(1+C(2)*PR+C(3)*RS^C(4))^2+RS^C(3)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	128.3866	4.400733	29.17392	0.0000
C(2)	-31.15451	0.129455	-240.6586	0.0000
C(3)	0.562147	0.802459	0.700530	0.4845
C(4)	-4.867467	13.91098	-0.349901	0.7268

R-squared	0.988415	Mean dependent var	445.0064
Adjusted R-squared	0.988218	S.D. dependent var	344.8315
S.E. of regression	37.42977	Akaike info criterion	10.10478
Sum squared resid	246573.8	Schwarz criterion	10.17574
Log likelihood	-905.4303	Hannan-Quinn criter.	10.13355

Figure 10.39 Statistical results based on the NLS model in (10.53)

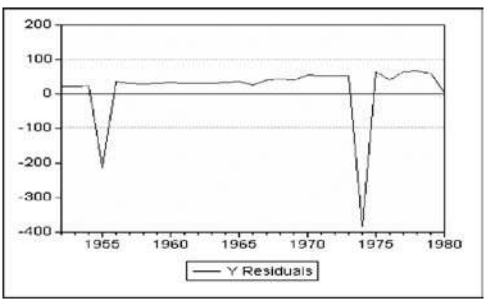

Figure 10.40 Residual graph of the model in Figure 10.39

Dependent Variable: M1
Method: Least Squares
Date: 01/18/08 Time: 19:33
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 16 iterations
M1=C(1)+(1+C(2)*PR+C(3)*RS)^2+RS*C(4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	133.7886	4.035558	33.15244	0.0000
C(2)	-32.35468	0.297846	-108.6289	0.0000
C(3)	0.278511	0.065310	4.264437	0.0000
C(4)	1.725458	0.119310	14.46200	0.0000

R-squared	0.990010	Mean dependent var	445.0064
Adjusted R-squared	0.989839	S.D. dependent var	344.8315
S.E. of regression	34.75879	Akaike info criterion	9.956713
Sum squared resid	212638.5	Schwarz criterion	10.02767
Log likelihood	-892.1042	Hannan-Quinn criter.	9.985482
F-statistic	5813.745	Durbin-Watson stat	0.071225
Prob(F-statistic)	0.000000		

Figure 10.41 Statistical results based on the NLS model in (10.54)

10.6.2 Cases based on the US_DPOC data

By applying similar models to the NLS models with acceptable statistical results based on Demo.wf1, acceptable statistical results might also be expected based on the US_DPOC data. In fact, it is recognized that there could be an error message. Do this as an exercise and find alternative NLS models in the following examples.

Example 10.25. (Quadratic objective function) Figure 10.42 presents statistical results based on a common quadratic function and its reduced model, with the objective function as follows:

$$P = c(1) + (1 + c(2)^*A + c(3)^*G)^2 + c(4)^*I \tag{10.55}$$

□

Dependent Variable: P
Method: Least Squares
Date: 01/18/08 Time: 19:16
Sample: 1951 1980
Included observations: 30
Convergence achieved after 1 iteration
P=C(1)+(1+C(2)*A+C(3)*G)^2+C(4)*I

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	7.159370	6.536460	1.095298	0.2834
C(2)	-0.001958	0.045459	-0.043080	0.9660
C(3)	0.002273	0.001194	1.903234	0.0681
C(4)	0.284634	0.074142	3.839035	0.0007

R-squared	0.924022	Mean dependent var	45.63967
Adjusted R-squared	0.915256	S.D. dependent var	21.74352
S.E. of regression	6.329725	Akaike info criterion	6.651957
Sum squared resid	1041.701	Schwarz criterion	6.838783
Log likelihood	-95.77935	Hannan-Quinn criter.	6.711724
F-statistic	105.4021	Durbin-Watson stat	1.250182
Prob(F-statistic)	0.000000		

Dependent Variable: P
Method: Least Squares
Date: 01/18/08 Time: 20:18
Sample: 1951 1980
Included observations: 30
Convergence achieved after 4 iterations
P=C(1)+(1+C(3)*G)^2+C(4)*I

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	6.962784	4.643324	1.499526	0.1453
C(3)	0.002222	0.000218	10.17333	0.0000
C(4)	0.286278	0.062780	4.560025	0.0001

R-squared	0.924018	Mean dependent var	45.63967
Adjusted R-squared	0.918390	S.D. dependent var	21.74352
S.E. of regression	6.211573	Akaike info criterion	6.585345
Sum squared resid	1041.758	Schwarz criterion	6.725465
Log likelihood	-95.78017	Hannan-Quinn criter.	6.630170
F-statistic	164.1743	Durbin-Watson stat	1.250382
Prob(F-statistic)	0.000000		

Figure 10.42 Statistical results based on the model in (10.55) and its acceptable reduced model

Example 10.26. (Quadratic NLS model) Figure 10.43 presents statistical results based on the following quadratic NLS model, with its residual graphs in Figure 10.44:

$$P = (c(1) + c(2)^{*}G^{(-c(3))} + c(4)^{*}A)^{2} \tag{10.56}$$

These residual graphs, as well as the DW-statistic, show the limitation of the model, since the model does not take into account the autocorrelation of the error terms. On the other hand, two out of four parameters have very large p-values. Try to modify this model as an exercise. □

Dependent Variable: P
Method: Least Squares
Date: 01/18/08 Time: 20:47
Sample: 1951 1980
Included observations: 30
Convergence achieved after 4 iterations
P=(C(1)+C(2)*G^(-C(3))+C(4)*A)^2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.738731	2.969229	1.259159	0.2192
C(2)	-5.96E-13	6.75E-12	-0.088268	0.9303
C(3)	-3.951039	1.415441	-2.791383	0.0097
C(4)	0.094599	0.124113	0.762201	0.4528

R-squared	-0.288071	Mean dependent var	45.63967
Adjusted R-squared	-0.436695	S.D. dependent var	21.74352
S.E. of regression	26.06226	Akaike info criterion	9.482420
Sum squared resid	17660.28	Schwarz criterion	9.669246
Log likelihood	-138.2363	Hannan-Quinn criter.	9.542187
Durbin-Watson stat	0.350099		

Figure 10.43 Statistical results based on the NLS model in (10.56)

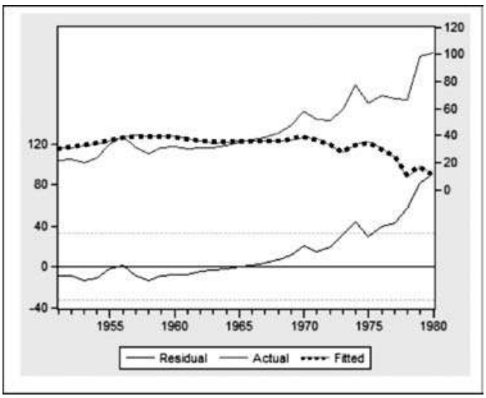

Figure 10.44 Residual graphs of the NLS model in (10.56)

Example 10.27. (Fifth-degree NLS model) Figure 10.45 presents statistical results based on a fifth-degree NLS model, with its residual graph in Figure 10.46. In a statistical sense, this model should be considered as a good fit model, even though its $DW = 1.06$, since three out of four parameters are significant at the level of 0.01 or 0.10, except the intercept $C(1)$. The objective function is

$$P = C(1) + (1 + C(2)^{*}G^{(-C(3))} + C(4)^{*}L)^{5} + C(4)^{*}L \tag{10.57}$$

□

Dependent Variable: P
Method: Least Squares
Date: 01/18/08 Time: 20:55
Sample: 1951 1980
Included observations: 30
Convergence achieved after 57 iterations
P= C(1)+ (1+C(2)*G^(-C(3))+C(4)*L)^5+C(4)*L

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.470910	9.805665	0.353970	0.7262
C(2)	0.248406	0.139480	1.780940	0.0866
C(3)	-0.192560	0.062856	-3.063497	0.0050
C(4)	0.000368	0.000115	3.206438	0.0035

R-squared	0.952668	Mean dependent var	45.63967
Adjusted R-squared	0.947206	S.D. dependent var	21.74352
S.E. of regression	4.995989	Akaike info criterion	6.178714
Sum squared resid	648.9575	Schwarz criterion	6.365540
Log likelihood	-88.68070	Hannan-Quinn criter.	6.238481
F-statistic	174.4356	Durbin-Watson stat	1.052565
Prob(F-statistic)	0.000000		

Figure 10.45 Statistical results based on the NLS model in (10.57)

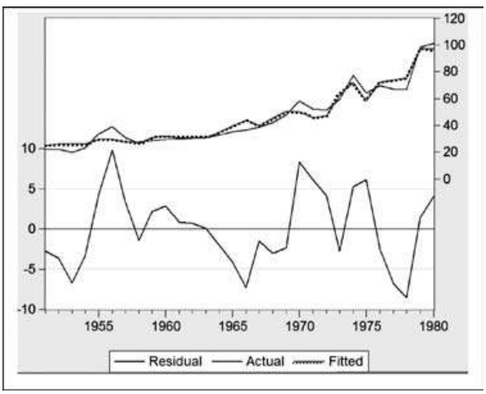

Figure 10.46 Residual graphs of the NLS model in (10.57)

Example 10.28. (TGARCH(1,1,0) NLS model) As an extension of the NLS model in (10.57), Figure 10.47 presents the statistical results based on the TGARCH(1,1,0) NLS model as follows:

$$P = C(1) + (1 + C(2)^{*}G^{(-C(3))} + C(4)^{*}L)^{5} + C(4)^{*}L \tag{10.58}$$

Dependent Variable: P
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/19/08 Time: 18:00
Sample: 1951 1980
Included observations: 30
Convergence achieved after 1 iteration (for starting values)
Convergence achieved after 30 iterations
Presample variance: backcast (parameter = 0.7)
P= C(1)+ (1+C(2)*G^(-C(3))+C(4)*L)^5+C(4)*L
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	3.475150	10.25185	0.338978	0.7346
C(2)	0.252431	0.147246	1.714350	0.0865
C(3)	-0.190538	0.066727	-2.855484	0.0043
C(4)	0.000375	0.000119	3.162273	0.0016
	Variance Equation			
C	10.14256	16.68997	0.607704	0.5434
RESID(-1)^2	-0.259534	0.232735	-1.115149	0.2648
GARCH(-1)	0.809519	0.576635	1.403868	0.1604

R-squared	0.951862	Mean dependent var	45.63967
Adjusted R-squared	0.939305	S.D. dependent var	21.74352
S.E. of regression	5.356822	Akaike info criterion	6.251052
Sum squared resid	659.9975	Schwarz criterion	6.577998
Log likelihood	-86.76577	Hannan-Quinn criter.	6.355644
F-statistic	75.79948	Durbin-Watson stat	1.036505
Prob(F-statistic)	0.000000		

Figure 10.47 Statistical results based on the TGARCH(1,1,0) NLS model in (10.58)

Dependent Variable: P
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/19/08 Time: 18:14
Sample: 1951 1980
Included observations: 30
Convergence achieved after 7 iterations (for starting values)
Convergence achieved after 29 iterations
Presample variance: backcast (parameter = 0.7)
P= C(1)+ (1+C(2)*G^(-C(3))+C(4)*L)^5+C(4)*L
GARCH = C(5) + C(6)*RESID(-1)^2*(RESID(-1)<0) + C(7)*GARCH(-1) +
C(8)*A + C(9)*H + C(10)*I

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	3.463529	10.29868	0.336308	0.7366
C(2)	0.244535	0.149725	1.633229	0.1024
C(3)	-0.200260	0.069465	-2.882910	0.0039
C(4)	0.000302	9.68E-05	3.123653	0.0018
	Variance Equation			
C	20.51892	34.91551	0.587673	0.5568
RESID(-1)^2*(RESID(-1)<0)	0.769087	0.664007	1.158252	0.2468
GARCH(-1)	0.708313	0.259138	2.733342	0.0063
A	-0.773563	0.924306	-0.836912	0.4026
H	-0.014332	0.016426	-0.872498	0.3829
I	0.270156	0.179644	1.503842	0.1326

R-squared	0.950949	Mean dependent var	45.63967
Adjusted R-squared	0.928875	S.D. dependent var	21.74352
S.E. of regression	5.798821	Akaike info criterion	6.115165
Sum squared resid	672.5265	Schwarz criterion	6.582231
Log likelihood	-81.72748	Hannan-Quinn criter.	6.264584
F-statistic	43.08171	Durbin-Watson stat	1.051380
Prob(F-statistic)	0.000000		

Figure 10.48 Statistical results based on the TGARCH(0,1,1) NLS model in (10.59)

Furthermore, Figure 10.48 presents the statistical results based on the TGARCH (0,1,1) NLS model with variance regressors, as follows:

$$\begin{aligned} P &= C(1) + (1 + C(2)^*G^{(-C(3))} + C(4)^*L)^5 + C(4)^*L \\ GARCH &= C(5) + C(6)^*RESID(-1)^2{}^*(RESID(-1) < 0) \\ &\quad + C(7)^*GARCH(-1) + C(8)^*A + C(9)^*H + C(10)^*I \end{aligned} \tag{10.59}$$

Since, all of the variance regressors are insignificant, modified models need to be found in order to produce an acceptable TGARCH NLS model. However, further modification will not be done. Do this as an exercise, since the author is very confident that readers can easily develop many alternative TGARCH(a, b, c) NLS models, as well as EGARCH, PARCH and CGARCH, based on the NLS model in (10.57). Likewise, a lot of ARCH models also can esily be developed based on any other NLS models. □

11

Nonparametric estimation methods

11.1 What is the nonparametric data analysis

By using the name 'distribution free statistics' instead of nonparametric estimation methods, it may be thought that there is no need for a distribution probability or density functions in testing hypotheses. In fact, any testing hypotheses should always be dependent on a specific probability distribution. The two simplest probability distributions, in nonparametric statistics, are the binomial and chi-squared distributions, which are widely used or applied in elementary data analysis. For this reason, it is suggested that the name 'distribution free statistics' should not be used for the nonparametric statistics.

When talking about the nonparametric statistics, Conover (1980, pp. 2–3) stated:

> *The nonparametric approach involved making and using simple and unsophisticated methods to find desired probabilities, or at least a good approximation to those probabilities, and the methods often involve less computational work, and therefore are easier and quicker to apply than other statistical methods.*

On the other hand, Hardle (1999, pp. 6–7) stated four main purposes for the nonparametric approach:

> *First, it provides a versatile method of exploring a general relationship between two variables. Second, it gives predictions of observations yet to be made without reference to a fixed parametric model. Third, it provides a tool for finding spurious observations by studying the influence of isolated points. Fourth, it constitutes a flexible method of substituting for missing values or interpolating between adjacent X-values.*

The nonparametric procedure or data analysis was introduced in the late 1930s, such as the Kendall τ, or Kendall-tau, index (1938), as a measure of nonparametric correlation, as well as the Spearman rank correlation. There is no

Time Series Data Analysis Using EViews IGN Agung

record of the binomial and chi-squared-statistics. Then nonparametric procedures were developed to nonparametric regression (Hardle, 1999) and nonparametric methods in multivariate analysis (Puri and Sen, 1993). Hardle presents smoothing techniques, which should be considered as sophisticated nonparametric estimation methods. The simplest one is the *kth nearest neighbor estimation method*, namely the *k-NN* estimate, which should be considered as an extension of the (very) basic moving average method, based on uncensored data sets. Three important theorems presented by Puri and Sen are the nonparametric multivariate central limit theorems, or NM_CTL, as an extension of the important central limit theorem (CTL). Since the CTL is considered as the base theorem for inferential statistical analysis of the parametric procedure, then the NM_CLTs should be considered as the important base theorems of the nonparametric statistical methods.

Furthermore, the nonparametric procedure has been extended to the analysis based on censored data sets, such as the Kendall-tau index has been extended to a *generalized-Kendall-tau* (*GKT*) or *Agung-Kendall-tau* (*AKT*), based on general right censored data. This AKT index and other nonparametric procedures for the general right censored data, such as the AKT in multivariate problems and alternative presentation of the multivariate Simon statistics, have been presented in the author's dissertation with a copyright (Agung, 1981).

This chapter presents the application of the nonparametric estimation methods provided by EViews 6. However, the presentation will begin with the basic or classical moving average, as presented in the following subsection. The theoretical concept of the moving average (MA) models are presented in Appendix A.

11.2 Basic moving average estimates

Alternative basic or classical moving average estimation methods based on the observed values of a time series $\{Y_t\}$, $t=1, 2, \ldots, T$, will be presented in the following subsections. The estimate(s) based on these estimation methods can easily be done or computed manually using Excel.

11.2.1 Simple moving average estimates

Basically, a moving average estimation method is to compute an average value of a set of observations at two time points or more. Corresponding to this idea, the following simple moving average estimation methods or nonparametric models are proposed:

(a) *The 2K Nearest Time Points* (*2K_NT*)

$$NT(Y_t) = \sum_{k=-K}^{k=+K} \frac{Y_{t+k}}{2K+1} + \mu_t \qquad (11.1)$$

Note that by using this estimation method, there will only be $(T - 2K)$ point estimates. As a result, there will not be estimates for the first K time points and the last K time points.

(b) *The K Previous Time Points* (*K_PT*)
As a modification of the model in (11.1), the following model is presented, namely the *K previous time points* (*K-PT*):

$$PT(Y_t) = \sum_{k=0}^{k=K} \frac{Y_{t-k}}{K+1} + \mu_t \tag{11.2}$$

Note that this model, in fact, represents the average of Y_t at $K + 1$ time points (i.e. the current time and the kth previous time points). Based on this model, the estimates from $t = K + 1$ up to $t = T$ can be obtained.

(c) *The K Nearest Time Forecast* (*K_NTF*)
Since the moving average estimation methods in (11.1) and (11.2) lack forecasting ability, such that the value of Y_t for $t = T + 1$ cannot be estimated, the following estimation method, namely the *K nearest time forecast* (*K-NTF*), is proposed:

$$NTF(Y_t) = \sum_{k=1}^{k=K} \frac{Y_{t-k}}{K} + \mu_t \tag{11.3}$$

Based on this method, the following estimate at the time $(T + 1)$, or one period ahead forecast, is found as follows:

$$\begin{aligned} NTF(Y_T) &= \sum_{k=1}^{k=K} \frac{Y_{T-k+1}}{K} + \mu_t \\ &= \frac{(Y_T + Y_{T+1} + \cdots + Y_{T-K+1})}{K} + \mu_t \end{aligned} \tag{11.4}$$

Then the time $(T + i)$, for $i > 1$, can be estimated recursively. For example, for $K = 3$:

$$\begin{aligned} NTF_{T+1} &= NTF(Y_{T+1}) = \frac{Y_T + Y_{T-1} + Y_{T-2}}{3} \\ NTF_{T+2} &= \frac{NTF_{T+1} + Y_T + Y_{T-1}}{3} \\ NTF_{T+3} &= \frac{NTF_{T+2} + NTF_{T+1} + Y_T}{3} \\ NTF_{T+4} &= \frac{NTF_{T+3} + NTF_{T+2} + NTF_{T+1}}{3} \\ &\ldots \\ NTF_{T+n} &= \frac{NTF_{T+n-1} + NTF_{T+n-2} + NTF_{T+n-3}}{3} \end{aligned} \tag{11.5}$$

The computation based on this model can easily be done manually or using Excel, as presented in the following example.

Example 11.1. (Classical moving average) Table 11.1 presents the basic moving average of $Y_t = M1_t$, based on a subsample of Demo.wf1, having 24 observations from 1990Q1 ($t = 1$) up to 1996Q4 ($t = 24$). This table presents the basic moving average estimation methods or nonparametric models in (11.1) to (11.3) for $K = 2$. Based on the basic statistics, namely the mean, SD, max and min, it could be said that the $2K$-NT estimate is the best fit.

The computation can easily be done using Excel. However, for a comparison, the moving average estimation method provided by Eviews will be considered in Section 11.4. Based on Table 11.1, note the following estimates:

(a) *The 2K-NT Moving Average or Estimate for K = 2*

$$MA_3 = ma(Y_3) = \frac{863.09 + 875.83 + 882.55 + 887.74 + 900.90}{5} = 882.02 \quad (11.6)$$

(b) *The K-PT Moving Average or Estimate, for K = 2*

$$MA_3 = ma(Y_3) = \frac{863.09 + 875.83 + 882.55}{3} = 873.82 \quad (11.7)$$

(c) *The K-NTF Estimate or Moving Average for K = 2*

$$\begin{aligned} MA_{T+1} &= MA_{29} = \frac{Y28 + Y27}{2} = \frac{1202.15 + 1218.99}{2} = 1210.57 \\ MA_{T+2} &= MA_{30} = (MA29 + Y28) = \frac{1210.57 + 1202.15}{2} = 1206.36 \quad (11.8) \\ MA_{T+3} &= MA_{31} = (MA30 + MA29) = \frac{1206.36 + 1210.57}{2} = 1208.465 \end{aligned}$$

□

11.2.2 The weighted moving average estimates

Corresponding to the *K-NTF* in (11.3), a weighted moving average estimate can be considered, as follows:

$$wma(Y_t) = \sum_{k=1}^{K} N(k)^* Y_{t-k} + \mu_t \quad (11.9)$$

where $N(1) = N_1 = 1 - K(K+1)\alpha/2$ and $N(k) = N_k = (k-1)\alpha$, for all $k > 1$.

Table 11.1 Illustration of the classical moving average estimates

Year/*Q*	*t*	*Yt*	*2k_NT*	Error1	*k_PT*	Error2	*Modk_PT*	Error3
1990Q1	1	863.09						
1990Q2	2	875.83						
1990Q3	3	882.55	882.02	0.53	873.82	8.73	869.46	13.09
1990Q4	4	887.74	892.27	−4.53	882.04	5.70	879.19	8.55
1991Q1	5	900.90	905.22	−4.33	890.40	10.50	885.15	15.75
1991Q2	6	914.36	920.66	−6.30	901.00	13.36	894.32	20.04
1991Q3	7	940.57	943.64	−3.08	918.61	21.96	907.63	32.94
1991Q4	8	959.75	966.46	−6.71	938.23	21.53	927.46	32.29
1992Q1	9	1002.64	993.77	8.87	967.65	34.99	950.16	52.48
1992Q2	10	1014.98	1023.55	−8.57	992.46	22.52	981.20	33.78
1992Q3	11	1050.91	1051.25	−0.33	1022.84	28.07	1008.81	42.10
1992Q4	12	1089.48	1077.86	11.62	1051.79	37.69	1032.94	56.53
1993Q1	13	1098.22	1108.59	−10.37	1079.54	18.69	1070.19	28.03
1993Q2	14	1135.69	1135.90	−0.21	1107.80	27.89	1093.85	41.84
1993Q3	15	1168.66	1160.06	8.60	1134.19	34.47	1116.96	51.70
1993Q4	16	1187.48	1182.72	4.75	1163.94	23.53	1152.17	35.30
1994Q1	17	1210.24	1197.78	12.46	1188.79	21.45	1178.07	32.17
1994Q2	18	1211.56	1204.92	6.64	1203.09	8.47	1198.86	12.70
1994Q3	19	1210.96	1209.27	1.69	1210.92	0.04	1210.90	0.06
1994Q4	20	1204.37	1211.11	−6.74	1208.96	−4.60	1211.26	−6.90
1995Q1	21	1209.24	1209.70	−0.47	1208.19	1.05	1207.66	1.57
1995Q2	22	1219.42	1207.03	12.39	1211.01	8.41	1206.80	12.62
1995Q3	23	1204.52	1205.32	−0.80	1211.06	−6.54	1214.33	−9.81
1995Q4	24	1197.61	1205.08	−7.47	1207.18	−9.57	1211.97	−14.36
1996Q1	25	1195.81	1204.99	−9.18	1199.31	−3.50	1201.06	−5.26
1996Q2	26	1208.03	1204.52	3.51	1200.48	7.54	1196.71	11.32
1996Q3	27	1218.99			1207.61	11.38	1201.92	17.07
1996Q4	28	1202.15			1209.72	−7.57	1213.51	−11.36
	$T+1$	—					1210.57	
		Mean		0.082		12.930		19.395
		SD		7.161		13.885		20.828
		Max		12.459		37.687		56.530
		Min		−10.370		−9.574		−14.361

In order to obtain the best estimate or the best fit, the trial-and-error methods should be used. Under the assumption (or a *rule of thumb*) that Y_{t-1} should contribute at least 90% to the estimate of $wma(Y_t)$, then α should be selected in the range of 0 up to $K(K+1)\alpha/2 < 0.10$ or $\alpha < 0.2/(K(K+1))$. For example, for $K = 4$, the values of α should be in the interval (0, 0.01].

For $K = 2$:

$$wma(Y_t) = (1-\alpha)Y_{t-1} + \alpha Y_{t-2} + \mu_t \tag{11.10}$$

Then the estimated or fitted values will be as follows:

$$\begin{aligned} \hat{Y}_t &= (1-\alpha)Y_{t-1} + \alpha Y_{t-2} = WMA_t \\ WMA_{t+1} &= (1-\alpha)\hat{Y}_t + \alpha Y_{t-1} = (1-\alpha)WMA_t + \alpha Y_{t-1} \end{aligned} \tag{11.11}$$

For $K = 3$:

$$wma(Y_t) = (1-3\alpha)Y_{t-1} + 2\alpha Y_{t-2} + \alpha Y_{t-3} + \mu_t \tag{11.12}$$

with the estimated or fitted values as follows:

$$\begin{aligned} \hat{Y}_t &= (1-3\alpha)Y_{t-1} + 2\alpha Y_{t-2} + \alpha Y_{t-3} = WMA_t \\ WMA_{t+1} &= (1-3\alpha)WMA_t + 2\alpha Y_{t-1} + \alpha Y_{t-2} \end{aligned} \tag{11.13}$$

For $\alpha = 0$:

$$wma(Y_t) = E(Y_t) = Y_{t-1} + \mu_t \tag{11.14}$$

which indicates that the expected value of Y_t is equal to the observed value of Y_{t-1}. This model or process could be extended to the conditional expectation as follows:

$$E(Y_t|Y_{t-1}, Y_{t-2}, \ldots, Y_{t-k}) = Y_{t-1} + \mu_t \tag{11.15}$$

11.3 Measuring the best fit model

In the basic regression, the best fit regression or curve should have the smallest mean squared error (MSE), which is measured as

$$MSE = \sum_{t=1}^{T} \frac{e_t^2}{T-k} \tag{11.16}$$

where T = total number of observations and k = number of parameters.

Since, corresponding to nonparametric curves, the number of the parameters is not known, then it is suggested that the mean absolute error (MAE) or the sum of squared error (SSE) should be used, which are computed as follows:

$$MAE = \sum_{t=1}^{T} \frac{|e_t|}{T} \tag{11.17}$$

$$SSE = \sum_{t=1}^{T} e_t^2 \tag{11.18}$$

On the other hand, Yaffee and McGee (2000, p. 17) proposed other measures of fit in comparing the fits of different time series models, namely the average percentage error in the entire series (MPE) and the mean absolute percentage error (MAPE), which are computed as follows:

$$MPE = \sum_{t=1}^{T} \frac{PE_t}{T} \tag{11.19}$$

$$MAPE = \sum_{t=1}^{T} \frac{|PE_t|}{T} \tag{11.20}$$

where

$$PE_t = \frac{100^* e_t}{\sum_{t=1}^{T} e_t} \tag{11.21}$$

indicates the percentage error at each time t.

11.4 Advanced moving average models

Corresponding to the basic moving average estimation methods based on a single time series $\{Y_t\}$, $t = 1, 2, \ldots, T$, presented in the previous subsections, in the following subsections the moving average models, the autoregressive moving average models, as well as the moving average models with covariates, will be presented using EViews 6.

11.4.1 The moving average models

EViews defines a moving average model by using the following equation specification.

$$Y\, C\, MA(1)\, MA(2) \cdots MA(k) \tag{11.22}$$

The corresponding model will be called the kth order or level moving average model, namely the MA(k) model. Experimentation has been done in order to study the characteristics of this model, with some of the results presented in the following examples.

Yaffee and McGee (2000, p. 137) stated that the corresponding functions of the moving average models are functions of the error terms. For example, the first-order moving average process, namely the MA(1) process, is a function of the current error and the previous error. Hence, the function corresponding to the model in (11.22) is a function of the current error and $(k - 1)$ previous errors.

Note that, if the equation specification is used or entered in the form of an explicit equation, such as $Y = C(1) + [MA(1) = C(2)]$, then the error message, as

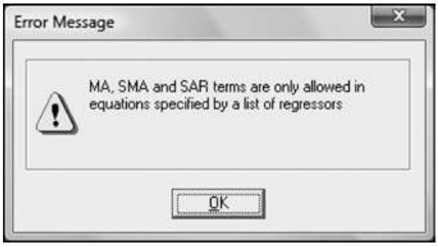

Figure 11.1 An error message for the MA models

presented in Figure 11.1, will be obtained. As a result, the MA model cannot be applied using the system estimation method (or the system function).

Example 11.2. (MA(*k*) for the endogenous variable *M*1) Table 11.2 presents a summary of statistical results based on an MA(5) model having the endogenous variable $M1$, its reduced model, namely an MA(4) model, and the MA(6) model.

Table 11.2 Statistical results summary based on an MA(5) model and its modified models

Dependent variable: $M1$
Method: least squares
Date: 01/12/08 Time: 17:02
Sample: 1952Q1 1996Q4
Included observations: 180
Convergence achieved after 33 iterations
MA backcast: 1950Q4 1951Q4

Variable	Coefficient	*t*-statistic	Coefficient	*t*-statistic	Coefficient	*t*-statistic
C	458.7634	14.699 27	455.9422	14.680 83	461.2874	14.026 02
$MA(1)$	2.277 701	51.157 81	2.549 964	60.553 64	2.336 831	54.878 96
$MA(2)$	3.214 210	38.928 13	3.238 417	41.470 32	3.549 493	45.449 91
$MA(3)$	3.117 261	33.267 36	2.347 540	30.899 98	3.980 295	37.562 08
$MA(4)$	2.030 308	25.590 73	0.810 940	20.222 06	3.349 814	32.598 63
$MA(5)$	0.790 707	18.593 27	—	—	2.098 819	29.981 29
$MA(6)$	—	—	—	—	0.805 527	21.203 13
R-squared	0.990 717		0.985 566		0.994 599	
Adjusted R^2	0.990 450		0.985 236		0.994 412	
SSR	197 587.3		307 221.8		114 948.5	
F-statistic	3713.956		2987.299		5310.147	
Prob(F-statistic)	0.000 000		0.000 000		0.000 000	
AIC	9.905 523		10.335 80		9.374 938	
SC	10.011 95		10.424 49		9.499 108	
HQC	9.948 676		10.371 76		9.425 284	
DW- statistic	0.895 234		0.995 158		1.102 522	

Based on this table the following notes and comments are presented:

(1) This table shows statistical results, which should be considered as unexpected results, since the indicator MA(5) is insignificant based on the MA(5) model, but it is significant based on the MA(6) model. In fact, it has also been found that it is significant based on the MA(7) and MA(8) models, with all MA(k) models being significant.
(2) The SSR (sum squared residual) decreases with increasing k of the MA(k) model. Should a model with the smallest value of SSE be used, as presented in Section 11.4? Corresponding to the results in Table 11.2, the MA(6) model is chosen as an acceptable or a good fit model, if it is based on the smallest SSR. Then if this is compared to the MA(8) model, the MA(8) model will be chosen as the best fit model.
(3) Corresponding to the DW-statistic, the MA(5) model has the greatest value. For this reason, should this model be chosen as a good fit model?
(4) These findings also show that (refer to the Section 2.14) a conclusion or decision should not be made that is highly dependent on only the sample statistics.
(5) For a comparison, Figure 11.2 presents the residual graphs of the MA(5) model and the $2K$-NT moving average estimate with $K=2$ presented in Example 11.1. Based on these graphs, it could be concluded that the $2K$-NT moving average estimate is better than the MA(5) model. This finding shows that the simple or classical moving average estimate could have a better estimate than complex estimation methods. □

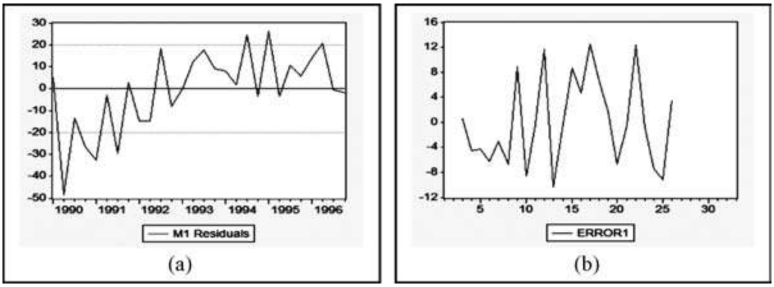

Figure 11.2 Residual graphs of (a) the MA(5) model and (b) the $2K$-NT moving average estimate for $K=2$, based on the subsample 1990Q1 1996Q4

Example 11.3. (Alternative simple MA models based on *M*1) Table 11.3 presents a summary of statistical results by using or entering the following equation specifications, based on the whole sample in Demo.wf1:

$$M1\ C\ MA(1) \tag{11.23}$$

$$\log(M1)\ C\ MA(1) \tag{11.24}$$

Table 11.3 Statistical results summary based on the four models

	$M1$		$\log(M1)$		$D(\log(M1))$		$D(\log(M1))$	
Variable	Coeff.	*t*-stat.	Coeff.	*t*-stat.	Coeff.	*t*-stat.	Coeff.	*t*-stat.
C	448.3	17.23	5.817	101.608	0.0126	10.131	0.0125	9.681
$MA(1)$	0.984	131.49	0.980	63.910	−0.0594	−0.790		
$MA(2)$	—	—	—	—	0.1501	1.997	0.1423	1.896
R-squared	0.741		0.737		0.034		0.031	
Adjusted R-squared	0.740		0.736		0.023		0.026	
SSR	175.922		26.804		0.041		0.041	
F-statistic	509.742		498.97		3.123		5.712	
Prob(F-statistic)	0.000 00		0.0000		0.046		0.018	
AIC	13.189		0.956		−5.515		−5.523	
SC	13.224		0.991		−5.462		−5.487	
HQC	13.203		0.970		−5.493		−5.509	
DW- statistic	0.034		0.098		1.947		2.062	

$$D(\log(M1))\ C\ MA(1)MA(2) \tag{11.25}$$

$$D(\log(M1)\ C\ MA(2) \tag{11.26}$$

Based on this summary, the DW-statistics and the SSE in particular, it could be said that the model in (11.26) has the best fit. Note that the MA model having the endogenous variable $D(\log(M1))$ is a return rate model (RRM) of the endogenous variable $M1$. Figure 11.3 presents its residual, actual and fitted graphs. Further residual analysis for this model can easily be done, as it has been presented in the previous chapter. Please do it for exercises.

However, corresponding to the R^2, as well the Adjusted R-squared, it is found that the model in (9.20) is the best one, since it has the largest R^2. For illustration purposes, Figure 11.3 presents the residuals graphs of the models in (9.20) and (9.23). Which one do you think is a better time series model? □

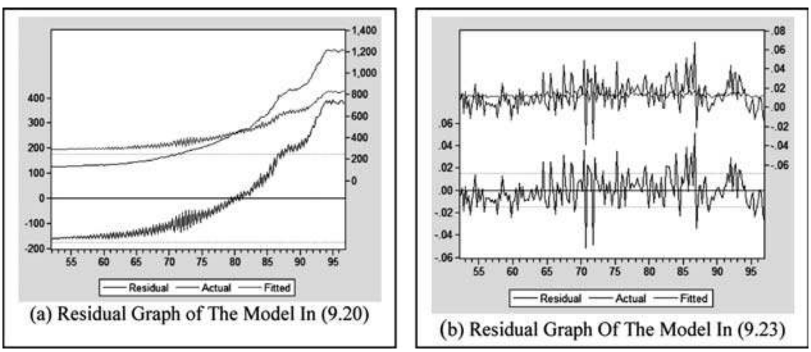

Figure 11.3 Residual graphs of the models in (a) (9.20) and (b) (9.23)

11.4.2 The autoregressive moving average models

Based on a single time series $\{Y_t\}$, $t=1, 2, \ldots, T$, presented in the previous subsection, an autoregressive moving average model is defined by using the following equation specification:

$$Y\,C\,AR(1)\cdots AR(p)\,MA(1)\cdots MA(q) \tag{11.27}$$

This model will be called the (p, q) autoregressive moving average model, namely the ARMA(p, q) model. In order to study the characteristics of this model, experimentation has been performed, with some of the results presented in the following examples.

Example 11.4. (ARMA(1,1) models based on *M*1) Table 11.4 presents a summary of statistical results based on three ARMA(1,1) models with endogenous variables $M1$, $\log(M1)$ and $D\log(M1)$ respectively. The statistical results are obtained by using the following equation specifications:

$$M1\,C\,AR(1)\,MA(1) \tag{11.28}$$

$$\log(M1)\,C\,AR(1)\,MA(1) \tag{11.29}$$

$$D(\log(M1))\,C\,AR(1)\,MA(1) \tag{11.30}$$

Table 11.4 Statistical results summary based on the models in (11.28) to (11.30)

	$M1$		$\log(M1)$		$D\log(M1)$	
Variable	Coefficient	t-stat.	Coefficient	t-stat.	Coefficient	t-stat.
C	−114.16	−0.835	634.0864	0.013	0.013	11.112
$AR(1)$	1.011	446.46	0.999 980	646.136	−0.981	−112.97
$MA(1)$	0.148	1.916	−0.021 524	−0.277	0.984	128.15
R-squared	0.999 331		0.999 582		0.060 367	
Adjusted R-squared	0.999 323		0.999 577		0.049 628	
Sum squared residual	14 180.86		0.042 234		0.039 676	
F-statistic	131 362.2		210 340.5		5.621 423	
Prob(F-statistic)	0.000 000		0.000 000		0.004 304	
AIC	7.243 659		−5.480 509		−5.537 202	
SC	7.297 079		−5.427 089		−5.483 576	
QHC	7.265 320		−5.458 848		−5.515 455	
DW- statistic	1.855 139		1.993 080		1.923 886	
Inverted AR roots	1.01[a]		1.00[b]		−0.98	
Inverted MA roots	−0.15		0.02		−0.98	

[a] With the error message ‘Estimated AR process is nonstationary’.
[b] Without the error message.

Based on this table, the following notes are given:

(1) The ARMA(1,1) model having the endogenous variable $M1$ has an inverted root that is strictly greater than one, and the output presents an error message '*Estimated AR process is nonstationary*.' This indicates that the model has heterogeneous error terms. Hence the model is an unstable model.
(2) The ARMA(1,1) model having the endogenous variable log($M1$) seems to be an unstable model, since it has an inverted root of one. However, its residual graph in Figure 11.4(a) does not clearly indicate its instability, compared to the model of $M1$. Since the result did not present the error message '*Estmated AR process is nonstationary*,' then it could be said that the root is in fact not outside the unit circle or the root is still less than one. However, it is presented as 1.00, because the number uses only two decimals. As a result, this model is an acceptable model.
(3) The ARMA (1,1) model of Dlog($M1$), with its residual graph in Figure 11.4(b), can be considered as a stable model, since its absolute inverted root is $|-0.98| < 1$.
(4) Finally, note that the three models considered have different dependent variables. Hence, each of them estimates different time series. However, the first model of $M1$ is an unstable model and has a very large $SSR = 14\,180.86$. For this reason, the last two ARMA(1,1) models should be considered as acceptable models, in a statistical sense, even though the model in (11.30) has a very low value of the adjusted R-squared. □

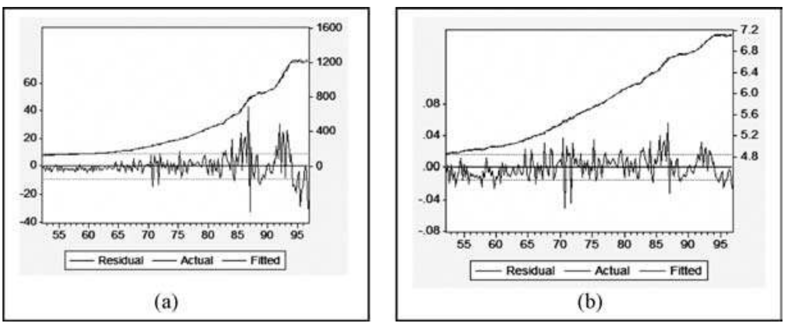

Figure 11.4 Residual graphs of (a) the ARMA(1,1) model of log($M1$) in (11.29) and (b) the ARMA(1,1) model of Dlog($M1$) in (11.30)

11.4.3 The ARMA models with covariates

Since many illustrative examples of the time series models having exogenous variables have been presented, here only simple autoregressive moving average models with a covariate or an exogenous variable, namely ARMA(p, q)_C models, will be discussed. The ARMA(p, q) model with multicovariate or multivariate exogenous variables can easily be derived from all models presented in the previous chapters.

In order to do the analysis based on ARMA models with k-covariates or exogenous variables, namely $X1, \ldots, Xk$, the following equation specification should be used:

$$Y\, C\, X1\, X2 \cdots Xk\, AR(1) \cdots AR(p)\, MA(1) \cdots MA(q) \tag{11.31}$$

Example 11.5. (The ARMA models with a covariate) For illustration purposes, Table 11.5 presents a summary of the statistical results based on Demo.wf1, by using the following equation specifications:

$$M1\, C\, GDP\, MA(1)\, MA(2)\, MA(3) \tag{11.32}$$

$$M1\, C\, GDP\, AR(1)\, MA(1) \tag{11.33}$$

$$M1\, C\, GDP\, AR(1)\, AR(2)\, MA(1)\, MA(2)\, MA(3) \tag{11.34}$$

Table 11.5 Statistical results summary based on the MA(3)_C, ARMA(1,1)_C and ARMA(2,3)_C models

Dependent variable: $M1$	MA(3)_C		ARMA(1,1)_C		ARMA(2,3)_C[a]	
Variable	Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic
C	61.588 22	11.336 31	24 754.78	0.013 578	19 104.02	0.023 036
GDP	0.605 294	96.152 92	0.341 532	3.049 041	0.289 908	2.626 587
$AR(1)$	—	—	0.999 901	136.6337	0.100 234	2.584 797
$AR(2)$	—	—	—	—	0.899 473	22.597 78
$MA(1)$	1.295 507	21.022 82	0.158 311	2.019 306	1.000 569	12.394 15
$MA(2)$	1.112 708	14.020 82			0.381 492	3.510 442
$MA(3)$	0.586 591	9.422 256			0.354 024	4.543 432
R-squared	0.998 794		0.999 316		0.999 392	
Adjusted R-squared	0.998 767		0.999 304		0.999 371	
SSR	25 667.73		14 499.07		12 807.18	
F-statistic	36 235.44		85 164.79		46 881.50	
Prob (F-statistic)	0.000 000		0.000 000		0.000 000	
AIC	7.853 465		7.277 024		7.192 506	
SC	7.942 159		7.348 251		7.317 633	
QHC	7.889 427		7.305 906		7.243 249	
DW- statistic	1.497 902		1.857 585		1.811 330	
Inverted AR roots	—		1.00		1.00	−0.90
Inverted MA roots	$-0.24 \pm 0.81i$	−0.81	−0.16		$-0.01 \pm 60i$	−0.0.98

[a]Convergence not achieved after 500 iterations.

Note that the first model is a third-order moving average model with a covariate, namely the MA(3)_C model, the second is an autoregressive moving average (1,1) model with a covariate, namely the ARMA(1,1)_C model, and the third is an autoregressive moving average (2,3) model with a covariate, namely the ARMA(2,3)_C model.

Even though the ARMA_C models have an inverted AR root of 1.00, the output of the three models do not present the error message '*Estimated AR process is nonstationary*.' Refer to the statistical results in the previous Example 11.3. For this reason, these ARMA_C models are considered as acceptable models, in a statistical sense. Since the ARMA(2,3)_C model has the smallest SSR (sum squared residual), this model should be considered as the best model.

Which one would you prefer? Remember that the three models in fact represent three distinct models, since they have different dependent variables. □

11.5 Nonparametric regression based on a time series

Without loss of generality the bivariate time series $\{(X_t, Y_t)\}_{t=1}^{T}$ can be written or considered as the ordered observations or cross-sectional data set as follows:

$$\{(X_i, Y_i)\}_{i=1}^{N} \quad \text{with} \quad X_1 \leq X_2 \leq \cdots \leq X_N \tag{11.35}$$

where $N = T$, since the scatter graph or plot in a two-dimensional coordinate system based on the time series $\{(X_t, Y_t)\}_{t=1}^{T}$ is exactly the same as the scatter graph based on the cross-sectional data set presented in (11.35).

In the following subsections the moving average models presented by Hardle (1999) will be reviewed, as well as some examples that can be done using EViews.

11.5.1 The Hardle moving average models

Hardle (1999) presents various nonparametric regressions or estimation methods based on a cross-sectional bivariate data set, namely the data set presented in (11.35). The general equation of the simplest nonparametric regression is

$$Y_i = m(X_i) + u_i, \quad i = 1, 2, \ldots, N \tag{11.36}$$

with the unknown regression function $m(X_i)$ and observation error u_i.

Instead of finding an explicit function of the exogenous variable X, a set of possible estimated values of $m(X_i)$ will be found using specific criteria, and then their values will be presented in the form of a curve.

The simplest technique to estimate all possible values of the function $m(X_i)$, say $X_i = x$ for all x, uses the following formula (Hardle, 1999):

$$\hat{m}_k(x) = N^{-1} \sum_{i=1}^{N} W_{ki}(x) Y_i \tag{11.37}$$

where $\{W_{ki}(x)\}_{i=1}^{N}$ is a weight sequence defined through the following set of indexes:

$$J_x = \{i : X_i \text{ is one of the } k \text{ nearest observations to } x\} \tag{11.38}$$

This method is called the *k-nearest neighbor estimation method*. Note that the *k-nearest neighbor* (*k-NN*) estimate of $m_k(x)$ is in fact a weighted average in a varying neighborhood. For a specific weighted average,

$$W_{ki}(x) = \frac{N}{k} \quad \text{if} \quad i \in J_x \quad \text{and} \quad W_{ki}(x) = 0 \quad \text{otherwise} \tag{11.39}$$

Therefore, the formula (11.37) can be written as

$$\hat{m}_k(x) = \sum_{i=1}^{N} \frac{J_x Y_i}{k} = k^{-1} \sum_{i \in J_x} Y_i \tag{11.40}$$

However, EViews provides different types of nonparametric regression, called the *nearest neighbor fit*, as presented in the following example. As a result, in the following subsections, empirical findings will be presented using the nonparametric estimation methods available in EViews 6.

11.5.2 The nearest neighbor fit

Based on the time series (X_i, Y_i) in the BASICS workfile, the stages of the estimation method are as follows:

(1) After opening the BASICS workfile, block the X-variable and then the Y-variable.
(2) Click *View/Show* ... and then click *OK*. The data of X_t and Y_t will appear on the screen.
(3) By selecting *View/Graph* ..., the options presented in Figure 11.5 will be available.
(4) By selecting *Scatter/Nearest Neighbor Fit* ... and clicking *Options*, the window in Figure 11.6 will appear with the default options, such as *Polynomial degree* = 1, *Bandwidth span* = δ = 0.3, *Local weighting*, *Cleveland subsampling* and *Number of evaluation* = 100.
(5) Finally, by clicking *OK*, the nonparametric regression curve is obtained, as presented later in Figure 11.7(a) in Example 11.6.
(6) Remember to insert a name in the '*Fitted series* (*Optional*)' to generate the fitted values variable for further statistical analysis, similar to generating the residual, and then do a detailed residual analysis. However, this option is available in EViews 5, but not in EViews 6, and is presented in Figure 11.6.

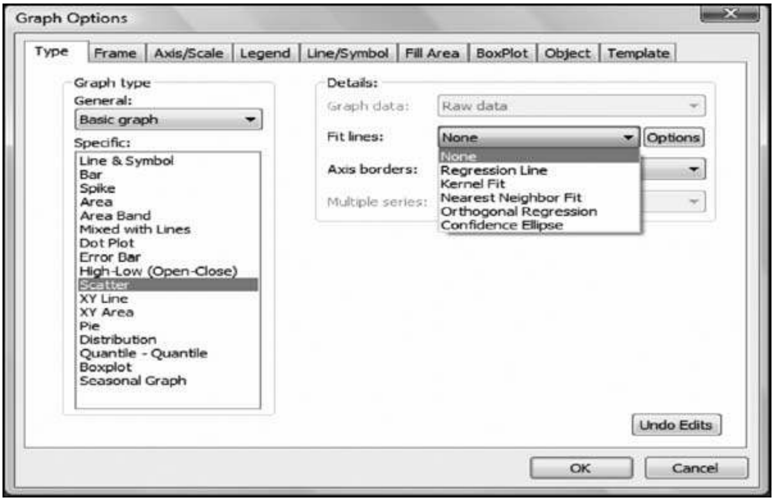

Figure 11.5 Alternative options of the scatter graphs

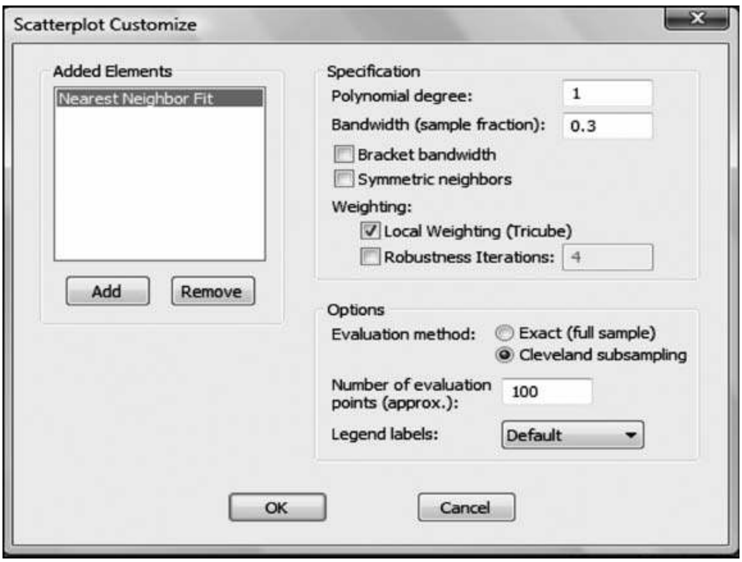

Figure 11.6 Default for the nearest neighbor fit options

11.5.3 Mathematical background of the nearest neighbor fit

The estimation method of the nearest neighbor fit, in fact, is based on a polynomial model as follows:

$$Y_i = a + b_1 X_i + b_2 X_i^2 + \cdots + b_n X_i^n + \mu_i \tag{11.41}$$

but the estimation method is using the weighted nonparametric regression. The weighted regression minimizes the weighted sum of squared residuals

$$WSSE = \sum_{i-1}^{N} W_i(Y_i - a - b_1X_i - b_2X_i^2 - \cdots - b_nX_i^n)^2 \tag{11.42}$$

where

$$W_i = \begin{cases} \left(1 - \left|\frac{d_i}{d[\delta N]}\right|^3\right)^3 & \text{for} \quad \left|\frac{d_i}{d[\delta N]}\right| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{11.43}$$

The *span* δ (=0.3) instructs EViews to include $[\delta N]$ observations nearest to the given point, where $[\delta N]$ is $100\delta\%$ of the total sample size, truncated to an integer k, as indicated above.

Polynomial degree specifies the degree of polynomial to fit in each local regression.

The *local weighting* (*Tricube*) weights the observations of each local regression, and $d_i = |x_i - x|$, as well as $d([\delta N])$, is the $[\delta N]$th smallest such distance. Observations that are relatively far from the point being evaluated are given small weights in the sum of squared residuals.

Example 11.6. (Alternative nearest neighbor fit estimates) Figure 11.7 presents four alternative nearest neighbor fit models (nonparametric or curves) based on the bivariate (X_t, Y_t) in BASICS.wf1. Based on these nonparametric regressions, the following notes are presented:

(1) Figure 11.7(a) presents the scatter graph of (X_i, Y_i) with its NN-Fit regression using the default options.
(2) Figure 11.7(b) is obtained by using a third-degree polynomial, with the same value of $\delta = 0.30$. Compared to the first curve, this curve has higher waves.
(3) By taking a smaller value of the span, namely $\delta = 0.10$, and a third-degree polynomial, the curve in Figure 11.7(c) is obtained, having several or many relative maximum and minimum fitted values.
(4) On the other hand, by using a larger span of $\delta = 0.80$, the curve in Figure 11.7(d) is obtained, which is very close to a straight line, even it uses the third-degree polynomial.
(5) Many more nonparametric regressions or curves based on this bivariate time series can be obtained, as well as any other bivariates, by using polynomials of various degrees and various bandwidth spans. There is a great problem that should be faced, which is how many curves should be obtained or developed in order to obtain the best fit nonparametric regression (refer to the special notes and comments in Section 2.14).
(6) However, it is suggested that default options should be used, if there is no other good reason to select the other alternative options. □

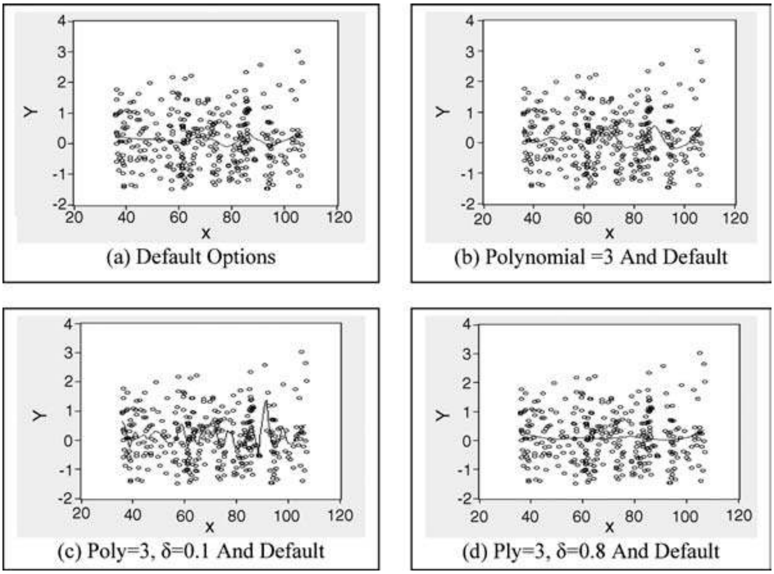

Figure 11.7 Scatter graph of (X, Y) with its alternative nearest neighbor fit curves

Example 11.7. (The nearest neighbor fit series using EViews 5) Note that the options of EViews 6 in Figure 11.6 do not provide the option for generating the *fitted series*, compared to EViews 5, presented in Figure 11.8. For this reason, in order to present additional illustrations based on the fitted series, it is best to use EViews 5.

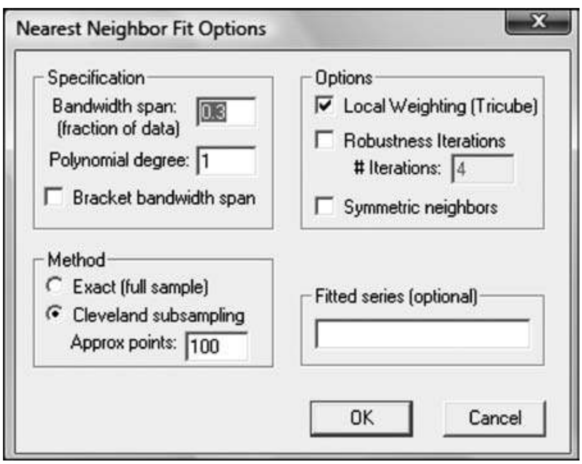

Figure 11.8 The fitted series option in EViews 5

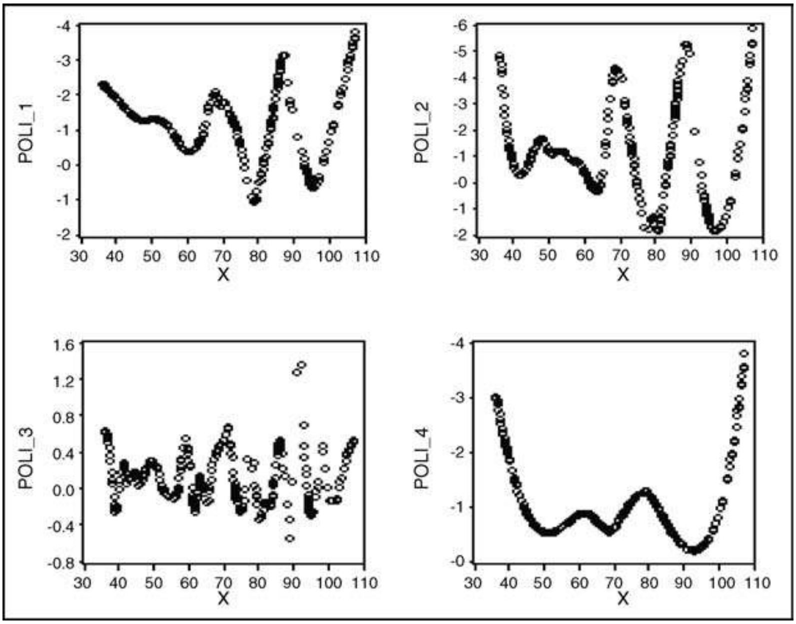

Figure 11.9 Scatter graphs of *Poli_k* on *X*, for $k = 1, 2, 3, 4$

Corresponding to the four alternative NN-Fit presented in Example 11.6, the four fitted series are defined as *Poli*_1 up to *Poli*_4. Figure 11.9 presents their scatter graphs on *X*. Further data analysis could be done by using these fitted series. After having these fitted series in the workfile, the data analysis using EViews 6 could then also be conducted. By using EView 6, multiple scatter graphs could be constructed directly for each *Poli_k* on *X*, with a parametric or nonparametric regression. Do this as an exercise. □

Example 11.8. (Data analysis based on the NN-Fit series using EViews 6) As an illustration, Figure 11.10 presents the statistical results based on an MAR(1) (a multivariate first-order autoregressive) model of the bivariate (*Poli*_1, *Poli*_2) on *X*. By using a fitted value variable as a dependent or an independent variable, various or many time series models could be developed, as presented in the previous chapters. This type of regression will be called a *switching regression*. Gulati, Lawrence and Puraman (2005) present switching regressions using the fitted value variables of a multinomial logit model.

Furthermore, Figure 11.11 presents the residual graphs of two AR(1) simple linear regressions, using the following equation specifications:

$$Poli\_1\ C\ X\ AR(1) \tag{11.44}$$

$$Poli\_2\ C\ X\ AR(1) \tag{11.45}$$

System: UNTITLED
Estimation Method: Iterative Least Squares
Date: 01/13/08 Time: 08:02
Sample: 1959M02 1989M12
Included observations: 337
Total system (balanced) observations 618
Convergence achieved after 13 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(11)	-1.096432	0.304853	-3.596589	0.0003
C(12)	0.014020	0.002114	6.632627	0.0000
C(13)	0.991701	0.004493	220.7026	0.0000
C(21)	-0.563403	0.339087	-1.661529	0.0971
C(22)	0.009312	0.004207	2.213293	0.0272
C(23)	0.966815	0.014130	68.42247	0.0000
Determinant residual covariance		7.17E-07		

Equation: POLI_1=C(11)+C(12)*X+[AR(1)=C(13)]
Observations: 309

R-squared	0.951431	Mean dependent var	0.120120
Adjusted R-squared	0.951114	S.D. dependent var	0.105952
S.E. of regression	0.023426	Sum squared resid	0.167930
Durbin-Watson stat	1.401932		

Equation: POLI_2=C(21)+C(22)*X+[AR(1)=C(23)]
Observations: 309

R-squared	0.907383	Mean dependent var	0.112122
Adjusted R-squared	0.906778	S.D. dependent var	0.182405
S.E. of regression	0.055692	Sum squared resid	0.949100
Durbin-Watson stat	1.326534		

Figure 11.10 Statistical results based on an MAR(1) model of (*Poli*_1, *Poli*_2) on *X*

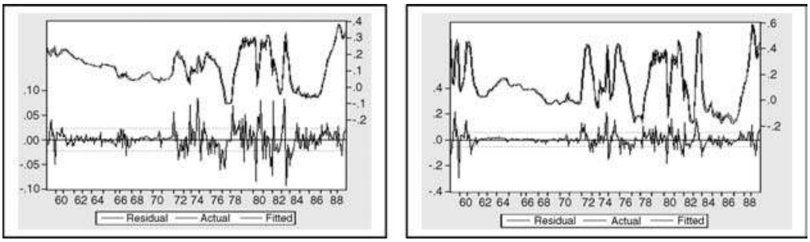

Figure 11.11 Residuals graphs of the AR(1) simple linear regressions of *Poli*_1 and *Poli*_2 on *X*

Based on either one of the residual graphs, the error terms can be identified to have heterogeneous variances. Hence, it is suggested that the Newey–West estimation method should be applied, as presented in the previous chapters. □

11.6 The local polynomial Kernel fit regression

The statistical method used to obtain a local kth degree polynomial Kernel fit of Y, at each value x, is the polynomial regression as follows:

$$Y_i = \beta_0 + \beta_1(x - X_i) + \beta_2(x - X_i)^2 + \cdots + \beta_k(x - X_i)^k + \mu_i \tag{11.46}$$

The model parameters are then estimated by minimizing the following weighted sum of squared residuals:

$$SS_{res}(x) = \sum_{i-1}^{N} (Y_i - \beta_0 - \beta_1(x - X_i) - \cdots - \beta_k(x - X_i)^k)^2 K\left(\frac{x - X_i}{h}\right) \tag{11.47}$$

where N is the number of observations, h is the bandwith (or smoothing parameter) and $K(x - X_i)/h)$ is a Kernel function that integrates to one. Note

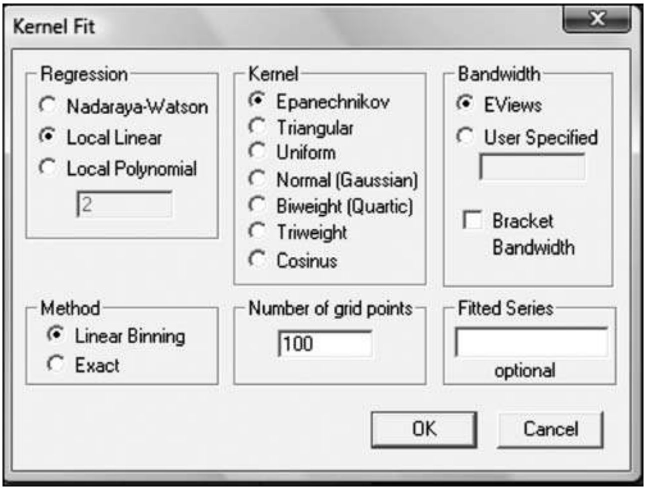

Figure 11.12 Kernel fit options in EViews 5

that the minimizing estimates of β will differ for each x. By default, EViews arbitrarily sets the bandwidth as

$$h = 0.15(X_U - X_L) \tag{11.48}$$

where $(X_U - X_L)$ is the range of observed values of X.

By selecting the option '*Scatter with Kernel Fit*,' using EViews 5 the *Kernel Fit* options presented in Figure 11.12 will appear. Note that EViews 6 does not provide the option to generate the fitted series.

This figure shows three options for the regressions and seven options for the Kernel fit beside the other options. Hence, there could be 21 types of Kernel fit regression, supported by the default case of the Method = Linear binning, Bandwith = EViews and Number of grid points = 100. As a result, there will be a great problem in selecting the best fit model(s).

Example 11.9. (Kernel fit regressions) For a comparison with the results in the previous examples, the first- and third-degree polynomials and the Epanechnikov (default) Kernel function are applied, as presented in Figure 11.13. The corresponding estimated values or fitted series are saved as the variables *KF*_1 and *KF*_2 for further analyses. Do this as an exercise.

Note that the analyses done using other Kernel functions can easily be obtained, and all the results can then be compared in order to select the best possible model. However, which one would you judge is the best fit model? Should the measuring fits presented in Section 11.3 be trusted or should best judgment be used (Turkey, 1962, quoted by Gifi, 1990), which was presented in Section 2.14? □

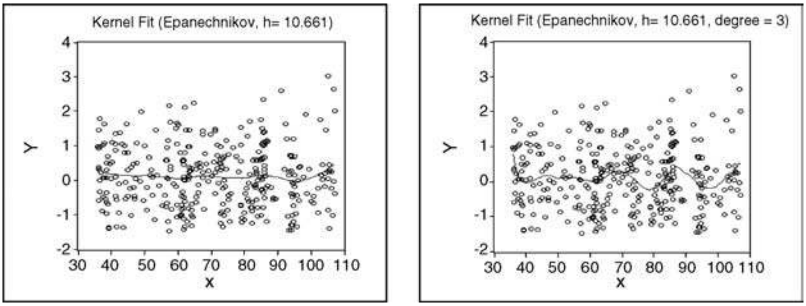

Figure 11.13 Two alternative kernel fit regressions of Y on X, using EViews 5

11.7 Nonparametric growth models

By considering the time series Y_t and the time t as the X-variable presented in the previous section, a nonparametric growth curve would be obtained, which should be considered as a modification of the classical continuous growth model in (2.3). The following examples present several nonparametric growth models or curves based on selected endogenous variables.

Example 11.10. (Nonparametric growth curve of $M1$) By using the endogenous variable log($M1$) with the exogenous variable t, the two graphs in Figure 11.14 are presented. The first graph on the left is based on the OLS simple linear regression of log($M1$) on t, namely log($m1$) = $c(1) + c(2)^* t + \mu$, and the second graph is the nearest neighbor fit regression using the default option of EViews 6. □

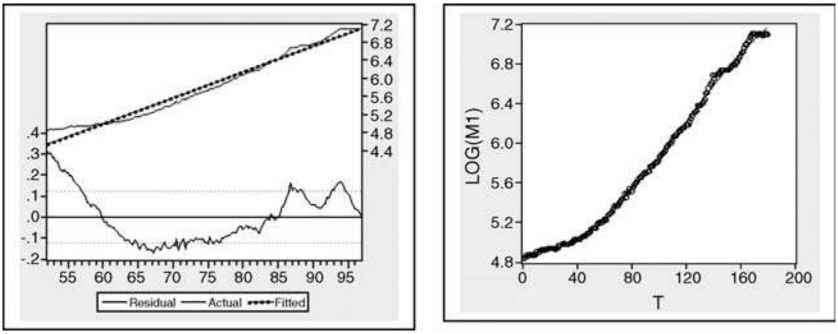

Figure 11.14 Graphical comparison between an OLS simple regression of log($M1$) on the time t and the nearest neighbor fit regression, using the default options

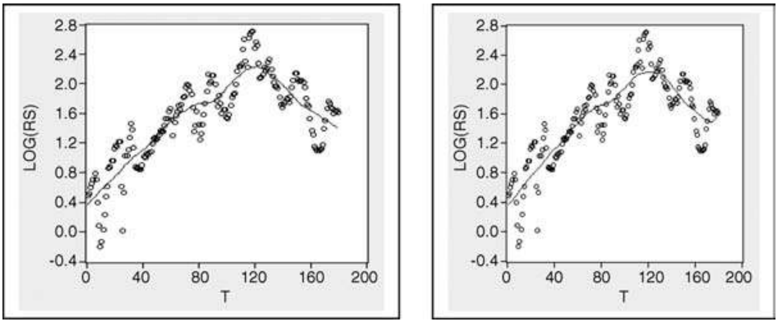

Figure 11.15 Graphical comparison between the nearest neighbor and kernel fit regressions of log(*RS*) on the time *t*, using the default options

Example 11.11. (Nonparametric growth curve of *RS*) By using the endogenous variable log(*RS*) and the exogenous variable *t*, the two graphs in Figure 11.15 are presented. The first graph on the left presents the nearest neighbor fit regression using the default options and the second graph on the right presents the Kernel fit regression using the default options. The two graphs look very similar. □

Example 11.12. (Nonparametric growth curve of the unemployment rate) By using the endogenous variable the natural logarithm of the variable *Urate* (i.e. *unemployment rate for all workers*, *16 years and over*) in the BASICS workfile and the exogenous variable *t*, the two graphs in Figure 11.16 are presented. The first graph on the left is based on the default option of the nearest neighbor fit regression and the second graph on the right is based on a Kernel fit regression of log(*Urate*) on the time *t*. It is very clear that the Kernel fit is a much better fit than the nearest neighbor fit.

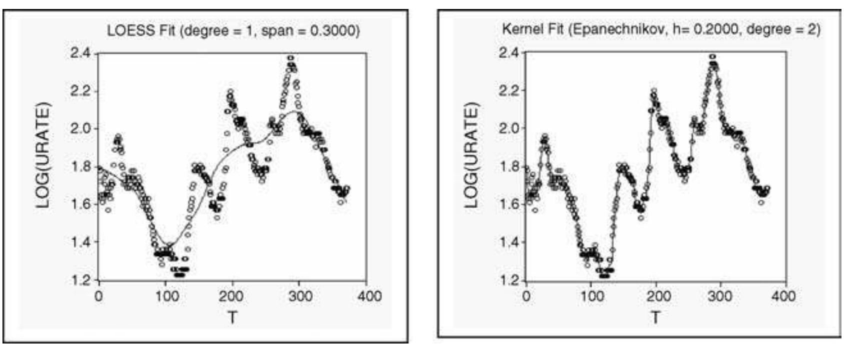

Figure 11.16 The nearest neighbor and kernel fit regressions of log(*Urate*) on the time *t*, using EViews 5

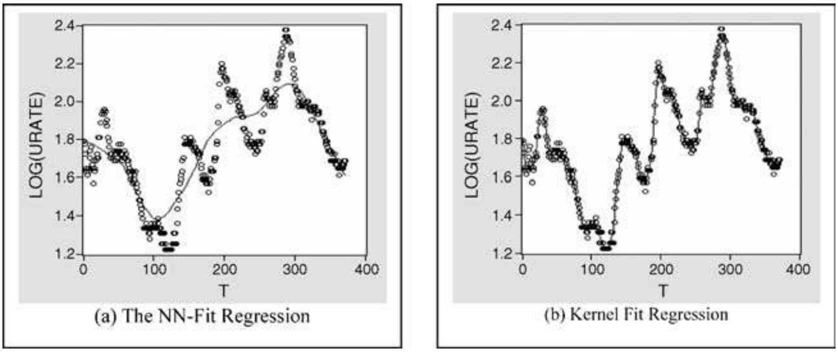

Figure 11.17 The nearest neighbor and kernel fit regressions of log(*Urate*) on the time *t*, using the default options in EViews 6

For a comparison, Figure 11.17 presents the nearest neighbor and Kernel fits using the default options in EViews 6. Note that EViews 5 should be used to generate the fitted value variable. By using EVeiws 5 and 6, many or infinitely many alternative nonparametric regressions can be obtained, besides these two nonparametric regressions.

Furthermore, if the SSE in (11.18) is being considered as a measure of a fit model, there could also be many SSEs. As a result, there would be many different choices in selecting the best fit model. However, based on other measures of fit models, there could be contradictory conclusions. □

Appendix A: Models for a single time series

Definition A.1: The univariate time series or process $\{Y_t\}_{t=1}^{T}$ is second-order stationary if and only if

(i) *the mean* $E(Y_t) = \mu$ *is independent of* t,
(ii) *the autocovariance* $\text{Cov}(Y_t, Y_{t-k}) = E(Y_t - \mu)(Y_{t-k} - \mu) = \gamma(k)$ *is independent of* t *for any* k; $\gamma(k) = \gamma_k$ *is the autocovariance function* (*ACF*) *of the process, with*

$$\gamma(-k) = \gamma(k) \tag{A.1}$$

□

Definition A.2: The second-order stationary process $\{\varepsilon_t\}_{t=1}^{T}$ is a weak white noise process if and only if

(i) *the mean* $E(\varepsilon_t) = 0, \quad \forall t$,
(ii) *the autocovariance*

$$\text{Cov}(\varepsilon_t, \varepsilon_{t-k}) = E(\varepsilon_t \varepsilon_{t-k}) = \gamma(k) = 0, \quad \forall k \neq 0 \tag{A.2}$$

□

A.1 The simplest model

Based on a single time series, namely $\{Y_t\}_{t=1}^{T}$, the simplest model, called the *mean model*, is obtained as follows:

$$Y_t = \mu + \varepsilon_t \tag{A.3}$$

where Y_t is an observable random variable, μ is the mean parameter, and ε_t is the unobserved random error term, for $t = 1,2,\ldots, T$.

Time Series Data Analysis Using EViews IGN Agung

A.1.1 OLS estimates

In order to obtain the ordinary least square (OLS) estimates of the parameter, not assumptions are needed on the error term. For this purpose, only the following quadratic function should be minimized as a function of μ based on a data set $\{y_t\}_{t=1}^{T}$ that happens to be selected by the researcher:

$$Q = Q(\mu) = \sum_{t=1}^{T} \varepsilon_t^2 = \sum_{t=1}^{T} (y_t - \mu)^2 \tag{A.4}$$

The necessary condition to minimize this function is

$$\frac{\partial Q}{\partial \mu} = 2\sum_{t=1}^{T} (y_t - \mu) = 0 \tag{A.5}$$

which is known as the normal equation of the model. Then an estimate is obtained of the parameter μ, namely $\hat{\mu}$, called the *sample mean*, as follows:

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} y_t = \bar{y} \tag{A.6}$$

Furthermore, this defines the following statistics:

1. The sample *error sum of square*:

$$SSE = \sum_{t=1}^{T} (y_t - \hat{\mu})^2 = \sum_{t=1}^{T} (y_t - \bar{y})^2 \tag{A.7}$$

2. The sample *mean error sum of squares*:

$$MSE = \frac{1}{T-1}\sum_{t=1}^{T} (y_t - \hat{\mu})^2 = \frac{1}{T-1}\sum_{t=1}^{T} (y_t - \bar{y})^2 \tag{A.8}$$

A.1.2 Properties of the error terms

For inferential statistical analysis, namely estimation and testing hypotheses, the following assumptions are required:

(a) The random error term ε_t has $E(\varepsilon_t) = 0$ and $\text{Var}(\varepsilon_t) = \sigma^2(\varepsilon_t) = \sigma^2$, for $t = 1, 2, \ldots, T$.
(b) The random error is normally distributed as $N(0, \sigma^2)$ or *Gaussian.*

Note that both of these assumptions indicate that ε_t is i.i.d. (independently and identically distributed) Gaussian or $N(0,\sigma^2)$, so that $E(\varepsilon_t\varepsilon_s) = 0$ for all $t \neq s$, since ε_t is the random error. Under these assumptions, it can be proven that the statistics $\hat{\mu}$ and MSE, not their sample values, are *unbiased estimators* of the corresponding parameters, which are indicated by the following expected values:

$$E(\hat{\mu}) = \mu \tag{A.9}$$

$$E(\text{MSE}) = \sigma^2 \tag{A.10}$$

A.1.3 Maximum likelihood estimates

Under the assumption(s) above, namely ε_t is i.i.d. Gaussian, the following normal density function is obtained:

$$f(\varepsilon_t) = [2\pi\sigma^2]^{-1/2} \exp\left[-\frac{\varepsilon_t^2}{2\sigma^2}\right] = [2\pi\sigma^2]^{-1/2} \exp\left[-\frac{(y_t-\mu)^2}{2\sigma^2}\right] \tag{A.11}$$

In order to estimate the model parameters, namely μ and σ^2, the following likelihood function is defined:

$$L = \prod_{t=1}^{T} f(\varepsilon_t) = [2\pi\sigma^2]^{-T/2} \exp\left[-\frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t-\mu)^2\right] \tag{A.12}$$

with its log likelihood function, namely $LL = \log(L) = \ln(L)$, as follows:

$$LL = -\frac{T}{2}\{\log(2\pi) + \log(\sigma^2)\} - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t-\mu)^2 \tag{A.13}$$

The necessary conditions for maximizing this *L function,* as well as *LL*, are

$$\begin{aligned} \frac{\partial(LL)}{\partial\mu} &= \frac{1}{\sigma^2}\sum_{t=1}^{T}(y_t-\mu) = 0 \\ \frac{\partial(LL)}{\partial\sigma^2} &= -\frac{T}{2\sigma^2} + \frac{1}{2\sigma^4}\sum \ (y_t-\mu)^2 = 0 \end{aligned} \tag{A.14}$$

which lead to the *normal equations* as follows:

$$\begin{aligned} &\sum_{t=1}^{T}(y_t-\mu) = 0 \\ &\sigma^2 = \frac{1}{T}\sum_{t=1}^{T}(y_t-\mu)^2 \end{aligned} \tag{A.15}$$

Then the following unique solutions are found:

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} y_t = \bar{y}$$
$$\hat{\sigma}_y^2 = \frac{1}{T}\sum_{t=1}^{T}(y_t - \hat{\mu})^2 = \frac{1}{T}\sum_{t=1}^{T}(y_t - \bar{y})^2 \quad \text{(A.16)}$$

Note that the first estimate is exactly the same as presented in (A.5). This result shows that both the OLS and ML estimation methods give the same estimates. Furthermore, the following expected value is obtained:

$$E(\hat{\sigma}_y^2) = \frac{T-1}{T} E\left[\frac{1}{T-1}\sum_{t=1}^{T}(y_t - \bar{y})^2\right] = \frac{T-1}{T} E(\text{MSE}) = \frac{T-1}{T}\sigma^2 \quad \text{(A.17)}$$

This indicates that the statistic $\hat{\sigma}_y^2$ is a biased estimator of the parameter σ^2. However, it is an asymptotical unbiased estimator, since for $T \to \infty$, $E\hat{\sigma}_y^2 = \sigma^2$.

A.2 First-order autoregressive models

The *first-order autoregressive* model, namely the AR(1) model based on a series$\{Y_t\}_{t=1}^{T}$, can be presented as follows:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t \quad \text{(A.18)}$$

where β_0 and β_1 are the model parameters and ε_t is i.i.d. Gaussian.

A.2.1 Properties of the parameters

Under the assumptions $E(\varepsilon_t) = 0$ and Var $(\varepsilon_t) = 0$, and the stationary condition $E(Y_t) = \mu$ and $\text{Var}(Y_t) = \sigma_y^2$ for all t, then

$$\mu = \beta_0 + \beta_1 \mu \quad \text{or} \quad E(Y_t) = \mu = \frac{\beta_0}{1 - \beta_1} \quad \text{(A.19)}$$

and

$$\text{Var}(Y_t) = \beta_1^2 \text{Var}(Y_{t-1}) + \text{Var}(\varepsilon_t)$$
$$(1-\beta_1^2)\text{Var}(Y_t) = \text{Var}(\varepsilon_t) = \sigma_\varepsilon^2$$
$$\text{Var}(Y_t) = \frac{\sigma_\varepsilon^2}{1-\beta_1^2} = \frac{\sigma^2}{1-\beta_1^2} = \gamma_0 \quad \text{(A.20)}$$

A.2.2 Autocorrelation function of an AR(1) model

By using $\beta_0 = \mu(1-\beta_1)$ in (A.18), the AR(1) model can be presented as

$$\begin{aligned} Y_t &= \mu(1-\beta_1) + \beta_1 Y_{t-1} + \varepsilon_t \\ Y_t - \mu &= \beta_1(Y_{t-1}-\mu) + \varepsilon_t \end{aligned} \tag{A.21}$$

Then, since $E[\varepsilon_t(Y_{t-1}-\mu)] = E(\varepsilon_t)E(Y_{t-1}-\mu) = 0$, the following expected value is obtained:

$$E[\varepsilon_t(Y_t-\mu)] = \beta_1 E[\varepsilon_t(Y_{t-1}-\mu)] + E(\varepsilon_t^2) = E(\varepsilon_t^2) = \sigma^2 \tag{A.22}$$

Furthermore, based on the model in (A.21), it is easy to derive the following *autocovariance function* or equation:

$$\begin{aligned} (Y_t-\mu)(Y_{t-k}-\mu) &= \beta_1(Y_{t-1}-\mu)(Y_{t-k}-\mu) + \varepsilon_t(Y_{t-k}-\mu) \\ \mathrm{Cov}(Y_t, Y_{t-k}) &= E[(Y_t-\mu)(Y_{t-k}-\mu)] \\ &= \beta_1 E[(Y_{t-1}-\mu)(Y_{t-k}-\mu)] + E[\varepsilon_t(Y_{t-k}-\mu)] \end{aligned} \tag{A.23}$$

Then by using the result in (A.22), and under the assumption that Y_t is *covariance stationary* or *weakly stationary,* namely

$$E[(Y_t-\mu)(Y_{t-k}-\mu)] = \gamma_k \quad \text{for all } t \text{ and any } k \tag{A.24}$$

the following results are obtained:

For $k = 0$,

$$\begin{aligned} &E[(Y_t-\mu)(Y_t-\mu)] = \beta_1 E[(Y_{t-1}-\mu)(Y_t-\mu)] + E[\varepsilon_t(Y_t-\mu)] \\ &\gamma_0 = \beta_1\gamma_1 + \sigma^2 \end{aligned} \tag{A.25}$$

For $k \neq 0$,

$$\begin{aligned} &E[(Y_t-\mu)(Y_{t-k}-\mu)] = \beta_1 E[(Y_{t-1}-\mu)(Y_{t-k}-\mu)] + E[\varepsilon_t(Y_{t-k}-\mu)] \\ &\gamma_k = \beta_1\gamma_{k-1} \end{aligned} \tag{A.26}$$

where $\gamma_k = \gamma_{-k}$ is used.

Furthermore, from (A.21) it is found that $\sigma^2 = (1-\beta_1^2)\gamma_0$. Inserting this value in (A.25) gives

$$\gamma_0 = \beta_1\gamma_1 + \sigma^2 = \beta_1\gamma_1 + (1-\beta_1^2)\gamma_0 \rightarrow \gamma_1 = \beta_1\gamma_0 \tag{A.27}$$

Then from (A.25) and (A.27), it is found that the *autocorrelation function* (*ACF*) of Y_t satisfies

$$\rho_k = \beta_1 \rho_{k-1,} = \beta_1^2 \gamma_{k-2} = \cdots = \beta_1^k \gamma_0 \quad \text{for} \quad k \geq 0 \tag{A.28}$$

Since $\gamma_0 = 1$ then $\gamma_k = \beta_1^k$.

A.2.3 Estimates of the parameters

Corresponding to the functions $Q(\mu)$ in (A.4) and LL in (A.13), for the model in (A.18), then

$$Q = Q(\beta_0, \beta_1) = \sum_{t=2}^{T} \varepsilon_t^2 = \sum_{t=2}^{T} (y_t - \beta_0 - \beta_1 y_{t-1})^2 \tag{A.29}$$

$$LL = -\frac{T-1}{2}\left\{\log(2\pi) + \log(\sigma^2)\right\} - \frac{1}{2\sigma^2}\sum_{t=2}^{T} (y_t - \beta_0 - \beta_1 y_{t-1})^2 \tag{A.30}$$

Here, only the LL function is considered when obtaining the estimates of the parameters β_0, β_1 and σ^2. Note that, in (A.29) and (A.30), the summation from $t = 2$ should be used, since the series y_{t-1} is used for the observed values $y_1, \ldots, y_T$.

EViews provides an iteration process or estimation method in order to obtain the estimates of these parameters. However, in a mathematical sense, the necessary conditions for maximizing this LL function are

$$\begin{aligned}
\frac{\partial(LL)}{\partial\beta_0} &= \frac{1}{\sigma^2}\sum_{t=2}^{T}(y_t - \beta_0 - \beta_1 y_{t-1}) = 0 \\
\frac{\partial(LL)}{\partial\beta_1} &= \frac{1}{\sigma^2}\sum_{t=2}^{T}(y_t - \beta_0 - \beta_1 y_{t-1}) y_{t-1} = 0 \\
\frac{\partial(LL)}{\partial\sigma^2} &= -\frac{T-1}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=2}^{T}(y_t - \beta_0 - \beta_1 y_{t-1})^2 = 0
\end{aligned} \tag{A.31}$$

As a result, the following normal equations are obtained:

$$\begin{aligned}
&\sum_{t=2}^{T}(y_t - \beta_0 - \beta_1 y_{t-1}) = 0 \\
&\sum_{t=2}^{T}(y_t - \beta_0 - \beta_1 y_{t-1}) y_{t-1} = 0 \\
&\sigma^2 = \frac{1}{T-1}\sum_{t=2}^{T}(y_t - \beta_0 - \beta_1 y_{t-1})^2
\end{aligned} \tag{A.32}$$

By using the notation

$$\ddot{y} = \frac{1}{T-1}\sum_{t=2}^{T} y_t \quad \text{and} \quad \dot{y} = \frac{1}{T-1}\sum_{t=2}^{T} y_{t-1} \tag{A.33}$$

the following estimates are obtained:

$$\begin{aligned} \hat{\beta}_1 &= \frac{\sum (y_t - \ddot{y})(y_{t-1} - \dot{y})}{\sum (y_{t-1} - \dot{y})^2} \\ \hat{\beta}_0 &= \ddot{y} - \hat{\beta}_1 \dot{y} \\ \hat{\sigma}^2 &= \frac{1}{T-1}\sum_{t=2}^{T} (y_t - \hat{\beta}_0 - \hat{\beta}_1 y_{t-1})^2 \end{aligned} \tag{A.34}$$

A.3 Second-order autoregressive model

The second-order autoregressive model, namely the AR(2) model based on a series$\{Y_t\}_{t=1}^{T}$, can be presented as follows:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \varepsilon_t \tag{A.35}$$

where β_0, β_1 and β_2 are the model parameters and ε_t is i.i.d. Gaussian.

A.3.1 Properties of the parameters

Corresponding to the expected value and variance of Y_t presented in (A.19) and (A.20), using the same technique gives

$$\mu = \beta_0 + \beta_1 \mu + \beta_2 \mu \quad \text{or} \quad E(Y_t) = \mu = \frac{\beta_0}{1 - \beta_1 - \beta_2} \tag{A.36}$$

and

$$\text{Var}(Y_t) = \frac{\sigma_\varepsilon^2}{1 - \beta_1^2 - \beta_2^2} = \frac{\sigma^2}{1 - \beta_1^2 - \beta_2^2} = \gamma_0 \tag{A.37}$$

A.3.2 Autocorrelation function of an AR(2) model

By using $\beta_0 = \mu(1 - \beta_1 - \beta_2)$ in (A.36), the AR(2) model in (A.35) can be presented as

$$\begin{aligned} Y_t &= \mu(1 - \beta_1 - \beta_2) + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \varepsilon_t \\ Y_t - \mu &= \beta_1 (Y_{t-1} - \mu) + \beta_2 (Y_{t-2} - \mu) + \varepsilon_t \end{aligned} \tag{A.38}$$

Using the same technique as for the AR(1) model gives

$$E[(Y_t-\mu)(Y_{t-k}-\mu)] = \beta_1 E[(Y_{t-1}-\mu)(Y_{t-k}-\mu)] + \beta_2 E[(Y_{t-2}-\mu)(Y_{t-k}-\mu)] + E[\varepsilon_t(Y_{t-k}-\mu)] \quad \text{(A.39)}$$

which leads to the following relationship:

$$\gamma_k = \beta_1\gamma_{k-1} + \beta_2\gamma_{k-2} \quad \text{for} \quad k>0 \quad \text{(A.40)}$$

Note that $\gamma_{-k} = \gamma_k$ By dividing both sides by γ_0, the relationship between the serial correlation or autocorrelation is obtained as follows:

$$\rho_k = \beta_1\rho_{k-1} + \beta_2\rho_{k-2} \quad \text{for} \quad k>0 \quad \text{(A.41)}$$

For the *ACF* of Y_t in particular,

$$\begin{aligned} \rho_1 &= \beta_1\rho_0 + \beta_2\rho_{-1} = \beta_1 + \beta_2\rho_1 \\ \rho_1 &= \frac{\beta_1}{1-\beta_2} \end{aligned} \quad \text{(A.42)}$$

Therefore, for the stationary AR(2) series Y_t,

$$\begin{aligned} \rho_0 &= 1 \\ \rho_1 &= \frac{\beta_1}{1-\beta_2} \\ \rho_k &= \beta_1\rho_{k-1} + \beta_2\rho_{k-2} \quad \text{for} \quad k \geq 2 \end{aligned} \quad \text{(A.43)}$$

A.3.3 Estimates of the parameters

Similar to the quadratic function $Q(\beta_0, \beta_1)$ in (A.19) and the log likelihood function *LL* in (A.20), for the model in (A.35),

$$Q = Q(\beta_0, \beta_1, \beta_2) = \sum_{t=3}^{T} \varepsilon_t^2 = \sum_{t=3}^{T} (y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 y_{t-2})^2 \quad \text{(A.44)}$$

$$LL = -\frac{T-2}{2}\left\{\log(2\pi) + \log(\sigma^2)\right\} - \frac{1}{2\sigma^2}\sum_{t=3}^{T} (y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 y_{t-2})^2 \quad \text{(A.45)}$$

With respect to the *LL* function, the following *normal equations* for estimating the parameters β_0, β_1, β_2 and σ^2 are

$$\begin{aligned}
&\sum_{t=3}^{T}(y_t-\beta_0-\beta_1 y_{t-1}-\beta_2 y_{t-2}) = 0\\
&\sum_{t=3}^{T}(y_t-\beta_0-\beta_1 y_{t-1}-\beta_2 y_{t-2})y_{t-1} = 0\\
&\sum_{t=3}^{T}(y_t-\beta_0-\beta_1 y_{t-1}-\beta_2 y_{t-2})y_{t-2} = 0\\
&\sigma^2 = \frac{1}{T-2}\sum_{t=3}^{T}(y_t-\beta_0-\beta_1 y_{t-1}-\beta_2 y_{t-2})^2
\end{aligned} \tag{A.46}$$

Here, the explicit solution of this normal equation will not be presented. However, it will be presented in Appendix C, by using the matrix equation based on a general linear model (GLM).

A.4 First-order moving average model

The *first-order moving average* model, namely the MA(1) model based on a series$\{Y_t\}_{t=1}^{T}$, can be presented as follows:

$$Y_t = \mu + \varepsilon_t + \delta\varepsilon_{t-1} \tag{A.47}$$

where μ and δ are the model parameters and ε_t is i.i.d. Gaussian or $N(0, \sigma^2)$.

Note that the series Y_t is constructed from a weighted sum of the most recent values of the error terms, namely ε_t and ε_{t-1}. Under the assumption above, the following expected values are obtained:

The expectation of Y_t is

$$E(Y_t) = E(\mu + \varepsilon_t + \delta\varepsilon_{t-1}) = \mu \tag{A.48}$$

The variance of Y_t is

$$\begin{aligned}
E(Y_t-\mu)^2 &= E(\varepsilon_t + \delta\varepsilon_{t-1})^2\\
&= E(\varepsilon_t^2 + \delta\varepsilon_t\varepsilon_{t-1} + \delta^2\varepsilon_{t-1}^2)\\
&= \sigma^2 + 0 + \delta^2\sigma^2\\
\gamma_0 &= (1+\delta^2)\sigma^2
\end{aligned} \tag{A.49}$$

By using $E(\varepsilon_t \varepsilon_s) = 0$ for all $t \neq s$, the *autocovariances* of Y_t and Y_{t-k} are obtained as follows:

$$\gamma_1 = E(Y_t - \mu)(Y_{t-1} - \mu) = E(\varepsilon_t + \delta\varepsilon_{t-1})(\varepsilon_{t-1} + \delta\varepsilon_{t-2}) = \delta\sigma^2 \tag{A.50}$$

$$\gamma_k = E(Y_t - \mu)(Y_{t-k} - \mu) = E(\varepsilon_t + \delta\varepsilon_{t-1})(\varepsilon_{t-k} + \delta\varepsilon_{t-k-1}) = 0 \quad \text{for} \quad k > 1 \tag{A.51}$$

From (A.49), (A.50) and (A.51), the ACF for the MA(1) model is

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\delta}{(1 + \delta^2)}, \quad \rho_k = 0 \quad \text{for} \quad k > 1 \tag{A.52}$$

A.5 Second-order moving average model

The *second-order moving average* model, namely the MA(2) model, can be presented as follows:

$$Y_t = \mu + \varepsilon_t + \delta_1\varepsilon_{t-1} + \delta_2\varepsilon_{t-2} \tag{A.53}$$

where μ, δ_1 and δ_2 are the model parameters and ε_t is i.i.d. Gaussian or $N(0, \sigma^2)$.

Under this assumption and by using the same technique or process as that for the MA(1) model, the following expected values are obtained:

The expectation of Y_t is

$$E(Y_t) = E(\mu + \varepsilon_t + \delta_1\varepsilon_{t-1} + \delta_2\varepsilon_{t-2}) = \mu \tag{A.54}$$

The variance of Y_t is

$$\begin{aligned} E(Y_t - \mu)^2 &= E(\varepsilon_t + \delta_1\varepsilon_{t-1} + \delta_2\varepsilon_{t-2})^2 \\ \gamma_0 &= (1 + \delta_1^2 + \delta_2^2)\sigma^2 \end{aligned} \tag{A.55}$$

The autocovariances of Y_t and Y_{t-k} are

$$\begin{aligned} E(Y_t - \mu)(Y_{t-1} - \mu) &= E(\varepsilon_t + \delta_1\varepsilon_{t-1} + \delta_2\varepsilon_{t-2})(\varepsilon_{t-1} + \delta_1\varepsilon_{t-2} + \delta_2\varepsilon_{t-3}) \\ &= \delta_1 E(\varepsilon_{t-1}^2) + \delta_1\delta_2 E(\varepsilon_{t-2}^2) \\ \gamma_1 &= (\delta_1 + \delta_1\delta_2)\sigma^2 \end{aligned} \tag{A.56}$$

$$\begin{aligned} E(Y_t - \mu)(Y_{t-2} - \mu) &= E(\varepsilon_t + \delta_1\varepsilon_{t-1} + \delta_2\varepsilon_{t-2})(\varepsilon_{t-2} + \delta_1\varepsilon_{t-3} + \delta_2\varepsilon_{t-4}) \\ \gamma_2 &= \delta_2 E(\varepsilon_{t-2}^2) = \delta_2\sigma^2 \end{aligned} \tag{A.57}$$

$$\gamma_k = E(Y_t - \mu)(Y_{t-k} - \mu) = 0 \quad \text{for} \quad k > 2 \tag{A.58}$$

From (A.55), (A.56) and (A.58), the *autocorrelation function* (*ACF*) for the MA(2) model is as follows:

$$\begin{aligned} \rho_1 &= \frac{\gamma_1}{\gamma_0} = \frac{\delta_1(1+\delta_2)}{(1+\delta_1^2+\delta_2^2)} \\ \rho_2 &= \frac{\gamma_2}{\gamma_0} = \frac{\delta_2}{(1+\delta_1^2+\delta_2^2)} \\ \rho_k &= 0 \quad \text{for} \quad k > 2 \end{aligned} \tag{A.59}$$

A.6 The simplest ARMA model

The *simplest autoregressive moving average* model is the ARMA(1,1) model, which can be presented as follows:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1} \tag{A.60}$$

where β_0, β_1 and δ_1 are the model parameters and ε_t is i.i.d. Gaussian or $N(0, \sigma^2)$.

Under this assumption, the following expectation of Y_t is obtained:

$$\begin{aligned} E(Y_t) &= E(\beta_0 + \beta_1 Y_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1}) \\ \mu &= \beta_0 + \beta_1 \mu + 0 + 0 \\ \mu &= \frac{\beta_0}{1-\beta_1} \end{aligned} \tag{A.61}$$

For further statistical analysis, it is suggested that the ARMA(1,1) model should be written in terms of the deviation from the mean (Hamilton, 1994, p. 60), as follows:

$$\begin{aligned} (Y_t - \mu) &= \beta_1(Y_{t-1} - \mu) + \varepsilon_t + \delta_1 \varepsilon_{t-1} \\ d_t &= \beta_1 d_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1} \end{aligned} \tag{A.62}$$

Therefore, $E(d_t) = E(Y_t - \mu) = 0$ and $\text{Var}(d_t) = E(d_t^2)$. The following results can then be derived by using or assuming that d_{t-1} and ε_t are uncorrelated (i.e. $E(d_{t-1}\varepsilon_t) = 0$:

$$\begin{aligned} E(d_t\varepsilon_t) &= \beta_1 E(d_{t-1}\varepsilon_t) + E(\varepsilon_t^2) + \delta_1 E(\varepsilon_t \varepsilon_{t-1}) \\ &= 0 + \sigma^2 + 0 = \sigma^2 \end{aligned} \tag{A.63}$$

The variance of d_t is given by

$$\begin{aligned}
\mathrm{Var}(d_t) &= \beta_1^2\mathrm{Var}(d_{t-1}) + \mathrm{Var}(\varepsilon_t) + \delta_1^2\mathrm{Var}(\varepsilon_{t-1}) + 2\beta_1\delta_1 E(d_{t-1}\varepsilon_{t-1}) \\
(1-\beta_1^2)\mathrm{Var}(d_t) &= (1+\delta_1^2+2\beta_1\delta_1)\sigma^2 \\
\mathrm{Var}(d_t) &= \frac{(1+\delta_1^2+2\beta_1\delta_1)\sigma^2}{1-\beta_1^2} = \gamma_0
\end{aligned} \tag{A.64}$$

In order to obtain the *autocovariance* function of d_t, the following expectation is considered:

$$E(d_t d_{t-k}) = \beta_1 E(d_{t-1}d_{t-k}) + E(\varepsilon_t d_{t-k}) + \delta_1 E(\varepsilon_{t-1}d_{t-k}) \tag{A.65}$$

Therefore,

$$\begin{aligned}
\gamma_1 &= \beta_1\gamma_0 + \delta_1\sigma^2 \quad \text{for} \quad k=1 \\
\gamma_k &= \beta_1\gamma_{k-1} \quad \text{for} \quad k>1
\end{aligned} \tag{A.66}$$

Hence, the ACF for the stationary ARMA(1,1) model is

$$\begin{aligned}
\rho_1 &= \frac{\gamma_1}{\gamma_0} = \beta_1 + \frac{\delta_1\sigma^2}{\gamma_0} = \beta_1 + \frac{(1-\beta_1^2)\delta_1}{1+\delta_1^2+2\beta_1\delta_1} \\
\rho_k &= \delta_1\rho_{k-1} \quad \text{for} \quad k>1
\end{aligned} \tag{A.67}$$

A.7 General ARMA model

A.7.1 Derivation of the ACF

A general *autoregressive moving average* model, namely the ARMA(p,q) model based on a series$\{Y_t\}_{t=1}^{T}$, is defined as follows:

$$Y_t = \beta_0 + \sum_{i=1}^{p}\beta_i Y_{t-i} + \varepsilon_t + \sum_{j=1}^{q}\delta_j\varepsilon_{t-j} \tag{A.68}$$

where β_0, β_1 and δ_j are the model parameters and ε_t is i.i.d. Gaussian or $N(0,\sigma^2)$.

Under this assumption,

$$E(Y_t) = \frac{\beta_0}{1-\sum_{i=1}^{p}\beta_i} \tag{A.69}$$

For further derivation of the statistical results, the deviation is used from the mean of the series Y_t, namely $d_t = Y_t - \mu$, giving the following ARMA model:

$$\begin{aligned} d_t &= \sum_{i=1}^{p}\beta_i d_{t-i} + \varepsilon_t + \sum_{j=1}^{q}\delta_j\varepsilon_{t-j} \\ &= \sum_{i=1}^{p}\beta_i d_{t-i} + \sum_{j=0}^{q}\delta_j\varepsilon_{t-j} \quad \text{with} \quad \delta_0 = 1 \end{aligned} \tag{A.70}$$

By using the same technique as that for the ARMA(1,1) model, the following results or statistics are obtained:

$$\begin{aligned} E(d_t\varepsilon_t) &= \sum_{i=1}^{p}\beta_i E(d_{t-i}\varepsilon_t) + \sum_{j=0}^{q}\delta_j(\varepsilon_t\varepsilon_{t-j}) \\ &= 0 + \delta_0 E(\varepsilon_t^2) = 1x\sigma^2 = \sigma^2 \end{aligned} \tag{A.71}$$

The variance of d_t is given by

$$\begin{aligned} \text{Var}(d_t) &= \sum_{i=1}^{p}\beta_i^2\text{Var}(d_{t-i}) + \sum_{j=0}^{q}\delta_j^2\text{Var}(\varepsilon_{t-j}) + 2\sum_{i=1}^{p}\sum_{j=0}^{q}\beta_i\delta_j E(d_{t-i}\varepsilon_{t-j}) \\ \left[1-\sum_{i=1}^{p}\beta_i^2\right]\text{Var}(d_t) &= \sum_{j=0}^{q}\delta_j^2\sigma^2 + 2\sum_{i=1}^{\min(p,q)}\beta_i\delta_i\sigma^2 \\ \text{Var}(d_t) &= \frac{\left[\sum_{j=0}^{q}\delta_j^2 2 + 2\sum_{i=1}^{\min(p,q)}\beta_i\delta_i\right]\sigma^2}{1-\sum_{i=1}^{p}\beta_i^2} = \gamma_0 \end{aligned} \tag{A.72}$$

Note that in deriving this result, $E(d_{t-1}\varepsilon_{t-j}) = 0$ for all $i \neq j$ and $E(d_{t-1}\varepsilon_{t-j}) = \sigma^2$ for all $i = j$ have been used. Therefore,

$$\sum_{i=1}^{p}\sum_{j=0}^{q}\beta_i\delta_j E(d_{t-i}\varepsilon_{t-j}) = \sum_{i=1}^{\min(p,q)}\beta_i\delta_i\sigma^2 \tag{A.73}$$

Then the autocovariance function is given by

$$\begin{aligned} \gamma_k &= E(d_t d_{t-k}) = \sum_{i=1}^{p}\beta_i E(d_{t-i}d_{t-k}) + \sum_{j=0}^{q}\delta_j E(\varepsilon_{t-j}d_{t-k}) \\ &= \sum_{i=1}^{p}\beta_i\gamma_{|i-k|} + \delta_k\sigma^2 \end{aligned} \tag{A.74}$$

For specific or selected values of (p,q), based on (A.73) and (A.74) an explicit form of γ_k for each k can be derived. Then the ACF of the ARMA(p, q) model could be obtained by using the general formula $\rho_k = \gamma_k/\gamma_0$. Furthermore, the following special cases can be derived:

1. *The Autocovariance Function of the AR(p) Model in* (A.23)
   For $q=0$, the variance of the series d_t and *ACF* of the AR(p) model is obtained, as follows:

$$\begin{aligned}\mathrm{Var}(d_t) &= \sum_{i=1}^{p}\beta_i^2\mathrm{Var}(d_{t-i}) + \mathrm{Var}(\varepsilon_t) + \sum_{i=1}^{p}\beta_i E(d_{t-i}\varepsilon_t)\\ \left[1-\sum_{i=1}^{p}\beta_i^2\right]\mathrm{Var}(d_t) &= \sigma^2 + 0\\ \mathrm{Var}(d_t) &= \frac{\sigma^2}{1-\sum_{i=1}^{p}\beta_i^2} = \gamma_0\end{aligned} \tag{A.75}$$

$$\begin{aligned}\gamma_k &= E(d_t d_{t-k}) = \sum_{i=1}^{p}\beta_i E(d_{t-i}d_{t-k}) + E(\varepsilon_t d_{t-k})\\ \gamma_0 &= \sum_{i=1}^{p}\beta_i\gamma_i + \sigma^2 \quad \text{for} \quad k=0\\ \gamma_k &= \sum_{i=1}^{p}\beta_i\gamma_{|i-k|} \quad \text{for} \quad k>0\end{aligned} \tag{A.76}$$

2. *The Autocovariance Function of the MA(q) Model*
   For $p=0$, the variance of the series d_t and *ACF* of the MA(q) model is obtained, as follows:

$$\begin{aligned}\mathrm{Var}(d_t) &= \beta_1^2\mathrm{Var}(d_{t-1}) + \sum_{j=0}^{q}\delta_j^2\mathrm{Var}(\varepsilon_{t-j}) + 2\beta_1\sum_{j=0}^{q}\delta_j E(d_{t-1}\varepsilon_{t-j})\\ [1-\beta_1^2]\mathrm{Var}(d_t) &= \sum_{j=0}^{q}\delta_j^2\sigma^2 + 2\beta_1\delta_1\sigma^2\\ \mathrm{Var}(d_t) &= \frac{\left[\sum_{j=0}^{q}\delta_j^2 + 2\beta_1\delta_1\right]\sigma^2}{1-\beta_1^2} = \gamma_0\end{aligned} \tag{A.77}$$

$$\begin{aligned}\gamma_k &= E(d_t d_{t-k}) = \beta_1 E(d_{t-1}d_{t-k}) + \sum_{j=0}^{q}\delta_j E(\varepsilon_{t-j}d_{t-k})\\ &= \beta_1\gamma_{|1-k|} + \delta_k\sigma^2 \quad \text{for} \quad k \le q \quad \text{with} \quad \delta_0 = 1\\ &= \beta_1\gamma_{|1-k|} \quad \text{for} \quad k>q\end{aligned} \tag{A.78}$$

A.7.2 Estimation method

EViews provides an iteration method to estimate directly the model parameters. In practice, there is not any difficulty in estimating the parameters, as well as testing hypotheses, since EViews will present error messages, including the unstationary process corresponding to the model considered, if the model cannot be estimated based on the data set used. Note that the error message does not directly mean that the model is a wrong or bad model, since its statistical results are highly dependent on the data. However, if there is an error message, the model needs to be modified in order to have an estimable model.

In this section, alternative conditional likelihood functions proposed by Hamilton (1994) will be presented, which can be used for an iteration estimation method. In order to write the likelihood function, the error term ε_t will be considered, as follows:

$$\varepsilon_t = Y_t - \beta_0 - \sum_{i=1}^{p} \beta_1 Y_{t-i} - \sum_{j=1}^{q} \delta_1 \varepsilon_{t-j} \tag{A.79}$$

which is i.i.d. Gaussian or $N(0, \sigma^2)$.

Then by taking $y_0 = (y_0, y_{-1}, y_{-p+1})$ and $\varepsilon_0 = (\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_{-q+1})$ as initial values, for $t = 1,2,\ldots, T$ the following conditional LL function is obtained for estimating the parameters by using the iteration method:

$$\begin{aligned} LL = \log f(y_T, \ldots, y_1 | y_0, \varepsilon_0) &= -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma^2) - \sum_{t=1}^{T} \frac{\varepsilon_t^2}{2\sigma^2} \\ &= -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma^2) \\ &\quad -\frac{1}{2\sigma^2}\sum_{t=1}^{T}\left[y_t - \beta_0 - \sum_{i=1}^{p}\beta_i y_{t-i} - \sum_{j=1}^{q}\delta_j\varepsilon_{t-j}\right]^2 \end{aligned} \tag{A.80}$$

Alternatively, Box and Jenkins (1976, quoted by Hamilton, 1994, p. 132) recommended setting ε to zero but the y values equal to their observed values. Hence, the iteration in (A.80) starts at date $t = p + 1$ with $y_1, y_2, \ldots, y_p$ set to observed values and $\varepsilon_p = \varepsilon_{p-1} = \cdots = \varepsilon_{p-q+1} = 0$.

Then the conditional LL function considered will be as follows:

$$\begin{aligned} LL &= \log f(y_T, \ldots, y_{P+1} | y_p, \ldots, y_1, \varepsilon_p = 0, \ldots, \varepsilon_{p-q+1} = 0) \\ &= \frac{T-p}{2}\log(2\pi) - \frac{T-p}{2}\log(\sigma^2) - \sum_{t=p+1}^{T}\frac{\varepsilon_t^2}{2\sigma^2} \\ &= \frac{T-p}{2}\log(2\pi) - \frac{T-p}{2}\log(\sigma^2) \\ &\quad -\frac{1}{2\sigma^2}\sum_{t=p+1}^{T}\left(y_t - \beta_0 - \sum_{t=p+1}^{T}\beta_i y_{t-i} - \sum_{t=p+1}^{T}\delta_j\varepsilon_{t-j}\right)^2 \end{aligned} \tag{A.81}$$

Appendix B: Simple linear models

B.1 The simplest linear model

Based on a pair of time series, namely Y_t and X_t, the simplest linear population (or true) model is defined as

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t \tag{B.1}$$

Based on a sample of size T, having observed values (x_t, y_t) for $t = 1, 2, \ldots, T$, the following T equations are obtained with unknown values of β_0, β_1 and ε_t:

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \quad t = 1, 2, \ldots, T \tag{B.2}$$

Note that this system of equations cannot have a unique solution, since this system has $(T + 2)$ unknown variables, namely two model parameters, and T of the error terms, which is greater than the number of equations. For this reason, in order to obtain the (estimated) values of the parameters β_0 and β_1, a quadratic function of β_0 and β_1 should be considered, as follows:

$$Q = Q(\beta_0, \beta_1) = \sum_{t=1}^{T} \varepsilon_t^2 = \sum_{t=1}^{T} (y_t - \beta_0 - \beta_1 x_t)^2 \tag{B.3}$$

B.1.1 Least squares estimators

According to the least squares (LS) estimation method, the estimators of β_0 and β_1 are the values that minimize Q. It is well known that the necessary conditions for minimizing Q are as follows:

$$\begin{aligned} \frac{\partial Q}{\partial \beta_0} &= -2\sum_{t=1}^{T}(y_t - \beta_0 - \beta_1 x_t) = 0 \\ \frac{\partial Q}{\partial \beta_1} &= -2\sum x_t(y_t - \beta_0 - \beta_1 x_t) = 0 \end{aligned} \tag{B.4}$$

Time Series Data Analysis Using EViews IGN Agung

or

$$\sum_{t=1}^{T} x_t(y_t - \beta_0 - \beta_1 x_t) = 0$$
$$\sum_{t=1}^{T} x_t(y_t - \beta_0 - \beta_1 x_t) = 0 \tag{B.5}$$

These equations are called the *normal equations*, which will, in general, have a unique solution called the *point estimators* of the parameters β_0 and β_1, namely $\hat{\beta}_0$ and $\hat{\beta}_1$ respectively, as follows:

$$\hat{\beta}_1 = \frac{\sum(X_t - \bar{X})(Y_t - \bar{Y})}{\sum(X_t - \bar{X})^2} = \frac{\text{Cov}(X,Y)}{\text{Var}(X)} = \frac{s_{xy}}{s_x^2}$$
$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \tag{B.6}$$

For these estimators, there will be a minimum value of the quadratic function Q, called the error sum of squares (SSE), based on a sample of size T, as follows:

$$SSE = SSE(\hat{\beta}_0, \hat{\beta}_1) = \sum_{t=1}^{T} \hat{\varepsilon}_t^2 = \sum_{t=1}^{T} (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)^2 \tag{B.7}$$

Furthermore, it has been known that this SSE has $(T-2)$ degrees of freedom, since two degrees of freedom are lost by estimating the two parameters β_0 and β_1.Therefore, the mean of squared errors, namely MSE, is calculated as follows:

$$MSE = \frac{SSE}{T-2} = \frac{1}{T-2}\sum_{t=1}^{T} (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)^2 = \hat{\sigma}^2 \tag{B.8}$$

Note that the point estimators of β_0 and β_1, as well as the values of SSE and MSE, can, in fact, be obtained without using any assumptions of the error terms. However, for making inferences there should be specific assumptions of the error term ε_t. Alternative cases will be presented in the following sections.

B.2 Linear model with basic assumptions

The basic simple linear model in (B.1) has the following assumptions:

A1. Y_t is an observable random variable.
A2. X_t is an observable nonrandom variable.
A3. β_0 and β_1 are unknown parameters, called the model parameters.

A4. ε_t is an unobserved random error term with mean $E(\varepsilon_t) = 0$, homogeneous variances, namely $\text{Var}(\varepsilon_t) = \sigma^2(\varepsilon_t) = \sigma^2$, and ε_t and ε_s are uncorrelated for all $t \neq s$, so that $\text{Cov}(\varepsilon_t, \varepsilon_s) = 0$. In other words, ε_t is i.i.d.

A5. ε_t has a normal distribution $N(0,\sigma^2)$, for $t = 1, 2, \ldots, T$.

Note that the assumptions A4 and A5 indicate that the error terms ε_t, $t = 1, 2, \ldots, T$, have independent identically normal distributions with $E(\varepsilon_t) = 0$ and $\text{Var}(\varepsilon_t) = \sigma^2(\varepsilon_t) = \sigma^2$, namely i.i.d. $N(0,\sigma^2)$. In order word, ε_t is i.i.d. *Gaussian* or is a *white noise process.*

Under these assumptions, the following statistics and results are given.

B.2.1 Sampling distributions of the model parameters

Since $E(\varepsilon_t) = 0$ and X_t is an observable nonrandom variable, then

$$E(Y_t) = E(\beta_0 + \beta_1 X_t) + E(\varepsilon_t) = E(\beta_0 + \beta_1 X_t) \tag{B.9}$$

Furthermore, the following results have been proved:

(i) The sampling distribution of the estimator $\hat{\beta}_1$ is normal with mean and variance

$$\begin{aligned} E(\hat{\beta}_1) &= \beta_1 \\ \text{Var}(\hat{\beta}_1) &= \frac{\sigma^2}{\sum_{t=1}^{T} (X_t - \bar{X})^2} \end{aligned} \tag{B.10}$$

where the first equation, $E(\hat{\beta}_1) = \beta_1$, indicates that $\hat{\beta}_1$ as a *statistic* (not a value computed based on the sample) is an *unbiased estimator* of the parameter β_1.

(ii) The sampling distribution of the estimator $\hat{\beta}_0$ is normal with mean and variance

$$\begin{aligned} E(\hat{\beta}_0) &= \beta_0 \\ \text{Var}(\hat{\beta}_0) &= \frac{\sigma^2 \sum_{t=1}^{T} X_t^2}{n \sum_{t=1}^{T} (X_t - \bar{X})^2} \end{aligned} \tag{B.11}$$

where the first equation, $E(\hat{\beta}_0) = \beta_0$, indicates that $\hat{\beta}_0$ as a *statistic* (not a value computed based on the sample) is an *unbiased estimator* of the parameter β_0.

(iii) The mean square error *MSE* as a *statistic* is an *unbiased estimator* of the corresponding population (or true) variance, which can be presented as

$$E(MSE) = \sigma^2 \tag{B.12}$$

B.2.2 Student's t-statistic

Since $\hat{\beta}_i$ is now assumed to be normally distributed for each $i=0$ and $i=1$, then for the model in (B.1)

$$\frac{\hat{\beta}_i - \beta_i}{s(\hat{\beta}_i)} \text{ is distributed as } t(n-2) \tag{B.13}$$

with

$$\begin{aligned} s^2(\hat{\beta}_1) &= \frac{MSE}{\sum_{t=1}^{T} (x_t - \bar{x})^2} \\ s^2(\hat{\beta}_0) &= \frac{MSE \sum_{t=1}^{T} x_t}{T \sum_{t=1}^{T} (x_t - \bar{x})^2} \end{aligned} \tag{B.14}$$

By using the t-statistic and the parameter β_1, we can test two- and one sided hypotheses on the linear effect of the independent (*source*, *cause* or *explanatory*) variable X on the dependent (*respond*, *impact* or *downstream*) variable Y. Table B.1 presents the criteria used in testing the hypotheses.

B.2.3 Analysis of variance table

Corresponding to the data analysis based on the model in (B.1), there will be an analysis of variance (ANOVA) table as presented in Table B.2. In addition to SSE and MSE, which have been presented above, this table presents the following statistics:

(1) The *regression sum of squares* (*SSR*) is defined or computed as

$$SSR = \sum_{t=1}^{T} (\hat{Y}_t - \bar{Y})^2 \tag{B.15}$$

where

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 X_t \quad \text{and} \quad \bar{Y} = \sum_{t=1}^{T} \frac{Y_t}{T} \tag{B.16}$$

Table B.1 Criteria used in testing hypotheses

Hypothesis	Types	Decision at the α significant level
H_1: $\beta_1 \neq 0$	Two-sided	If Prob $= P(\lvert t \rvert > t_0) < \alpha$, data support the hypothesis
H_1: $\beta_1 > 0$	Right-sided	(1) If $t_0 > 0$ and Prob/2 $< \alpha$, data support the hypothesis
		(2) If $t_0 < 0$, data do not support the hypothesis
H_1: $\beta_1 < 0$	Left-sided	(1) If $t_0 < 0$ and Prob/2 $< \alpha$, data support the hypothesis
		(2) If $t_0 > 0$, data do not support the hypothesis
		*) t_0 is the observed value of the t-statistic

Table B.2 ANOVA table for a simple linear regression

Source of variation	*SS*	*df*	*MS*
Regression	$SSR = \sum(\hat{Y}_t - \bar{Y})^2$	1	$MSR = SSR/1$
Error	$SSE = \sum(Y_t - \hat{Y}_t)^2$	$T-2$	$MSE = SSE/(T-2)$
Total	$SST = \sum(Y_t - \bar{Y})^2$	$T-1$	

with one degree of freedom, which is equal to the number of independent variables of the model. In this case, the *mean of squares regression* $MSR = SSR/1$.

(2) The *total sum of squares* (SST) is defined or computed as

$$SST = \sum (Y_t - \bar{Y})^2 \tag{B.17}$$

with $(T-1)$ degrees of freedom, and it has a special characteristic as follows:

$$SST = SSR + SSE \tag{B.18}$$

(3) The *chi-squared-statistic* corresponds to the MSR and MSE, giving the following values:

$$\chi_1^2 = MSR = \frac{SSR}{1} \sim \sigma^2\chi^2(1) \tag{B.19}$$

$$\chi_2^2 = MSE = \frac{SSE}{T-2} \sim \sigma^2\chi^2(T-2) \tag{B.20}$$

(4) The *Fisher F-statistic* is defined or computed as

$$F_0 = \frac{MSR}{MSE} = \frac{SSR/1}{SSE/(T-2)} \quad \text{is distributed as } F(1, T-2) \tag{B.21}$$

This F-statistic can be used to test the following two-sided hypothesis, where large values of F_0, which should be greater than one, support H_1, and values of F_0 near 1(one) support H_0:

$$\begin{aligned} H_0 &: \beta_1 = 0 \\ H_1 &: \beta_1 \neq 0 \end{aligned} \tag{B.22}$$

B.2.4 Coefficient of determination

The coefficient of determination of the model in (B.1), namely r^2, is computed as

$$r^2 = \frac{SST - SSE}{SST} = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} \tag{B.23}$$

Since $0 \leq SSE \leq SST$, then $0 \leq r^2 \leq 1$, which indicates the proportion of the total variation of Y that can be explained by using variation in the independent variable X, and the value of $(1 - r^2) = SSE/SST$ indicates the proportion of the total variation that cannot be explained by variation in X.

Furthermore, the following statistics and comments may be considered:

(1) The *correlation coefficient* of X and Y variables:

$$r(X, Y) = \pm\sqrt{r^2} \tag{B.24}$$

where the positive sign or $r(X, Y) > 0$ indicates that the observed values of Y_t has a positive trend, with respect to the observed values of X_t. In other words, the regression line has a positive slope and the regression line has a negative slope if $r(X, Y) < 0$.

(2) The *relationship between* $r(X, Y)$ *and* $\hat{\beta}_1$:

$$r(X, Y) = \hat{\beta}_1 \frac{s_x}{s_y} \tag{B.25}$$

where s_x and s_y are the standard deviations of X_t and Y_t respectively. This relationship indicates that the correlation coefficient of X and Y is in fact a measure of the linear association between the two variables, and it could also be used to test the *causal effect* of X on Y. Refer to the *standardized coefficient regression* of Y on X, which can be presented as $zy = \hat{\rho}^*zx = r(x, y)^*zx$, where zy and zx are the z-scores of Y and X respectively.

B.3 Maximum likelihood estimation method

Under the assumption that the error terms ε_t have a normal distribution with mean $E(\varepsilon_t) = 0$ and $\text{Var}(\varepsilon_t) = \sigma^2$, then ε_t has the density function

$$f(\varepsilon_t) = (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{\varepsilon_t^2}{2\sigma^2}\right) = (2\pi\sigma^2)^{-1/2} \exp\left[-\frac{(Y_t - \beta_0 - \beta_1 X_t)^2}{2\sigma^2}\right] \tag{B.26}$$

Furthermore, under the assumption that the error terms ε_t, $t = 1, 2, \ldots, T$, have independent identical normal distributions, the following *likelihood function* is defined as follows:

$$L = f(\varepsilon_1, \ldots, \varepsilon_T) = (2\pi\sigma^2)^{-T/2} \prod_{t=1}^{T} \exp\left[-\frac{(Y_t - \beta_0 - \beta_1 X_t)^2}{2\sigma^2}\right] \tag{B.27}$$

In order to obtain the estimators of the model parameters, the following natural logarithm function, called the *log-likelihood function*, should be considered:

$$LL = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(Y_t - \beta_0 - \beta_1 X_t)^2 \tag{B.28}$$

The necessary conditions to obtain the maximum value of LL are as follows:

$$\begin{aligned}
\frac{\partial(LL)}{\partial\beta_0} &= \frac{1}{\sigma^2}\sum_{t=1}^{T}(Y_t - \beta_0 - \beta_1 X_t) = 0 \\
\frac{\partial(LL)}{\partial\beta_1} &= \frac{1}{\sigma^2}\sum_{t=1}^{T}(Y_t - \beta_0 - \beta_1 X_t)X_t = 0 \\
\frac{\partial(LL)}{\partial\sigma^2} &= -\frac{T}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}(Y_t - \beta_0 - \beta_1 X_t)^2 = 0
\end{aligned} \tag{B.29}$$

As a result, the following normal equations are obtained:

$$\begin{aligned}
&\sum_{t=1}^{T}(y_t - \beta_0 - \beta_1 x_t) = 0 \\
&\sum_{t=1}^{T}x_t(y_t - \beta_0 - \beta_1 x_t) = 0 \\
&\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}(y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)^2
\end{aligned} \tag{B.30}$$

Note that the first two equations are exactly the same as the normal equations based on the LS estimation method in (B.5). Therefore, it is easy to write the estimators of the parameters as follows:

$$\begin{aligned}
\hat{\beta}_1 &= \frac{\sum(x_t - \bar{x})(y_t - \bar{y})}{\sum(x_t - \bar{x})^2} = \frac{\mathrm{Cov}(x,y)}{\mathrm{Var}(x)} = \frac{s_{xy}}{s_x^2} \\
\hat{\beta}_0 &= \bar{y} - \hat{\beta}_1\bar{x} \\
\sigma^2 &= \frac{1}{T}\sum(y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)^2
\end{aligned} \tag{B.31}$$

Furthermore, note that both estimation methods give the same estimate values for the β parameters. However, the estimators based on the OLS are obtained without using the normality assumption, but the maximum likelihood (ML) should use the independent identical normal distributions, which lead to the likelihood function in (B.27). On the other hand, in order to have unbiased estimators, the normality assumption that error terms should be taken for granted is not proven or tested. They differ only in estimating σ^2.

B.4 First-order autoregressive linear model

For the time series variables X_t and Y_t, the independent assumption of the error terms of the model in (B.1) is not realistic. For this reason, here the simplest model is considered by taking into account the autocorrelation or serial correlation between the error terms, namely the first-order autoregressive linear model, or AR(1) model, which is presented as follows:

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 X_t + \mu_t \\ \mu_t &= \rho\mu_{t-1} + \varepsilon_t \end{aligned} \tag{B.32}$$

where ρ is the *autocorrelation* or *serial correlation parameter* such that $|\rho| < 1$, and $\varepsilon_t, t = 1, 2, \ldots, T$, are i.i.d. $N(0, \sigma^2)$.

Compared to the autoregressive model presented in Appendix A, note that the autocorrelations in this model are related to the series of the error term μ_t. The autocorrelation for the models presented in Appendix A are related to the endogenous variable Y_t. Therefore, this AR(1) model is, in fact, a model with *first-order autoregressive errors*.

B.4.1 Two-stage estimation method

To estimate the model parameters, namely β_0, β_1 and ρ, two stages of regression analyses should be performed, as follows:

(1) The first stage is to apply the model

$$Y_t = \beta_0 + \beta_1 X_t + \mu_t \tag{B.33}$$

In this stage, there could be a variable of the error terms or residuals, namely $\hat{\mu}_t$, by using the LS estimation method. Furthermore, the variable $\hat{\mu}_{t-1}$ could be created.

(2) Then a model having two independent variables is applied, as follows:

$$Y_t = \beta_0 + \beta_1 X_t + \rho\hat{\mu}_{t-1} + \varepsilon_t \tag{B.34}$$

Under the basic assumptions A1 to A5 for ε_t, presented above, the unbiased estimators of the three parameters in the model in (B.32) are found, namely $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\rho}$. Therefore, the hypothesis on each parameter can be tested by using the usual t-statistic. Furthermore, refer to the following Durbin–Watson test statistic.

B.4.2 Durbin–Watson statistic

The Durbin–Watson statistic for testing the first-degree or first-order serial correlation or autocorrelation of the error terms μ_t is defined as

$$DW = \frac{\sum_{t=2}^{T} (\hat{\mu}_t - \hat{\mu}_{t-1})^2}{\sum_{t=1}^{T} \hat{\mu}_t} \tag{B.35}$$

The usual hypothesis considered in business and economics is a right-sided hypothesis as follows:

$$\begin{aligned} H_0 &: \rho = 0 \\ H_1 &: \rho > 0 \end{aligned} \tag{B.36}$$

Durbin and Watson produced lower and upper bounds d_L and d_U, which should be used for making a decision for a model having $(p - 1)$ independent variables.

If $DW > d_U$, the first-order autocorrelation is insignificant.
If $DW < d_L$, the first-order autocorrelation is significantly positive.
If $d_L \leq DW \leq d_U$, then the test is inconclusive.

In practice, however (by *rule of thumb*), if the value of a DW-statistic is closed to 2 (two), then the first-order autoregressive model is not used.

B.4.3 Properties of the error term μ_t

Based on the equation $\mu_t = \rho\mu_{t-1} + \varepsilon_t$, the following series can be derived:

$$\mu_t = \rho^n \mu_{t-n} + \varepsilon_t(1 + \rho + \rho^2 + \cdots + \rho^n) \tag{B.37}$$

In the long run, for $n \to \infty$ and $|\rho| < 1$,

$$\mu_t = \sum_{n=0}^{\infty} \rho^n \varepsilon_t \tag{B.38}$$

Since ε_t is normally distributed $N(0,\sigma^2)$, then μ_t is also normally distributed with mean and variance

$$\begin{aligned} E(\mu_t) &= \sum_{n=0}^{\infty} \rho^n E(\varepsilon_t) = 0 \\ \mathrm{Var}(\mu_t) &= \sum_{n=0}^{\infty} \mathrm{Var}(\rho^n \varepsilon_t) = \mathrm{Var}(\varepsilon_t) \sum_{n=0}^{\infty} \rho^{2n} = \frac{\sigma^2}{1 - \rho^2} \end{aligned} \tag{B.39}$$

Furthermore, the covariance between μ_t and μ_{t+s} for all $s > 0$, as well as their coefficient correlations, can be derived as follows:

$$\begin{aligned}\mathrm{Cov}(\mu_t, \mu_{t-s}) &= E(\mu_t \mu_{t-s}) \\ &= \rho^s E(\mu_{t-s}^2) + \sum_{k=0}^{s} \rho^k E(\varepsilon_t \mu_{t-s}) \\ &= \rho^s \mathrm{Var}(\mu_{t-s}) + 0 = \rho^s \frac{\sigma^2}{1-\rho^2}\end{aligned} \tag{B.40}$$

$$\mathrm{Cor}(\mu_t, \mu_{t-s}) = \frac{\mathrm{Cov}(\mu_t, \mu_{t-s})}{\sqrt{[\mathrm{Var}(\mu_t)\mathrm{Var}(\mu_{t-s})]}} = \rho^s \tag{B.41}$$

B.4.4 Maximum likelihood estimation method

Based on the model in (B.32), the error terms ε_t can be presented as follows:

$$\varepsilon_t = \mu_t - \rho\mu_{t-1} = (Y_t - \beta_0 - \beta_1 X_t) - \rho(Y_{t-1} - \beta_0 - \beta_1 X_{t-1}) \tag{B.42}$$

Since ε_t is normally distributed as $N(0,\sigma^2)$, the following density function is found:

$$\begin{aligned}f(\varepsilon_t) &= (2\pi\sigma^2)^{-1/2}\exp\left(-\frac{\varepsilon_t^2}{2\sigma^2}\right) \\ &= (2\pi\sigma^2)^{-1/2}\exp\left\{-\frac{[(Y_t - \beta_0 - \beta_1 X_t) - \rho(Y_{t-1} - \beta_0 - \beta_1 X_{t-1})]^2}{2\sigma^2}\right\}^d\end{aligned} \tag{B.43}$$

for $t = 2,3,\ldots,T$. Therefore, the log-likelihood function can be written as follows:

$$\begin{aligned}LL &= -\frac{T-1}{2}\ln(2\pi) - \frac{T-1}{2}\ln(\sigma^2) \\ &\quad -\frac{1}{2\sigma^2}\sum_{t=2}^{T}[(y_t - \beta_0 - \beta_1 x_t) - \rho(y_{t-1} - \beta_0 - \beta_1 x_{t-1})]^2\end{aligned} \tag{B.44}$$

The approach is then to maximize (B.42) numerically (by using the iterative process) with respect to β_0, β_1, ρ and σ^2. In a mathematical sense, however, the necessary conditions for maximizing this LL function are $\partial(LL)/\partial\nu = 0$ with respect to all parameters. Then the following normal equations could be obtained, but it is very difficult to obtain an explicit solution:

$$\begin{aligned}
&\sum_{t=2}^{T}[(y_t-\beta_0-\beta_1x_t)-\rho(y_{t-1}-\beta_0-\beta_1x_{t-1})]=0\\
&\sum_{t=2}^{T}[(y_t-\beta_0-\beta_1x_t)-\rho(y_{t-1}-\beta_0-\beta_1x_{t-1})](x_t-\rho x_{t-1})=0\\
&\sum_{t=2}^{T}[(y_t-\beta_0-\beta_1x_t)-\rho(y_{t-1}-\beta_0-\beta_1x_{t-1})](y_{t-1}-\beta_0-\beta_1x_{t-1})=0\\
&\sigma^2=\frac{1}{T-1}\sum_{t=2}^{T}[(y_t-\beta_0-\beta_1x_t)-\rho(y_{t-1}-\beta_0-\beta_1x_{t-1})]^2
\end{aligned} \tag{B.45}$$

For this reason, Hamilton (1994, p. 22) estimated the first-order autocorrelation ρ by using the first iteration alone, namely $\hat{\mu}_t = y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t$. He presented the following estimate by renormalizing the number of observations in the original sample to $(T+1)$, denoted by $y_0, y_1, \ldots, y_T$:

$$\hat{\rho} = \frac{(1/T)\sum_{t=1}^{T}\hat{\mu}_t\hat{\mu}_{t-1}}{(1/T)\sum_{t=1}^{T}\hat{\mu}_{t-1}^2} = \frac{\sum_{t=1}^{T}\hat{\mu}_t\hat{\mu}_{t-1}}{\sum_{t=1}^{T}\hat{\mu}_{t-1}^2} \tag{B.46}$$

It has been proved that

$$(\hat{\rho}-\rho)\sqrt{T} \text{ is asymptotic normally distributed as } N[0,(1-\rho^2)]. \tag{B.47}$$

Based on this normal distribution, an alternative statistic can be obtained for testing the null hypothesis '*no first-order autocorrelation of the error terms*', in addition to the Durbin–Watson test.

B.5 AR(p) linear model

An extension of the AR(1) model in (B.22) is an AR(p) model, described as follows:

$$\begin{aligned}
Y_t &= \beta_0 + \beta_1 X_t + \mu_t\\
\mu_t &= \sum_{i=1}^{p}\rho_i\mu_{t-i} + \varepsilon_t
\end{aligned} \tag{B.48}$$

where ρ_i are the ith autocorrelation or serial correlation parameters such that $|\rho_i| < 1$ and ε_t is i.i.d Gaussian or $N(0,\sigma^2)$.

B.5.1 Estimation method

Based on this model, a series of the error terms can be considered, as follows:

$$\varepsilon_t = \mu_t - \sum_{i=1}^{p} \rho_i \mu_{t-i} = (Y_t - \beta_0 - \beta_1 X_t) - \sum_{i=1}^{p} \rho_i (Y_{t-i} - \beta_0 - \beta_1 X_{t-i}) \quad \text{(B.49)}$$

Since the error terms ε_t, $t = 1,2, \ldots, T$, are i.i.d. $N(0,\sigma^2)$, then, similar to the *LL* function in (B.43), the following *LL* function is obtained:

$$\begin{aligned} LL = & -\frac{T-p}{2}\ln(2\pi) - \frac{T-p}{2}\ln(\sigma^2) \\ & -\frac{1}{2\sigma^2}\sum_{t=p+1}^{T}(y_t - \beta_0 - \beta_1 x_t) - \sum_{i=1}^{p}\rho_i (y_{t-i} - \beta_0 - \beta_1 x_{t-i})^2 \end{aligned} \quad \text{(B.50)}$$

Then the approach is to maximize this function numerically (by the iteration process) with respect to $\beta_0, \beta_1, \sigma^2$ and ρ_i, $i = 1, 2, \ldots, p$. The simplest approach is known as the *grid search* method (Hamilton, 1994, pp. 133–145), using the numerical process.

Alternatively, the following stages of regressions may be used:

(1) Regress Y_t on X_t in order to generate the series of residuals, namely $\hat{\mu}_{t-i}$, for $i = 0, 1, \ldots, p$.
(2) Regress Y_t on X_t and $\hat{\mu}_{t-i}$, $i = 1, \ldots, p$, to obtain the estimates of β_0, β_1 and ρ_i, as well as the residual ε_t. Then $\hat{\sigma}^2 = \sum_{t=1}^{T} \hat{\varepsilon}_t^2$.

B.5.2 Properties of μ_t

By considering only the equation or model of the error term, namely

$$\mu_t = \sum_{i=1}^{q} \rho_i \mu_{t-i} + \varepsilon_t \quad \text{(B.51)}$$

it can be seen that this model is in fact similar to or the same as the AR(p) model of a single series Y_t as presented in Appendix A. Therefore, the properties of μ_t can easily be derived by using the same steps, as follows:

$$\begin{aligned} \text{Var}(\mu_t) &= \sum_{i=1}^{p}\rho_i^2 \text{Var}(\mu_{t-i}) + \text{Var}(\varepsilon_t) + \sum_{i=1}^{p}\rho_i E(d_{t-i}\varepsilon_t) \\ \left[1 - \sum_{i=1}^{p}\rho_i^2\right]\text{Var}(\mu_t) &= \sigma^2 + 0 \\ \text{Var}(\mu_t) &= \frac{\sigma^2}{1 - \sum_{i=1}^{p}\rho_i^2} = \gamma_0 \end{aligned} \quad \text{(B.52)}$$

$$\gamma_k = E(\mu_t\mu_{t-k}) = \sum_{i=1}^{p}\rho_i E(\mu_{t-i}\mu_{t-k}) + E(\varepsilon_t\mu_{t-k})$$
$$\gamma_0 = \sum_{i=1}^{p}\rho_i\gamma_i + \sigma^2 \qquad \text{for} \quad k = 0 \tag{B.53}$$
$$\gamma_k = \sum_{i=1}^{p}\rho_i\gamma_{|i-k|} \qquad \text{for} \quad k > 0$$

B.6 Alternative models

Refer to the exogenous variable X_t of the model presented in the previous sections. In general, there are many choices for the exogenous variable, such as the time t-variable, the lagged dependent variable and the transformation of a variable. For this reason, in the following subsections, selected simple models are presented that have been presented in this book.

B.6.1 Alternative 1: The simplest model with trend

The simplest linear model with trend or the *simplest trend model* is defined as

$$Y_t = \beta_0 + \beta_1 t + \varepsilon_t \tag{B.54}$$

Under the assumption that ε_t is i.i.d. *non-Gaussian* (i.e. independent identically distributed as nonnormal) with mean zero, variance σ^2 and finite fourth moment, Hamilton (1994, pp. 458–459) derived statistics that are asymptotically normal (Gaussian), supported by the central limit theorem. Two of those statistics are univariate statistics as follows:

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\varepsilon_t \xrightarrow{L} N(0,\sigma^2) \tag{B.55}$$

and

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\frac{t}{T}\right)\varepsilon_t \xrightarrow{L} \left(0,\frac{\sigma^2}{3}\right) \tag{B.56}$$

B.6.2 Alternative 2: The classical growth model

This model can be considered as the model in (B.54) with the following equation (please refer to the model in (2.3), Chapter 2):

$$\log(Y_t) = \beta_0 + \beta_1 t + \varepsilon_t \tag{B.57}$$

This model could be extended to the AR(p) growth model, as a special case of the AR(p) model in (B.48), with the following equation:

$$\begin{aligned}\log(Y_t) &= \beta_0 + \beta_1 t + \mu_t \\ \mu_t &= \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t\end{aligned} \tag{B.58}$$

B.6.3 Alternative 3: The AR(p) polynomial model

This model is defined as

$$\begin{aligned}Y_t &= \beta_0 + \sum_{i=1}^{k} \beta_i X_t^i + \mu_t \\ \mu_t &= \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t\end{aligned} \tag{B.59}$$

B.6.4 Alternative 4: The AR(p) return rate model

This model is defined as

$$\begin{aligned}d\log(Y_t) &= \beta_0 + \beta_1 X_t + \mu_t \\ \mu_t &= \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t\end{aligned} \tag{B.60}$$

B.6.5 Alternative 5: The bounded translog linear (Cobb-Douglas) AR(p) model

This model is defined as

$$\begin{aligned}\log\frac{Y_t - L}{U - Y_t} &= \beta_0 + \beta_1 \log(X_t) + \mu_t \\ \mu_t &= \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t\end{aligned} \tag{B.61}$$

where U and L are the upper and lower bounds of the expected values of the series Y_t.

B.7 Lagged-variable model

A qth lagged endogenous variable model, namely the LV(q) model, with an exogenous variable X_t is defined as

$$Y_t = \beta_0 + \sum_{j=1}^{q} \beta_j Y_{t-j} + \delta X_t + \varepsilon_t \tag{B.62}$$

Based on this model, the error sum of squares function is defined as follows:

$$Q = \sum_{t=1}^{T} \varepsilon_t^2 = \sum_{t=1}^{T} \left(Y_t - \beta_0 - \sum_{j=1}^{q} \beta_j Y_{t-j} - \delta X_t \right)^2 \tag{B.63}$$

Furthermore, under the assumption that ε_t is i.i.d. Gaussian or $N(0,\sigma^2)$, the following log-likelihood function is obtained:

$$\begin{aligned} LL &= -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma^2) - \sum_{t=1}^{T} \frac{\varepsilon_t^2}{2\sigma^2} \\ LL &= -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma^2) - \frac{1}{2\sigma^2} \sum_{t=1}^{T} \left(y_t - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-j} - \delta x_t \right)^2 \end{aligned} \tag{B.64}$$

To estimate the parameters β_0, β_j, δ and σ^2, in a mathematical sense, the following normal equation is considered:

$$\begin{aligned} &\sum_{t=q+1}^{T} \left(y_t - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-j} - \delta x_t \right) = 0 \\ &\sum_{t=q+1}^{T} \left(y_t - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-j} - \delta x_t \right) y_{t-j} = 0, \quad j = 1, \ldots, q \\ &\sum_{t=q+1}^{T} \left(y_t - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-j} - \delta x_t \right) x_t = 0 \\ &\sigma^2 = \sum_{t=q+1}^{T} \left(y_t - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-j} - \delta x_t \right)^2 \end{aligned} \tag{B.65}$$

B.8 Lagged-variable autoregressive models

B.8.1 The simplest lagged-variable autoregressive model

The simplest lagged-variable autoregressive model, namely the LVAR(1,1) model, with an exogenous variable is defined as

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 Y_{t-1} + \delta X_t + \mu_t \\ \mu_t &= \rho \mu_{t-1} + \varepsilon_t \end{aligned} \tag{B.66}$$

Under the assumption that ε_t is i.i.d. $N(0,\sigma^2)$, then

$$\begin{aligned} \varepsilon_t &= (Y_t - \beta_0 - \beta_1 Y_{t-1} - \delta X_t) - \rho \mu_{t-1} \\ &= (Y_t - \beta_0 - \beta_1 Y_{t-1} - \delta X_t) - \rho (Y_{t-1} - \beta_0 - \beta_1 Y_{t-2} - \delta X_{t-1}) \end{aligned} \tag{B.67}$$

with the normal density function as follows:

$$f(\varepsilon_t) = (2\pi\sigma^2)^{-1/2}\exp\left\{\frac{[(Y_t-\beta_0-\beta_1 Y_{t-1}-\delta X_t)-\rho(Y_{t-1}-\beta_0-\beta_1 Y_{t-2}-\delta X_{t-1})]^2}{2\sigma^2}\right\} \tag{B.68}$$

for $t=3,\ldots,T$, since Y_{t-2} is on the right-hand side and the series considered is Y_t, for $t=1,\ldots,T$.

In order to estimate the parameters β, δ, ρ and σ^2, either the error sum of squares function or the *LL* function may be used, as follows.

The error sum of squares function is given by

$$\begin{aligned} Q &= Q(\beta_0,\ldots,\beta_q,\rho,\delta) = \sum_{t=3}^{T}\varepsilon_t^2 \\ &= \sum_{t=3}^{T}[(Y_t-\beta_0-\beta_1 Y_{t-1}-\delta X_t)-\rho(Y_{t-1}-\beta_0-\beta_1 Y_{t-2}-\delta X_{t-1})]^2 \end{aligned} \tag{B.69}$$

The *LL* function is given by

$$\begin{aligned} LL &= -\frac{T-2}{2}\log(2\pi)-\frac{T-2}{2}\log(\sigma^2)-\sum_{t=3}^{T}\left(\frac{\varepsilon_t^2}{2\sigma^2}\right) \\ LL &= -\frac{T-2}{2}\log(2\pi)-\frac{T-2}{2}\log(\sigma^2) \\ &\quad -\frac{1}{2\sigma^2}\sum_{t=3}^{T}[(y_t-\beta_0-\beta_1 y_{t-1}-\delta x_t)-\rho(y_{t-1}-\beta_0-\beta_1 y_{t-2}-\delta x_{t-1})]^2 \end{aligned} \tag{B.70}$$

Based on this *LL* function the following *normal equations* would be derived:

$$\begin{aligned} &\sum_{t=3}^{T}[(y_t-\beta_0-\beta_1 y_{t-1}-\delta x_t)-\rho(y_{t-1}-\beta_0-\beta_1 y_{t-2}-\delta x_{t-1})] = 0 \\ &\sum_{t=3}^{T}[(y_t-\beta_0-\beta_1 y_{t-1}-\delta x_t)-\rho(y_{t-1}-\beta_0-\beta_1 y_{t-2}-\delta x_{t-1})](y_{t-1}-\rho y_{t-2}) = 0 \\ &\sum_{t=3}^{T}[(y_t-\beta_0-\beta_1 y_{t-1}-\delta x_t)-\rho(y_{t-1}-\beta_0-\beta_1 y_{t-2}-\delta x_{t-1})](x_t-\rho x_{t-1}) = 0 \\ &\sum_{t=3}^{T}[(y_t-\beta_0-\beta_1 y_{t-1}-\delta x_t)-\rho(y_{t-1}-\beta_0-\beta_1 y_{t-2}-\delta x_{t-1})] \\ &\qquad (y_{t-1}-\beta_0-\beta_1 y_{t-2}-\delta x_{t-1}) = 0 \\ &\sigma^2 = \frac{1}{T-2}\sum_{t=3}^{T}[(y_t-\beta_0-\beta_1 y_{t-1}-\delta x_t)-\rho(y_{t-1}-\beta_0-\beta_1 y_{t-2}-\delta x_{t-1})]^2 \end{aligned} \tag{B.71}$$

Note that in (B.71) there are five equations with five unknowns or parameters, so that, in general, a unique solution would be expected. However, it is very difficult to present an explicit solution for each parameter. Therefore, EViews provides an iteration estimation process, as presented in Section A.7.

B.8.2 General lagged-variable autoregressive model

A general *lagged-variable autoregressive* model, namely the LVAR(p, q) model, with an exogenous variable, is defined as

$$\begin{aligned} Y_t &= \beta_0 + \sum_{j=1}^{q} \beta_j Y_{t-j} + \delta X_t + \mu_t \\ \mu_t &= \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t \end{aligned} \tag{B.72}$$

Compared to the AR(p) model presented in Section A.7, where the term AR(p) is related to the endogenous variable Y_t, in the model (B.72) the term AR(p) is related to the error term or residual $\mu_{t-1}, \ldots, \mu_{t-p}$.

Under the assumption that ε_t is i.i.d. $N(0,\sigma^2)$, then the model parameters can be estimated by using either the error sum of squares function or the *LL* function, as follows.

The error or residual sum of squares function is

$$\begin{aligned} Q &= \sum_{t=k+1}^{T} \varepsilon_t^2 \\ &= \sum_{i=k+1}^{T} \left[(y_t - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-j} - \delta x_t) - \sum_{i=1}^{p} \rho_i (y_{t-i} - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-i-j} - \delta x_{t-i}) \right]^2 \end{aligned} \tag{B.73}$$

where $k = p + q = \max\{i + j, \forall i \text{ and } j\}$. Then the log-likelihood function is given by

$$\begin{aligned} LL &= -\frac{T-k}{2}\log(2\pi) - \frac{T-k}{2}\log(\sigma^2) - \sum_{t=k+1}^{T} \frac{\varepsilon_t^2}{2\sigma^2} \\ LL &= -\frac{T-k}{2}\log(2\pi) - \frac{T-k}{2}\log(\sigma^2) \\ &\quad - \frac{1}{2\sigma^2} \sum_{t=k+1}^{T} \left[(y_t - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-j} - \delta x_t) - \sum_{i=1}^{p} \rho_i (y_{t-i} - \beta_0 - \sum_{j=1}^{q} \beta_j y_{t-i-j} - \delta x_{t-i}) \right]^2 \end{aligned} \tag{B.74}$$

By using the same technique as presented in Section A.7, it is easy to obtain the estimates of the parameters, as well as testing hypotheses, using EViews.

B.9 Special notes and comments

Considering the application of the basic model in (B.1), namely $Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$, the following notes and comments are made:

(1) This model represents the *linear trend* of an endogenous variable Y_t with respect to an exogenous variable X_t in the population. Even though their pattern of relationship could be nonlinear, the true population model will never be known. However, the linear trend of Y_t with respect to X_t can always be considered. Therefore, it could be said that this model can be defined as a *true population model with trend* of Y_t with respect to X_t.

(2) It is well known that the moment product correlation in the population, namely $\rho = \rho(X_t, Y_t)$, is a measure of a linear correlation. Therefore, this moment product correlation can also be used to present the linear trend of Y_t with respect to X_t. Refer to the standardized regression of Y_t on X_t, which can be presented as $ZY_t = \rho ZX_t + \varepsilon_t$, where ZY_t and ZX_t are the Z-scores of the variable Y_t and X_t respectively. Hence, testing the null hypothesis H_0: $\beta_1 = 0$ is exactly the same as testing the null hypothesis H_0: $\rho = \rho(X_t, Y_t) = 0$.

(3) On the other hand, to study their pattern of relationship in more detail, as well as the growth curve of Y_t with respect to X_t, there should be a high dependence on the data set that happens to be available. In this case, the scatter graph or plot of the bivariate (X_t, Y_t) with a regression or kernel fit should be observed, as presented in this book. Then personal judgment should be used to define a model or alternative models, as presented in Section 2.6. Refer to Section 2.14 for more detailed comments.

(4) In order to present the causal effect of an exogenous variable X_t on an endogenous variable Y_t, it is suggested that X_{t-i} should be used for some selected $i > 0$, instead of X_t, since a cause factor needs to be measured prior or before the impact factor. However, in general, researchers have been using X_{t-1}.

(5) It has been recognized that any time series models should be using either the lag(s) of the endogenous variable or the autoregressive errors, or both.

(6) Finally, whatever model is used, it is suggested that an additional residual analysis should be done in order to find out the limitation of the final model(s).

Appendix C: General linear models

C.1 General linear model with i.i.d. Gaussian disturbances

As an extension of the basic model presented in Appendix B, a (univariate) *general linear model* (*GLM*) is presented as

$$y_t = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_{k-1} x_{(k-1)t} + \mu_t = \sum_{i=0}^{k-1} \beta_i X_{it} + \mu_t \tag{C.1}$$

for $t = 1, \ldots, T$, which can be presented in matrix form as

$$\underset{(Tx1)}{y} = \underset{(Txk)}{X} \underset{(kx1)}{\beta} + \underset{(Tx1)}{\mu} \tag{C.2}$$

where

$$\underset{(Tx1)}{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \vdots \\ \vdots \\ y_t \end{bmatrix} \quad \underset{(Txk)}{X} = \begin{bmatrix} X'_1 \\ X'_2 \\ \vdots \\ \vdots \\ \vdots \\ X'_t \end{bmatrix} \quad \text{with} \quad \underset{1x(k-1)}{X} = \begin{bmatrix} x_{0t} = 1 \\ x_{1t} \\ \vdots \\ \vdots \\ \vdots \\ x_{(k-1)t} \end{bmatrix} \quad \underset{(kx1)}{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \vdots \\ \vdots \\ \beta_{k-1} \end{bmatrix} \quad \underset{(Tx1)}{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \vdots \\ \vdots \\ \mu_t \end{bmatrix} \tag{C.3}$$

Note that for $k = 2$, the model is in the form presented in (B.1). Furthermore, also note that the independent variables x_{it}, $i = 0, 1, 2, \ldots, (k-1)$, could be any set of exogenous variables, such as pure exogenous variables and their lags, the time t-variable, as well as selected two-way or higher interactions of the independent variables. By selecting a set of relevant independent variables from all possible types of those variables, it is expected that the error term μ_t is i.i.d. distributed.

C.1.1 The OLS estimates

Under the basic assumptions A1 to A5 presented in Appendix B, namely the multivariate X is deterministic and the error term is an i.i.d. Gaussian disturbance, the following *OLS* estimates are obtained:

1. *The unbiased estimator of the vector parameter* β:

$$y = Xb \rightarrow X'y = X'Xb \rightarrow (X'X)^{-1}X'y = (X'X)^{-1}(X'X)b \tag{C.4}$$

If the matrix $X'X$ is nonsingular then the estimator is

$$\underset{(kx1)}{\hat{\beta}} = b = (X'X)^{-1}X'y \tag{C.5}$$

or

$$b = (X'X)^{-1}X'(X\beta + \mu) = \beta + (X'X)^{-1}X'\mu \tag{C.6}$$

with its expected value

$$E(b) = b \tag{C.7}$$

which indicates that b is an unbiased estimator of β.

2. *The unbiased estimator of the population variance* σ^2:
The estimate of the error term vector can be written as

$$\underset{(Tx1)}{\hat{\mu}} = u = y - X(X'X)^{-1}X'y = [I_T - X(X'X)^{-1}X']y = M_x y \tag{C.8}$$

Therefore, the sum of squared errors (SSE) and the mean of squared errors (MSE) can be written as:

$$SSE = u'u = \sum (y_t - X_t'b)^2 \tag{C.9}$$

$$MSE = s^2 = \frac{SSE}{T-k} \tag{C.10}$$

where $X_t'b$ indicates $\sum_{i=0}^{k-1} x_{it}b_i$. Furthermore,

$$E(MSE) = E(s2) = \sigma^2 \tag{C.11}$$

which indicates that the MSE is an unbiased estimator for the population variance.

3. *The uncentered and centered R-squared, namely R_u^2 and R_c^2 respectively*:

$$R_u^2 = \frac{SSE}{\sum_{t=1}^{T} y_t^2} \tag{C.12}$$

$$R_c^2 = \frac{SSE - T\bar{y}^2}{\sum_{t=1}^{T} y_t^2 - T\bar{y}^2} \tag{C.13}$$

4. *The variance–covariance matrix of b*:

$$E[(b-b)(b-\beta)'] = \sigma^2 (X'X)^{-1} \tag{C.14}$$

5. *The normal distribution of b*:

$$b \sim N(\beta, \sigma^2 (X'X)^{-1}) \tag{C.15}$$

C.1.2 Maximum likelihood estimates

Under the assumption that the error term $\mu_t = Y_t - X_t'\beta$ is i.i.d. Gaussian, the following density function is obtained (compare with the density function in (B.3)):

$$f(\mu_t) = (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{\mu_t^2}{2\sigma^2}\right) = (2\pi\sigma^2)^{-1/2} \exp\left[-\frac{(y_t - X_t'\beta)^2}{2\sigma^2}\right] \tag{C.16}$$

where $X_t'\beta = \sum_{i=0}^{k-1} \beta_i x_{it}$. Therefore, the log likelihood function considered for estimation purposes is

$$LL = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - X_t'\beta)^2 \tag{C.17}$$

The necessary conditions to obtain the maximum value of LL are as follows:

$$\begin{aligned}
\frac{\partial(LL)}{\partial\beta_0} &= \frac{1}{\sigma^2}\sum_{t=1}^{T}(Y_t - X_t'\beta) = 0 \\
\frac{\partial(LL)}{\partial\beta_i} &= \frac{1}{\sigma^2}\sum_{t=1}^{T}(Y_t - X_t'\beta)x_{it} = 0, \quad i = 1, 2, \ldots, (k-1) \\
\frac{\partial(LL)}{\partial\sigma^2} &= -\frac{T}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}(Y_t - X_t'\beta)^2 = 0
\end{aligned} \tag{C.18}$$

As a result, the following normal equations are obtained:

$$\sum_{t=1}^{T}(y_t - X_t'\beta) = 0$$
$$\sum_{t=1}^{T}x_{it}(y_t - X_t'\beta) = 0, \quad \text{for } i = 0, 1, \ldots, (k-1) \qquad \text{(C.19)}$$
$$\sigma^2 = \frac{1}{T}\sum_{t=1}^{T}(y_t - X_t'\beta)^2$$

It is well known that the first two sets of equations can also be obtained by using the OLS estimation method. Therefore, in a mathematical sense, the same estimates of the vector parameter β can be obtained by using either one of the estimation methods. As a result, based on the last equation,

$$\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\left(y_t - \sum_{i=0}^{k-1} b_i x_{it}\right)^2 \qquad \text{(C.20)}$$

C.1.3 Student's t-statistic

Corresponding to the multivariate distribution of the vector $b = [b_0, b_1, \ldots, b_{k-1}]$ as $N(\beta,\ \sigma^2(X'X)^{-1})$ in (C.15), each of its components $b_i = \hat{\beta}_i$ is normally distributed as $N(\beta_i, \sigma_{ii}^2)$, where σ_{ii}^2 is the element in row i and column i of $[\sigma^2(X'X)^{-1}]$. By using $s^2(b_i) = \hat{\sigma}_{ii}^2$, Student's t-statistic can be presented as

$$\frac{b_i - \beta_i}{s(b_i)} \text{ is distributed as } t(T-k) \qquad \text{(C.21)}$$

C.1.4 The Wald form of the OLS F-test

C.1.4.1 Testing the Hypothesis

$$H_0 : C\beta = c \text{ and } H_1 : \text{Otherwise} \qquad \text{(C.22)}$$

where C is a constant $(m \times k)$ matrix representing the particular linear combinations of the model parameter β and c is an $(m \times 1)$ vector of defined values that are believed or judged to be the true values of the corresponding linear combinations.

From (C.15) it is found that, under H_0,

$$Cb \sim N(c, \sigma^2(X'X)^{-1}C') \qquad \text{(C.23)}$$

Furthermore, under H_0, the chi-squared test is found to be

$$(Cb-c)'[\sigma^2(X'X)^{-1}C']^{-1}(Cb-c) \sim \chi^2(m) \qquad \text{(C.24)}$$

By replacing σ^2 with its estimate $s^2 = SSE/(T-k)$, the Wald form of the OLS F-test is obtained:

$$\frac{(Cb-c)'[\sigma^2(X'X)^{-1}C']^{-1}(Cb-c)}{m} \sim F(m, T-k) \tag{C.25}$$

or

$$\frac{(Cb-c)'[(X'X)^{-1}C']^{-1}(Cb-c)}{ms^2} \sim F(m, T-k) \tag{C.26}$$

The hypothesis (C.22) can be represented as

$$\begin{aligned} H_0 &: \text{ Restricted model} \\ H_1 &: \text{ Unrestricted model} \end{aligned} \tag{C.27}$$

Then the Wald form of the OLS F-test can be written as

$$F = \frac{(SSE_\mathrm{R}-SSE_\mathrm{U})/m}{SSE_U/(T-k)} \sim F(m, T-k) \tag{C.28}$$

where SSE_R indicates the sum of squared errors of the restricted model (i.e. if the null hypothesis $C\beta = c$ is true) and SSE_U indicates the sum of squared errors of the unrestricted or full model. Furthermore, it is well known that the numerator and denominator of the F-test are the chi-squared tests as follows:

$$\chi_1^2 = (SSE_\mathrm{R}-SSE_\mathrm{U})/m \sim \sigma^2\chi^2(m) \tag{C.29}$$

$$\chi_2^2 = \frac{SSE_\mathrm{U}}{(T-k)} \sim \sigma^2\chi^2(T-k) \tag{C.30}$$

C.2 AR(1) general linear model

Corresponding to the basic model in (C.1), the AR(1) model, without lag of the endogenous variable, should be considered as follows:

$$\begin{aligned} y_t &= X\beta + \mu_t \\ \mu_t &= \rho\mu_{t-1} + \varepsilon_t \end{aligned} \tag{C.31}$$

with the assumptions that $\mu = [\mu_1, \mu_2, \ldots, \mu_\mathrm{T}] \sim N(0, \sigma^2 V)$, where V is a known $(T \times T)$ positive definite matrix and $|\rho| < 1$.

Compared to the AR(1) model in (A.18) in Appendix A, this model is in fact a linear model with first-order autoregressive errors. However, the same terminology is used here, namely the AR(1) model. Note that the AR(1) model in (B.19) and (C.31) have different characteristics, and similarly for the AR(p) models presented in Appendix A and the AR(p) model presented in the following section.

C.2.1 Properties of μ_t

Under the assumption that ε_t is i.i.d. $N(0, \sigma^2)$, the residual μ_t has exactly the same properties as presented in Section B.4.3.

C.2.2 Estimation method

By presenting the model in (C.31) as

$$y_t = X_t\beta + \rho\mu_{t-1} + \varepsilon_t \tag{C.32}$$

then under the assumption that the error term of this model, namely ε_t, is i.i.d. Gaussian, $\varepsilon = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T] \sim N(0, \sigma^2 I)$.

Furthermore, based on the model in (C.32), the error term is as follows:

$$\varepsilon_t = (y_t - X_t\beta) - \rho(y_{t-1} - X_{t-1}\beta) \tag{C.33}$$

By using the same process as in Appendix B, the following log-likelihood function is obtained:

$$\begin{aligned} LL &= -\frac{T-p}{2}\ln(2\pi) - \frac{T-p}{2}\ln(\sigma^2) \\ &= -\frac{1}{2\sigma^2}\sum_{t=2}^{T}[(y_t - X'_t\beta) - \rho(y_{t-1} - X'_{t-1}\beta)]^2 \end{aligned} \tag{C.34}$$

where $X'_t\beta = \sum_{i=0}^{k-1}\beta_i x_{it}$. Then the approach is to maximize this function numerically with respect to β_0, β_1, σ^2 and ρ.

In fact, corresponding to the model in (C.31), the following normal equation is considered for the estimation process, for $|\rho| < 1$:

$$\sum_{t=2}^{T}\left[\left(y_t - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{it}\right) - \rho\left(y_{t-1} - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{i(t-1)}\right)\right] = 0$$

$$\sum_{t=2}^{T}\left[\left(y_t - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{it}\right) - \rho\left(y_{t-1} - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{i(t-1)}\right)\right](x_{it} - \rho x_{i(t-1)}) = 0$$

for $i = 1, 2, \ldots, k-1$.

$$\sum_{t=2}^{T}\left[\left(y_t - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{it}\right) - \rho\left(y_{t-1} - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{i(t-1)}\right)\right]\left(y_{t-1} - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{i(t-1)}\right) = 0$$

$$\sigma^2 = \frac{1}{T-1}\sum_{t=2}^{T}\left[\left(y_t - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{it}\right) - \rho\left(y_{t-1} - \beta_0 - \sum_{i=1}^{k-1}\beta_i x_{i(t-1)}\right)\right]^2 \tag{C.35}$$

Alternatively, instead of using the numerical iteration method, the following regression may be used:

$$(y_t - \rho y_{t-1}) = \beta_0 + \sum_{i=1}^{k-1} \beta_i (x_{it} - \rho x_{i(t-1)}) + \varepsilon_t \tag{C.36}$$

for various values of ρ, such as 0.05, 0.10, ..., 0.95. Then a model could be chosen having the smallest sum of squared errors or other measures of fit, as presented in Section 11.3.

C.3 AR(p) general linear model

As an extension of the AR(1) model in (C.31) or the model in (2.8), this is an AR (p) GLM, without lag of the endogenous variable, as follows:

$$\begin{aligned} y_t &= X\beta + \mu_t \\ \mu_t &= \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t \end{aligned} \tag{C.37}$$

where ρ_i are the ith autocorrelation or serial correlation parameter such that $|\rho_i| < 1$ and ε_t, $t = 1, 2, \ldots, T$, are i.i.d. Gaussian or $N(0, \sigma^2)$.

In order to estimate the parameters, the following LL function should be considered:

$$\begin{aligned} LL = & -\frac{T-p}{2}\ln(2\pi) - \frac{T-p}{2}\ln(\sigma^2) \\ & -\frac{1}{2\sigma^2}\sum_{t=p+1}^{T}\left[(y_t - X'_t\beta) - \sum_{i=1}^{p}\rho_i(y_{t-i} - X'_{t-i}\beta)\right]^2 \end{aligned} \tag{C.38}$$

where $X'_t\beta = \sum_{i=0}^{k-1} \beta_i x_{it}$(compare this to the LL function in (C.17).

C.4 General lagged-variable autoregressive model

As an extension of the LVAR(p, q) model in (C.31) with an exogenous variable, a general *lagged-variable autoregressive model*, namely the LVAR(p, q) model with multivariate exogenous variables, is defined as

$$\begin{aligned} Y_t &= \beta_0 + \sum_{i=1}^{q} \beta_i Y_{t-i} + \sum_{i=1}^{k} \delta_i X_{it} + \mu_t \\ \mu_t &= \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t \end{aligned} \tag{C.39}$$

Note that the AR(p) model in (C.39) is in fact a model with autoregressive errors, which is indicated by the error terms $\mu_t = \sum_{i=1}^{p} \rho_i \mu_{t-i} + \varepsilon_t$, compared to the AR($p$) model in Appendix A, $y_t = \beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i} + \varepsilon_t$, with respect to the endogenous variable y_t.

In order to estimate the parameters, the following LL function should be considered:

$$LL = -\frac{T-p-q}{2}\ln(2\pi) - \frac{T-p-q}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=p+q+1}^{T}\left[\begin{array}{l}\left(y_t - \beta_0 - \sum_{i=1}^{q}\beta_i y_{t-i} - \sum_{i=1}^{k}\delta_i x_{it}\right) \\ -\sum_{i=1}^{p}\rho_i\left(y_{t-i} - \beta_0 - \sum_{j=1}^{q}\beta_j y_{t-i-j} - \sum_{i=1}^{k}\delta_i x_{i(t-1)}\right)\end{array}\right]^2 \tag{C.40}$$

C.5 General models with Gaussian errors

C.5.1 Gaussian errors with a known variance covariance matrix

Corresponding to the general linear model in (C.1), namely

$$y = X\beta + \mu \tag{C.41}$$

the following assumptions are made:

A1. X is stochastic.
A2. Conditional on the full matrix X, the error vector μ is $N(0, \sigma^2 V)$.
A3. V is a known positive definite matrix.

Recall from (C.6) that

$$(b - \beta) = (X'X)^{-1}X'\mu \tag{C.42}$$

Under the assumption A2, the conditional expectation is

$$E[(b-\beta)|X] = (X'X)^{-1}X'E(\mu) = 0 \tag{C.43}$$

and by the law of iterated expectation (Hamilton, 1994, p. 217),

$$E(b-\beta) = E_x\{E[(b-\beta)|X] = 0 \tag{C.44}$$

The variance of the vector b conditional on X is given by

$$\begin{aligned} E[(b-\beta)(b-\beta)'|X] &= E[\{(X'X)^{-1}X'\mu\mu'X(X'X)^{-1}\}|X] \\ &= \sigma^2(X'X)^{-1}X'VX(X'X)^{-1} \end{aligned} \tag{C.45}$$

As a result, the vector b conditional on X is multivariate normally distributed with $E(b)=\beta$ and $\mathrm{Var}(b)=\sigma^2(X'X)^{-1}X'VX(X'X)^{-1}$, which can be presented as

$$b|X \sim N(\beta, \sigma^2(X'X)^{-1}X'VX(X'X)^{-1}) \tag{C.46}$$

C.5.2 Generalized least squares with a known covariance matrix

Under the assumptions A1 to A3 above, namely $\mu|X \sim N(0, \sigma^2 V)$, where V is a known symmetric and positive $(T \times T)$ matrix, there exists a nonsingular $(T \times T)$ matrix G such that

$$V^{-1} = G'G \tag{C.47}$$

Then the model (C.39) should be transformed to

$$Gy = (GX)\beta + G\mu \tag{C.48}$$

with $G\mu|X \sim N(0, \sigma^2 I_T)$. Under this condition, the estimator of β is as follows:

$$\begin{aligned} (GX)'Gy &= (GX)'(GX)b_G = (GX)'(GX)\underline{b} \\ X'(G'G)y &= X'(G'G)X)\underline{b} \\ X'V^{-1}y &= X'V^{-1}X\underline{b} \\ \underline{b} &= (X'V^{-1}X)^{-1}X'V^{-1}y \end{aligned} \tag{C.49}$$

which is known as the *generalized least squares* (*GLS*) estimator, with

$$\mathrm{Cov}(\underline{b}) = \sigma^2(X'V^{-1}X)^{-1} \tag{C.50}$$

Furthermore, similar to the estimator of the vector b in (C.46), the conditional distribution of the vector estimator in (C.49) is

$$\underline{b}|X \sim N(\beta, \sigma^2(X'V^{-1}X)^{-1}) \tag{C.51}$$

Similarly, the sum of squared errors has a conditional chi-squared distribution,

$$\underline{s}^2 = y'[V^{-1} - V^{-1}X(X'V^{-1}X)^{-1}X'V^{-1}]y \sim \sigma^2.\chi^2(T-k) \tag{C.52}$$

Then under the null hypothesis $C\beta = c$ in (C.19), the Wald form of the F-test is given by

$$\frac{[C\underline{b}-c]'[\underline{s}^2 C(X'V^{-1}X)^{-1}C']^{-1}[C\underline{b}-c]}{m} \sim F(m, T-k) \qquad \text{(C.53)}$$

C.5.3 GLS and ML estimations

Under the assumption that $\mu|X \sim N(0, \sigma^2 V)$, then, based on the model in (C.41),

$$y|X \sim N(X\beta, \sigma^2 V) \qquad \text{(C.54)}$$

The log-likelihood function of y conditioned on X is given by

$$LL = \left(\frac{-T}{2}\right)\log(2\pi) - \left(\frac{1}{2}\right)\log|\sigma^2 V| - \left(\frac{1}{2}\right)(y - X\beta)'(\sigma^2 V)^{-1}(y - X\beta)$$

$$LL = \left(\frac{-T}{2}\right)\log(2\pi) - \left(\frac{1}{2}\right)\log|\sigma^2 V| - \frac{1}{2\sigma^2}(y - X\beta)'V^{-1}(y - X\beta) \qquad \text{(C.55)}$$

Since $V^{-1} = G'G$, then

$$LL = \left(\frac{-T}{2}\right)\log(2\pi) - \left(\frac{1}{2}\right)\log|\sigma^2 V| - \frac{1}{2\sigma^2}(y - X\beta)'G'G(y - X\beta)$$

$$LL = \left(\frac{-T}{2}\right)\log(2\pi) - \left(\frac{1}{2}\right)\log|\sigma^2 V| - \frac{1}{2\sigma^2}(Gy - GX\beta)'(Gy - GX\beta) \qquad \text{(C.56)}$$

This equation shows that the log likelihood function is maximized with respect to β by an OLS regression of Gy on $GX\beta$ (refer to the model in (C.48). Hence the GLS estimate is also the maximum likelihood estimate.

C.5.4 The variance of the error is proportional to the square of one of the explanatory variables

Under the assumption that $\text{Var}(\mu_t) = x_{1t}^2\sigma^2$, then

$$\sigma^2 V = \sigma^2 \text{Diag}[x_{11}^2, x_{12}^2, \ldots, x_{1T}^2] \qquad \text{(C.57)}$$

where $\mathrm{Diag}[a_{ij}]$ is a square matrix whose off-diagonal elements are zeros. It is then easy to verify that

$$G = \mathrm{Diag}\left[\frac{1}{|x_{11}|}, \frac{1}{|x_{12}|}, \ldots, \frac{1}{|x_{1T}|}\right] \tag{C.58}$$

Furthermore, to estimate the model parameters, the regression

$$\frac{y_t}{|x_{1t}|} = \sum_{k=0}^{p} \beta_k \frac{x_{kt}}{|x_{1t}|} + \varepsilon_t \tag{C.59}$$

can be used, where $\varepsilon_t = \mu_t/|x_{it}|$, and $\mathrm{Var}(\varepsilon_t) = \sigma^2$.

C.5.5 Generalized least squares with an unknown covariance matrix

In this case the model in (C.41) will be presented as

$$y_t = \sum_{k=0}^{p} \beta_k x_{kt} + \mu_t \tag{C.60}$$

for $t = 1, \ldots, T$, where $x_{0t} = 1$.

Under the assumption that $\mu|X \sim N(0, \sigma^2 V)$ and $\mu_t = y_t - \sum_{k=0}^{p} \beta_k x_{kt}$, the PDF of a T-variate normal distribution is observed (Wilks, 1962, p. 164), as follows:

$$\begin{aligned} f(\mu_1, \ldots, \mu_T) &= (2\pi)^{-T/2}\sqrt{|\sigma^{ij}|}\exp\left[-\frac{1}{2}Q(\mu_i, \ldots, \mu_T)\right] \\ Q(\mu_1, \ldots, \mu_T) &= \sum_{i,j=1}^{T} \sigma^{ij}\left(y_i - \sum_{k=0}^{p}\beta_k x_{ki}\right)\left(y_j - \sum_{k=0}^{p}\beta_k x_j\right) \end{aligned} \tag{C.61}$$

where $||\sigma^{ij}||$ is the *inverse of the covariance matrix* $\sigma^2 V = ||\sigma_{ij}||$ and it is assumed that it is positive definite, which implies that the determinants $|\sigma^{ij}| \neq 0$ and $|\sigma_{ij}| \neq 0$.

To estimate the model parameters, namely β and σ_{ij}, the following log likelihood function should be considered:

$$LL = -\frac{T}{2}\log(2\pi) + \frac{1}{2}\log(|\sigma^{ij}|) - \sum_{i,j=1}^{T} \sigma^{ij}\left(y_i - \sum_{k=0}^{p} \beta_k x_{ki}\right)\left(y_j - \sum_{k=0}^{p} \beta_k x_j\right) \tag{C.62}$$

Therefore, in general, $(p + 1)$ of the β parameters and $T(T - 1)$ of the σ_{ij} parameters are obtained. However, under some restrictions on the series or in special cases, the model would have less parameters, such that $V = V(\theta)$, where θ is a small dimensional vector parameter that could be estimated by using the OLS or GLS regressions.

Appendix D: Multivariate general linear models

Definition D.1: The multivariate or N-dimensional process $\{Yt = (Y_{1t}, Y_{2t}, \ldots, Y_{Nt}\}t = 1T$ is second-order stationary if and only if

(i) *the mean* $E(\boldsymbol{Y}_t) = \mu$ *is independent of* t,
(ii) *the autocovariance* $(N \times N)$ *matrix* $\text{Cov}(\boldsymbol{Y}_t, \boldsymbol{Y}t_{-h}) = E(\boldsymbol{Y}_t - \mu)(\boldsymbol{Y}_{t-h} - \mu) = \boldsymbol{\Gamma}(t,\ t-h) = \boldsymbol{\Gamma}(h)$ *is independent of* t *for any* h; $\boldsymbol{\Gamma}(h)$ *is the autcovariance function* (*ACF*) *of the process, with* $\boldsymbol{\Gamma}(-h) = \boldsymbol{\Gamma}(h)'$. □

Definition D.2: The N-dimensional second-order stationary process $\{\varepsilon_t\}_{t=1}^{T}$ is a weak white noise process if and only if

(i) *the mean* $E(\varepsilon_t) = 0,\ \forall t$,
(ii) *the autocovariance* $\text{Cov}(\varepsilon_t,\ \varepsilon_{t-h}) = E(\varepsilon_t \varepsilon_{t-h}) = \boldsymbol{\Gamma}(t,\ t-h) = 0,\ \forall h \neq 0$. □

D.1 Multivariate general linear models

A *multivariate general linear model* (*MGLM*), in EViews, is presented or considered as a *system equations* or system of equations (i.e. a set of univariate linear models). Furthermore, EViews provides an option called '*System*,' which can be used to estimate any type of MGLM, such as the following special types of MGLM:

(1) The first special type of MGLM is the VAR (*vector autoregressive*) model, where all regressions of the model have the same set of lagged endogenous variables and the same set of exogenous variables. Refer to Chapter 6. On the other hand, since the term '*VAR*', in EViews, is used as an option, function or estimation method for this special type of MGLM, then the term 'VAR' is not appropriate to represent a general multivariate time series model. For this

Time Series Data Analysis Using EViews IGN Agung

reason, the term MAR (*multivariate autoregressive*) model is proposed to represent the general multivariate autoregressive model.

(2) The second types are the VMA (*vector moving average*) model and VARMA (*vector autoregressive moving average*) model.
(3) The third type of MGLM is the VEC (*vector error correction*) model, where all regressions in a VEC model have the same sets of exogenous variables (refer to Chapter 6, where it is shown that the VEC model can also be estimated by using the VAR function or estimation method).
(4) The fourth type is the *simultaneous causal models*, where at least two of the endogenous variables are defined to have simultaneous causality.
(5) Finally, the fifth type is the *structural equation model* (*SEM*), where all regressions in an MGLM can have unequal sets of exogenous variables, either additive or interaction models, including the multivariate models with trend and time-related effects, and multivariate seemingly causal models, as well as the multivariate models with dummy variables.

D.2 Moments of an endogenous multivariate

Let $\boldsymbol{Y_t} = (Y_{1t}, Y_{2t}, \ldots, Y_{Nt})$ be an N-dimensional multivariate time series. However, note that in some equations the symbol $\boldsymbol{Y_t}$ should be presented or written as Y_t (i.e. not a bold letter), for simplicity.

The mean or the first moment of the multivariate process $Y_{\boldsymbol{t}} = (Y_{1t}, Y_{2t}, \ldots, Y_{Nt})$ is defined as

$$\mu_t = \boldsymbol{E}(Y_{\mathbf{t}}) = [E(Y_{1t}), \ldots, E(Y_{Nt})]' \tag{D.1}$$

which is a N-dimensional column vector.

The variance–covariance of $Y_{\boldsymbol{t}}$ is an $(N \times N)$ symmetric positive matrix, namely $\boldsymbol{V}(Y_{\boldsymbol{t}}) = \boldsymbol{\Gamma}(t,t)$, as follows:

$$\boldsymbol{V}(Y_t) = \Gamma(t,t) = ||\sigma_{ij}(t)|| \tag{D.2}$$

where

$$\sigma_{ij}(t) = \begin{cases} \mathrm{Var}(Y_{it}) \text{ for } i = j \\ \mathrm{Cov}(Y_{it}, Y_{jt}) \text{ for } i \neq j \end{cases} \tag{D.3}$$

Then the correlation between Y_{it} and Y_{jt} is given by

$$\mathrm{Corr}(Y_{it}, Y_{jt}) = \frac{\mathrm{Cov}(Y_{it}, Y_{jt})}{\sqrt{\mathrm{Var}(Y_{it})\mathrm{Var}(Y_{jt})}} \tag{D.4}$$

For any N-dimensional vector $\boldsymbol{C}$, the following identity is obtained:

$$\boldsymbol{C}'\boldsymbol{V}(\boldsymbol{Y_t})\boldsymbol{C} = \boldsymbol{V}(\boldsymbol{C}'\boldsymbol{Y_t}) \geq 0 \tag{D.5}$$

The *multivariate autocovariance function* in matrix form is defined as

$$\mathbf{Cov}(\boldsymbol{Y_t}, \boldsymbol{Y}_{\mathrm{t-h}}) = \Gamma(\mathrm{t}, \mathrm{t-h}) = \boldsymbol{E}(\boldsymbol{Y}_{\mathrm{t}}\boldsymbol{Y}'_{\mathrm{t-h}}) - \boldsymbol{E}(\boldsymbol{Y}_{\mathrm{t}})\boldsymbol{E}(\boldsymbol{Y}'_{\mathrm{t-h}}) = ||\sigma_{\mathrm{ijt,t-h}}||, \forall h \tag{D.6}$$

where $\sigma_{ijt,t-h} = \mathrm{Cov}(Y_{it},\ Y_{j,t-h})$ for $i,\ j = 1,\ \ldots,\ N$. Since the covariance is a symmetric matrix, then

$$\mathbf{Cov}(\boldsymbol{Y}_{\mathrm{t}}, \boldsymbol{Y}_{\mathrm{t+h}})' = \mathbf{Cov}(\boldsymbol{Y}_{\mathrm{t}}, \boldsymbol{Y}_{\mathrm{t-h}}) \quad \text{or} \quad \Gamma(\mathrm{h})' = \Gamma(-\mathrm{h}) \tag{D.7}$$

Furthermore, it is also possible to define a *lagged effect of* Y_i *on* Y_j, namely $Y_{i,t-h}$ on Y_{jt}, for $i \neq j$, which is measured as

$$\mathrm{Corr}(Y_{i,t-h}, Y_{jt}) = \frac{\mathrm{Cov}(Y_{i,t-h}, Y_{jt})}{\sqrt{\mathrm{Var}(Y_{i,t-h})\mathrm{Var}(Y_{jt})}} \tag{D.8}$$

D.3 Vector autoregressive model

Based on the N-dimensional multivariate time series, $Y_t = (Y_{1t},\ Y_{2t},\ \ldots,\ Y_{Nt})$, a *vector autoregressive* model of order p, namely the VAR(p) model, is defined as

$$Y_t = \Theta_0 + \sum_{i=1}^{p} \Theta_i Y_{t-i} + U_t \tag{D.9}$$

Note that this model represents the following N regressions, which is an extension of the AR(p) model in Appendix A:

$$\begin{cases} Y_{1t} = \beta_{10} + \beta_{11}Y_{t-1} + \cdots + \beta_{1p}Y_{t-p} + \mu_{1t} \\ \vdots \\ Y_{Nt} = \beta_{N0} + \beta_{N1}Y_{t-1} + \cdots + \beta_{Np}Y_{t-p} + \mu_{Nt} \end{cases} \tag{D.10}$$

for each time point t. This model also can be considered as a member of the MGLMs, which has the following general equation:

$$\boldsymbol{Y_t} = \boldsymbol{X_t}\beta + \boldsymbol{\mu_t} \tag{D.11}$$

In this case, $\boldsymbol{Y_t}$ is an $(N \times 1)$ vector of the observed endogenous variable, $\boldsymbol{X_t} = [1,\ \boldsymbol{Y}_{t-1},\ \ldots,\ \boldsymbol{Y}_{t-p}]$ is an $[N \times (p+1)]$ matrix of the lagged endogenous variables, β is a $[(p+1) \times 1]$ vector of model parameters and $\boldsymbol{\mu_t}$ is an $(N \times 1)$ vector of random errors.

Finally, if there are T-values of observations at time points $t = 1, 2, \ldots, T$, then there will be a system of $(N \times T)$ equations to estimate the model parameters, which can be presented as the following matrix equation:

$$\underset{NTx1}{\boldsymbol{Y}} = \underset{NTx(p+1)}{\boldsymbol{X}} * \underset{(p+1)x1}{\boldsymbol{B}} + \underset{NTx1}{\boldsymbol{E}} \tag{D.12}$$

Note that this system has NT equations with $[N \times (p + 1)]$ *model parameters*, β_{nk} for $n = 1, \ldots, N; k = 0, 1, \ldots, p$, and N *error terms*, μ_{nt}, for $n = 1, 2, \ldots, N$. The general estimation process will be presented later.

D.4 Vector moving average model

Based on the N-dimensional multivariate time series above, a *vector moving average* model of order q, namely the MA(q) model, is defined as

$$Y_t = \varepsilon_t + \sum_{l=1}^{q} \Psi_i \varepsilon_{t-l} \tag{D.13}$$

where Y_t is an $(N \times 1)$ vector of the observed endogenous variables and Ψ is an $(N \times N)$ matrix of the model parameters.

A moving average model can be derived from the first-order autoregressive model in (D.9) for $p = 1$. This gives the following derivation:

$$\begin{aligned} Y_t &= \Theta_0 + \Theta_1 Y_{t-1} + U_t \\ &= \Theta_0 + \Theta_1^2 Y_{t-2} + U_t + \Theta_1 U_{t-1} \\ &\vdots \\ &= \Theta_0 + \Theta_1^k Y_{t-k} + U_t + \Theta_1 U_{t-1} + \cdots + \Theta_1^{k-1} U_{t-k+1} \end{aligned} \tag{D.14}$$

Under the condition that $\text{Lim}_{k \to \infty} \Theta_1^k = 0$, i.e. all eigenvalues of $\Theta_1 < 1$, then from (D.14), the following *multivariate infinite moving average* series can be obtained:

$$Y_t = \Theta_0 + \sum_{k=0}^{\infty} \Theta_i^k U_{t-k} \tag{D.15}$$

D.5 Vector autoregressive moving average model

Finally, based on the multivariate endogenous variables, $Y_t = (Y_{1t}, Y_{2t}, \ldots, Y_{Nt})$, a *vector autoregressive moving average* model of order (p, q), namely VARMA (p, q), is defined as

$$Y_t = \Theta_0 + \sum_{i=1}^{p} \Theta_i Y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \Psi_j \varepsilon_{t-j} \tag{D.16}$$

For each component of Y_t, namely $Y_{nt}, n = 1, 2, \ldots, N$, there will be an ARMA (p, q) model, as presented in (A.68), Appendix A, as follows:

$$Y_{nt} = \theta_{n0} + \sum_{i=1}^{p} \theta_{ni} Y_{n(t-i)} + \varepsilon_{nt} + \sum_{j=1}^{q} \psi_{nj} \varepsilon_{n(t-j)} \tag{D.17}$$

The derivation of the corresponding statistics, such the Var(Y_{nt}) and the *autocovariance* function Cov($Y_{nt} Y_{n(t-h)}$) are exactly the same as those presented in (A.72) and (A.74), Appendix A, as well as for each regression in the VAR(p) model, i.e. for $q = 0$, and each regression in the VMA(q) model, i.e. for $p = 0$. Therefore, they will not be presented again in this section.

D.6 Simple multivariate models with exogenous variables

In the following subsections, two simple models based on two endogenous variables will be presented, namely $\{Y_{1t}, Y_{2t}\}_{t=1}^{T}$, with a single exogenous (independent or source) variable and a multidimensional exogenous variable.

D.6.1 The simplest multivariate model

The simplest multivariate model is a bivariate linear model having a single exogenous variable, which is an extension of the model in (B.1), Appendix B, as follows:

$$\begin{aligned} Y_{1t} &= \beta_{10} + \beta_{11} X_t + \mu_{1t} \\ Y_{2t} &= \beta_{20} + \beta_{21} X_t + \mu_{2t} \end{aligned} \tag{D.18}$$

for $t = 1, 2, \ldots, T$. Therefore, in the estimation process, this bivariate model represents a system of $2T$ equations based on a time series data set, namely $\{x_t, y_{1t}, y_{2t}\}_{t=1}^{T}$. Similarly, there are two sets of the error terms as follows:

$$\begin{aligned} \mu_{1t} &= y_{1t} - \beta_{10} - \beta_{11} x_t \\ \mu_{2t} &= y_{2t} - \beta_{20} - \beta_{21} x_t \end{aligned} \tag{D.19}$$

The parameters β_{10} and β_{11} as well as β_{20} and β_{21} can be estimated by using the OLS estimation method. However, in general, the residuals μ_{1t} and μ_{2t} are correlated, so their (2×2) covariance matrix should be considered. An estimate of the covariance matrix can easily be computed based on the OLS estimators $\{\hat{\mu}_{1t}, \hat{\mu}_{2t}\}_{t=1}^{T}$. Refer to the examples previously presented, specifically the covariance analysis.

On the other hand, for the normal linear regression, the bivariate (μ_{1t}, μ_{2t}) is assumed to have a bivariate normal density function, as follows:

$$f(\mu_{1t}, \mu_{2t}) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}} \exp\left[-\frac{1}{2} Q(\mu_{1t}, \mu_{2t})\right]$$

where

$$Q(\mu_{1t},\mu_{2t}) = \frac{1}{1-\rho^2}\left[\begin{array}{c}\dfrac{(y_{1t}-\beta_{10}-\beta_{11}x_t)^2}{\sigma_1^2} + \dfrac{(y_{2t}-\beta_{20}-\beta_{21}x_t)^2}{\sigma_2^2} \\ -\rho\dfrac{(y_{1t}-\beta_{10}-\beta_{11}x_t)(y_{2t}-\beta_{20}-\beta_{21}x_t)}{\sigma_1\sigma_2}\end{array}\right] \tag{D.20}$$

Therefore, the following log likelihood function is obtained:

$$LL = -T\log\left[2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}\right] - \frac{1}{2}\sum_{t=1}^{T} Q(\mu_{1t},\mu_{2t}) \tag{D.21}$$

D.6.2 Simple model with a multidimensional exogenous variable

A simple model with a multidimensional exogenous variable can be presented as

$$\begin{aligned} Y_{1t} &= \sum_{i=0}^{k}\beta_{1i}X_{it} + \mu_{1t} \\ Y_{2t} &= \sum_{i=0}^{k}\beta_{2i}X_{it} + \mu_{2t} \end{aligned} \tag{D.22}$$

where $X_{0t} = 1, \forall t$.

Similar to the simplest model in (D.18) the parameters β_{1i}s and β_{2i}s could be estimated by using the OLS estimation method. Then the covariance matrix of μ_{1t} and μ_{2t} should be estimated using the OLS estimators $\{\hat{\mu}_{1t},\hat{\mu}_{2t}\}_{t=1}^{T}$.

Corresponding to the *LL* function in (D.21), the *LL* function for the model in (D.22) is

$$LL = -T\log(2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}) - \frac{1}{2}\sum_{t=1}^{T} Q(\mu_{1t},\mu_{2t})$$

where

$$Q(\mu_{1t},\mu_{2t}) = \frac{1}{1-\rho^2}\left[\begin{array}{c}\dfrac{\left(y_{1t}-\sum_{i=0}^{k}\beta_{1i}x_{it}\right)^2}{\sigma_1^2} + \dfrac{\left(y_{2t}-\sum_{i=0}^{k}\beta_{2i}x_{it}\right)^2}{\sigma_2^2} \\ -\rho\dfrac{\left(y_{1t}-\sum_{i=0}^{k}\beta_{1i}x_{it}\right)\left(y_{2t}-\sum_{i=0}^{k}\beta_{2i}x_{it}\right)}{\sigma_1\sigma_2}\end{array}\right] \tag{D.23}$$

In a mathematical sense, the parameters β, σ_1^2, σ_2^2 and ρ can be estimated. However, here an explicit estimator will not be presented. Refer to the general estimation method presented in Section D.7 below.

D.6.3 A more general model

Note that the two regressions in the model in (D.22) have the same set of exogenous variables. As an extension of this model is a model where each of the two regressions have unequal sets of exogenous variables, which can be presented as

$$
\begin{aligned}
Y_{1t} &= \sum_{i=0}^{k} \beta_{1i} X_{1it} + \mu_{1t} \\
Y_{2t} &= \sum_{j=0}^{m} \beta_{2i} X_{2jt} + \mu_{2t}
\end{aligned}
\qquad \text{(D.24)}
$$

where $X_{10t} = X_{2ot} = 1, \forall t$.

The estimation method can easily be done by using the OLS or ML estimation methods, as mentioned above. Furthermore, note that the independent variables $X_{1i}s$ and $X_{2j}s$ can be any types of variables, such as the lags of endogenous variables, other endogenous variables, pure exogenous variables, as well as their lags, the time t-variable and selected two-way or three-way interactions of the main independent variables, as well as the power of selected exogenous variables, and dummy variables. Therefore, the model in (D.24) could represent all types of time series models. Some selected models are presented in the following subsection.

D.6.4 Selected bivariate time series models

D.6.4.1 A VAR model with a multivariate endogenous variable

Note that, in EViews, the term VAR indicates a special case of the multivariate time series models, where all regressions have the same set of independent variables, including the general model as follows:

$$
\begin{aligned}
Y_{1t} &= \sum_{i=1}^{p} \beta_{1i} Y_{1(t-i)} + \sum_{i=1}^{p} \delta_{1i} Y_{2(t-i)} + C_1 + \sum_{j=1}^{k} \gamma_{1j} X_{jt} + \mu_{1t} \\
Y_{2t} &= \sum_{i=1}^{p} \beta_{2i} Y_{1(t-i)} + \sum_{i=1}^{p} \delta_{2i} Y_{2(t-i)} + C_2 + \sum_{j=1}^{k} \gamma_{2j} X_{jt} + \mu_{2t}
\end{aligned}
\qquad \text{(D.25)}
$$

for $p > 0$, where the X_j are other exogenous variables, besides the lagged endogenous variable.

D.6.5 Bivariate Granger causality tests

Note that the model in (D.26) below is in fact a bivariate VAR model without an exogenous variable. However, here X_t and Y_t are used as endogenous or dependent variables, instead of Y_{1t} and Y_{2t}. By using this VAR model, the *Granger causality* of the X and Y variables needs to be investigated:

$$\begin{aligned} X_t &= \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i} + \sum_{i=1}^{p} \beta_i Y_{t-i} + \mu_{1t} \\ Y_t &= \delta_0 + \sum_{i=1}^{p} \delta_i X_{t-i} + \sum_{i=1}^{p} \gamma_i Y_{t-i} + \mu_{2t} \end{aligned} \tag{D.26}$$

In order to test the hypothesis of whether X does or does not give Granger causality of Y, the following hypothesis is considered:

$$\begin{aligned} &H_0 : \delta_1 = \delta_2 = \cdots = \delta_p = 0 \\ &H_1 : \text{Otherwise} \end{aligned} \tag{D.27a}$$

or

$$\begin{aligned} &H_0 : \text{Restricted/Reduced model} : Y_t = \delta_0 + \sum_{i=1}^{p} \gamma_i Y_{t-i} + \varepsilon_t \\ &H_1 : \text{Full/Unstricted model} : Y_t = \delta_0 + \sum_{i=1}^{p} \delta_i X_{t-i} + \sum_{i=1}^{p} \gamma_i Y_{t-i} + \mu_t \end{aligned} \tag{D.27b}$$

Note that the first model is a nested model of the second, so the usual lack of fit F-test can be used, or the Wald form of the F-test as provided by EViews. If the null hypothesis is rejected then X does give Granger causality of Y.

Furthermore, the following hypothesis should be considered for testing whether Y does or does not give Granger causality of X:

$$\begin{aligned} &H_0 : \beta_1 = \beta_2 = \cdots = \beta_p = 0 \\ &H_1 : \text{Otherwise} \end{aligned} \tag{D.28a}$$

or

$$\begin{aligned} &H_0 : \text{Restricted/Reduced model} : X_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i} + \varepsilon_t \\ &H_1 : \text{Full/Unstricted model} : X_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i} + \sum_{i=1}^{p} \beta_i Y_{t-i} + \mu_t \end{aligned} \tag{D.28b}$$

The F-test can be computed as

$$F = \frac{(SSE_{\text{R}} - SSE_{\text{UR}})/p}{SSE_{\text{UR}}/(T-2p-1)} \sim F(p, T-2p-1) \tag{D.29}$$

with an asymptotically equivalent test (Hamilton, 1994, p. 305)

$$\chi^2 = \frac{T(SSE_{\mathrm{R}} - SSE_{\mathrm{UR}})}{SSE_{\mathrm{UR}}} \sim \chi^2(p) \tag{D.30}$$

where SSE_{R} and SSE_{UR} are the sum of squared errors of the reduced/restricted and unrestricted/full models respectively, which can be computed as follows:

$$SSE_{\mathrm{R}} = \sum_{t=1}^{T} \hat{\varepsilon}_t^2 \text{ and } SSE_{\mathrm{UR}} = \sum_{t=1}^{T} \hat{\mu}_t^2 \tag{D.31}$$

D.6.6 Simultaneous causal model

$$\begin{aligned} Y_{1t} &= \varphi_1 Y_{2t} + \sum_{i=0}^{k} \beta_{1i} X_{1it} + \mu_{1t} \\ Y_{2t} &= \varphi_2 Y_{1t} + \sum_{j=0}^{m} \beta_{2i} X_{2jt} + \mu_{2t} \end{aligned} \tag{D.32}$$

where the independent variables X_{1i} and X_{2j} could be any types of exogenous variables, including the lagged endogenous variables. Note that this model is a special case of the model in (D.22) and shows that the series Y_1 and Y_2 have simultaneous causal effects.

D.6.7 Additional bivariate models

Furthermore, all types of the previous models can easily be extended by using the transformed variables, as presented in the examples of data analysis, such as the bounded growth models, models with time-related effects, seemingly causal additive as well as interaction models, bounded semilog models, the bounded translog linear (i.e. Cobb–Douglas) model, the bounded CES (i.e. constant elasticity of substitution) model and bivariate models with dummy variables.

D.7 General estimation methods

A basic multivariate general linear model, namely the MGLM, based on the time series data set, can be presented in matrix notation as

$$\underset{(TxN)}{\boldsymbol{Y}} = \underset{(TxK)}{\boldsymbol{X}} \underset{(KxN)}{\boldsymbol{\beta}} + \underset{(TxN)}{\boldsymbol{\varepsilon}} \tag{D.33}$$

where

$$\underset{(TxN)}{\boldsymbol{Y}} = \begin{bmatrix} Y'_1 \\ Y'_2 \\ \vdots \\ Y'_T \end{bmatrix} \underset{(TxK)}{\boldsymbol{X}} = \begin{bmatrix} X'_1 \\ X'_2 \\ \vdots \\ X'_T \end{bmatrix} \underset{(KxN)}{\beta} = \begin{bmatrix} \beta'_0 \\ \beta'_1 \\ \vdots \\ \beta'_{K-1} \end{bmatrix} \underset{(TxN)}{\varepsilon} = \begin{bmatrix} \varepsilon'_1 \\ \varepsilon'_2 \\ \vdots \\ \varepsilon'_T \end{bmatrix} \tag{D.34}$$

$Y'_t = [y_{1t}, y_{2t}, \ldots, y_{Nt}],\ t = 1, 2, \ldots, T$
$X'_t = [x_{ot} = 1, x_{1t}, \ldots, x_{(K-1)t}],\ t = 1, 2, \ldots, T$
$\beta'_t = [\beta_{i0}, \beta_{i1}, \ldots, \beta_{i(K-1)}],\ i = 0,\ 1, \ldots, (p-1)$
$\varepsilon'_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt}]$

This model, in fact, is a set of N univariate linear models or multiple regressions, where all multiple regressions have the same sets of exogenous variables, namely the $(K-1)$ dimensional exogenous variable $\boldsymbol{X}$.

D.7.1 The OLS estimates

Under the basic assumptions A1 to A5 presented in Appendix B, namely the multivariate $\boldsymbol{X}$ is deterministic and ε_t is i.i.d Gaussian disturbance or multivariate normally distributed, the following properties of the OLS parameter estimates are obtained:

(1) *The unbiased estimator of the vector parameter* β:
To estimate the element of the $(K \times N)$ parameter matrix β by least squares, the following function should be minimized

$$Q(\beta) = Tr[(\boldsymbol{Y}-\boldsymbol{X}\beta)'(\boldsymbol{Y}-\boldsymbol{X}\beta)] \tag{D.35}$$

This gives the *normal equations*

$$(\boldsymbol{X}'\boldsymbol{X})\boldsymbol{B} = \boldsymbol{X}'\boldsymbol{Y} \tag{D.36}$$

If the matrix $\boldsymbol{X}'\boldsymbol{X}$ is a nonsingular matrix then the following estimator is obtained:

$$\underset{(KxN)}{\hat{\beta}} = \boldsymbol{B} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} \tag{D.37}$$

Note that if the matrix $X'X$ is a singular matrix, then there will not be a unique solution or estimators. In this case, EViews will present an error message '*Near singular matrix.*' Therefore, the model needs to be modified in order to obtain an estimable model. However, as there is no law or general rule to overcome the error message, the trial-and-error methods should be used.

(2) *Unbiased estimator of parameter β is given by*:

$$\boldsymbol{E}(\boldsymbol{B}) = \beta \tag{D.38}$$

(3) *The variance–covariance matrix of* ***B***:

$$E[(\boldsymbol{B}-\beta)'(\boldsymbol{B}-\beta)] = \sigma^2(\boldsymbol{X'X})^{-1} \tag{D.39}$$

(4) *The unbiased estimator of the population covariance matrix* ***V***:
The estimates of the error term matrix can be written as

$$\underset{(TxN)}{\hat{\varepsilon}} = \boldsymbol{e} = (\boldsymbol{Y}-\boldsymbol{B}) = \boldsymbol{Y}-\boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'Y} = [\boldsymbol{I}-\boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'}]\boldsymbol{Y} \tag{D.40}$$

Therefore, the $(N \times N)$ matrix of sum of squared errors is given by

$$\boldsymbol{Q}_e = \boldsymbol{e'e} = (\boldsymbol{Y}-\boldsymbol{B})'(\boldsymbol{Y}-\boldsymbol{B}) \tag{D.41}$$

with the unbiased estimate of the covariance matrix

$$\boldsymbol{S} = \frac{(\boldsymbol{Y}-\boldsymbol{B})'(\boldsymbol{Y}-\boldsymbol{B})}{(\boldsymbol{T}-\boldsymbol{K})} \tag{D.42}$$

D.8 Maximum likelihood estimation for an MGLM

Recall the multivariate model in (D.33), where for each component of the N-dimensional endogenous times series $Y_t = (Y_{1t}, \ldots, Y_{Nt})$ at time point t, the following multiple regression for the estimation process was considered:

$$Y_{nt} = \boldsymbol{X_t\beta_n} + \varepsilon_{nt} \tag{D.43}$$

Therefore, a set of N equation specifications, for $n = 1, \ldots, N$, is considered. Under the assumption that the error term $\varepsilon_{nt} = Y_{nt} - \boldsymbol{X_t\beta_n}$ is i.i.d. Gaussian, then ε_{nt} has a normal density function, as follows:

$$f(\varepsilon_{nt}) = (2\pi\sigma^2)^{-1/2}\exp\left(-\frac{\varepsilon_{nt}^2}{2\sigma^2}\right) = (2\pi\sigma^2)^{-1/2}\exp\left[-\frac{(y_{nt}-X_t{*}\beta_n)^2}{2\sigma^2}\right] \tag{D.44}$$

where the symbol $X_t{*}\beta_n = \sum_{k=0}^{K} \beta_{nk} X_{kt}$, with a joint density function or the likelihood function

$$L = f(\varepsilon_{n1}, \varepsilon_{n2}, \ldots, \varepsilon_{nT}) = (2\pi\sigma^2)^{-T/2} \prod_{t=1}^{T} \exp\left[-\frac{(y_{nt} - X_t{*}\beta_n)^2}{2\sigma^2} \right] \tag{D.45}$$

Therefore, the log likelihood function considered for estimation purposes is

$$LL = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_{nt} - X_t{*}\beta_n)^2 \tag{D.46}$$

Note that this LL function has exactly the same form as the LL function of the univariate GLM in (C.17), so that the model (D.43) has exactly the same characteristics and properties as the model in (C.1), as well as all statistics related to this model, including the t-test and the Wald form of the OLS F-test, as follows.

D.8.1 Student's t-test

Let $\boldsymbol{B_n} = [B_{n0}, B_{n1}, \ldots, B_{n,K-1}]$, the estimator of the model parameter $\boldsymbol{\beta_n}$, be multivariate normally distributed, which can be presented as

$$\boldsymbol{B_n} \sim N(\boldsymbol{\beta_n}, \sigma^2(\boldsymbol{X_t'X_t})^{-1}) \tag{D.47}$$

Then each of its component B_{nk} for $k = 0, 1, \ldots, (K-1)$ is normally distributed as $N(\beta_{nk}, \sigma_{kk}^2)$, where σ_{kk}^2 is the element in row k and column k of the covariance matrix $[\sigma^2(\boldsymbol{X_t'X_t})^{-1}]$. By using $s^2(B_{nk}) = \hat{\sigma}_{kk}^2$, then the Student's t-test considered is

$$\frac{(B_{nk} - \beta_{nk})}{sB_{nk}} \text{ is distributed as } t(T-K) \tag{D.48}$$

D.8.2 The Wald form of the OLS F-test

D.8.2.1 Testing the hypothesis

$$H_0: \boldsymbol{C\beta_n} = \boldsymbol{c} \tag{D.49}$$

where $\boldsymbol{C}$ is a constant $(m \times K)$ matrix representing the particular linear combination of the model parameter $\boldsymbol{\beta_i}$ and $\boldsymbol{c}$ is an $(m \times 1)$ vector of defined values that are believed or judged to be the true values of the corresponding linear combinations.

The hypothesis (D.49) can be presented as

$$\begin{aligned} &H_0: \text{Restricted model}: Y_{it} = \boldsymbol{X_t\beta_i} + \varepsilon_{it} \text{ with } \boldsymbol{C\beta_i} = \boldsymbol{c} \\ &H_1: \text{Unrestricted model}: Y_{it} = \boldsymbol{X_t\beta_i} + \varepsilon_{it} \end{aligned} \tag{D.50}$$

This hypothesis can be tested using the Wald form of the OLS F-test as follows:

$$F = \frac{(SSE_{\mathrm{R}} - SSE_{\mathrm{U}})/m}{SSE_{\mathrm{U}}/(T-K)} \sim F(m, T-K) \tag{D.51}$$

where SSE_{R} indicates the error sum of squares of the restricted model (i.e. if the null hypothesis $\boldsymbol{C\beta_n} = \boldsymbol{c}$ is true) and SSE_{U} indicates the error sum of squares of the unrestricted or full model. Furthermore, it is well known that the numerator and denominator of the F-test are the chi-squared tests as follows:

$$\chi_1 = \frac{(SSE_{\mathrm{R}} - SSE_{\mathrm{U}})}{m} \sim \sigma^2 \chi^2(m) \tag{D.52}$$

$$\chi_2 = \frac{SSE_{\mathrm{U}}}{(T-K)} \sim \sigma^2 \chi^2(T-2) \tag{D.53}$$

D.8.3 Residual analysis

Corresponding to the multivariate model in (D.33), there are N time series of residuals or N-dimensional error terms, namely $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt})$. These N series of residuals can easily be generated by using EViews, as well as presented in the form of graphs. Then it is easy to conduct a detailed analysis based on these series of residuals, in order to study or investigate whether or not the models are acceptable models, in a statistical sense.

Based on the experimentation, it has been found that the models need to be modified in most cases, and trial-and-error methods should be used. Refer to the special notes and comments in Section 2.14.

D.9 MGLM with autoregressive errors

This section will present two types of AR(p) MGLMs. The first type is the AR(p) MGLM, where all multiple regressions have an equal set of exogenous or independent variables, and the second type has unequal sets of exogenous variables. However, since in EView an MGLM is presented as system equations, here the AR(p) MGLM is presented as the following system equations.

D.9.1 AR(p) MGLM with equal sets of exogenous variables

As an extension of the AR(p) model in (B.19), this is an MGLM with autoregressive errors, namely the AR(p) MGLM, where all regressions have equal sets of exogenous variables. However, as an extension of the model in (D.43), the AR(p) MGLM is ddefined as follows:

$$Y_{nt} = \boldsymbol{X_t \beta_n} + \mu_{nt}, \text{ for } n = 1, \ldots, N \tag{D.54a}$$

$$\mu_{nt} = \rho_{n1}\mu_{n,t-1} + \rho_{n2}\mu_{n,t-2} + \cdots + \rho_{np}\mu_{n,t-p} + \varepsilon_{nt} \tag{D.54b}$$

where ρ_{ni} is the ith autocorrelation or serial correlation parameter, corresponding to the nth component of the multivariate endogenous variable, such that $|\rho_{ni}| < 1$, for all n and i, and ε_{nt}, $t = 1, 2, \ldots, T$, are i.i.d. Gaussian or normally distributed with $E(\varepsilon_{nt}) = 0$, and $\text{Var}(\varepsilon_{nt}) = E(\varepsilon_{nt}^2) = \sigma_n^2$ and $\text{Cov}(\varepsilon_{nt}, \varepsilon_{t,t-h}) = E(\varepsilon_{nt}, \varepsilon_{n,t-h}) = (h) = \gamma_h$ are independent on t.

Note that the AR(p) model with the endogenous variable μ_{nt}, for $n = 1, \ldots, N$, in (D.54b) has exactly the same form, as well as characteristics, as the AR(p) model with the endogenous variable Y_t in (D.10). Then similar statistics can easily be derived, as well as tested, based on the model in (D.54b) by using the same process in deriving the statistics based on the model in (D.10). Therefore, this will not be presented again in this section.

Furthermore, in order to estimate the model parameters, for each component of the N-dimensional endogenous variable, the LL function considered is given by

$$\begin{aligned} LL &= -\frac{T-p}{2}\ln(2\pi) - \frac{T-p}{2}\ln(\sigma_n^2) \\ &\quad - \frac{1}{2\sigma_n^2}\sum_{t=p+1}^{T}\left[(y_{nt} - X_t{*}\beta_n) - \sum_{i=1}^{p}\rho_{ni}(y_{n,t-i} - X_t{*}\beta_n)\right]^2 \end{aligned} \qquad \text{(D.55)}$$

where $X_t{*}\beta_n = \sum_{k=0}^{K}\beta_{nk}x_{kt}$ (compare this with the LL function in (D.46)).

D.9.2 AR(p) MGLM with unequal sets of exogenous variables

An AR(p) MGLM with unequal sets of exogenous variables can easily be derived from the model in (D.54). In this case, the sets of independent variables of the regressions are highly dependent on or closely related to the endogenous variables. The model can be considered as an extension of the model in (D.32), with the following system equations:

$$Y_{nt} = \boldsymbol{X_{nt}\beta_n} + \mu_{nt}, \text{ for } n = 1, \ldots, N \qquad \text{(D.56a)}$$

$$\mu_{nt} = \rho_{n1}\mu_{n,t-1} + \rho_{n2}\mu_{n,t-2} + \cdots + \rho_{np}\mu_{n,t-p} + \varepsilon_{nt} \qquad \text{(D.56b)}$$

where the multimensional exogenous variable $\boldsymbol{X}_n = (X_{n0}, \ldots, X_{nK(n)})$ is dependent on Y_n.

For each component of the multivariate endogenous series $\boldsymbol{Y}_t$, namely y_{nt}, the following LL function is obtained:

$$\begin{aligned} LL &= -\frac{T-p}{2}\ln(2\pi) - \frac{T-p}{2}\ln(\sigma_n^2) \\ &\quad - \frac{1}{2\sigma_n^2}\sum_{t=p+1}^{T}\left[(y_{nt} - X_{nt}{*}\beta_n) - \sum_{i=1}^{p}\rho_{ni}(y_{n,t-i} - X_{nt}{*}\beta_n)\right]^2 \end{aligned} \qquad \text{(D.57)}$$

where $X_{nt}*\beta_n = \sum_{k=0}^{K_n} \beta_{nk}x_{nkt}$. Note that the multidimensional exogenous variables $\boldsymbol{X_n}$ are dependent on the endogenous variable $\boldsymbol{Y_n}$.

By using a matrix equation, the whole set of regressions in (D.56a) will be presented as

$$\underset{NTx1}{\boldsymbol{Y}} = \underset{NTxK}{\boldsymbol{X}} * \underset{Kx1}{\beta} + \underset{NTx1}{\mu} \tag{D.58}$$

where $K = K_1 + \cdots + K_n$, with K_n the number of exogenous variables in the nth multiple regression in (D.56).

D.9.3 Special notes and comments

(1) For $N = 1$, the model in (D.56) can represent any time series univariate regressions, either the AR(p) models as presented in Appendix B or the models with autoregressive errors as presented in Appendices B and C and the models with trend, as well as two-way or three-way interaction models, including the models with time-related effects and the dummy variables models.
(2) Furthermore, for $N > 1$, the model in (D.56) can represent multiple association time series models or seemingly causal models, such as additive and interaction structural equation models and simultaneous causal models, with or without the time t as an endogenous or independent variable.
(3) For each of those models, any linear combinations of the model parameters, namely the K-column vector of the parameter β in model (D.58), can be tested as a univariate or multivariate hypothesis, as follows:

$$\begin{aligned} &H_0 : \boldsymbol{C}\beta = \boldsymbol{c} \text{ or Restricted model} \\ &H_1 : \text{Unrestricted model} \end{aligned} \tag{D.59}$$

where $\boldsymbol{C}$ is an $(m \times K)$ constant matrix representing a univariate hypothesis for $m = 1$ and a multivariate hypothesis for $m > 1$ and β is a K-dimensional column vector of the model parameters. The test can easily be done by using the Wald form of the OLS F-statistic, as presented in (D.51) (refer to the examples).

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Index

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